Multi-Particle SUSY Simulations at LHC & ILC —

Off-Shell Effects, interferences and radiative corrections

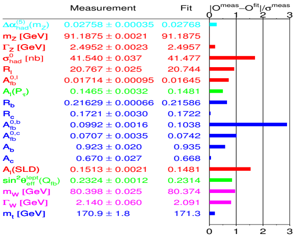
Jürgen Reuter

Albert-Ludwigs-Universität Freiburg

Hagiwara/Kilian/Krauss/Ohl/Plehn/Rainwater/JR/Schumann **PRD 73** (2006), 055005; JR et al., hep-ph/0512012; Kilian/Robens/JR, **EPJ C48** (2006), 389; Ohl/JR, **EPJ C30** (2003), 525; work in progress: Alboteanu/Alwall/Kilian/Plehn/JR; Kalinowski/Kilian/JR/Robens/Rolbiecki

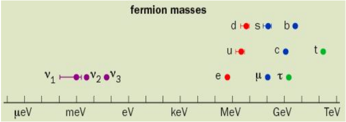
Uppsala, May 16th, 2008

Challenging the Standard Model



– describes microcosm (too well?)

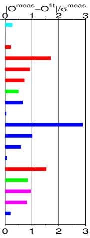
– 28 free parameters



– Form of Higgs potential ?

Challenging the Standard Model

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3



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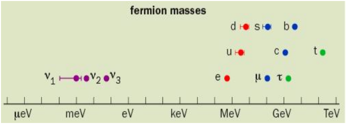

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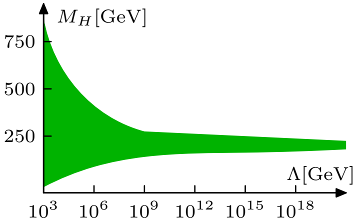


Hierarchy problem

chiral symmetry: $\delta m_f \propto v \ln(\Lambda^2/v^2)$

no symmetry for quantum corrections to Higgs mass

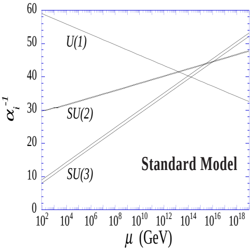
$$\delta M_H^2 \propto \Lambda^2 \sim M^2_{\text{Planck}} = (10^{19})^2\,\text{GeV}^2$$

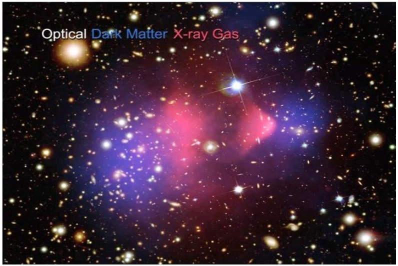
20000 GeV^2 = (100000000000000000000000000000000020000 −

1000000000000000000000000000000000000) GeV^2

Open questions & Loose ends

- Unification of all interactions (?)
- Baryon asymmetry $\Delta N_B - \Delta N_{\bar{B}} \sim 10^{-9}$
  missing CP violation
- Flavour: three generations
- Tiny neutrino masses: $m_\nu \sim \frac{v^2}{M}$
- Dark Matter:
  - stable
  - weakly interacting
  - $m_{DM} \sim 100\,\mathrm{GeV}$
- Quantum theory of gravitation
- Cosmic inflation
- Dark Energy
  (Cosmological constant?)





Old (and New) Ideas for New Physics (since $\sim$ 1970)

(1) **Substructure/New strong interactions**

- Technicolour: Higgs as a bound state of strongly-interacting partons

(2) **Symmetries for cancellation of quantum corrections**

- Supersymmetrie: Spin-statistics $\Rightarrow$ corrections from bosons and fermions cancel each other
- Little-Higgs/Moose models: global symmetries $\Rightarrow$ corrections from like-statistics particles cancel each other

(3) **Non-trivial space-time structure eliminates hierarchy**

- Large extra dimensions: Gravity appears only weak
- Warped extra dimensions (Randall-Sundrum): Gravity only weak in our world

(4) **Ignoring the hierarchy**

- Anthropic principle: parameters have their values, *because* we (can) measure them

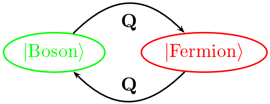
Supersymmetry (SUSY)

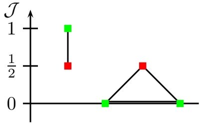
Gelfand/Likhtman, 1971; Akulov/Volkov, 1973; Wess/Zumino, 1974

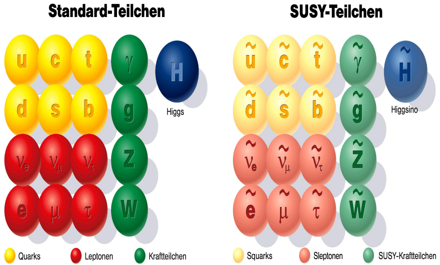
– (uniquely) connects gauge and space-time symmetries

– Multiplets of fermions and bosons with equal masses

⇒ SUSY broken in nature







– To each particle add a superpartner

– Minimal Supersymmetric Standard Model (MSSM)

– Mass eigenstates:
Charginos: $\tilde{\chi}^\pm = \tilde{H}^\pm, \tilde{W}^\pm$
Neutralinos: $\tilde{\chi}^0 = \tilde{H}, \tilde{Z}, \tilde{\gamma}$

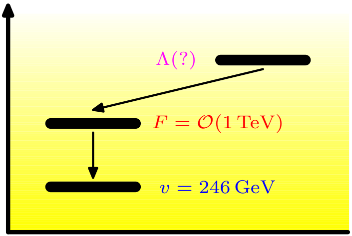
SUSY: Successes and Drawbacks

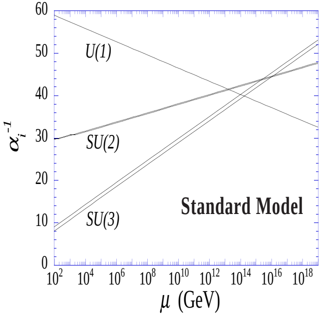
spontaneous SUSY breaking in the MSSM ↯ (MeV superpartners)

Breaking in "hidden sector"

Breaking mechanism induces 100 new parameters

solves hierarchy problem: $\delta M_H \propto F \log(\Lambda^2)$





- Fundamental scalars natural
- Form of Higgs potential
- light Higgs ($M_H = 90 \pm 50\,$GeV)
- discrete R-parity
  - SM particles even, SUSY partners odd
  - prevents too rapid proton decay
  - lightest SUSY particle (LSP) stable
    Dark matter: $\tilde{\chi}_1^0$
- Unification of coupling constants

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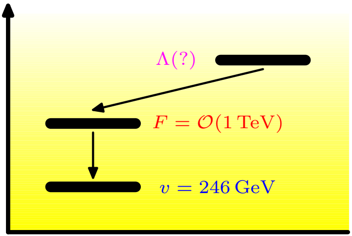

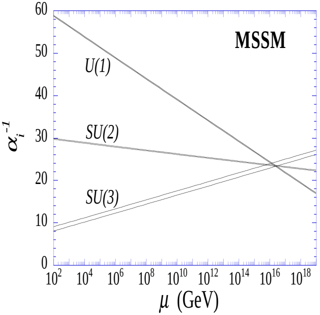

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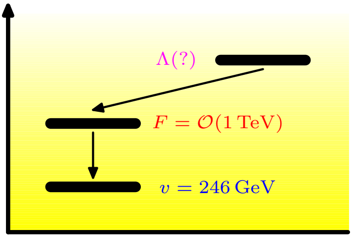

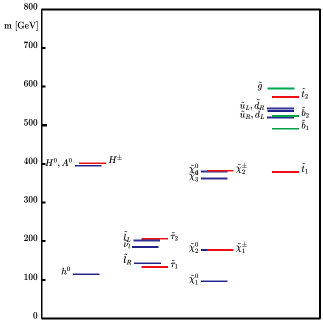

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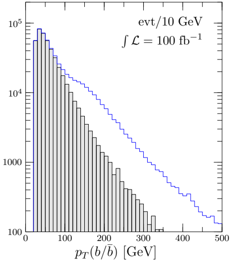
Orthodoxist: "SUSY will be discovered, even if non-existent"

Revolutionist: "SUSY is already dead"

LHC search for new particles



Decay products from heavy particles:

- high-p_T jets
- many hard leptons

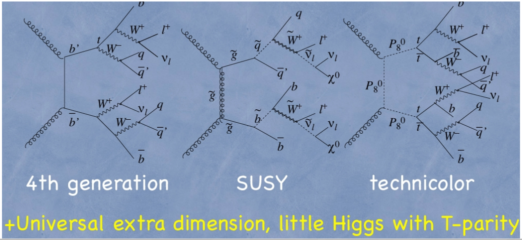
Production of colored states

weakly interact. particles only in decays

Dark matter $\Leftrightarrow$ discrete parity $(R,\, T,\, KK)$

- new particles only in pairs $\Rightarrow$ high energies, (long) decay chains
- Dark matter $\Rightarrow$ much missing energie in the detector ($\not{E}_T$)

Different models/decay chains — identical signatures



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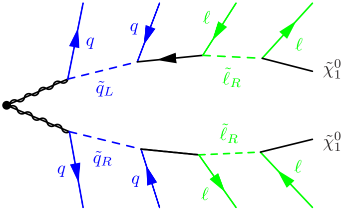

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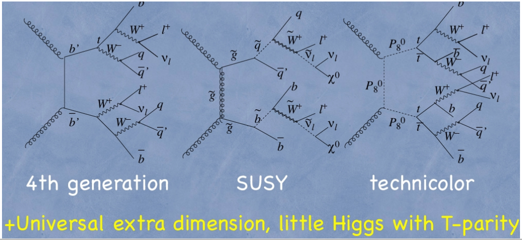
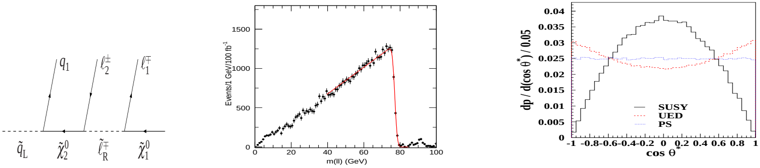


(SUSY) Observables/Precision Measurements

- Mass (difference) of new particles: endpoints of decay spectra



- Spin of new particles : angular distributions, correlations . . .
- Model discrimination: (Precise) measurements of coupling constants
  Reverse RG evolution: get a handle on GUT parameters
  ⇒ **SPA project** `http://spa.desy.de/spa`

⇒ Precise prediction for signals and backgrounds:
background to other (more difficult) SUSY processes

- Consideration of kinematical cuts
- Exclusive multi-particle final states: $2 \to 4$ to $2 \to 10$
- Quantum corrections: real and virtual

Classification of corrections to (SUSY) processes

Corrections to the SUSY processes fall into six categories:

- off-shell kinematics for the signal process
  Hagiwara ea. PRD 73 (2006), 055005; JR ea., Snowmass 2005 Proc. see also Berdine ea., 2007
- irreducible background from all other SUSY processes see above
- reducible, experimentally indistinguishable SM background processes
  see above
- Loop corrections to SUSY production and decay processes
  Kilian/JR/Robens, EPJ C48 (2006), 389, Kalinowski/Kilian/JR/Robens/Rolbiecki, in prep.
- nonfactorizable, maximally resonant gluon [photon] exchange between production and decay
- real radiation of gluons [photons] see above

Classification of approximations in (SUSY) processes

Some generic SUSY process:

$e^+e^- \to b\bar{b}e^+e^-\tilde{\chi}_1^0\tilde{\chi}_1^0$ 66478 diagrams. (It's just $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0$!)

- Entanglement of different signal diagrams ($e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0, \tilde{b}_i\tilde{b}_j, \tilde{e}_i\tilde{e}_j$)
- Need for cuts to disentangle those (experimentally/simulation)
- Add SM backgrounds ($e^+e^- \to b\bar{b}e^+e^-\nu_i\bar{\nu}_i$)
- Much more complicated processes for LHC, and even also for ILC

Process $A_1A_2 \to P^{(*)} \to F_1F_2$, 3 different levels:

Narrow width	$\sigma(A_1A_2 \to P) \times \mathrm{BR}(P \to F_1F_2)$
Breit-Wigner	$\sigma(A_1A_2 \to P) \times \frac{M_P^2\Gamma_P^2}{(s-M_P^2)^2+\Gamma_P^2M_P^2} \times \mathrm{BR}(P \to F_1F_2)$
Full matrix element	$\sigma(A_1A_2 \to F_1F_2)$

last level *not* featured by ISAJET, PYTHIA, HERWIG, SUSYGEN

The Multi-Particle Generator WHIZARD

Kilian/Ohl/JR, 07

- Optimized helicity amplitudes: completely avoids redundancies
- Iterative adaptive multi-channel phase space (viable for $2 \to 10$)
- Unweighted events (formats: binary, HEPEVT, ATHENA, LHA, STDHEP)
- Graphical analysis tool
- Very high level of Complexity:

$ee \to t\bar{t}H \to b\bar{b}b\bar{b}jj\ell\nu$ (110,000 diagrams)
$ee \to ZHH \to ZWWWW \to bb + 8j$ (12,000,000 diagrams)
$pp \to \ell\ell + nj,\, n = 0, 1, 2, 3, 4, \ldots$ (2,100,000 diagrams with 4 jets + flavors)
$pp \to \tilde{\chi}^0_1 \tilde{\chi}^0_1 bbbb$ (32,000 diagrams, 22 color flows, $\sim 10,000$ PS channels)
$pp \to VVjj \to jj\ell\ell\nu\nu$ incl. anomalous TGC/QGC
Test case $gg \to 9g$ (224,000,000 diagrams)

- WHiZard 1.51 / O'Mega 000.011beta Ω $\to$ joint version:
  WHIZARD 1.92 release date: 2008, April, 29th

  one grand unified package (incl. VAMP, Circe, Circe 2, WHiZard, O'Mega)

New web address: `http://whizard.event-generator.org`
Standard Reference for 1.92 + new versions: Kilian/Ohl/JR, 0708.4233

- Major upgrade this summer: WHIZARD 2.0.0

Implemented Physics Content

Structured beams:

- For Tevatron/LHC: PDFs from LHAPDF (or PDFLIB)
- Parton Shower ($k_\perp$ ordered)
- For ILC physics:

  ISR, polarization, beamstrahlung, photon collider spectra (CIRCE/CIRCE 2)

external (user-defined) beam spectra can be read in

Supported Physics Models:

- Test models: QED, QCD
- SM (R_ξ gauge, . . .)
- Littlest/Simplest Little Higgs, Little Higgs Models with T parity
- Moose models: 3-site model
- MSSM, NMSSM, extended SUSY models, incl. gravitinos (SLHA/SLHA2)
- Graviton resonances, Universal extra dimensions, Randall-Sundrum
- Noncommutative Standard Model
- Higher-dimensional operators, SM effective field theory extensions
- Anomalous triple and quartic gauge couplings
- K-matrix/Padé unitarization, unitarized resonances Alboteanu/Kilian/JR

Tests and Checks of MSSM implementation

JR et al., 2005; Hagiwara/Kilian/Krauss/Ohl/Plehn/Rainwater/JR/Schumann, 2006

- MSSM: doubled spectrum, 100 parameters, 5000 vertices
- Unitarity checks: $\sigma(2 \to 2, s), \sigma(2 \to 3, s) \sim const$ or $1/s$ ✓
- Gauge invariance: Ward- and Slavnov-Taylor identities ✓
- Supersymmetry: Ward-/Slavnov-Taylor identities ✓ JR, 2002; Ohl/JR, 2002
- Comparison of codes ($\mathcal{O}(600)$ processes): JR et al., 2005; K. Hagiwara/.../JR/..., 2006
  Reference: http://whizard.event-generator.org/susy_comparison.html

$\tau^+\tau^- \to X$

Process	status	Madgraph/Helas 0.5 TeV	Madgraph/Helas 2 TeV	Whizard/O'Mega 0.5 TeV	Whizard/O'Mega 2 TeV	Sherpa/A'Megic 0.5 TeV	Sherpa/A'Megic 2 TeV
$\tilde{\tau}_1 \tilde{\tau}_1^*$	●	257.57(7)	79.63(4)	257.32(1)	79.636(4)	257.30(1)	79.638(4)
$\tilde{\tau}_2 \tilde{\tau}_2^*$	●	46.55(1)	66.86(2)	46.368(2)	66.862(3)	46.372(2)	66.862(3)
$\tilde{\tau}_1 \tilde{\tau}_2^*$	●	95.50(3)	19.00(1)	94.637(3)	19.0015(8)	94.645(5)	19.000(1)
$\tilde{\nu}_\tau \tilde{\nu}_\tau^*$	●	502.26(7)	272.01(8)	502.27(2)	272.01(1)	502.30(3)	272.01(1)
$\tilde{\chi}_1^0 \tilde{\chi}_1^0$	●	249.94(2)	26.431(1)	249.954(9)	26.431(1)	249.96(1)	26.431(1)
$\tilde{\chi}_1^0 \tilde{\chi}_2^0$	●	69.967(3)	9.8940(3)	69.969(2)	9.8940(4)	69.968(3)	9.8937(5)
$\tilde{\chi}_1^0 \tilde{\chi}_3^0$	●	17.0387(3)	0.7913(1)	17.0394(1)	0.79136(2)	17.040(1)	0.79137(5)
$\tilde{\chi}_1^+ \tilde{\chi}_1^-$	●	185.09(3)	45.15(1)	185.093(6)	45.147(2)	185.10(1)	45.151(2)
$h^0 h^0$	●	0.3533827(3)	0.0001242(2)	0.35339(2)	0.00012422(3)	0.35340(2)	0.000124218(6)
$A^0 A^0$	●	—	0.07975(3)	—	0.079758(6)	—	0.079744(4)
$Z h^0$	●	59.591(3)	3.1803(8)	59.589(3)	3.1802(1)	59.602(3)	3.1829(2)
$Z A^0$	●	2.9915(4)	4.682(5)	2.99162(9)	4.6821(3)	2.9917(2)	4.6817(2)

Parameter point under consideration

Following discussions do not depend on the special parameter point

SUGRA-inspired point, non-universal right-handed scalar masses

$\tan\beta = 20$

Particle	M [GeV]	Γ [GeV]	Particle	M [GeV]	Γ [GeV]
h	114.45	0.0050	$\tilde\chi_1^0$	46.84	—
H	300.15	2.2924	$\tilde\chi_2^0$	112.41	0.00005
A	300.00	2.7750	$\tilde\chi_3^0$	148.09	0.01162
$H^\pm$	310.96		$\tilde\chi_4^0$	236.77	1.0947
$\tilde b_1$	295.36	0.5395	$\tilde\chi_1^\pm$	106.60	
$\tilde b_2$	399.92	3.4956	$\tilde\chi_2^\pm$	237.25	
$\tilde e_L$	205.02		$\tilde t_1$	413.84	
$\tilde e_R$	205.65		$\tilde t_2$	978.88	

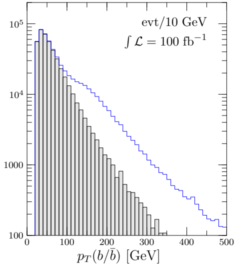
- (Very) light Higgs, directly above LEP limit
- $h \sim 47$ % invisible decays to LSP
- $m_{\tilde q} \sim 430$ GeV
- Light sbottoms accessible at the ILC
- Low-energy data-compatible: $b \to s\gamma$, $B_s \to \mu^+\mu^-$, $\Delta\rho$, $g_\mu - 2$, CDM

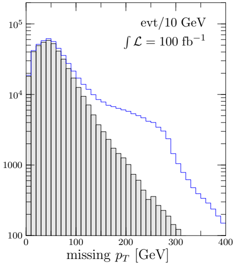
- Focus on BR$(\tilde b_1 \to b\tilde\chi_1^0) = 43.2\%$

Sbottom production at the LHC

Hagiwara/.../JR/..., 2006

$\tilde{b}_1$ production with subsequent decay $\tilde{b}_1 \to \tilde{\chi}^0_1 b$





Parton-level distributions

$pp \to b\bar{b}\tilde{\chi}^0_1\tilde{\chi}^0_1$

Cuts: $p_{T,b} > 20$ GeV, $|\eta_b| < 4$, and $\Delta R_{bb} > 0.4$.

Main bkgd: $gg \to b\bar{b}\nu\bar{\nu}$

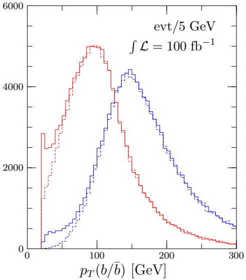
Signal jets harder

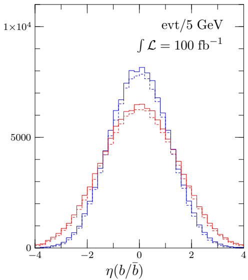
Off-Shell Effects at the LHC:

PS: harder jet more central

Off-Shell effects ($b\bar{b}Z^*$): only low-$p_{T,b}$ $\longrightarrow$ is cut out

(Un)lucky case !!





Real corrections: Bottom-jet radiation

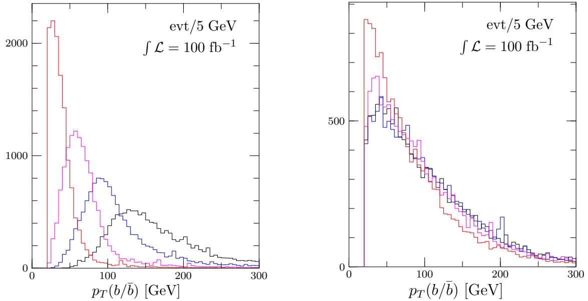
K. Hagiwara/.../JR/..., 2006

$g \to b\bar{b}$-splitting, b-ISR as combinatorial background

$pp \to \tilde{\chi}_1^0 \tilde{\chi}_1^0 b\bar{b}b\bar{b}$: 32112 diagrams, 22 color flows, $\sim$ 4000 PS channels

$\sigma(pp \to b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0) = 1177\,\text{fb} \quad \longrightarrow \quad \sigma(pp \to b\bar{b}b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0) = 130.7\,\text{fb}$

Forward discrimination of ISR and decay jets difficult:



Only the most forward jet considerably softer

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$g \to b\bar{b}$-splitting, b-ISR as combinatorial background

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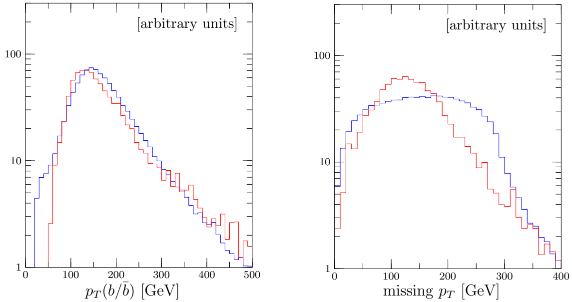
$\sigma(pp \to b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0) = 1177\,\text{fb} \quad \longrightarrow \quad \sigma(pp \to b\bar{b}b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0) = 130.7\,\text{fb}$

Only marginal differences in $p_{T,b}$, PDF: Maximum at lower value



shifted to lower $\not{p}_T$: light particles balance out event

Big Off-Shell/Interference Effects at LHC

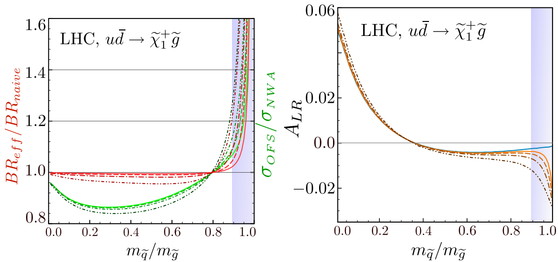
Berdine et al., 2007; Alwall/.../JR, in prep.

Off-Shell Effects in SUSY/general BSM:

- Decay chains w/ nearly-degenerate mother/daughter
- Decay matrix element: shifts q^2-dependence of propagators
- Decay thresholds: Γ/M enhanced by $1/\beta^n$
- Interference of non-resonant contributions

Large effects occur:

- Effective BRs (defined via excl. final states) deviate by up to $\mathcal{O}(100\,\%)$
- Charge asymmetries in decays: e.g. $\tilde{g}\tilde{g} \to b\tilde{b}_1^* b\tilde{b}_1^* \,/\, \bar{b}\tilde{b}_1\bar{b}\tilde{b}_1$
- Chirality asymmetries: e.g. $\tilde{g}\tilde{g} \to \tilde{q}_L\tilde{q}_L jj \,/\, \tilde{q}_R\tilde{q}_R jj$



Sbottom production at the ILC

Cross sections for $\sqrt{s} = 800$ GeV

- In contrast to the LHC: Electroweak production
- More channels contribute to $e^+e^- \to b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0$:

  $e^+e^- \to Zh, ZH, Ah, HA, \tilde{\chi}_1^0\tilde{\chi}_2^0, \tilde{\chi}_1^0\tilde{\chi}_3^0, \tilde{\chi}_1^0\tilde{\chi}_4^0, \tilde{b}_1\tilde{b}_1^*, \tilde{b}_1\tilde{b}_2^*$ (412 diagrams)
- Irreducible SM background: $e^+e^- \to b\bar{b}\nu_i\bar{\nu}_i$ (WW fusion, Zh, ZZ) (47 diagrams)

Channel	$\sigma_{2\to 2}$ [fb]	$\sigma \times$ BR [fb]	$\sigma_{\rm BW}$ [fb]
Zh	20.574	1.342	1.335
ZH	0.003	0.000	0.000
hA	0.002	0.001	0.000
HA	5.653	0.320	0.314
$\tilde{\chi}_1^0\tilde{\chi}_2^0$	69.109	13.078	13.954
$\tilde{\chi}_1^0\tilde{\chi}_3^0$	24.268	3.675	4.828
$\tilde{\chi}_1^0\tilde{\chi}_4^0$	19.337	0.061	0.938
$\tilde{b}_1\tilde{b}_1$	4.209	0.759	0.757
$\tilde{b}_1\tilde{b}_2$	0.057	0.002	0.002
Sum		19.238	22.129
Exact			19.624
w/ISR			22.552

Channel	$\sigma_{2\to 2}$ [fb]	$\sigma \times$ BR [fb]	$\sigma_{\rm BW}$ [fb]
ZZ	202.2	12.6	13.1
Zh	20.6	1.9	1.9
ZH	0.0	0.0	0.0

Channel	$\sigma_{2\to 3}$ [fb]	$\sigma \times$ BR [fb]	$\sigma_{\rm BW}$ [fb]
$Z\bar{\nu}\nu$	626.1	109.9	111.4
$h\bar{\nu}\nu$	170.5	76.5	76.4
$H\bar{\nu}\nu$	0.0	0.0	0.0
Sum		186.5	187.7
Exact			190.1
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- Use widths to the same order as your process

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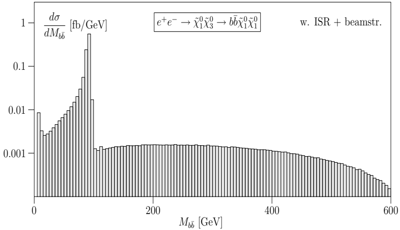
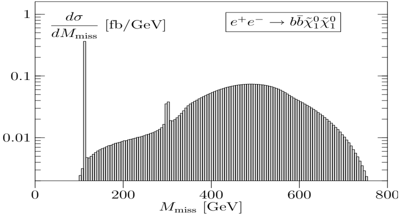
Results

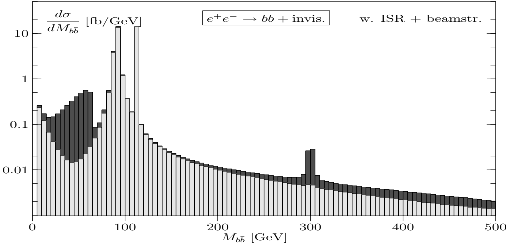
Off-shell decay $\tilde{\chi}_3^0 \to (\tilde{b}_1)_{off} \bar{b} \to b\bar{b}\tilde{\chi}_1^0$ gives broad continuum





ISR/beamstrahlung: corrections of same order (effects all p_{miss} observables)

$b\bar{b}$ invariant mass with SM background:



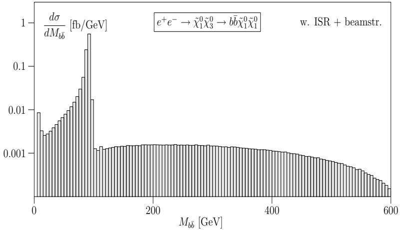
Cut out the resonances

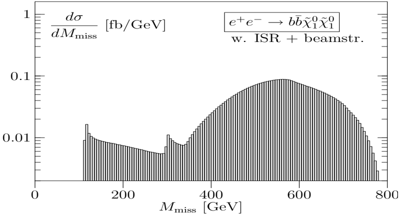
$M_{b\bar{b}} < 150$ GeV

$250 \text{ GeV} < M_{b\bar{b}} < 350$ GeV

Results

Off-shell decay $\tilde{\chi}^0_3 \to (\tilde{b}_1)_{off}\bar{b} \to b\bar{b}\tilde{\chi}^0_1$ gives broad continuum





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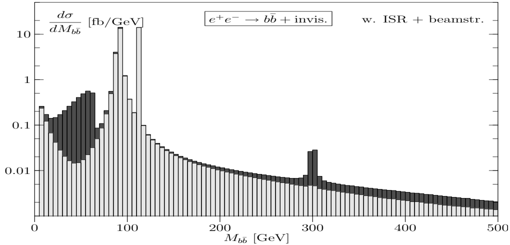

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$Z\bar{\nu}\nu$	111.4	2.114
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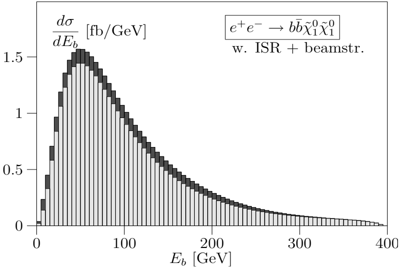
Sum	187.7	2.117
Exact	190.1	1.765
w/ISR	174.2	1.609

Channel	$\sigma_{\rm BW}$ [fb]	$\sigma^{\rm cut}_{\rm BW}$ [fb]
Zh	1.335	0.009
HA	0.314	0.003
$\tilde\chi^0_1\tilde\chi^0_2$	13.954	0.458
$\tilde\chi^0_1\tilde\chi^0_3$	4.828	0.454
$\tilde\chi^0_1\tilde\chi^0_4$	0.938	0.937
$\tilde b_1\tilde b_1$	0.757	0.451
$\tilde b_1\tilde b_2$	0.002	0.001
Sum	22.129	**2.314**
Exact	19.624	**0.487**
w/ISR	22.552	**0.375**

$\tilde b_1 \to b\tilde\chi^0_1$ decay kinematics affected



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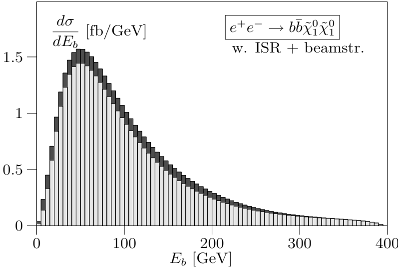
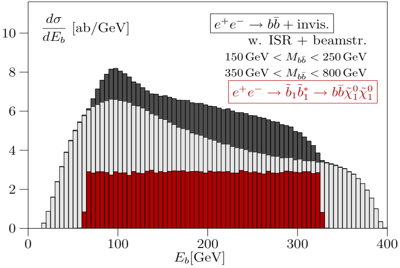




Going to Next-to-Leading Order

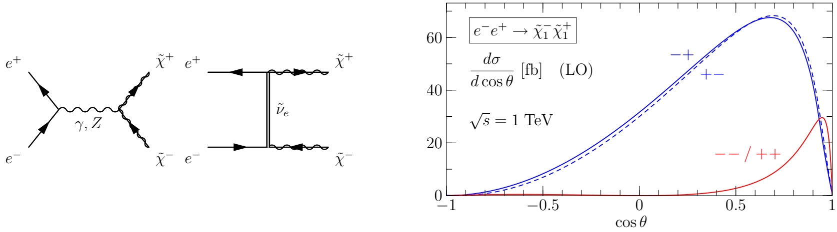
For the rest: always SPS1a' SUGRA-scenario with

$m_0 = 70\,\text{GeV}$ $m_{1/2} = 250\,\text{GeV}$ $\tan\beta = 10$ $\text{sgn}\,\mu = 1$ $A_0 = -300\,\text{GeV}$

Chargino masses and widths:

	M	Γ	Γ/M
$\tilde\chi_1^+$	183.7 GeV	0.077 GeV	0.00042
$\tilde\chi_2^+$	415.4 GeV	3.1 GeV	0.0075

SPS1a'-preferred decay (2-step cascade): $\tilde\chi_1^+ \to \tilde\tau_1 \nu_\tau \to \tau^+ \tilde\chi_1^0 \nu_\tau$



$\cos\theta$ angle between e^- and $\tilde\chi_1^-$

- Born helicity amplitudes known analytically   Choi et al., 9812236, 0002033
- Implemented in narrow width approximation in many programs

Virtual Corrections

Virtual corrections from SUSY and SM particles: self energies, vertex corrections, box diagrams (as usual)

(Semi-)automatized calculation with `FeynArts/FormCalc`
Hahn et al., 9807565, 0012260, 0105349 ; Fritzsche, ; Fritzsche/Hollik, 0407095

Independent check of numerical results Öller/Eberl/Majerotto, 0504109

Regulators:

- Electron mass m_e for collinear photon radiation
- Fictitious photon mass λ for infrared divergencies

Interference of Born and virtual corrections

$$\sigma_{\text{virt}}(s, \lambda^2, m_e^2) = \int d\Gamma_2 \left[2\text{Re} \left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2) \right) \right]$$

Eliminate dependence on λ by

- neglecting power corrections in λ
- Adding real (1st order) photon radiation with $E_\gamma < \Delta E_\gamma$
- Correction (terms $\propto \log \Delta E_\gamma$) is shifted into soft-photon factor

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Virtual corrections from SUSY and SM particles: self energies, vertex corrections, box diagrams (as usual)

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Soft-photon factor:

$$f_{\text{soft}} = -\frac{\alpha}{2\,\pi} \sum_{i,j\,=\,e^\pm,\widetilde{\chi}^\pm} \int_{|\mathbf{k}|\leq \mathbf{\Delta E}_\gamma} \frac{d^3k}{2\omega_k} \, \frac{(\pm) p_i p_j \, Q_i \, Q_j}{(p_i k)(p_j k)}$$

Real and Collinear Photons

"Virtual + Soft"

$$\sigma_{\text{v+s}}(s, \Delta E_\gamma, m_e^2) = \int d\Gamma_2 \left[f_{\text{soft}}(\tfrac{\Delta E_\gamma}{\lambda}) \, |\mathcal{M}_{\text{Born}}(s)|^2 + 2\text{Re} \left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2) \right) \right]$$

for simulation choose $\Delta E_\gamma \leq \Delta E_\gamma^{\text{exp}}$

Real radiation (i.e. the process $e^- e^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^- \gamma$):

$$\sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2) = \int_{\Delta E_\gamma} d\Gamma_3 \, |\mathcal{M}_{2\to 3}(s, m_e^2)|^2 .$$

"Total" cross section (fixed order):

$$\sigma_{\text{tot}}(s, m_e^2) = \sigma_{\text{Born}}(s) + \sigma_{\text{v+s}}(s, \Delta E_\gamma, m_e^2) + \sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2)$$

should not depend on ΔE_γ, but power corrections only in $\sigma_{2\,\to\,3}$, not in $\sigma_{\text{v+s}}$

As usual, split $2 \to 3$ cross section:

$$\sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2) = \sigma_{\text{hard,non-coll}}(s, \Delta E_\gamma, \Delta\theta_\gamma) + \sigma_{\text{hard,coll}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)$$

$x = 1 - 2E_\gamma/\sqrt{s}$ electron energy fraction after radiation

Approximate collinear radiation by convoluting the Born cross section with a structure function

$$\begin{aligned}\sigma_{\text{hard,coll}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2) &= \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3 \, |\mathcal{M}_{2\to 3}(s, m_e^2)|^2 \\ &= \int_0^{x_0} dx \, f(x; \Delta\theta_\gamma, \tfrac{m_e^2}{s}) \int d\Gamma_2 \, |\mathcal{M}_{\text{Born}}(xs, m_e^2)|^2.\end{aligned}$$

collinear structure functions (helicity conserving and helicity flip):

Böhm/Dittmaier, 1993

$$\begin{aligned} f^+(x) &= \tfrac{\eta}{4} \tfrac{1+x^2}{1-x} \\ f^-(x) &= \tfrac{\alpha}{2\pi}(1-x) \end{aligned} \qquad \eta := \frac{2\alpha}{\pi}\left[\log\left(\frac{s}{4m_e^2}(\Delta\theta_\gamma)^2\right) - 1\right]$$

Cutoff $\Delta E_\gamma \to x_0 = 1 - 2\Delta E_\gamma/\sqrt{s}$ (no power corrections in $\Delta\theta_\gamma$)

Simulation

Combining all parts:

$$\sigma_{\text{tot}}(s, m_e^2) = \int dx\, f_{\text{eff}}(x_1,\ x_2; \Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2\, |\mathcal{M}_{\text{eff}}(s, x_1,\ x_2; m_e^2)|^2 + \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3\, |\mathcal{M}_{2\to 3}(s)|^2,$$

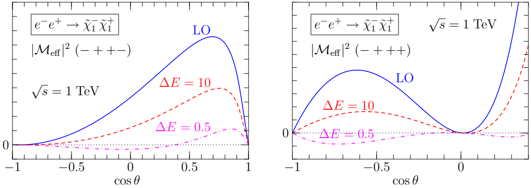
with

$$\begin{aligned} f_{\text{eff}}(x_1,\ x_2; \Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s}) = {} & \delta(1-x_1)\,\delta(1-x_2) \\ & + \delta(1-x_1)\, f(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s})\, \theta(x_0 - x_2) \\ & + f(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s})\, \delta(1-x_2)\, \theta(x_0 - x_1) \end{aligned}$$

$$\begin{aligned} |\mathcal{M}_{\text{eff}}(s, x_1,\ x_2; m_e^2)|^2 = {} & \left[1 + f_{\text{soft}}(\Delta E_\gamma, \lambda^2)\, \theta(x_1, x_2))\right]\ |\mathcal{M}_{\text{Born}}(s)|^2 \\ & + 2\text{Re}\left[\mathcal{M}_{\text{Born}}(s)\, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2)\right]\, \theta(x_1 - x_0)\theta(x_2 - x_0) \end{aligned}$$

All corrections defined as a generalized structure function
$\Rightarrow$ suitable for implementation in an event generator

Resumming photons



Experimental resolution drives one into negative weights region

Soft-collinear region: $E_\gamma < \Delta E_\gamma$, $\Delta\theta_\gamma < \theta_\gamma$: double logs $\frac{\alpha}{\pi} \log \frac{E_\gamma^2}{s} \log \theta_\gamma$ invalidate perturbative series

In that region resummation of all orders is possible

$$\sigma_{\text{Born+ISR}}(s, \Delta\theta_\gamma, m_e^2) = \int dx\, f_{\text{ISR}}(x; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2\, |\mathcal{M}_{\text{Born}}(xs)|^2,$$

f_{ISR} includes all order soft-photon radiation (LLA), hard-collinear up to 3rd order Skrzypek/Jadach, 1991

For collinear photons cancellation of infrared divergencies built in, main source of negative weights removed

Matching with NLO

Combine ISR-resummed LO with NLO, avoid double-counting

Subtract contribution of one soft photon (already in soft-photon factor)

$$f_{\text{soft,ISR}}(\Delta E_\gamma, \Delta\theta_\gamma, m_e^2) = \frac{\eta}{4}\int_{x_0}^1 dx \left(\frac{1+x^2}{1-x}\right)_+ = \frac{\eta}{4}\left(2\ln(1-x_0) + x_0 + \frac{1}{2}x_0^2\right).$$

After this subtraction we have

$$|\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2 = \left[1 + f_{\text{soft}}(\tfrac{\Delta E_\gamma}{\lambda}) - 2 f_{\text{soft,ISR}}(\Delta E_\gamma, \Delta\theta_\gamma, \tfrac{m_e^2}{s})\right] |\mathcal{M}_{\text{Born}}(\hat{s})|^2 + 2\text{Re}\left[\mathcal{M}_{\text{Born}}(\hat{s})\, \mathcal{M}_{\text{1-loop}}(\hat{s}, \lambda^2, m_e^2)\right],$$

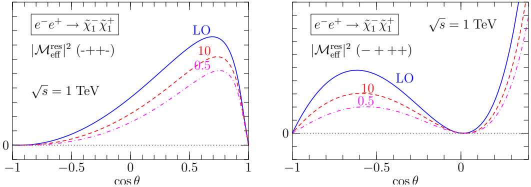
contains Born, virtual + soft contr. with LL part of virtual and soft-coll. removed

New "s+v" term (contains also soft/coll. corrections to Born/1-loop interference)

$$\sigma_{\text{v+s,ISR}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2) = \int dx_1\, f_{\text{ISR}}(x_1; \Delta\theta_\gamma, \tfrac{m_e^2}{s}) \int dx_2\, f_{\text{ISR}}(x_2; \Delta\theta_\gamma, \tfrac{m_e^2}{s}) \int d\Gamma_2\, |\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2$$

Simulation

Resummation approach eliminates problem of negative weights:



Only source for negative weights: soft-noncollinear region, does not cause problems

Final improvement:

- convoluting $2 \to 3$ part with ISR structur function
- add $2 \to 4$ part

$$\sigma_{\text{tot,ISR+}}(s, m_e^2) = \int dx_1\, f_{\text{ISR}}(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int dx_2\, f_{\text{ISR}}(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s})$$

$$\times \left(\int d\Gamma_2\, |\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2 + \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3\, |\mathcal{M}_{2\to 3}(\hat{s})|^2 \right)$$

$$+ \int_{\Delta E_{\gamma,i}, \Delta\theta_{\gamma,i}} d\Gamma_4\, |\mathcal{M}_{2\to 4}(s)|^2$$

Results and Distributions

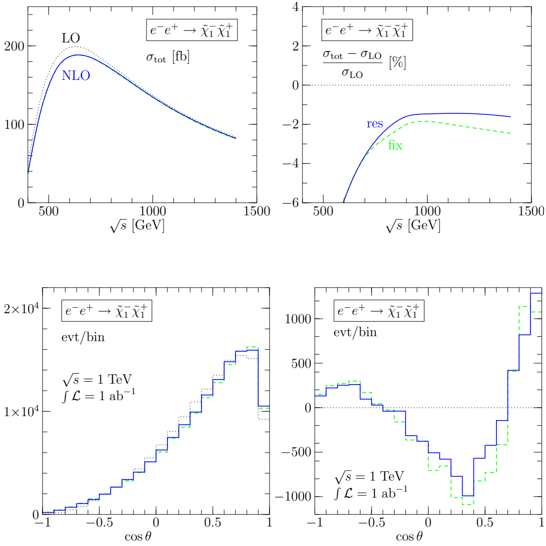
NLO corrections -5% (Xsec max.)

-2% (-1.5%) fixed-order (resummed) @ 1 TeV

Binned distribution of chargino scattering angle

Cutoffs: $\Delta\theta_\gamma = 1°$, $\Delta E_\gamma = 3$ GeV (fixed-order)

K-factor approach insufficient



Summary and Outlook

- LHC: new era of physics
- Precision predictions for SUSY pheno are important
  - Higher orders: virtual corrections
  - Higher orders: real corrections
- Factorization in $2 \to 2$ production and decay insufficient/wrong
- Off-shell effects and interferences affect results (especially with cuts)
- Use full matrix elements
- Tools are available for LHC/ILC: WHIZARD Kilian/Ohl/JR, 0708.4233

  `http://whizard.event-generator.org`
  - First BSM signal vs. bkgd. jet studies
  - Next step(s): Reconsider all edge structures [LHC]:

    Alboteanu/Alwall/Plehn/JR/Schumann
- Extended WHIZARD: 1st NLO SUSY MC Generator for the ILC

  `FeynArts/FormCalc` interface: all MSSM $2 \to 2$ processes for ILC
- QCD NLO simulations: GOLEM/WHIZARD collab. Binoth/Guffanti/Reiter/JR