Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Supersymmetrie

WS 07/08

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Blatt 10

Hausaufgabe 9 Invariant Terms for Model Building

a) Show that there are two $SU(2)$ invariants one can construct from spinors:

$$\psi_i^\star \phi_i \quad \text{and} \quad \phi_i \epsilon_{ij} \phi_j .$$

Use

$$\phi_i \stackrel{\text{inf.}}{\to} \phi_i + i\alpha^a \frac{\sigma_{ij}^a}{2} \phi_j , \quad \text{with} \quad \sigma_{ij}^{a\star} = \sigma_{ji}^a ,$$

as well as

$$\left(\sigma^2 \sigma^a \sigma^2\right)_{ij} = -\sigma_{ji}^a \quad \text{and} \quad \epsilon_{ij} = i\sigma_{ij}^2 .$$

b) For $SU(3)$ use the transformation laws

$$\delta\phi_i = i\alpha^a T_{ij}^a \phi_j \quad \text{and} \quad \delta\phi_i^c = -i\alpha^a \phi_j^c T_{ji}^a$$

for fields in the fundamental and anti-fundamental representation 3 and $\bar{3}$, respectively.

Where does the second transformation law for the complex conjugate representation come from?

Show that $\psi_i^c \phi_i$ is invariant.

Prove the invariance of the antisymmetric product of three (anti-)fundamentals

$$\epsilon_{ijk} \phi_i \psi_j \chi_k \quad \text{and} \quad \epsilon_{ijk} \phi_i^c \psi_j^c \chi_k^c ,$$

by employing that

$$\left(T^a \epsilon\right)_{ijk} = T_{il}^a \epsilon_{ljk} + T_{jl}^a \epsilon_{ilk} + T_{kl}^a \epsilon_{ijl}$$

is totally antisymmetric in ijk. Then, evaluate the expression for $ijk = 123$.

Präsenz-Aufgabe 11 The Superpotential of the MSSM

d	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q	3	2	$1/6$
L	1	2	$-1/2$
H_u	1	2	$1/2$
H_d	1	2	$-1/2$
u^c	$\bar{3}$	1	$-2/3$
d^c	$\bar{3}$	1	$1/3$
e^c	1	1	1

a) Construct the most general superpotential from these chiral superfields, subject to the following constraints:

- renormalizability
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance.

What follows from those constraints for the general structure of the allowed terms?
Which interaction terms do appear?

b) Which terms seem to be strange? Why should they not arise?
Consider the combination

$$P_R = (-)^{3(B-L)}$$

with the baryon and lepton numbers B and L, respectively. This matter parity is often rephrased as R parity acting on the component fields, $P_R = (-)^{3(B-L)+2s}$, where s denotes the component field's spin. Which interactions have to be excluded from the superpotential in order to make it even under R parity?