Ward Identities as consistency checks of automatized calculations in supersymmetric field theories

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Talk, 2001/2002

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Problem: High energy processes at colliders need precise calculation of processes with up to 8 final state particles.

The number of Feynman diagrams grows with the factorial of the number of external particles.

ϕ^3 theory unflavored: $(2N-5)!!$

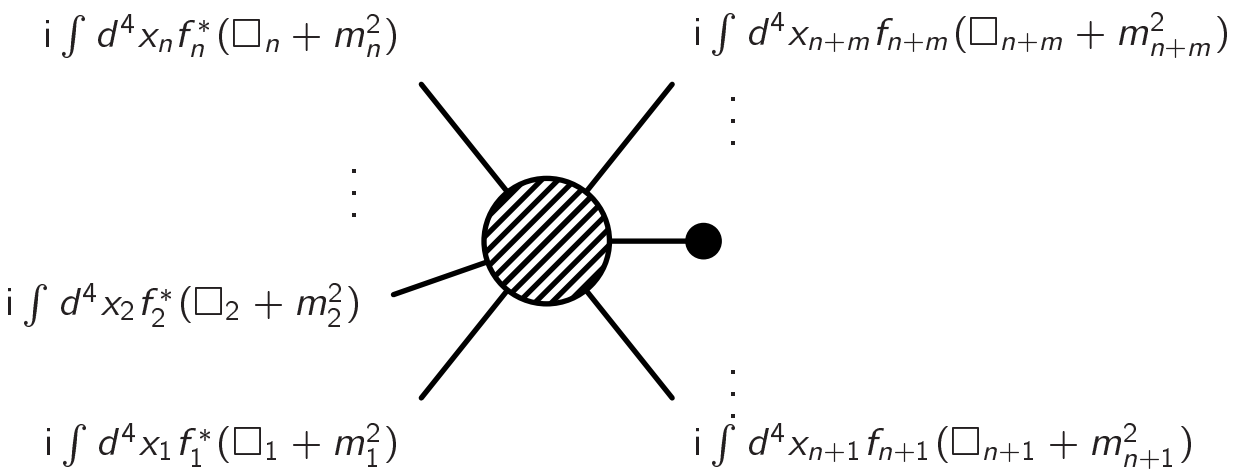
Solution:

O'Mega: An Optimizing Matrix Element Generator for Amplitudes, invented by Thorsten Ohl at TU Darmstadt.

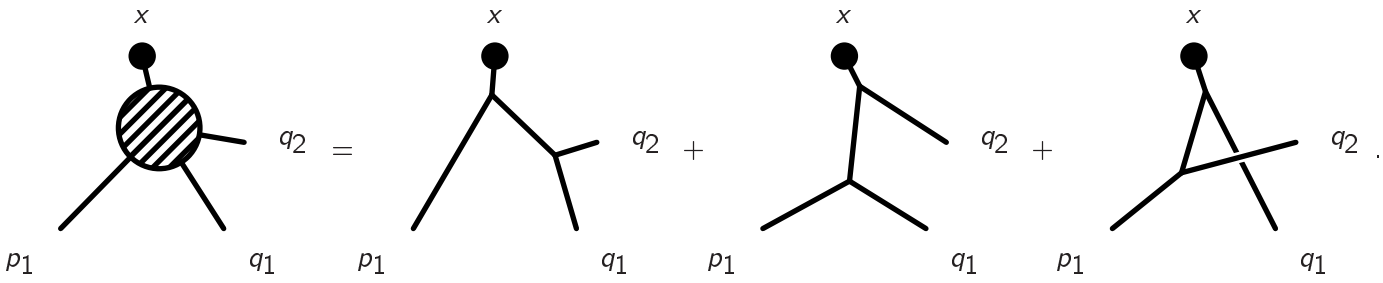
O'Mega reduces the growth in calculational effort from $\mathcal{O}(N!)$ to $\mathcal{O}(e^N)$, N: number of external particles

process	Diagrams		O'Mega	
	#	vertices	#prop.	vertices
$e^+e^- \to e^+\nu_e d\bar{u}$	20	80	14	45
	20	80	14	45
$e^+e^- \to e^+\nu_e d\bar{u}\gamma$	146	730	36	157
	142	710	33	151
$e^+e^- \to e^+\nu_e d\bar{u}\gamma\gamma$	1256	7536	80	462
	1174	7044	71	441
$e^+e^- \to e^+\nu_e d\bar{u}\gamma\gamma\gamma$	12420	86940	168	1343
	11058	77406	147	1284
$e^+e^- \to e^+\nu_e d\bar{u}\gamma\gamma\gamma\gamma$	138816	1110528	344	3933
	116832	$O(10^6)$	299	3784

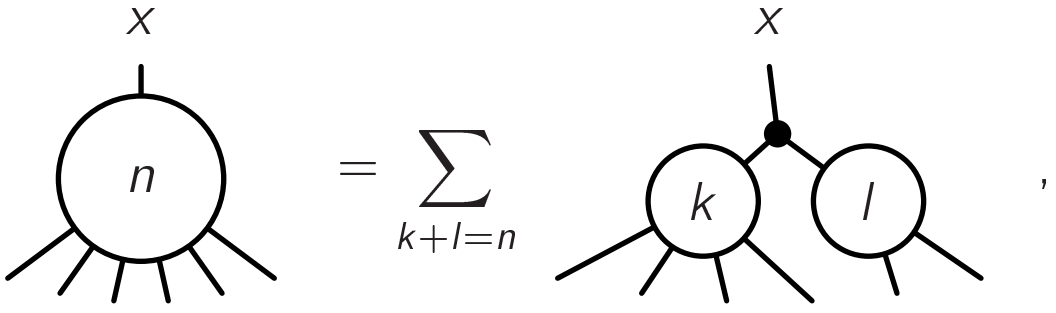
Basic ingredients are the One particle off shell wavefunctions (1POWs), which are constructed by the LSZ reduction applied to all but one external legs:



with the abbreviation $f_i \equiv e^{-\mathrm{i}k_i x_i}/\sqrt{(2\pi)^3 2k_i^0}$



At tree-level, the set of all 1POWs for a given set of external momenta can be constructed recursively



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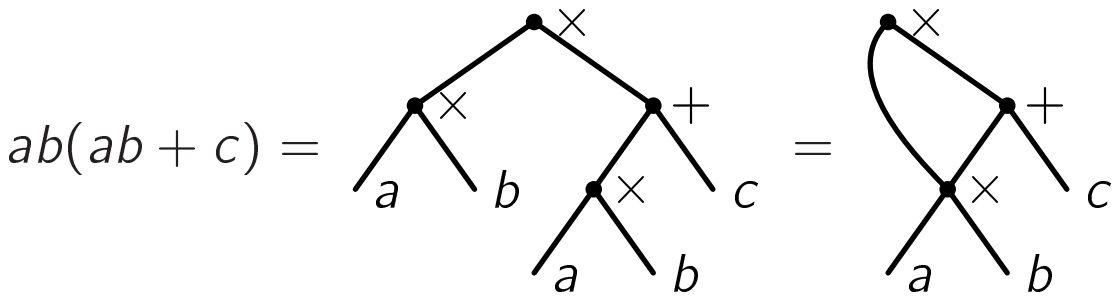
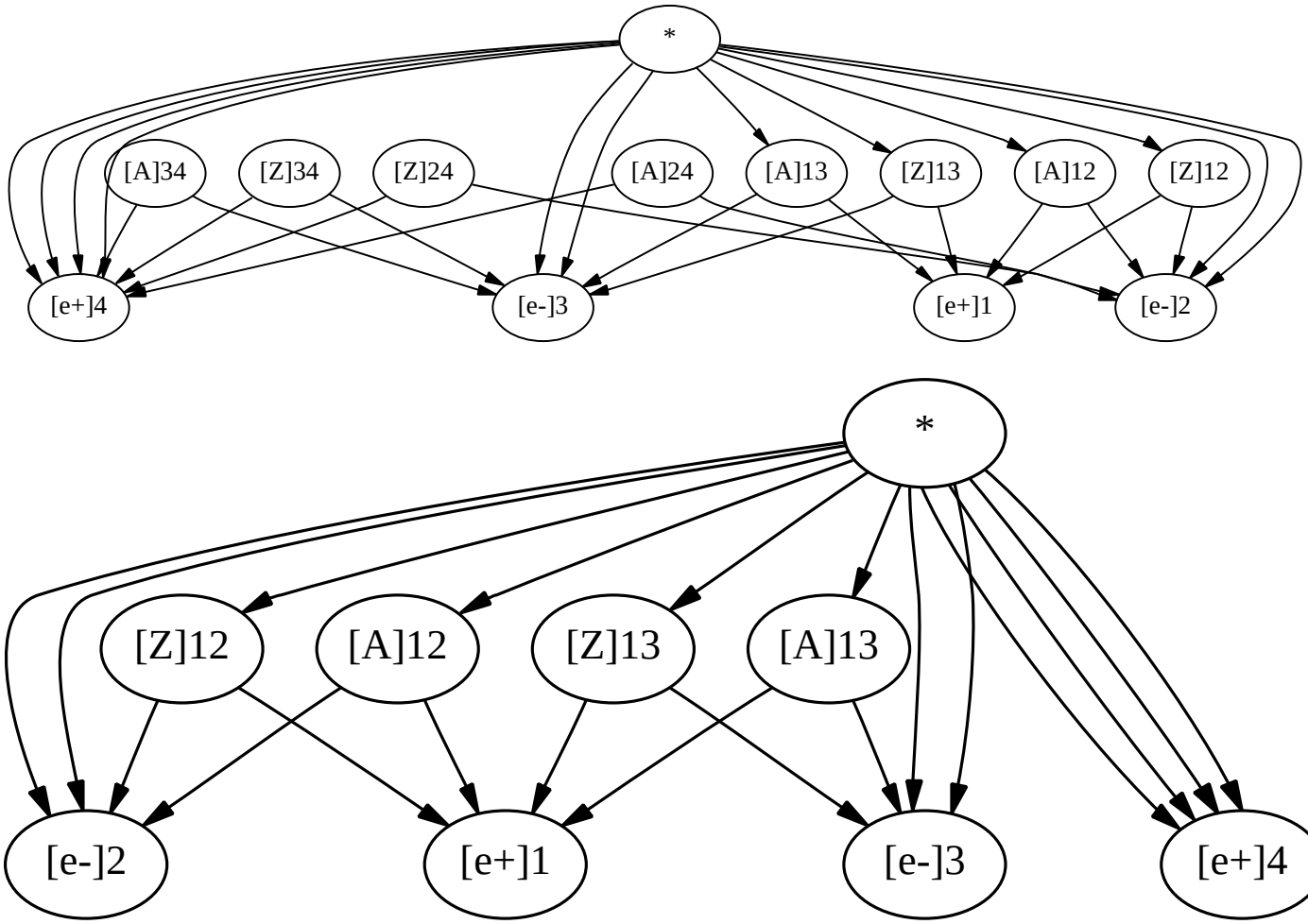
Directed Acyclical Graphs (DAGs) are the most effective representation of tree Feynman diagrams, because they eliminate the redundacies:

Trivial Example:



Examples for physical DAGs:



Grow: starting from the external particles, build the tower of *all* 1POWs up to a given height (the height is less than the number of external lines for asymmetric keystones and less than half of that for symmetric keystones) and translate it to the equivalent DAG D.

Select: from D, determine *all* possible *flavored keystones* for the process under consideration and the 1POWs appearing in them.

Harvest: construct a sub-DAG $D^* \subseteq D$ consisting *only* of nodes that contribute to the 1POWs appearing in the flavored keystones.

Calculate: multiply the 1POWs as specified by the keystones and sum the keystones.

VERTICES	#
Gauge-IA: $WW\gamma$, WWZ, ggg	3
Gauge-Lepton-IA: $\ell^+\ell^-\gamma$, $\ell^+\ell^-Z$, $\ell^+\nu W^-$, $\ell^-\bar{\nu}W^+$, $\nu\bar{\nu}Z$	$5\,G \to 15$
Gauge-Quark-IA: $q\bar{q}\gamma$, $q\bar{q}Z$, $u\bar{d}W^-$, $d\bar{u}W^+$, $q\bar{q}g$	$2\,G(\,G\,+\,3) \to 36$
Higgs-IA: HHH	1
Higgs-Gauge-IA: HW^+W^-, HZZ	2
Higgs-Lepton-IA: $\ell^+\ell^-H$	$G \to 3$
Higgs-Quark-IA: $q\bar{q}H$	$2\,G \to 6$
Higgs-Gst.-IA: $H\phi^+H\phi^-$, $H\phi\phi$	2
Gst.-Gauge-IA: $\phi^+\phi^-\gamma$, $\phi^+\phi^-Z$, $\phi^\pm\phi W^\mp$, $\phi^\pm HW^\mp$, ϕHZ, $\phi^\pm W^\mp Z$, $\phi^\pm W^\mp\gamma$	11
Gst.-Lepton-IA: $\ell^+\ell^-\phi$, $\ell^-\bar{\nu}\phi^+$, $\ell^+\nu\phi^+$	$3\,G \to 9$
Gst.-Quark-IA: $q\bar{q}\phi$, $u\bar{d}\phi^-$, $d\bar{u}\phi^+$	$2G(G+1) \to 24$

3-Vertices: $\left(2G^2+14G+6\right)+\left(2G^2+5G+13\right) \to 66+46$

VERTICES	#
Gauge-IA: $WW\gamma\gamma$, $WWZ\gamma$, $WWZZ$, $WWWW$, $gggg$	5
Higgs-IA: $HHHH$	1
Higgs-Gauge-IA: HHW^+W^-, $HHZZ$	2
Higgs-Gst.-IA: $HH\phi^+\phi^-$, $HH\phi\phi$, $\phi^+\phi^-\phi^+\phi^-$, $\phi^+\phi^-\phi\phi$, $\phi\phi\phi\phi$	5
Gst.-Gauge-IA: $\phi\phi W^+W^-$, $\phi\phi ZZ$, $\phi^+\phi^-WW$, $\phi^+\phi^-ZZ$, $\phi^+\phi^-Z\gamma$, $\phi^+\phi^-\gamma\gamma$, $H\phi^\pm W^\mp Z$, $H\phi^\pm W^\mp\gamma$, $\phi\phi^\pm W^\mp Z$, $\phi\phi^\pm W^\mp\gamma$	14

4-Vertices: $8+19$

VERTICES	#
Gauge-IA: $WW\gamma$, WWZ, ggg	3
Gauge-Lepton-IA: $\ell^+\ell^-\gamma$, $\ell^+\ell^-Z$, $\ell^+\nu W^-$, $\ell^-\bar{\nu}W^+$, $\nu\bar{\nu}Z$	$5\,G \to 15$
Gauge-Quark-IA: $q\bar{q}\gamma$, $q\bar{q}Z$, $u\bar{d}W^-$, $d\bar{u}W^+$, $q\bar{q}g$	$2\,G(G+3) \to 36$
Higgs-IA: H^+H^-H, H^+H^-h, HHH, HHh, Hhh, hhh, AAH, AAh	8
Higgs-Gauge-IA: $AW^\pm H^\mp$, HAZ, hAZ, W^+W^-H, W^+W^-h, $W^\pm H^\mp H$, $W^\mp H^\pm h$, ZZH, ZZh, $H^+H^-\gamma$, H^+H^-Z	14
Higgs-Lepton-IA: $H^+\bar{\nu}\ell^-$, $H^-\nu\ell^+$, $\ell^+\ell^-H$, $\ell^+\ell^-h$, $\ell^+\ell^-A$	$5\,G \to 15$
Higgs-Quark-IA: $q\bar{q}H$, $q\bar{q}h$, $q\bar{q}A$, $u\bar{d}H^-$, $d\bar{u}H^+$	$2\,G(G+3) \to 36$
Higgs-Chargino-Neutralino-IA: $\tilde{\chi}\tilde{\chi}H$, $\tilde{\chi}\tilde{\chi}h$, $\tilde{\chi}\tilde{\chi}A$, $\tilde{\chi}^+\tilde{\chi}^-H$, $\tilde{\chi}^+\tilde{\chi}^-h$, $\tilde{\chi}^+\tilde{\chi}^-A$, $\tilde{\chi}^\pm\tilde{\chi}H^\mp$	52
Slepton-Gauge-IA: $\tilde{\ell}^+\tilde{\ell}^-\gamma$, $\tilde{\ell}^+\tilde{\ell}^-Z$, $\tilde{\nu}\tilde{\nu}^*Z$, $\tilde{\ell}^+\tilde{\nu}W^-$, $\tilde{\ell}^-\tilde{\nu}W^+$	$11\,G \to 33$
Squark-Gauge-IA: $\tilde{q}\tilde{q}^*\gamma$, $\tilde{q}\tilde{q}^*Z$, $\tilde{d}\tilde{u}^*W^+$, $\tilde{u}\tilde{d}^*W^-$, $\tilde{q}\tilde{q}^*g$	$8\,G(G+2) \to 120$
Chargino-Neutralino-Gluino-Gauge-IA: $\tilde{\chi}\tilde{\chi}Z$, $\tilde{\chi}^+\tilde{\chi}^-Z$, $\tilde{\chi}^+\tilde{\chi}^-\gamma$, $\tilde{\chi}^+\tilde{\chi}W^-$, $\tilde{\chi}^-\tilde{\chi}W^+$, $\tilde{g}\tilde{g}g$	27
Other Chargino-Neutralino-Gluino-IA: $q\tilde{g}\tilde{q}^*$, $\bar{q}\tilde{q}\tilde{q}$, $\tilde{\chi}^+\ell^-\tilde{\nu}^*$, $\tilde{\chi}^-\ell^+\tilde{\nu}$, $\tilde{\chi}^+\bar{\nu}\tilde{\ell}^-$, $\tilde{\chi}^-\nu\tilde{\ell}^+$, $\tilde{\chi}^+\bar{u}\tilde{d}$, $\tilde{\chi}^-u\tilde{d}^*$, $\tilde{\chi}^+d\tilde{u}^*$, $\tilde{\chi}^-\bar{d}\tilde{u}$, $\tilde{\chi}\nu\tilde{\nu}^*$, $\tilde{\chi}\bar{\nu}\tilde{\nu}$, $\tilde{\chi}\ell^\mp\tilde{\ell}^\pm$, $\tilde{\chi}q\tilde{q}^*$, $\tilde{\chi}\bar{q}\tilde{q}$	$4G(4G+19) \to 372$
Higgs-Slepton-IA: $H\tilde{\nu}\tilde{\nu}^*$, $h\tilde{\nu}\tilde{\nu}^*$, $H\tilde{\ell}^+\tilde{\ell}^-$, $h\tilde{\ell}^+\tilde{\ell}^-$, $A\tilde{\ell}^+\tilde{\ell}^-$, $H^+\tilde{\ell}^-\tilde{\nu}^*$, $H^-\tilde{\ell}^+\tilde{\nu}$	$18\,G \to 54$
Higgs-Squark-IA: $\tilde{q}\tilde{q}^*H$, $\tilde{q}\tilde{q}^*h$, $\tilde{q}\tilde{q}^*A$, $\tilde{u}\tilde{d}^*H^-$, $\tilde{u}^*\tilde{d}H^+$	$8\,G(3\,G+1) \to 240$

3-Vertices: $52\,G^2 + 151\,G + 104 \to 1025$

VERTICES	#
Gauge-IA: $WW\gamma\gamma$, $WWZ\gamma$, $WWZZ$, $WWWW$, $gggg$	5
Higgs-IA: $H^+H^-H^+H^-$, H^+H^-HH, H^+H^-Hh, H^+H^-hh, H^+H^-AA, $HHHH$, $HHHh$, $HHhh$, $Hhhh$, $hhhh$, $HHAA$, $HhAA$, $hhAA$, $AAAA$	14
Higgs-Gauge-IA: $HHZZ$, $hhZZ$, $AAZZ$, H^+H^-ZZ, $H^+H^-Z\gamma$, $H^+H^-\gamma\gamma$, $H^\pm HW^\mp\gamma$, $H^\pm hW^\mp\gamma$, $H^\pm HW^\mp Z$, $H^\pm hW^\mp Z$, HHW^+W^-, hhW^+W^-, AAW^+W^-, $H^\pm AW^\mp\gamma$, $H^\pm AW^\mp Z$	21
Slepton-Gauge-IA: $\tilde\ell^+\tilde\ell^-\gamma\gamma$, $\tilde\ell^+\tilde\ell^-Z\gamma$, $\tilde\ell^+\tilde\ell^-ZZ$, $\tilde\nu\tilde\nu^*ZZ$, $\tilde\ell^+\tilde\ell^-W^+W^-$, $\tilde\nu\tilde\nu^*W^+W^-$, $\tilde\ell^-\tilde\nu^*W^+\gamma$, $\tilde\ell^+\tilde\nu W^-\gamma$, $\tilde\ell^-\tilde\nu^*W^+Z$, $\tilde\ell^+\tilde\nu W^-\gamma$	$24G \to 72$
Squark-Gauge-IA: $\tilde q\tilde q^*\gamma\gamma$, $\tilde q\tilde q^*Z\gamma$, $\tilde q\tilde q^*ZZ$, $\tilde q\tilde q^*W^+W^-$, $\tilde u\tilde d^*W^-\gamma$, $\tilde u^*\tilde dW^+\gamma$, $\tilde u\tilde d^*W^-Z$, $\tilde u^*\tilde dW^+Z$, $\tilde q\tilde q^*gg$, $\tilde q\tilde q^*g\gamma$, $\tilde q\tilde q^*gZ$, $\tilde u\tilde d^*gW^-$, $\tilde u^*\tilde dgW^+$	$4G(6G+11)$ $\to 348$
Slepton-Slepton-IA: $\tilde\nu\tilde\nu^*\tilde\nu\tilde\nu^*$, $\tilde\nu\tilde\nu^*\tilde\ell^-\tilde\ell^+$, $\tilde\ell^-\tilde\ell^+\tilde\ell^-\tilde\ell^+$	$\frac{25}{2}G^2+\frac{1}{2}G+1$ $\to 115$
Squark-Squark-IA: $\tilde q\tilde q^*\tilde q\tilde q^*$, $\tilde u\tilde u^*\tilde d\tilde d^*$	$2G(8G^3+8G$ $+1) \to 1446$
Slepton-Squark-IA: $\tilde q\tilde q^*\tilde\nu\tilde\nu^*$, $\tilde q\tilde q^*\tilde\ell^+\tilde\ell^-$, $\tilde u\tilde d^*\tilde\ell^-\tilde\nu^*$, $\tilde u^*\tilde d\tilde\ell^+\tilde\nu$	$8G^2(2G+5)$ $\to 792$
Higgs-Slepton-IA: $HH\tilde\nu\tilde\nu^*$, $Hh\tilde\nu\tilde\nu^*$, $hh\tilde\nu\tilde\nu^*$, $AA\tilde\nu\tilde\nu^*$, $H^+H^-\tilde\nu\tilde\nu^*$, $HH\tilde\ell^+\tilde\ell^-$, $Hh\tilde\ell^+\tilde\ell^-$, $hh\tilde\ell^+\tilde\ell^-$, $AA\tilde\ell^+\tilde\ell^-$, $H^+H^-\tilde\ell^+\tilde\ell^-$, $HH^+\tilde\ell^-\tilde\nu^*$, $hH^+\tilde\ell^-\tilde\nu^*$, $AH^+\tilde\ell^-\tilde\nu^*$, $HH^-\tilde\ell^+\tilde\nu$, $hH^-\tilde\ell^+\tilde\nu$, $AH^-\tilde\ell^+\tilde\nu$	$37G \to 111$
Higgs-Squark-IA: $\tilde q\tilde q^*HH$, $\tilde q\tilde q^*Hh$, $\tilde q\tilde q^*hh$, $\tilde q\tilde q^*AA$, $\tilde q\tilde q^*H^+H^-$, $\tilde u^*\tilde dH^+H$, $\tilde u^*\tilde dH^+h$, $\tilde u^*\tilde dH^+A$, $\tilde u\tilde d^*H^+H$, $\tilde u\tilde d^*H^+h$, $\tilde u\tilde d^*H^+A$	$8G(3G+5)$ $\to 336$

4-Vertices: $16G^4+16G^3+\frac{233}{2}G^2+\frac{295}{2}G+41 \to 3260$

VERTICES	#
Higgs-Gst.-IA: $HA\phi$, $hA\phi$, $H^\pm H\phi^\mp$, $H^\pm h\phi^\mp$, $H^\pm A\phi^\mp$, $H\phi\phi$, $h\phi\phi$, $H\phi^+\phi^-$, $h\phi^+\phi^-$	12
Higgs-Gst.-Gauge-IA: $Z\phi^+\phi^-$, $\gamma\phi^+\phi^-$, $W^\pm\phi^\mp\phi$, $ZH\phi$, $Zh\phi$, $W^\pm\phi^\mp H$, $W^\pm\phi^\mp h$, $W^\pm Z\phi^\mp$, $W^\pm\gamma\phi^\mp$	14
Gst.-Lepton-IA: $\ell^+\ell^-\phi$, $\ell^-\bar{\nu}\phi^+$, $\ell^+\nu\phi^-$	$3G \to 9$
Gst.-Quark-IA: $q\bar{q}\phi$, $u\bar{d}\phi^-$, $d\bar{u}\phi^+$	$2G(G+1)$ $\to 24$
Gst.-C/N-ino-IA: $\tilde{\chi}\tilde{\chi}\phi$, $\tilde{\chi}^+\tilde{\chi}^-\phi$, $\tilde{\chi}^+\tilde{\chi}\phi^-$, $\tilde{\chi}^-\tilde{\chi}\phi^+$	30
Gst.-Slepton-IA: $\tilde{\ell}^+\tilde{\ell}^-\phi$, $\tilde{\ell}^-\tilde{\nu}^*\phi^+$, $\tilde{\ell}^+\tilde{\nu}\phi^-$	$8G \to 24$
Gst.-Squark-IA: $\tilde{q}\tilde{q}^*\phi$, $\tilde{u}\tilde{d}^*\phi^-$, $\tilde{u}^*\tilde{d}\phi^+$	$8G(G+1)$ $\to 96$

3-Vertices: $10G^2 + 21G + 56 \to 209$

VERTIZES	#
Higgs-Gst.-IA: $HHA\phi$, $HhA\phi$, $hhA\phi$, $AAA\phi$, $H^+H^-A\phi$, $H^\pm HH\phi^\mp$, $H^\pm Hh\phi^\mp$, $H^\pm hh\phi^\mp$, $H^\pm AA\phi^\mp$, $H^\pm HA\phi^\mp$, $H^\pm hA\phi^\mp$, $H^\pm H^+H^-\phi^\mp$, $HH\phi^+\phi^-$, $Hh\phi^+\phi^-$, $hh\phi^+\phi^-$, $AA\phi^+\phi^-$, $H^\pm H\phi^\mp\phi$, $H^\pm h\phi^\mp\phi$, $H^\pm A\phi^\mp\phi$, $HH\phi^+\phi^-$, $Hh\phi^+\phi^-$, $hh\phi^+\phi^-$, $AA\phi^+\phi^-$, $H^+H^-\phi^+\phi^-$, $H^\pm H^\pm\phi^\mp\phi^\mp$, $A\phi\phi\phi$, $H^\pm\phi^\mp\phi\phi$, $A\phi^+\phi^-\phi$, $H^\pm\phi^\mp\phi^+\phi^-$, $\phi\phi\phi\phi$, $\phi^+\phi^-\phi\phi$, $\phi^+\phi^-\phi^+\phi^-$	45
Higgs-Gst.-IA: $ZZ\phi\phi$, $ZZ\phi^+\phi^-$, $Z\gamma\phi^+\phi^-$, $\gamma\gamma\phi^+\phi^-$, $W^+W^-\phi^+\phi^-$, $W^+W^-\phi\phi$, $W^\pm Z\phi^\mp\phi$, $W^\pm\gamma\phi^\mp\phi$, $W^\pm Z\phi^\mp H$, $W^\pm\gamma\phi^\mp H$, $W^\pm Z\phi^\mp h$, $W^\pm\gamma\phi^\mp h$	18
Slepton-Gst.-IA: $\tilde{\nu}\tilde{\nu}^*A\phi$, $\tilde{\nu}\tilde{\nu}^*H^\pm\phi^\mp$, $\tilde{\nu}\tilde{\nu}^*\phi^+\phi^-$, $\tilde{\nu}\tilde{\nu}^*\phi\phi$, $\tilde{\ell}^+\tilde{\ell}^-A\phi$, $\tilde{\ell}^+\tilde{\ell}^-H^\pm\phi^\mp$, $\tilde{\ell}^+\tilde{\nu}h\phi^-$, $\tilde{\ell}^-\tilde{\nu}^*h\phi^+$, $\tilde{\ell}^+\tilde{\nu}A\phi^-$, $\tilde{\ell}^-\tilde{\nu}^*\phi\phi^+$, $\tilde{\ell}^+\tilde{\nu}H\phi^-$, $\tilde{\ell}^-\tilde{\nu}^*\phi\phi^+$, $\tilde{\ell}^+\tilde{\ell}^-\phi^+\phi^-$, $\tilde{\ell}^+\tilde{\ell}^-\phi\phi$, $\tilde{\ell}^+\tilde{\nu}H^-\phi$, $\tilde{\ell}^-\tilde{\nu}^*H^+\phi$, $\tilde{\ell}^+\tilde{\nu}H\phi^-$, $\tilde{\ell}^-\tilde{\nu}^*H\phi^+$	$45G$ $\to 135$
Squark-Gst.-IA: $\tilde{q}\tilde{q}^*A\phi$, $\tilde{q}\tilde{q}^*H^\pm\phi^\mp$, $\tilde{q}\tilde{q}^*\phi^+\phi^-$, $\tilde{q}\tilde{q}^*\phi\phi$, $\tilde{u}^*\tilde{d}H^+\phi$, $\tilde{u}\tilde{d}^*H^-\phi$, $\tilde{u}^*\tilde{d}H\phi^+$, $\tilde{u}\tilde{d}^*H\phi^-$, $\tilde{u}^*\tilde{d}h\phi^+$, $\tilde{u}\tilde{d}^*h\phi^-$, $\tilde{u}^*\tilde{d}A\phi^+$, $\tilde{u}\tilde{d}^*A\phi^-$, $\tilde{u}^*\tilde{d}\phi^+\phi$, $\tilde{u}\tilde{d}^*\phi^-\phi$	$40G \cdot$ $(G+1)$ $\to 480$

4-Vertices: $40G^2 + 85G + 63 \to 678$

Ward Identities (WI):

WI are fulfilled for linearly realized symmetries on-shell *and* off-shell, for gauge symmetries and supersymmetries after eliminating the auxiliary fields *only* on-shell. (Assumption: $[\mathcal{J}^0(x), \mathcal{O}(y)]\Big|_{x^0=y^0} = \delta^3(\vec{x}-\vec{y})[Q, \mathcal{O}(y)]$):

$$\mathrm{i}\partial_\mu^x \left\langle 0\middle|\mathrm{T}\left[\mathcal{J}^\mu(x)\mathcal{O}^1(y_1)\ldots\mathcal{O}^n(y_n)\right]\middle|0\right\rangle =$$
$$\sum_{i=1}^{N} \left\langle 0\middle|\mathrm{T}\left[\delta^4(x-y_i)\mathcal{O}^1(y_1)\ldots(\delta\mathcal{O}^i(y_i))\ldots\mathcal{O}^n(y_n)\right]\middle|0\right\rangle$$

On shell the Schwinger contact terms (RHS) vanish. (Total amputation - there is one propagator on the RHS with "wrong" momentum)

Slavnov-Taylor Identities (STI):

BRS invariance of the gauge fixed theory. STI are valid off-shell.

$$0 = \langle 0|\mathrm{T}\left[\{\mathrm{i}Q_{\mathrm{BRS}}, \mathcal{O}(x_1)\ldots\mathcal{O}(x_n)\}\right]|0\rangle =$$
$$\sum_{i=1}^{n} (-)^{\sum_{j=1}^{i-1}|j|} \langle 0|\mathrm{T}\left[\mathcal{O}(x_1)\ldots s\mathcal{O}(x_i)\ldots\mathcal{O}(x_n)\right]|0\rangle$$

Example: (C. Schwinn, TU Darmstadt, 2002)

Ward-Identities for spontaneously broken gauge symmetries produce numerical consistency checks:

$$\left| \mathrm{i}\frac{p^\mu}{m} W_\mu + \phi \right| \leq \|\tfrac{p}{m}\| \cdot \|W\| \epsilon, \qquad \epsilon = 3500\, \epsilon_M$$

Test reveals deviations in the coupling constants (e.g. g_{HHZZ}) of order 10^{-8}.

Our Aim:

In supersymmetric field theories can Ward identities of the supersymmetry be used as another consistency check.

Off-Shell identities provide more stringent tests for the theory. $\Longrightarrow$ Next step: Slavnov-Taylor identities

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We consider the simplest Abelian model with one superfield:

$$\frac{1}{2}\left[\hat{\Phi}^\dagger \exp(-\mathcal{V})\hat{\Phi}\right]_D + \frac{1}{2}\Re\left[\overline{W_R}W_L\right]_F$$

with $W := -\frac{1}{4}(\overline{\mathcal{D}}\mathcal{D})\mathcal{D}V, \qquad \mathcal{D} := \frac{\partial}{\partial\overline{\theta}} + (\mathrm{i}\gamma^\mu\theta)\partial_\mu$

Gauge symmetry forbids a superpotential; $\rightarrow$ all fields are massless.

Lagrangean density in components:

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2}(\partial_\mu A)(\partial^\mu A) + \frac{1}{2}(\partial_\mu B)(\partial^\mu B) + \frac{\mathrm{i}}{2}\overline{\Psi}\partial\!\!\!/\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + \frac{\mathrm{i}}{2}\overline{\lambda}\partial\!\!\!/\lambda + eG_\mu\left(B\partial^\mu A - A\partial^\mu B\right) + \frac{e^2}{2}G_\mu G^\mu\left(A^2+B^2\right) \\ & - e\overline{\Psi}\left(A + \mathrm{i}\gamma^5 B\right)\lambda - \frac{e}{2}\overline{\Psi}G\!\!\!/\gamma^5\Psi - \frac{e^2}{8}\left(A^4 + B^4 + 2A^2B^2\right)\end{aligned}$$

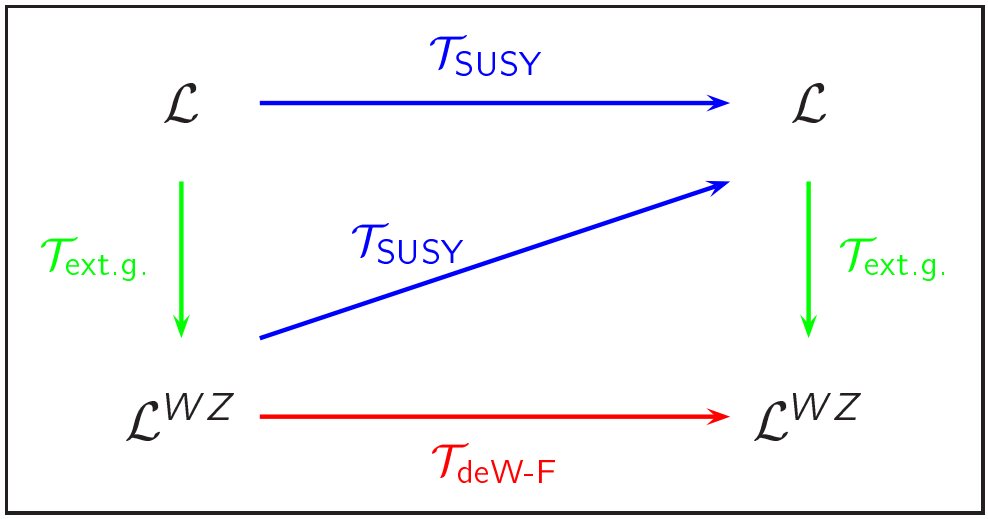
Supersymmetric current:

$$\begin{aligned}\mathcal{J}^\mu = {} & -(\partial\!\!\!/ A)\gamma^\mu\Psi - \mathrm{i}(\partial\!\!\!/ B)\gamma^\mu\gamma^5\Psi + \mathrm{i}eAG\!\!\!/\gamma^\mu\gamma^5\Psi - eBG\!\!\!/\gamma^\mu\Psi \\ & + \frac{1}{2}[\gamma^\alpha,\gamma^\beta]\gamma^\mu\gamma^5(\partial_\alpha G_\beta)\lambda - \frac{\mathrm{i}e}{2}\left(A^2+B^2\right)\gamma^\mu\lambda\end{aligned}$$

Used in the sequel: $\phi \equiv \frac{1}{\sqrt{2}}(A + \mathrm{i}B)$.

SUSY transformation:

$$\begin{aligned}
\delta_\xi \phi &= \sqrt{2}\,(\overline{\xi_R}\Psi_L) \\
\delta_\xi \Psi_L &= -\sqrt{2}\,\mathrm{i}(\not{\partial}\phi)\xi_R + \sqrt{2}\mathcal{F}\xi_L \\
\delta_\xi \mathcal{F} &= -\sqrt{2}\,\mathrm{i}\,(\overline{\xi_L}\not{\partial}\Psi_L)
\end{aligned}$$



de Wit-Freedman transformation:

$$\begin{aligned}
\tilde{\delta}_\xi \phi &= \sqrt{2}\,(\overline{\xi_R}\Psi_L) \\
\tilde{\delta}_\xi \Psi_L &= -\sqrt{2}\,\mathrm{i}(\not{D}\phi)\xi_R + \sqrt{2}\mathcal{F}\xi_L \\
\tilde{\delta}_\xi \mathcal{F} &= -\sqrt{2}\,\mathrm{i}\,(\overline{\xi_L}\not{D}\Psi_L) \\
&\quad -2\mathrm{i}Q\phi\,(\overline{\xi_L}\lambda_R)
\end{aligned}$$

Inserting the equations of motion of the auxiliary fields:

"On-Shell" de Wit-Freedman transformation:

$$\begin{aligned}
\tilde{\delta}'_\xi \phi &= \sqrt{2}\,(\overline{\xi_R}\Psi_L) \\
\tilde{\delta}'_\xi \Psi_L &= -\sqrt{2}\,\mathrm{i}(\not{D}\phi)\xi_R \\
&\quad -\sqrt{2}\left(\frac{\partial f(\phi)}{\partial \phi}\right)^* \xi_L
\end{aligned}$$

There are also changes in the vector superfield of the gauge bosons:

SUSY transformation:

$$\begin{aligned}
\delta_\xi G_\mu &= -\left(\bar{\xi}\gamma_\mu\gamma^5\lambda\right) \\
\delta_\xi \lambda &= -\frac{\mathrm{i}}{2}[\gamma^\alpha,\gamma^\beta]\gamma^5\left(\partial_\alpha G_\beta - \partial_\beta G_\alpha\right) + \mathcal{D}\xi \\
\delta_\xi \mathcal{D} &= -\mathrm{i}\left(\bar{\xi}\not{\partial}\lambda\right)
\end{aligned}$$

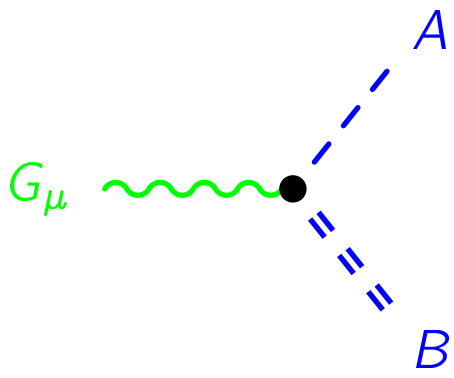
de Wit-Freedman transformations:

$$\begin{aligned}
\tilde{\delta}_\xi G_\mu &= -\left(\bar{\xi}\gamma_\mu\gamma^5\lambda\right) \\
\tilde{\delta}_\xi \lambda &= -\frac{\mathrm{i}}{2}[\gamma^\alpha,\gamma^\beta]\gamma^5 F_{\alpha\beta} + \mathcal{D}\xi \\
\tilde{\delta}_\xi \mathcal{D} &= -\mathrm{i}\left(\bar{\xi}\not{D}\lambda\right)
\end{aligned}$$

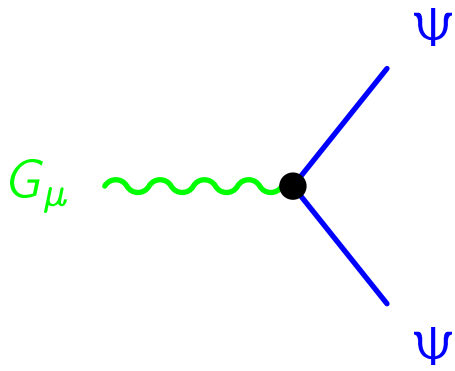
"On-Shell" de Wit-Freedman transformations:

$$\begin{aligned}
\tilde{\delta}'_\xi G_\mu &= -\left(\bar{\xi}\gamma_\mu\gamma^5\lambda\right) \\
\tilde{\delta}'_\xi \lambda &= -\frac{\mathrm{i}}{2}[\gamma^\alpha,\gamma^\beta]\gamma^5 F_{\alpha\beta} - e\left(\phi^\dagger Q\phi\right)\xi
\end{aligned}$$

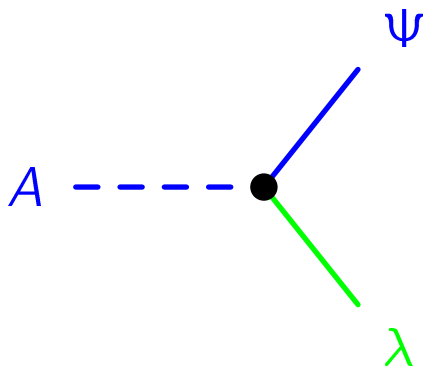
All momenta are incoming.



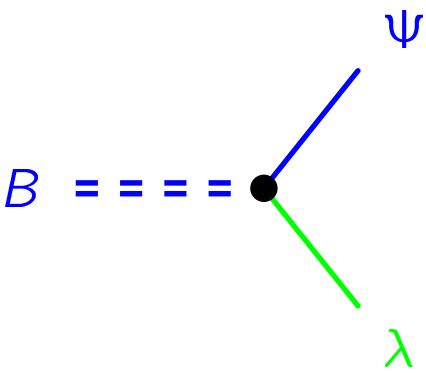
$$= e(p_A - p_B)_\mu$$



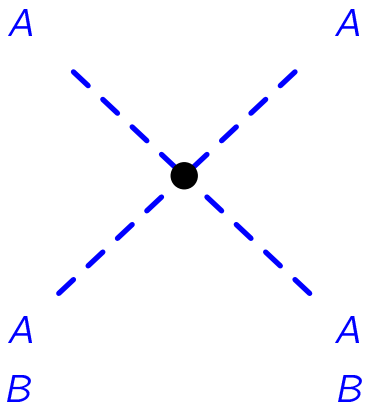
$$= \mathrm{i}e\gamma^5\gamma_\mu$$



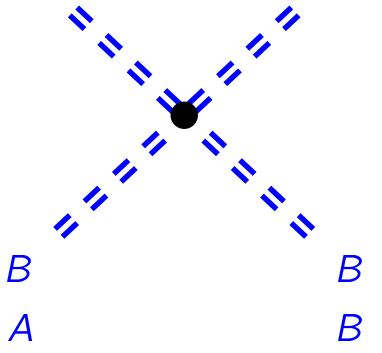
$$= -\mathrm{i}e$$



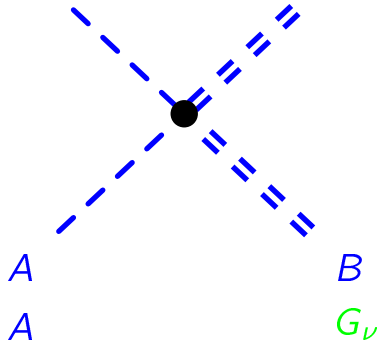
$$= e\gamma^5$$



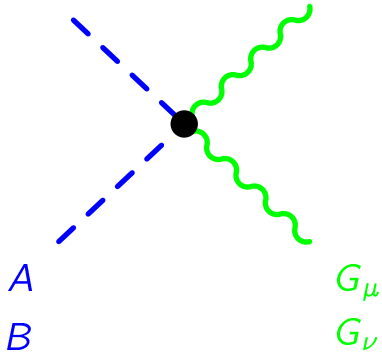
$= -3ie^2$



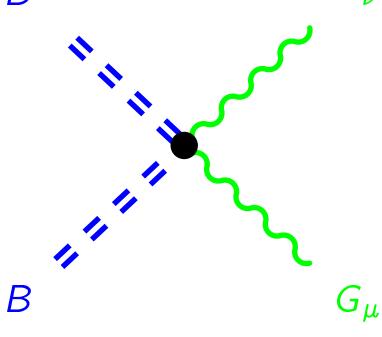
$= -3ie^2$



$= -ie^2$

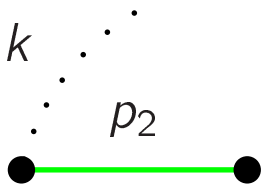


$= 2ie^2\eta_{\mu\nu}$

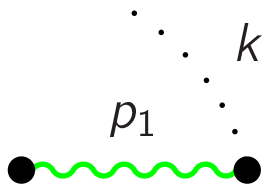


$= 2ie^2\eta_{\mu\nu}$

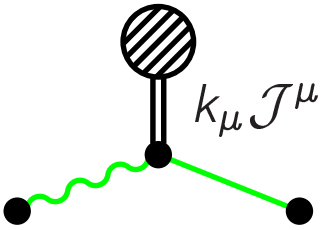
$$
\begin{aligned}
\mathrm{i}\partial_\mu^x \left\langle 0\middle|\mathsf{T}\left[\overline{\xi}\mathcal{J}^\mu(x)G_\nu(y_1)\lambda(y_2)\right]\middle|0\right\rangle = \\
\delta^4(x-y_1)\left\langle 0\middle|\mathsf{T}\left[(\tilde{\delta}'_\xi G_\nu(y_1))\lambda(y_2)\right]\middle|0\right\rangle \\
+\delta^4(x-y_2)\left\langle 0\middle|\mathsf{T}\left[G_\nu(y_1)(\tilde{\delta}'_\xi\lambda(y_2))\right]\middle|0\right\rangle \\
+\text{Off-shell terms}
\end{aligned}
$$



$$= \frac{-1}{\not{p}_2}\gamma_\nu\gamma^5\xi$$



$$= -\frac{1}{2}\frac{1}{p_1^2}[\not{p}_1,\gamma_\nu]\gamma^5\xi$$



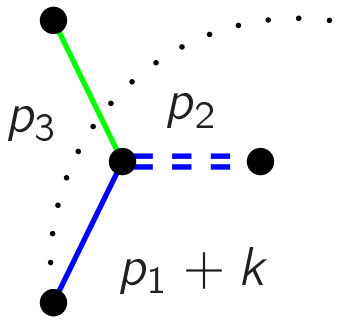
$$= \frac{1}{2}\frac{1}{\not{p}_2}\gamma^5(\not{p}_1+\not{p}_2)[\not{p}_1,\gamma_\nu]\frac{1}{p_1^2}\xi$$

The two sides are equal up to the off-shell term on the RHS:

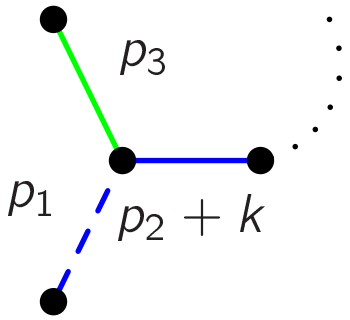
$$\frac{1}{\not{p}_2}\frac{\not{p}_1}{p_1^2}p_{1,\nu}\gamma^5\xi$$

After LSZ reduction and multiplication with the on shell wavefunctions this term vanishes.

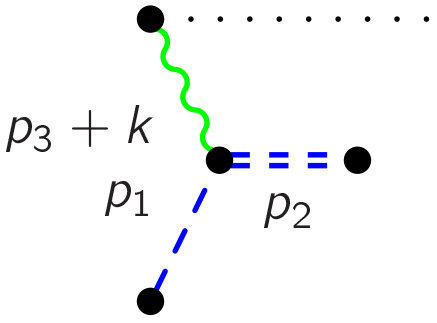
$$\partial_y^\mu \left\langle 0\middle|\mathrm{T}\left[\bar\xi \mathcal{J}_\mu(y) A(x_1) B(x_2)\lambda(x_3)\right]\middle|0\right\rangle =$$
$$\delta^4(y-x_1)\left\langle 0\middle|\mathrm{T}\left[\mathrm{i}(\bar\xi\Psi(x_1))B(x_2)\lambda(x_3)\right]\middle|0\right\rangle$$
$$-\delta^4(y-x_2)\left\langle 0\middle|\mathrm{T}\left[A(x_1)(\bar\xi\gamma^5\Psi(x_2))\lambda(x_3)\right]\middle|0\right\rangle$$
$$+\delta^4(y-x_3)\frac{1}{2}\left\langle 0\middle|\mathrm{T}\left[A(x_1)B(x_2)\partial_\alpha G_\beta(x_3)[\gamma^\alpha,\gamma^\beta]\gamma^5\xi\right]\middle|0\right\rangle$$
$$+ \text{Off-Shell terms}$$



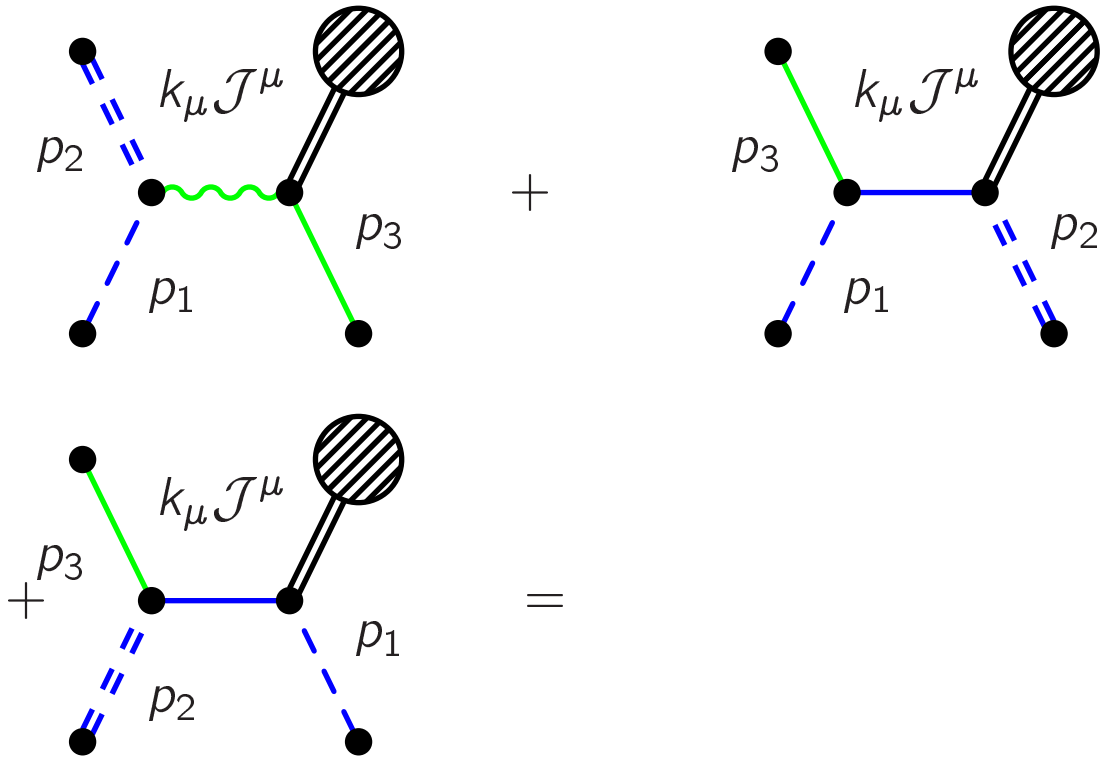
$$= -\frac{e}{p_2^2 \not p_3}\frac{1}{\not p_2+\not p_3}\gamma^5\xi$$



$$= \frac{e}{p_1^2 \not p_3}\frac{1}{\not p_1+\not p_3}\gamma^5\xi$$



$$= -\frac{e}{p_1^2 p_2^2 (p_1+p_2)^2}[\not p_1,\not p_2]\gamma^5\xi$$



$$= (\text{Contact terms}) -$$

$$- \frac{e}{p_1^2 p_2^2 (p_1+p_2)^2 \not{p}_3} (\not{p}_1 + \not{p}_2)(p_1^2 - p_2^2)\gamma^5 \xi$$

On shell the additional term vanishes again, the WI is fulfilled.

- SUSY Ward Identities are valid only on shell!
- **Gauge fixing à la BRS is not SUSY-invariant**
- Closed BRS Algebra only by adding of (global) SUSY transformations *and* (global) translations
- Ghosts of global symmetries are constants (Filtration: Functionals split in sectors of fixed ghost number)
- New ghosts: SUSY ghost ϵ, bosonic Majorana spinor, Translation ghost ω^μ, fermionic vector

Ghost	Dim.	Grassmann P.	Charge	Ghost Number
c	s	1	0	$+1$
$\bar{c}$	$2-s$	1	0	-1
ϵ	$s-\frac{1}{2}$	0	0	$+1$
ω^μ	$s-1$	1	0	$+1$

Gauge transformation with parameter $\theta^{a\,*} = \theta^a$, a gauge index

Ghost is established after extraction of a Grassmann odd, constant parameter λ:

$$\mathbb{R} \ni \theta^a = \lambda c^a, \quad (\lambda c^a)^* = c^a \lambda^* = -\lambda^* c^a \Rightarrow \boxed{\lambda^* = -\lambda}$$

Analogously: Transformation parameter $\xi^\alpha, \bar{\xi}_{\dot\alpha} \longrightarrow$ SUSY Ghosts $\epsilon^\alpha, \bar{\epsilon}_{\dot\alpha}$: $\xi^\alpha = \lambda \epsilon^\alpha$,

Property of transformation parameter $(\xi^\alpha)^* = \bar{\xi}^{\dot\alpha}$ motivates the ansatz $(\epsilon^\alpha)^* = \bar{\epsilon}^{\dot\alpha}$

$$(\xi^\alpha)^* = (\lambda\epsilon^\alpha)^* = \lambda^* (\epsilon^\alpha)^* = -\lambda \bar{\epsilon}^{\dot\alpha} \stackrel{!}{=} \bar{\xi}^{\dot\alpha},$$

analogously $\xi^\alpha = \lambda \epsilon^\alpha$, $\bar{\epsilon}_{\dot\alpha} = -\lambda \bar{\epsilon}_{\dot\alpha}$. in bispinor formalism:

$$\xi \equiv \left(\xi_\alpha,\ \bar{\xi}^{\dot\alpha}\right)^T, \qquad \epsilon \equiv \left(\epsilon_\alpha,\ \bar{\epsilon}^{\dot\alpha}\right),$$

This results in the important relation:

$$\boxed{\xi = -\lambda \gamma^5 \epsilon}$$

Infinitesimal translation of a function

$$\delta_a f(x) = a^\mu \partial_\mu f(x)$$

Again: Transformation parameter $a^\mu \longrightarrow$ Translation ghost ω^μ: Convention $a^\mu = \mathrm{i}\lambda\omega^\mu$

$$\mathbb{R}^4 \ni a^\mu \Rightarrow (\mathrm{i}\lambda\omega^\mu)^* = -\mathrm{i}\omega^{\mu\,*}\lambda^* = +\mathrm{i}\lambda^*\omega^{\mu\,*}$$

$$= -\mathrm{i}\lambda\omega^{\mu\,*} \stackrel{!}{=} \mathrm{i}\lambda\omega^\mu \Longrightarrow \boxed{\omega^{\mu\,*} = -\omega^\mu}$$

$$
\begin{aligned}
sA(x) &= -ec(x)B(x) - \bar{\epsilon}\gamma^5\Psi(x) - \mathrm{i}\omega^\nu\partial_\nu A(x) \\
sB(x) &= +ec(x)A(x) - \mathrm{i}\bar{\epsilon}\Psi(x) - \mathrm{i}\omega^\nu\partial_\nu B(x) \\
s\Psi(x) &= -\mathrm{i}ec(x)\gamma^5\Psi(x) + \mathrm{i}(\partial\!\!\!/ - \mathrm{i}eA\!\!\!/\gamma^5)(A(x)\gamma^5 + \mathrm{i}B(x))\epsilon \\
&\quad - \mathrm{i}\omega^\nu\partial_\nu\Psi(x) \\
s\lambda(x) &= \frac{\mathrm{i}}{2}F_{\alpha\beta}(x)\gamma^\alpha\gamma^\beta\epsilon + \frac{e}{2}\left(A^2(x) + B^2(x)\right)\gamma^5\epsilon \\
&\quad - \mathrm{i}\omega^\nu\partial_\nu\lambda(x) \\
sG_\mu(x) &= \partial_\mu c(x) - \bar{\epsilon}\gamma_\mu\lambda(x) - \mathrm{i}\omega^\nu\partial_\nu G_\mu(x) \\
sc(x) &= \mathrm{i}\bar{\epsilon}\gamma^\mu\epsilon G_\mu(x) - \mathrm{i}\omega^\nu\partial_\nu c(x) \\
s\bar{c}(x) &= \mathrm{i}\tilde{B}(x) - \mathrm{i}\omega^\nu\partial_\nu\bar{c}(x) \\
s\tilde{B}(x) &= \bar{\epsilon}\gamma^\mu\epsilon\partial_\mu\bar{c}(x) - \mathrm{i}\omega^\nu\partial_\nu\tilde{B}(x) \\
s\epsilon &= 0 \\
s\omega^\mu &= \bar{\epsilon}\gamma^\mu\epsilon
\end{aligned}
$$

BRS algebra without auxiliary fields is on shell nilpotent.

$$
s^2A = s^2B = s^2G_\mu = s^2c = s^2\bar{c} = s^2\tilde{B} = s^2\epsilon = s^2\omega_\mu = 0,
$$

$$
s^2\Psi = -\frac{1}{2}(\bar{\epsilon}\gamma^\mu\epsilon)\gamma_\mu\frac{\delta\mathcal{L}}{\delta\bar{\Psi}}, \qquad s^2\lambda = -\frac{1}{4}(\bar{\epsilon}\gamma^\mu\epsilon)\gamma_\mu\frac{\delta\mathcal{L}}{\delta\bar{\lambda}}
$$

Off shell nilpotency can be achieved by the Batalin-Vilkovisky formalism.

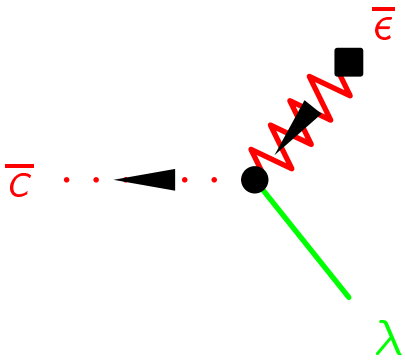
$$S_{\mathrm{GF+FP}} = -\mathrm{i}\int d^4x\, s(\bar{c}F) = -\mathrm{i}\int d^4x\, [(s\bar{c})F - \bar{c}(sF)]$$

F is the so called "gauge fermion": $F = \partial_\mu G^\mu + \frac{\alpha}{2}\tilde{B}$.

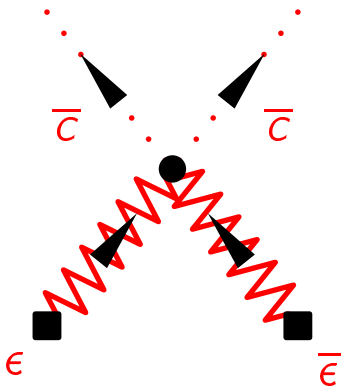
$$\boxed{\mathcal{L}_{\mathrm{GF+FP}} = -\frac{1}{2\alpha}(\partial_\mu G^\mu)^2 + \mathrm{i}\bar{c}\Box c - \mathrm{i}\bar{c}(\bar{\epsilon}\not{\partial}\lambda) + \mathrm{i}\frac{\alpha}{2}\bar{c}(\bar{\epsilon}\gamma^\mu\epsilon)\partial_\mu\bar{c}}$$

$\tilde{B}(x)$: Nakanishi-Lautrup field, $\tilde{B}(x) = -\frac{1}{\alpha}\partial_\mu G^\mu(x)$

Ghost Vertices: (Momenta incoming)

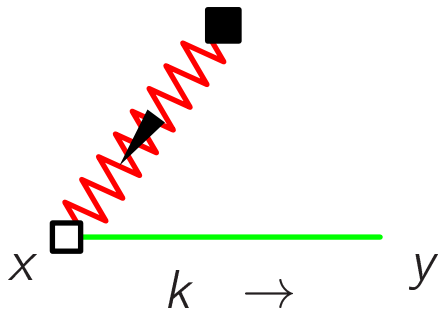


$= -\mathrm{i}\not{p}$

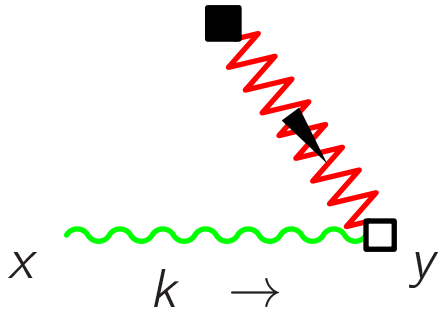


$= \alpha\not{p}$

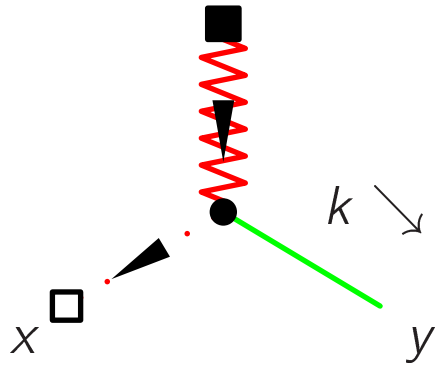
$$\boxed{\langle 0|\mathrm{T}\left[\{Q_{\mathrm{BRS}}, G_\nu(x_1)\lambda(x_2)\}\right]|0\rangle = 0}$$



$$= \frac{\mathrm{i}}{\not{k}}\gamma_\nu\epsilon$$



$$= -\frac{\mathrm{i}}{2}\frac{1}{k^2}[\not{k}, \gamma_\nu]\epsilon$$

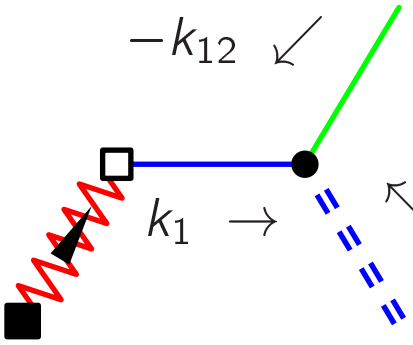


$$-\frac{\mathrm{i}k_\nu}{k^2}$$

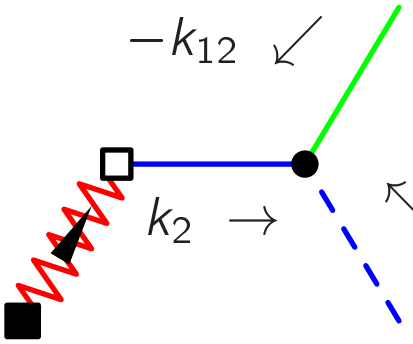
This sums up to zero:

$$\frac{\mathrm{i}}{\not{k}}\gamma_\nu\epsilon - \frac{\mathrm{i}}{2}\frac{1}{k^2}[\not{k}, \gamma_\nu]\epsilon - \frac{\mathrm{i}k_\nu}{k^2} = 0$$

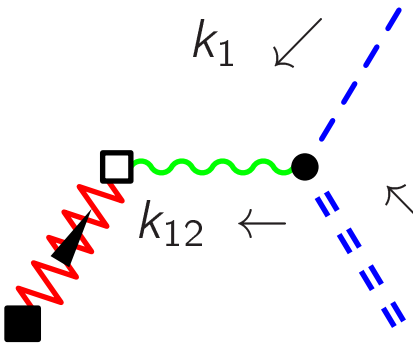
$$
\begin{aligned}
0 = \langle 0|\mathrm{T}\left[\{\mathrm{i}Q_{\mathrm{BRS}}, A(x_1)B(x_2)\lambda(x_3)\}\right]|0\rangle \\
= \quad \langle 0|\mathrm{T}\left[[\mathrm{i}Q_{\mathrm{BRS}}, A(x_1)]B(x_2)\lambda(x_3)\right]|0\rangle \\
+ \langle 0|\mathrm{T}\left[A(x_1)[\mathrm{i}Q_{\mathrm{BRS}}, B(x_2)]\lambda(x_3)\right]|0\rangle \\
+ \langle 0|\mathrm{T}\left[A(x_1)B(x_2)\{\mathrm{i}Q_{\mathrm{BRS}}, \lambda(x_3)\}\right]|0\rangle
\end{aligned}
$$



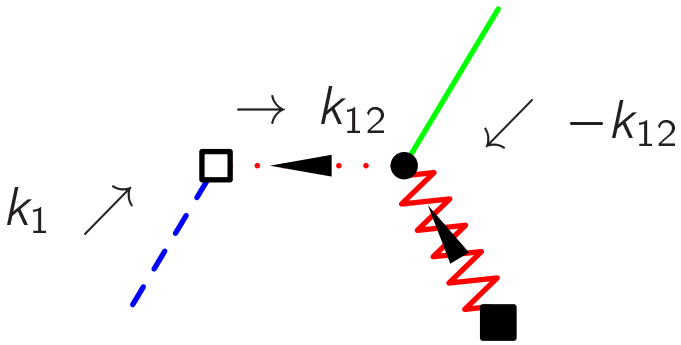
$$= \quad -\frac{\mathrm{i}e}{k_2^2}\frac{1}{\not{k}_1+\not{k}_2}\frac{1}{\not{k}_1}\epsilon$$



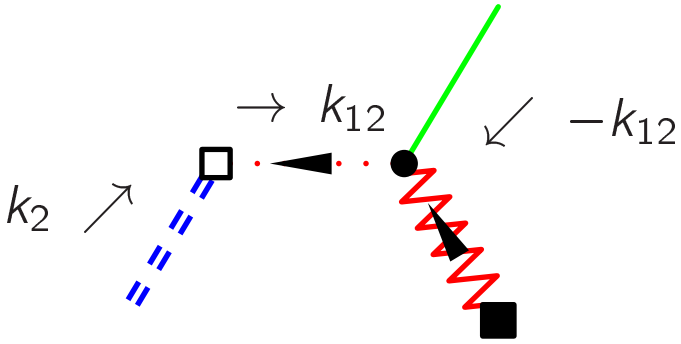
$$= \quad \frac{\mathrm{i}e}{k_1^2}\frac{1}{\not{k}_1+\not{k}_2}\frac{1}{\not{k}_2}\epsilon$$



$$= \quad \frac{\mathrm{i}e}{2k_1^2k_2^2(k_1+k_2)^2}\cdot [\not{k}_1+\not{k}_2, \not{k}_1-\not{k}_2]\epsilon$$



$$= -\frac{ie}{k_1^2(k_1+k_2)^2}\epsilon$$



$$= \frac{ie}{k_2^2(k_1+k_2)^2}\epsilon$$

Again, by adding all terms one gets zero:

$$\frac{ie}{k_1^2k_2^2(k_1+k_2)^2}\cdot\Big\{-(\not k_1+\not k_2)\not k_1 + (\not k_1+\not k_2)\not k_2 - [\not k_1,\not k_2] - k_2^2 + k_1^2\Big\}\epsilon = 0$$

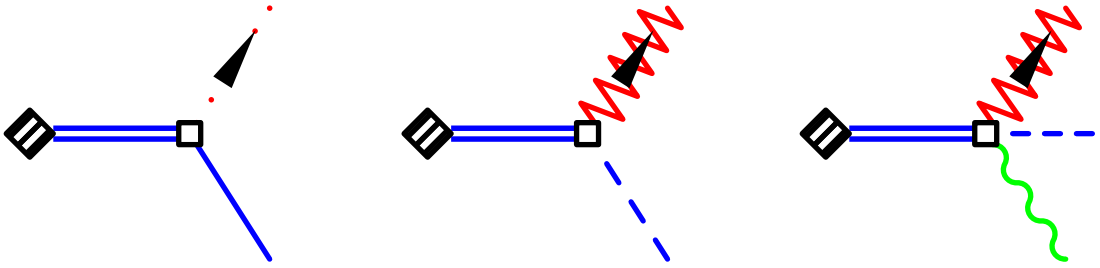
STI in supersymmetric gauge theories are fulfilled off-shell; of crucial importance is the vertex which couples antighost, susy ghost and gaugino.

Problem: BRS vertizes include two-point vertices, how can we handle them in *O'Mega*, which works with n-ary topologies for $n \geq 3$? More generally, how can we generate BRS transformations?

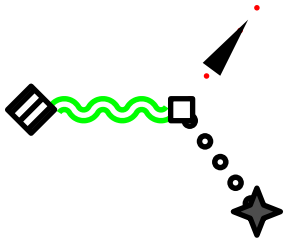
Answer: Establish sources for the BRS transformations $BRS\ \Phi$ for the fields Φ of the model as nonpropagating insertions.

Example: (without the translation ghost)

$$s\Psi(x) = -ie c(x)\gamma^5\Psi(x) + i(\partial\!\!\!/ - ie A\!\!\!/\gamma^5)(A(x)\gamma^5 + iB(x))\epsilon$$



For the BRS transformation of the gauge boson one must introduce a special nonpropagating local operator.



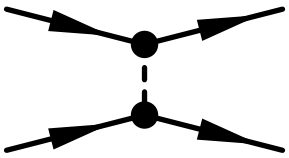
One can absorb the inverse propagator from amputating the identity into the source for the BRS transformation.

For the correct calculation of amplitudes including fermions one has to evaluate the relative signs of diagrams.

Juxtaposition:

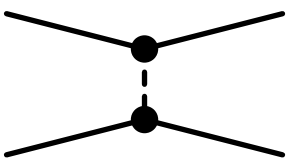
Only Dirac Fermions (SM): (T. Ohl, 2000)

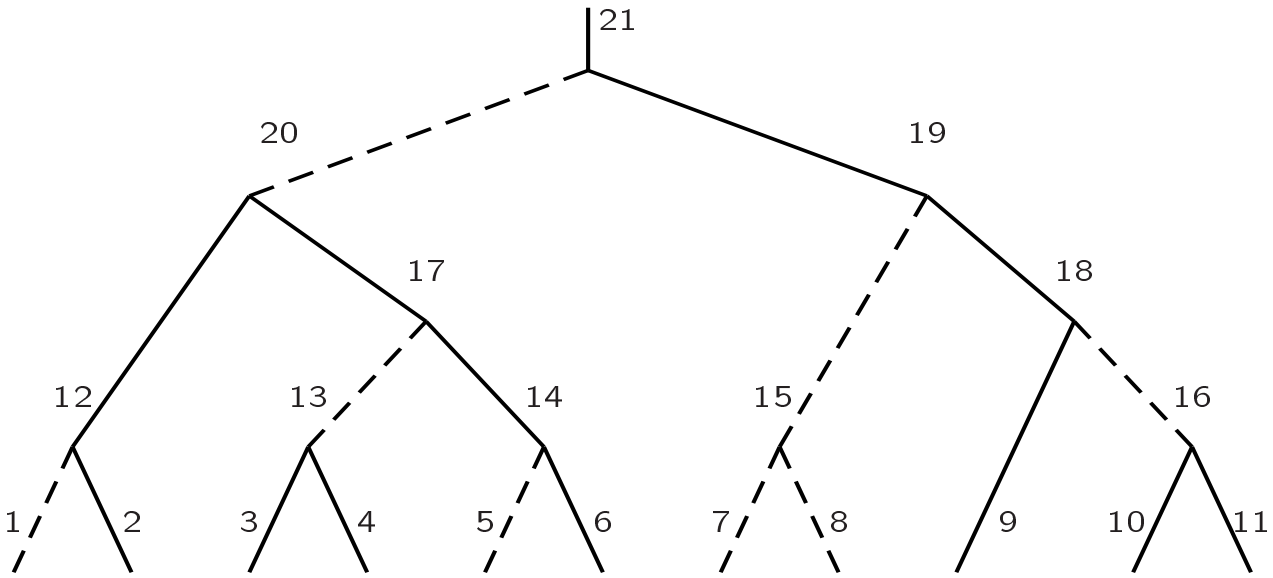
Fermion lines have a definite direction (pairs of $\overline{\Psi}\Psi$)



General Fermions (MSSM): (JR, 2000)

A direction is no longer defineable (Majorana fermions, "clashing arrows"). Use the property of fermion conservation instead of fermion number conservation (A. Denner et al., 1992). Pairings: $(\overline{\Psi}\Psi)$, $(\Psi\overline{\Psi})$, $(\Psi\Psi)$, $(\overline{\Psi\Psi})$.





In our ansatz for general fermions fusion always happens from left to right. $\Longrightarrow \{(2,6),(3,4),(9,21),(10,11)\}$

Examples:

Sign:	$\{(2,6),(3,4)\} \leftrightarrow \{(6,2),(3,4)\}$,
No Sign:	$\{(2,6),(3,4)\} \leftrightarrow \{(3,4),(2,6)\}$,
Sign:	$\{(2,6),(3,4)\} \leftrightarrow \{(2,4),(3,6)\}$,
No Sign:	$\{(2,6),(3,4)\} \leftrightarrow \{(2,4),(6,3)\}$

Calculate the total sign of that diagram from the permutation of these pairs and from the fusion at the vertices.

Dividing by the sign of subamplitudes makes this evaluation compatible with *O'Mega* factorization.

Slavnov-Taylor identities of the supersymmetry (SSTI) provide a check for the consistency of calculations in a model in addition to the analogous identities of the gauge symmetry (GSTI).

STI can be implemented in the *same* formalism for SUSY and gauge symmetries.

Numerical checks for SSTI not yet made.

There is no further difficulty compared to the tests of the gauge symmetries.

Future Plans:

Simulation of explicit (soft) supersymmetry breaking with a spurion formalism.

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Debug all MSSM Feynman rules implemented in *O'Mega* with GSTI and SSTI.