Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Supersymmetrie

WS 07/08

Prof. Dr. J. Reuter
(Hochhaus/803), 8407

Blatt 3

Hausaufgabe 2 spinor acrobatics continued

The following relations are supposed to be part of a formulary to the lecture. One should feel encouraged to prove as many of them as possible, but this is not strictly required.

a) Using two different approaches, show the following:

$$\theta^\alpha \, \theta^\beta \; = \; -\frac{1}{2} \, \epsilon^{\alpha\beta} \, (\theta\theta) \, , \quad \theta_\alpha \, \theta_\beta \; = \; \frac{1}{2} \, \epsilon_{\alpha\beta} \, (\theta\theta) \, ,$$

$$\bar\theta^{\dot\alpha} \, \bar\theta^{\dot\beta} \; = \; \frac{1}{2} \, \epsilon^{\dot\alpha\dot\beta} \, \left(\bar\theta\bar\theta\right) , \quad \bar\theta_{\dot\alpha} \, \bar\theta_{\dot\beta} \; = \; -\frac{1}{2} \, \epsilon_{\dot\alpha\dot\beta} \, \left(\bar\theta\bar\theta\right) ,$$

$$\left(\theta\sigma^\mu\bar\theta\right) \left(\theta\sigma^\nu\bar\theta\right) = -\frac{1}{2} g^{\mu\nu} \, (\theta\theta) \left(\bar\theta\bar\theta\right)$$

b) **Schouten Identity** make use of the symmetry properties in $(\beta\gamma\delta)$:

$$\epsilon^{\alpha\beta}\epsilon^{\gamma\delta} + \epsilon^{\alpha\gamma}\epsilon^{\delta\beta} + \epsilon^{\alpha\delta}\epsilon^{\beta\gamma} = 0$$

$$(\phi\psi) \, (\xi\chi) + (\phi\xi) \, (\chi\psi) + (\phi\chi) \, (\psi\xi) = 0$$

c)

$$(\theta\phi) \, (\theta\psi) = -\frac{1}{2} \, (\phi\psi) \, (\theta\theta)$$

$$\left(\bar\theta\bar\phi\right) \left(\bar\theta\bar\psi\right) = -\frac{1}{2} \left(\bar\phi\bar\psi\right) \left(\bar\theta\bar\theta\right)$$

d)

$$\epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha}$$

$$\epsilon^{\alpha\beta} \frac{\partial}{\partial\theta^\alpha} \frac{\partial}{\partial\theta^\beta} \, (\theta\theta) = 4$$

$$\epsilon_{\dot\alpha\dot\beta} \frac{\partial}{\partial\bar\theta_{\dot\alpha}} \frac{\partial}{\partial\bar\theta_{\dot\beta}} \left(\bar\theta\bar\theta\right) = 4$$

Präsenz-Aufgabe 3 SUSY transformations, component fiels

We look at a complex scalar field and a complex Weyl-spinor field. How many degrees of freedom do these fields have on- and off-shell. What follows for the off-shell case?

a) Since $\{Q, \bar{Q}\} \propto P$, Q and $\bar{Q}$ have mass dimension $1/2$. Conclude that the SUSY transformations are determined to be (up to a normalization factor)

$$\delta_\xi \phi(x) = \sqrt{2}\xi\psi(x)$$

$$\delta_\xi \psi(x) = i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\phi(x).$$

b) Employ the equations of motion of the free fields (how do they look like in the massless limit?) to show that the above transformations obey the SUSY algebra, i.e.

$$(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta)\,\phi(x) = -2i\left(\eta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\eta}\right)\partial_\mu\phi(x) \tag{1}$$

$$(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta)\,\psi(x) = -2i\left(\eta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\eta}\right)\partial_\mu\psi(x) \tag{2}$$

c) For the off-shell case make the ansatz

$$\delta_\xi \psi(x) = i\sqrt{2}\sigma^\mu\bar{\xi}\partial_\mu\phi(x) + \sqrt{2}\xi F(x).$$

Show that eq.(1) remains valid and that

$$\delta_\xi F(x) = i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu\partial_\mu\psi(x)$$

has to hold in order to satisfy eq.(2).

d) Verify that

$$(\delta_\eta\delta_\xi - \delta_\xi\delta_\eta)\,F(x) = -2i\left(\eta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\eta}\right)\partial_\mu F(x)$$