COMPOSITE HIGGS

Literature:
Giuliano Panico / Andrea Wulzer
The Composite NG Higgs, LNP 913,
Springer, 2016.

Composite Higgs is sort of an upscaled version of QCD, where there is a strong interaction creating bound states from constituents. In QCD, there are the chiral symmetries (independent rotations of left- and right-handed light quarks) that are explicitly broken by the (different) quark masses. Those symmetries generate multiplets of (pseudo-) Nambu-Goldstone bosons (pNGBs) with certain quantum numbers.

Nambu-Goldstone theorem:

> For each spontaneously broken global symmetry generator there is a massless boson with those quantum numbers in the spectrum.

$\phi_i \to i\theta^a T^a_{ik}\phi_k \Rightarrow \mathcal{L}$ invariant, means $\mathcal{L}_{kin} \equiv T$ invariant as well as $V(\phi)$

$$V_{inv} \Rightarrow 0 = \delta V = \frac{\partial V}{\partial \phi_i}\delta\phi_i = \frac{\partial V}{\partial \phi_i} T^a_{ij}\phi_j \xrightarrow[\partial/\partial\phi_j(\dots)]{}$$

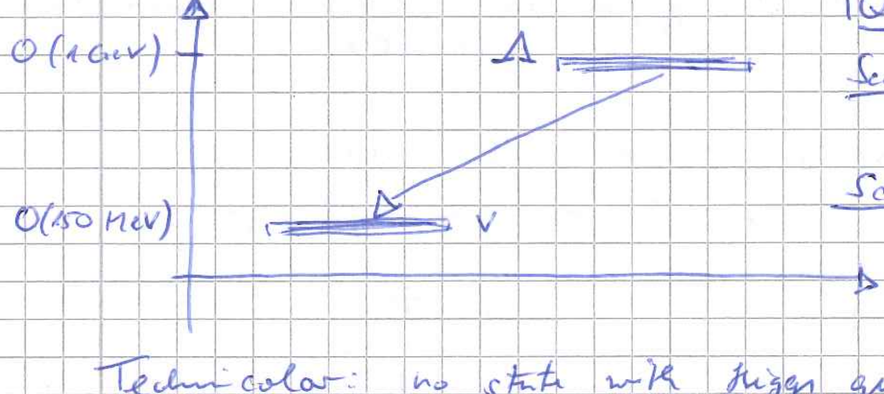
and take everything at the VEV of ϕ, $\langle\phi\rangle$

$$\underbrace{\left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|_{\langle\phi\rangle}}_{(m^2)_{ij}} T^a_{jk}\langle\phi_k\rangle + \underbrace{\left.\frac{\partial V}{\partial\phi_j}\right|_{\langle\phi\rangle}}_{=0,\ \text{minimization condition of vacuum}} T^a_{ji} = 0$$

$$\boxed{(m^2)_{ij} T^a_{jk}\langle\phi_k\rangle = 0}$$

must have zero eigenvalue for eigenvector with quantum numbers a, if $T^a\langle\phi\rangle \neq 0$.

Old Idea: Georgi / Pais, 1974;



QCD:

Scale Λ: chiral symmetry breaking, quarks, $SU(3)_c$

Scale v: pions, kaons.

Technicolor: no state with Higgs quantum numbers at all
Dimopoulos / Susskind NPB 155 (1979) 237
Susskind, PR D20 (1979) 2619.

Generating an ~~except~~ exponentially suppressed scale by dimensional transmutation

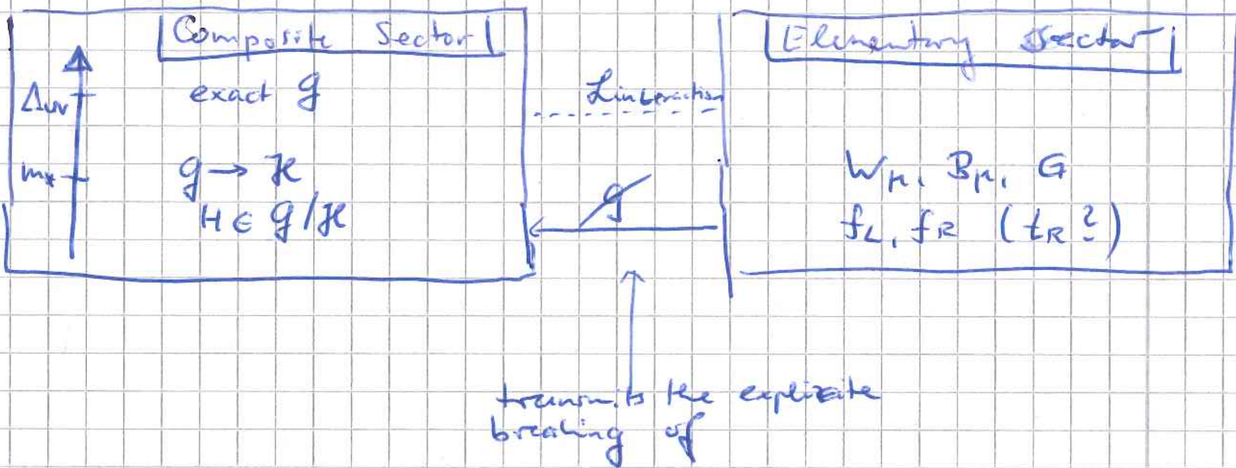
$$\log\left(\frac{\Lambda_{UV}}{m}\right) = c\left(\frac{4\pi}{g_s}\right)^2 \Rightarrow \Lambda_{UV} = m\cdot e^{4\pi/\alpha_s} \longrightarrow \boxed{m = \Lambda_{UV}\, e^{-4\pi/\alpha_s}}$$

Start at Λ_{UV} ("Planck scale" etc., "GUT scale") with a UV fixpoint, e.g. the fixpoint of the free YM/QCD theory. This generates an exponentially suppressed QCD confinement scale m_*.

The SM fermions need to be elementary at the TeV scale, but could be partially composite.

Elementary sector is only a small perturbation of the new strong (compositeness) dynamics (in order to keep the pNGB picture)

$\boxed{\text{Composite Higgs Scenario}}$



Note that the best (only?) examples for composite Higgs models with partial fermion compositeness are 5-dim. gauge theories on (truncated) anti-de Sitter spaces. They are not UV complete.

$\boxed{\text{Vacuum misalignment}}$

How can a composite Higgs act as an elementary impostor?

Composite sector $\mathcal{G} \to \mathcal{H}$, with pNGBs $\in \mathcal{G}/\mathcal{H}$

$G_{EW} = SU(2)_L \times U(1)_Y \subseteq \mathcal{H}$ $\quad \mathcal{G}/\mathcal{H}$ should at least contain one Higgs doublet.

$$\mathcal{G}: \quad \{T^A\} = \{\underbrace{T^a}_{\substack{\text{unbroken} \\ 1,\dots,\dim[\mathcal{H}]}}, \underbrace{\hat{T}^{\hat{a}}}_{\substack{1,\dots,\dim[\mathcal{G}/\mathcal{H}] \\ \text{broken}}}\}$$

$\vec{F}$ one vacuum representative $\qquad T^a \vec{F} = 0, \quad \hat{T}^{\hat{a}} \vec{F} \neq 0$

$$\vec{\Phi}(x) = e^{i\theta_A(x) T^A} \vec{F}$$

$$g[\alpha_A] = e^{i\alpha_A T^A} = e^{i f_{\hat{a}}[\alpha] \hat{T}^{\hat{a}}} \cdot \underbrace{e^{i f_a[\alpha] T^a}}_{\text{leaves } \vec{F} \text{ invariant}}$$

$$\Rightarrow \quad \vec{\Phi}(x) = U(\Pi)\vec{F} \quad \text{with} \quad U[\Pi] = e^{i\frac{\sqrt{2}}{f}\Pi_{\hat{a}}(x)\hat{T}^{\hat{a}}}$$

(canonically normalized).

CCWZ construction Callan/Coleman/Wess/Zumino, 1969.

Symmetry transformation: $g\cdot U[\Pi] = U[\Pi^{(g)}]\cdot h[\Pi;g]$

$\Rightarrow \quad U[\Pi] \to U[\Pi^{(g)}] = g\cdot U[\Pi]\cdot h^{-1}[\Pi;g]$, which induces

$\vec{\Phi}(x) \to g\,\vec{\Phi}(x)$ Note $\Pi^{(g_1\cdot g_2)} = \left(\Pi^{(g_2)}\right)^{(g_1)}$

$\Pi^{(g)}$ has non-linear dependence on Π, hence: non-linear realization of $\mathcal{G}$.

$$[T^a, T^b] = i f^{ab}{}_c T^c + \overset{\text{subgroup!}}{\cancel{i f^{ab}{}_{\hat c} T^{\hat c}}} \equiv T^c (t^a_{Ad})_c{}^b$$

$$[T^a, \hat T^{\hat b}] = i f^{a\hat b}{}_{\hat c} \hat T^{\hat c} + \underbrace{\cancel{i f^{a\hat b}{}_c T^c}}_{\text{1. line and antisymmetry of } f_{abc}} \equiv \hat T^{\hat c}(t_\pi{}^a)_{\hat c}{}^{\hat b}$$

$$[\hat T^{\hat a}, \hat T^{\hat b}] = i f^{\hat a\hat b}{}_c T^c + i f^{\hat a\hat b}{}_{\hat c} \hat T^{\hat c}$$

$\mathrm{Ad}_{\mathcal{G}} = \mathrm{Ad}_{\mathcal{H}} \oplus r_\pi$ $\quad$ $\mathcal{H}$ representation.

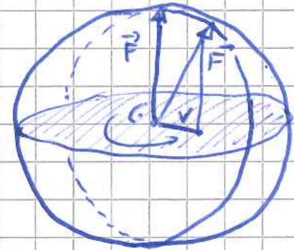
for $SO(N)/SO(N-1)$, r_π is the fundamental representation.

NGB: $\vec{\Phi}(x) = e^{i\theta^{\hat a}(x)\hat T^{\hat a}}\vec{F}$

$\mathcal{G}$ exact: θs exact NGBs, $\langle\theta_{\hat a}\rangle$ arbitrary/unobservable (can be rotated away by $\exp[-i\langle\theta_{\hat a}\rangle\hat T^{\hat a}]\vec{\Phi}$) $\Big\}\to \langle\theta_{\hat a}\rangle\equiv 0$

$\mathcal{G}$ explicitly broken: NGBs $\to$ pNGBs, θ develop potential, VEV not arbitrary any more.
$\langle\theta\rangle \longrightarrow$ EWSB

e.g. $SO(3)\to SO(2)$



$v = f\sin\langle\theta\rangle$

$f = |\vec{F}|$ is scale of $\mathcal{G}\to\mathcal{H}$ breaking

$\boxed{\text{Vacuum misalignment.}}$

$v\sim f$: Technicolor, i.e. maximal breaking of EW symmetry.

$\xi \equiv \frac{v^2}{f^2} = \sin^2\langle\theta\rangle \ll 1$ $\quad$ Misalignment condition.
Dugan/Georgi/Kaplan/Dimopoulos, 1984/85

$\xi\to 0$ for v fixed: decoupling of composite sector (SM limit)

Generically, there will be large misalignment $\xi\sim 1$.
Naturally small misalignment is e.g. achieved in Little Higgs Models (LHM).

<u>Example:</u> Abelian Composite Higgs Model:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\vec{\Phi}^T\partial^\mu\vec{\Phi} - \frac{g_*^2}{8}\left(\vec{\Phi}^T\vec{\Phi} - f^2\right)^2$$

invariant under $SO(3)$ rotations $\vec{\Phi}\to U\vec{\Phi}$, $U = e^{i\theta^A T^A}\in SO(3)$

$$\mathrm{tr}\left[T^A T^B\right] = \delta^{AB}$$

$$T = \frac{1}{\sqrt{2}} \left(\begin{array}{cc|c} 0 & -i & 0 \\ i & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad \hat{T}^i = \left\{ \frac{1}{\sqrt{2}} \left(\begin{array}{cc|c} 0 & 0 & -i \\ 0 & 0 & 0 \\ \hline i & 0 & 0 \end{array}\right), \frac{1}{\sqrt{2}} \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & -i \\ \hline 0 & i & 0 \end{array}\right) \right\}$$

unbroken $SO(2)$ broken generators

Minimization: $\langle \vec{\Phi}^T \rangle \langle \vec{\Phi} \rangle \stackrel{!}{=} f^2$, hence we get the sphere picture.

$$\text{Vacuum } \begin{array}{c}(\text{o.B.d.A})\\(\text{w.l.o.g.})\end{array} \quad \vec{F} = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

"Radial" and "angular" field variables:

$$\vec{\Phi} = \underbrace{e^{i \frac{\sqrt{2}}{f} \pi^i(x) \hat{T}^i}}_{U(\pi) \text{ Goldstone matrix}} \begin{pmatrix} 0 \\ 0 \\ f + \sigma(x) \end{pmatrix}$$

$$U(\pi) = e^{i \sqrt{2}/f \, \pi_i(x) \hat{T}^i} = \begin{pmatrix} \mathbb{1} - \left(1 - \cos \frac{\pi}{f}\right) \frac{\vec{\pi}\vec{\pi}^T}{\pi^2} & \sin \frac{\pi}{f} \frac{\vec{\pi}}{\pi} \\ -\sin \pi \cdot \frac{\vec{\pi}^T}{\pi} & \cos \pi/f \end{pmatrix}$$

for $\pi = \sqrt{\vec{\pi}^T \vec{\pi}}$. This is general for $SO(N) \to SO(N-1)$ breaking with $\vec{F} = (0, 0, \ldots, 0, f)^T$. $\quad \vec{\Phi} = (f + \sigma) \begin{pmatrix} \sin \pi/f \cdot \vec{\pi}/\pi \\ \cos \pi/f \end{pmatrix}$

$$\boxed{\begin{aligned} \mathcal{L} = {} & \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{(g_* f)^2}{2} \sigma^2 - \frac{g_*^2 f}{2} \sigma^3 - \frac{g_*^2}{8} \sigma^4 \\ & + \frac{1}{2} \left(1 + \frac{\sigma}{f}\right)^2 \left[\frac{f^2}{\pi^2} \sin^2\left(\frac{\pi}{f}\right) (\partial_\mu \vec{\pi}^T) \partial^\mu \vec{\pi} + \frac{f^2}{4\pi^4} \left(\frac{\pi^2}{f^2} - \sin^2 \frac{\pi}{f}\right) \cdot \right. \\ & \left. \cdot \, \partial_\mu \pi^2 \partial^\mu \pi^2 \right] \end{aligned}}$$

Taylor expansion around $\pi = 0$:

$\Rightarrow$ infinite set of arbitrary number of NGBs, but only two derivatives. each leg of NGB comes with f.

$f \equiv$ "Higgs decay constant".

σ field mass: $m = g^* f$, $\vec{\pi}$s are massless

$SO(2)$ symmetry: $\vec{\pi} \to e^{i \alpha \sigma_2} \vec{\pi}$ define: $H = \frac{\pi_1 - i \pi_2}{\sqrt{2}}$

with $Q = \mathbb{1}$ under $U(1) = SO(2)$

This is a rotation along the unbroken generator $\vec{\Phi} \to e^{i\sqrt{2}\alpha T} \vec{\Phi}$

the broken generators induce the symmetry transformations:

$$\vec{\pi} \to \vec{\pi} + \pi \cot \frac{\pi}{f} \vec{\alpha} + \left(\frac{f}{\pi} - \cot \frac{\pi}{f}\right) (\vec{\alpha}^T \vec{\pi}) \frac{\vec{\pi}}{\pi} \quad \Longleftrightarrow$$

$$\vec{\Phi} \to \vec{\Phi} + i \alpha_i \hat{T}^i \vec{\Phi}$$

$0 \neq \langle \vec{\pi} \rangle$ can be transformed into $\langle \vec{\pi} \rangle = 0 \iff$ Composite Higgs (NGB)
VEV has no physical meaning in the case of exact symmetries.

Now: gauging unbroken $U(1)$

$$\partial_\mu \vec{\Phi} \to D_\mu \vec{\Phi} = \left(\partial_\mu - i\sqrt{2}\, e\, A_\mu T\right) \vec{\Phi}$$

Gauging selects one of the generators $\to$ explicitly breaks $SO(3) \to SO(2)$

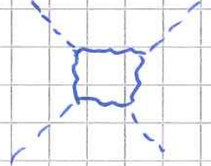
Composite Higgs $\longrightarrow$ pNGB.

$$D_\mu \vec{\pi} = \left(\partial_\mu - ie A_\mu \sigma_2\right) \vec{\pi}$$

Complex notation: $\quad D_\mu H = \partial_\mu H - ie A_\mu H$

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{\sigma}{f}\right)^2 \left\{ \frac{f^2}{|H|^2} \sin^2 \frac{\sqrt{2}|H|}{f} D_\mu H^\dagger D^\mu H + \frac{f^2}{4|H|^4}\left(2\frac{|H|^2}{f^2} - \sin^2\frac{\sqrt{2}|H|}{f}\right)\left(\partial_\mu |H|^2\right)^2 \right\}$$

Now a potential for H arises, no longer forbidden by the non-linearly realized symmetry. Even if absent at tree-level will be generated by loops like (Coleman-Weinberg mechanism) [Erick]



Higgs get a VEV: $\quad \langle H \rangle = \frac{V}{\sqrt{2}}$ which generates a $U(1)$ gauge boson mass:

$$\boxed{m_A = e f \sin\frac{V}{f} \equiv e v}$$

$$v = f \cdot \sin\frac{V}{f} \implies \boxed{\xi = \frac{v^2}{f^2} = \sin^2 \frac{V}{f}}$$

This corresponds to the misalignment projection.

The Minimal Composite Higgs Model (MCHM)

Consider $SO(5) \to SO(4) \qquad SO(4) = SU(2)_L \times SU(2)_R$.

identify $Y = t^3_R$

Technicalities: $SO(4)$: $\frac{4 \cdot 3}{2} = 6$ generators, traceless imaginary 4×4 Hermitian matrices as generators

$$SO(4) \simeq SU(2)_L \times SU(2)_R \qquad \Pi^a \text{ real 4-vector}$$

$$\Sigma = \frac{1}{\sqrt{2}}\left(i\sigma_a \Pi^a + \mathbb{1}_{2\times2} \Pi^4\right) \equiv \frac{1}{\sqrt{2}} \bar{\sigma}_i \Pi^i \qquad \bar{\sigma}_i = \{i\sigma_a, \mathbb{1}_{2\times2}\}$$

$$\mathrm{Tr}[\bar\sigma_i^\dagger \bar\sigma_j] = 2\delta_{ij}, \qquad \sum_{i=1}^{4} (\bar\sigma_i^\dagger)_a{}^b (\bar\sigma_i)_c{}^d = 2\,\delta_a{}^d \delta_c{}^b$$

$$(\bar\sigma_i)^* = \sigma_2 \bar\sigma_i \sigma_2 \qquad \bar\sigma_i \bar\sigma_j^\dagger - \bar\sigma_j \bar\sigma_i^\dagger = 2\,\bar\sigma_i \bar\sigma_j^\dagger - 2\delta_{ij}\, \mathbb{1}_{2\times 2}$$

Σ is pseudoreal: $\quad \Sigma^* = \sigma_2 \Sigma \sigma_2$

$SU(2)_L \times SU(2)_R$: $\quad \Sigma \longrightarrow g_L \Sigma g_R^\dagger$ preserves pseudo-reality condition

Σ is a bi-doublet $\quad (2,2)$

$\mathrm{Tr}[\Sigma^\dagger \Sigma] = |\vec\pi|^2$ trace invariant under $\Sigma \to g_L \Sigma g_R^\dagger$,
hence norm of $\vec\pi$ invariant under chiral transformations.

$\Longrightarrow \quad SU(2)_L \times SU(2)_R = SO(4)$ as $SO(4)$ has no subalgebras.

$$t^a = \{t_L^\alpha, t_R^\alpha\}, \quad a = 1,2,3$$

$$[t_L^\alpha, t_L^\beta] = i\varepsilon^{\alpha\beta\gamma} t^\gamma, \quad [t_R^\alpha, t_R^\beta] = i\varepsilon^{\alpha\beta\gamma} t_R^\gamma, \quad [t_L^\alpha, t_R^\beta] = 0$$

$$g_{L/R} \simeq \mathbb{1} + i\varepsilon_\alpha^{L/R} \frac{\sigma^\alpha}{2} \qquad \delta_L \vec\pi = i\varepsilon_\alpha^L t_L^\alpha \vec\pi$$
$$\delta_R \vec\pi = i\varepsilon_\alpha^R t_R^\alpha \vec\pi$$

corresponds to

$$\left.\begin{aligned} \delta_L \Sigma &= i\varepsilon_\alpha^L \frac{\sigma^\alpha}{2}\Sigma \\ \delta_R \Sigma &= -i\varepsilon_\alpha^R \Sigma \frac{\sigma^\alpha}{2} \end{aligned}\right\} \Rightarrow \begin{aligned} (t_L^\alpha)_{ij} &= \tfrac14 \mathrm{tr}[\bar\sigma_i^\dagger \sigma^\alpha \bar\sigma_j] = -\tfrac{i}{2}\left\{\varepsilon_{\alpha\beta\gamma}\delta_i^\beta \delta_j^\gamma + (\delta_i^\alpha \delta_j^4 - \delta_j^\alpha \delta_i^4)\right\} \\ (t_R^\alpha)_{ij} &= \tfrac14 \mathrm{tr}[\bar\sigma_i \sigma^\alpha \bar\sigma_j^\dagger] = -\tfrac{i}{2}\left\{\varepsilon_{\alpha\beta\gamma}\delta_i^\beta \delta_j^\gamma - (\delta_i^\alpha \delta_j^4 - \delta_j^\alpha \delta_i^4)\right\} \end{aligned}$$

This implies:

$$\mathrm{Tr}[t_L^\alpha t_L^\beta] = \mathrm{Tr}[t_R^\alpha t_R^\beta] = \delta^{\alpha\beta}, \quad \mathrm{Tr}[t_L^\alpha t_R^\beta] = 0$$

$$\sum_{\alpha=1}^{3} \left\{ (t_L^\alpha)_{ij} (t_L^\alpha)_{kl} + (t_R^\alpha)_{ij} (t_R^\alpha)_{kl} \right\} = -\frac12 (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})$$

$$\sum_{\alpha=1}^{3} \left\{ (t_L^\alpha)_{ij} (t_L^\alpha)_{kl} - (t_R^\alpha)_{ij} (t_R^\alpha)_{kl} \right\} = -\frac12 \varepsilon_{ijkl}$$

$$\Sigma = (H^c, H) = (i\sigma_2 H^*, H) \qquad \text{(pseudo-reality condition)}$$

$$H = \frac{1}{\sqrt2}\begin{pmatrix} \pi^2 + i\pi^1 \\ \pi^4 - i\pi^3 \end{pmatrix} \equiv \begin{pmatrix} h_u \\ h_d \end{pmatrix} \qquad 4 = (2,2) \longrightarrow [2_{\frac12} \text{ or }]\, 2_1$$

(according to diff. hypercharge definitions)

$SO(4)$ fourplet for matter fermions

$$\Psi = \frac{1}{\sqrt2}(\psi^4 + i\sigma_\alpha \psi^\alpha) = \frac{1}{\sqrt2}\bar\sigma_i \psi^i \qquad (2,2)_c \quad \text{this is a complex doublet}$$

$$\Psi = \frac{1}{\sqrt2}\begin{pmatrix} \psi^4 + i\psi^3 & \psi^2 + i\psi^1 \\ -\psi^2 + i\psi^1 & \psi^4 - i\psi^3 \end{pmatrix} \equiv (\Psi_-, \Psi_+) \equiv \begin{pmatrix} \Psi_-^u & \Psi_+^u \\ \Psi_-^d & \Psi_+^d \end{pmatrix}$$

$$4_c = (2,2)_c \longrightarrow 2_1 \oplus 2_{-1}$$

$$\vec\Psi = \frac{1}{\sqrt2}\begin{pmatrix} -i\Psi_+^u - i\Psi_-^d \\ \Psi_+^u - \Psi_-^d \\ i\Psi_+^d - i\Psi_-^u \\ \Psi_+^d + \Psi_-^u \end{pmatrix} \qquad \begin{aligned} (2,1) &\longrightarrow 2_0 \\ (1,2) &\longrightarrow 1_1 \oplus 1_{-1} \end{aligned}$$

$$\begin{array}{ccc} SO(4) & SU(2)_L & SU(2)_R \\ 6 = & (3,1) & \oplus\ (1,3) \\ & \downarrow & \downarrow \\ & 3_0 & 1_0 \oplus 1_2 \oplus 1_{-2} \end{array}$$

$$9 = (3,3) \longrightarrow 3_0 \oplus 3_2 \oplus 3_{-2}.$$

Composite sector Lagrangian: $\vec{\Phi}$ 5-plet under $SO(5)$

4-sphere manifold of equivalent vacua, representative vacuum along 5th component.

$$\vec{\Phi} = e^{i\frac{\sqrt{2}}{f}\Pi_i(x)\hat{T}^i}\begin{pmatrix}\vec{0} \\ f+\sigma(x)\end{pmatrix} = (f+\sigma)\begin{bmatrix}\sin\frac{\Pi}{f}\cdot\vec{\Pi}/\Pi \\ \cos\Pi/f\end{bmatrix}$$

Linearly-realized group $SO(4)$ rotations of $\vec{\Pi}$ 4-plet correspond to rotation with unbroken generators t^a on $\vec{\Phi}$:

$$\vec{\Pi} \to e^{i\alpha_a t^a}\vec{\Pi} \Longleftrightarrow \vec{\Phi} \to e^{i\alpha_a T^a}\vec{\Phi}$$

$$\vec{\Pi} = \begin{pmatrix}\Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}-i(h_u - h_u^+) \\ h_u + h_u^+ \\ i(h_d - h_d^+) \\ h_d + h_d^+\end{pmatrix}$$

EW interactions: gauging $SU(2)_L \times U(1)_Y$ subgroup of $SO(4)$

$$D_\mu\vec{\Phi} = (\partial_\mu - igW_\mu^a T_L^a - ig' B_\mu T_R^3)\vec{\Phi}$$

At leading order in ξ, EW boson propagators and self-interactions vertices are identical to the SM ones:

$$\mathcal{L}_E = -\frac{1}{4}W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

SM: Higgs + EW bosons + resonance σ with mass $m_* = g_* f$

Strong sector coupling g_* large $\in (1, 4\pi)$ to make resonances heavy.

$$D_\mu H = \left(\partial_\mu - igW_\mu^a\frac{\sigma_a}{2} - ig' B_\mu\frac{\mathbb{1}}{2}\right)H$$

Higgs-Lagrangian of MCHM:

$$\boxed{\mathcal{L}_{MCHM} = \frac{f^2}{2|H|^2}\sin^2\left(\frac{\sqrt{2}|H|}{f}\right)(D_\mu H)^+(D^\mu H) + \frac{f^2}{8|H|^4}\left[2\frac{|H|^2}{f^2} - \sin^2\left(\frac{\sqrt{2}|H|}{f}\right)\right](\partial_\mu|H|^2)^2}$$

Going to unitarity gauge:

$$H = \begin{pmatrix}0 \\ \frac{1}{\sqrt{2}}(V + H(x))\end{pmatrix} \Rightarrow \boxed{\mathcal{L}_{MCHM,\,unit.g.} = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4}f^2\sin^2\left(\frac{V+h}{f}\right)\cdot\left(|W|^2 + \frac{1}{2c_w^2}Z^2\right)}$$

$$m_W = c_w m_Z = \frac{1}{2}gf\sin\frac{V}{f} = \frac{1}{2}gv.$$

$v \simeq 246$ GeV is not directly related to the Higgs VEV, but via an algebraic expression.

Taylor-expanding around $h=0$:

$$\frac{g^2v^2}{4}\left(|W|^2+\frac{1}{2c_W^2}Z^2\right)\left[2\sqrt{1-\xi}\,\frac{h}{v}+(1-2\xi)\frac{h^2}{v^2}-\frac{4}{3}\xi\sqrt{1-\xi}\,\frac{h^3}{v^3}+\dots\right]$$

$$\xi \equiv v^2/f^2 \qquad k_V \equiv \frac{g^{MCHM}_{hVV}}{g^{SM}_{hVV}}=\sqrt{1-\xi}<1 \qquad \frac{g^{MCHM}_{hhVV}}{g^{SM}_{hhVV}}=1-2\xi$$

There are higher-dimensional vertices with more Higgs fields

SM limit: $\xi\to 0$, with $f\to\infty$, v fixed. (Higgs becomes elementary)

Alternative description — expansion in inverse powers of f up to dim. 6

$$(D_\mu H)^\dagger(D^\mu H)-\frac{2}{3f^2}|H|^2(D_\mu H)^\dagger(D^\mu H)+\frac{1}{6f^2}\partial_\mu(H^\dagger H)\partial^\mu(H^\dagger H)+\dots$$

Modifications of hVV and $hhVV$ couplings at level ξ.

The relation $\rho=1$ for $\rho\equiv\frac{m_W^2}{c_W^2m_Z^2}$ is not violated. The operator

$$O_T=\frac{1}{2f^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\left(H^\dagger\overleftrightarrow{D}^\mu H\right) \qquad \left(A\overleftrightarrow{\partial}_\mu B\equiv A\partial_\mu B-(\partial_\mu A)B\right)$$

leading to $\rho-1\sim\frac{v^2}{f^2}=\xi$ is absent. $SO(4)\simeq SU(2)_L\times SU(2)_R$

Gauging of $SU(2)_L$ preserves $SU(2)_R$, only hypercharge Y breaks custodial symmetry. The Higgs VEV breaks $SO(4)\longrightarrow SO(3)_c$

Sikivie / Susskind / Voloshin / Zakharov, NP B173 (1980) 189.

— Solution of Naturalness — strong interactions of underlying UV model; Higgs mass stabilized through dim. transmutation

— Higgs naturally light — Goldstone symmetry

Additional resonances ("bound states") can be introduced, transforming linearly under the unbroken group h in a certain representation

$$\Psi_i\longrightarrow\Psi^{(g)}_i=\left(e^{i\xi_a[\Pi;g]T^a_{r_\Psi}}\right)_i{}^j\,\Psi_j\equiv h_{r_\Psi}[\Pi;g]_i{}^j\,\Psi_j$$

E.g. in $SO(5)/SO(4)$ MCHM one can introduce $\underline{4}$ and $\underline{1}$ $SO(4)$ resonances.

(Cartan-Maurer form)

$$iU[\Pi]^{-1}\cdot\partial_\mu U[\Pi]=d_{\mu,\hat a}[\Pi]\,T^{\hat a}+e_{\mu,a}[\Pi]\,T^a\equiv d_\mu+e_\mu$$

$$d_\mu[\Pi]\longrightarrow h[\Pi;g]\cdot d_\mu[\Pi]\cdot h[\Pi;g]^{-1}$$

$$e_\mu[\Pi]\longrightarrow h[\Pi;g]\cdot\left(e_\mu[\Pi]+i\partial_\mu\right)\cdot h[\Pi;g]^{-1}$$

$$E_{\mu\nu}\equiv E_{\mu\nu,a}T^a=i\left[D_\mu,D_\nu\right]=\partial_\mu e_\nu-\partial_\nu e_\mu-i\left[e_\mu,e_\nu\right]$$

$$E_{\mu\nu}[\Pi,A]\longrightarrow h[\Pi;g]\cdot E_{\mu\nu}[\Pi,A]\cdot h[\Pi;g]^{-1}$$

Define resonances with proper covariant derivatives according to

$$D_\mu \Psi_i = \partial_\mu \Psi_i - i e_{\mu,a} (t_{r_\Psi}^{a})_i{}^j \Psi_j .$$

$$\mathcal{L} = i \bar{\Psi} \gamma^\mu D_\mu \Psi - m_\Psi \bar{\Psi}\Psi$$

Partial Fermion Compositeness

Orig. Ref.: D. B. Kaplan NP **B365** (1991) 259
Contino / Pomarol JHEP **0411** (2004), 058

Compare technicolor with partial compositeness.

$$\boxed{\mathcal{L}_{int} = \frac{\lambda_t}{\Lambda_{UV}^{d-1}} \left(\bar{q}_L \mathcal{O}_S^c t_R\right) + \frac{\lambda_b}{\Lambda_{UV}^{d-1}} \left(\bar{q}_L \mathcal{O}_S b_R\right) + h.c.}$$ TECHNI-COLOR

SM fermions are elementary, external to strong-dynamics sector

$\mathcal{O}_S, \mathcal{O}_S^c$: Lorentz-scalar operator composed of strong-sector fields.

In two-flavor technicolor: $(\mathcal{O}_S)_i{}^j = (\mathbb{1}\, \mathcal{O}_S^4 + i \sigma_a \mathcal{O}_S^a)_i{}^j = \bar{\Psi}_R{}^j \Psi_{L_i}$

with the techniquark fields with dimension $d = 3$

E		Technicolor-like	Partial Compositeness
Λ_{UV}	$\mathcal{L}_{int}[\Lambda_{UV}]$	$\frac{\lambda_t}{\Lambda_{UV}^{d-1}} \bar{q}_L \mathcal{O}_S^c t_R + \dots$	$\frac{\lambda_{tL}}{\Lambda_{UV}^{d_L - 5/2}} \bar{q}_L \mathcal{O}_F^L + \frac{\lambda_{tR}}{\Lambda_{UV}^{d_R - 5/2}} \bar{t}_R \mathcal{O}_F^R + \dots$
m_*	$\mathcal{L}_{int}[m_*]$	$\frac{\lambda_t[m_*]}{m_*^{d-1}} \bar{q}_L \mathcal{O}_S^c t_R + \dots$	$\frac{\lambda_{tL}[m_*]}{m_*^{d_L - 5/2}} \bar{q}_L \mathcal{O}_F^L + \frac{\lambda_{tR}[m_*]}{m_*^{d_R - 5/2}} \bar{t}_R \mathcal{O}_F^R + \dots$
EW			

at Λ_{UV}

g_{ETC}: extended technicolor gauge coupling.

$$y_{t,b} = \sqrt{2}\, \frac{m_{t,b}}{v} \simeq \lambda_{t,b}[m_*] \simeq \lambda_{t,b} \left(\frac{m_*}{\Lambda_{UV}}\right)^{d-1} \quad (\text{RG running})$$

$\lambda_{t,b} \sim g_{ETC}^2$ cannot be arbitrarily large as it is a small perturbation of the strong sector TC dynamics

Bound $\quad \Lambda_{UV} \simeq \left(\frac{\lambda_t}{y_t}\right)^{\frac{1}{d-1}} m_* < \lambda_{max}^{\frac{1}{d-1}}$ TeV.

Scale separation is in contradiction to problems from flavor-violating bounds.

If $\dim[\mathcal{O}_S^a] < 4$, this might exactly re-introduce the naturalness problem, $d = 1$ is the dimension of the elementary Higgs field.

Partial compositeness is identical except for the right-handed top quark:

Difference: Interaction terms are linear in the elementary fields rather than bilinear, composite operators are fermionic, and not bosonic:

$$\boxed{\mathcal{L}_{int} = \frac{\lambda_{tL}}{\Lambda_{UV}^{d_L - 5/2}} \overline{q_L}\, \mathcal{O}_F^L + \frac{\lambda_{tR}}{\Lambda_{UV}^{d_R - 5/2}} \overline{t_R}\, \mathcal{O}_F^R + \dots}$$

Partial compositeness because these operators lead to mixtures of elementary (SM) fermions with (composite) fermionic resonances.

$$\boxed{\lambda_{t_{L/R}} \simeq \lambda_{t_{L/R}} \left(\frac{m_*}{\Lambda_{UV}}\right)^{d_{L/R} - 5/2}}$$

Sizable Yukawas are possible for $d_{L/R} \sim 5/2$ even for $\Lambda_{UV} \gg m_*$.

1) $|\mathcal{O}_F|^2 = \dim = 5 \rightarrow$ not reintroducing Naturalness problem

2) Explicit 5-dim. AdS constructions for such models exist.

Individual flavors $\Longrightarrow$ individual scaling dimensions $\longrightarrow$ small Yukawa couplings possible

$$|\text{Phys.}\rangle = \cos\theta_i\, |\text{Elementary}_i\rangle + \sin\theta_i\, |\text{Composite}_i\rangle$$

Composite fermion partners are "vector-like", i.e. they can have Dirac mass terms

$$\langle 0 | \mathcal{O}_F^L | Q \rangle \neq 0, \quad \langle 0 | \mathcal{O}_F^R | \tilde{T} \rangle \neq 0 \qquad -M_Q \overline{Q} Q, \quad - M_{\tilde{T}} \overline{\tilde{T}} \tilde{T}.$$

$$\boxed{\begin{aligned} \mathcal{L}_{mass}^L &= -m_* \overline{Q} Q - \frac{\lambda_{fL}}{g_*} m_* \left(\overline{q_L} Q + h.c.\right) \\ \mathcal{L}_{mass}^R &= -m_* \overline{\tilde{T}} \tilde{T} - \frac{\lambda_{fR}}{g_*} m_* \left(\overline{t_R} \tilde{T} + h.c.\right) \end{aligned}}$$

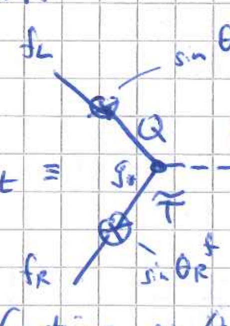
g_*: composite sector coupling.

Light states partially composite with compositeness fractions:

$$\sin\theta_L^f = \frac{\lambda_{fL}}{\sqrt{g_*^2 + (\lambda_{fL})^2}} \simeq \frac{\lambda_{fL}}{g_*}; \quad \sin\theta_R^f = \frac{\lambda_{fR}}{\sqrt{g_*^2 + (\lambda_{fR})^2}} \simeq \frac{\lambda_{fR}}{g_*}$$

with $\lambda_{f_{L/R}} \ll g_*$ for light flavors.

Generation of Yukawa couplings:

$Y_t =$ 

$= g_* \sin\theta_L^f \sin\theta_R^f \simeq \frac{\lambda_{fL}\lambda_{fR}}{g_*}$

Light quarks/leptons have tiny compositeness fractions $\sin\theta_{L/R}^f \ll 1$, top quark a big one.

Embedding all SM fermions [quarks] necessitates an additional $U(1)_X$ group for the MCHM:

$$SO(5) \times U(1)_X \times SU(3)_c \longrightarrow SO(4) \times U(1)_X \times SU(3)_c$$

Different hypercharge embedding: $Y = T_3^R + X$, X: $U(1)_X$ generator.

$$\underline{5}_{2/3} \xrightarrow[\text{decomposes}]{} \underline{4}_{2/3} \oplus \underline{1}_{2/3} \longrightarrow \underbrace{\underline{2}_{7/6}}_{} \oplus \underbrace{\underline{2}_{1/6}}_{Q_L} \oplus \underbrace{\underline{1}_{2/3}}_{\tilde{T}_R}$$

Couplings to SM fermions can be written with incomplete 5-plets (e.g.), as the elementary (SM) d.o.f.s do not obey the strong-sector symmetries:

$$\mathcal{L}_{int}^{t_R} = \lambda_{t_R} (\overline{T}_R^I)(\mathcal{O}_F^R)_I + h.c. \qquad , \quad T_R = (0,0,0,0,t_R)^T \quad (**)$$

Dressing with the Goldstone boson matrix effectively trades an index transforming under $SO(5)$ $[\mathcal{G}]$ into one under $SO(4)$ $[\mathcal{H}]$.

$$\underset{\underline{4}_{2/3}}{\underset{\cap}{(T_R^4}}, \underset{\underline{1}_{2/3}}{\underset{\cap}{T_R^1}})^T = U[\pi]^{-1} \cdot T_R .$$

$$\mathcal{L}_{int}^{q_L} = \lambda_{t_L} (\overline{Q}_{t_L}^I)(\mathcal{O}_F^L)_I + h.c.$$

$$Q_{t_L} = \frac{1}{\sqrt{2}} (-ib_L, -b_L, -it_L, t_L, 0)^T \quad (**)$$

$$\underset{\underline{4}_{2/3}}{\underset{\cap}{(Q_{t_L}^4}}, \underset{\underline{1}_{2/3}}{\underset{\cap}{Q_{t_L}^1}})^T = U[\pi]^{-1} \cdot T_L$$

There is a unique operator for a generalized top Yukawa coupling:

$$(\overline{Q}_{t_L}^4)^i (T_R^4)_i + \overline{Q}_{t_L}^1 T_R^1 = (\overline{Q}_{t_L})^I [U_I^i U_i^{\dagger J} + U_I^5 U_5^{\dagger J}](T_R)^J$$

$$= (\overline{Q}_{t_L})^I (T_R)_I = 0 \quad \text{(as they are orthogonal, as independent irreps)}$$

$$\mathcal{L}_{Yuk}^t = -c^t \frac{\lambda_{t_L}\lambda_{t_R}}{g_*^2} m_* \overline{Q}_{t_L}^1 T_R^1 + h.c.$$

$$= -c^t \frac{\lambda_{t_L}\lambda_{t_R}}{g_*^2} m_* \frac{1}{2\sqrt{2}|H|} \sin\left(\frac{2\sqrt{2}|H|}{f}\right) \overline{q}_L H^c t_R + h.c.$$

with $H^c = i\sigma^2 H^*$ — explained later.

$$m_t = c^t \frac{\lambda_{t_L}\lambda_{t_R}}{g_*^2} m_* \frac{\sqrt{\xi(1-\xi)}}{\sqrt{2}} \longrightarrow$$

$$\mathcal{L}_{Yuk}^t = -\frac{m_t}{2} \frac{1}{\sqrt{\xi(1-\xi)}} \sin\left(\frac{2(V+h)}{f}\right) \bar{t}t = -m_t \bar{t}t - \kappa_t \frac{m_t}{v} h\bar{t}t$$

$$+ c_2 \frac{m_t}{v^2} h^2 \bar{t}t + \dots \qquad \kappa_t^S \equiv \frac{g_{h\bar{t}t}^{\text{composite}}}{g_{h\bar{t}t}^{SM}} = \frac{1-2\xi}{\sqrt{1-\xi}}$$

Bottom quarks: $\underline{5}_{-1/3} \longrightarrow \underline{4}_{-1/3} \oplus \underline{1}_{-1/3} \longrightarrow \underline{2}_{1/6} \oplus \underline{2}_{5/6} \oplus \underline{1}_{-1/3}$.

$$\longrightarrow \mathcal{L}^{b}_{Yuk} = -m_b \bar{b}b - k_b \frac{m_b}{v} h\bar{b}b + \dots$$

$$k_b^5 \equiv \frac{g^{composite}_{hbb}}{g^{SM}_{hbb}} = \frac{1-2\xi}{\sqrt{1-\xi}}$$

Spinorial rep. $\underline{4} = (\underline{2},\underline{1}) \oplus (\underline{1},\underline{2})$ $\quad (SO(4)\simeq SU(2)_L\times SU(2)_R)$

$$\underline{4}_{1/6} \to (\underline{2},\underline{1})_{1/6}\oplus(\underline{1},\underline{2})_{1/6} = \underline{2}_{1/6}\oplus \underline{1}_{2/3}\oplus\underline{1}_{-1/3}.$$

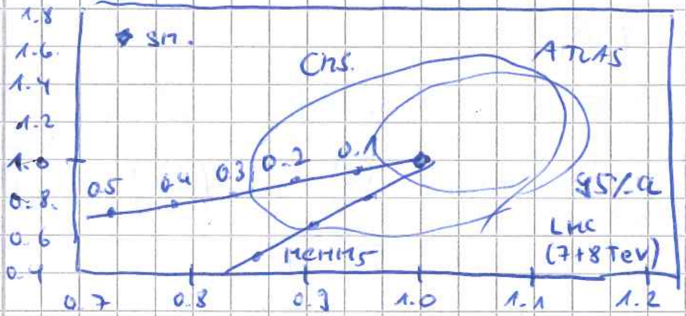
$$Q_{t_L}=Q_{b_L} = (t_L,b_L,0,0)^T,\quad T_R=(0,0,t_R,0)^T,\quad B_R=(0,0,0,b_R)^T$$

↳ Just a different embedding compared to the one above.

$$k_t^4 = k_b^4 = \sqrt{1-\xi}.$$

These two models are dubbed $MCHM_5$ and $MCHM_4$, respectively.

$\xi \gtrsim 0.2$ excluded by Higgs data.



Using the rep.

$$\underline{14}_{2/3} \to (\underline{3},\underline{3})_{2/3}\oplus(\underline{2},\underline{2})_{2/3}\oplus(\underline{1},\underline{1})_{2/3}$$

it is possible to show that the t_R can be completely composite.

Irrelevant for the lecture: explicit CCWZ for $SO(5)/SO(4)$

$$[T^a,\hat T^i] = i f^{ai}{}_j \hat T^j = \hat T^j (t^a)_j{}^i \qquad t^a=(t^\alpha_L,t^\alpha_R)\ SO(4) \text{ generators for the } \underline{4}$$

$$[\hat T^i,\hat T^j] = i f^{ij}{}_a T^a = (t_a)^{ji} T^a$$

$$\mathrm{Tr}[T^A T^B] = \delta^{AB},\quad \sum_{A=1}^{10} (T^A)_{IJ}(T^A)_{KL} = -\frac{1}{2}\left(\delta_{IK}\delta_{JL}-\delta_{IL}\delta_{JK}\right)$$

$$U = e^{i\frac{\sqrt{2}}{f}\pi_i(x)\hat T^i} = \left(\begin{array}{c|c} \mathbb{1}-\left(1-\cos\frac{\pi}{f}\right)\frac{\vec\pi\,\vec\pi^T}{\pi^2} & +\sin\left(\frac{\pi}{f}\right)\frac{\vec\pi}{\pi} \\ \hline -\sin\left(\frac{\pi}{f}\right)\frac{\vec\pi^T}{\pi} & \cos\left(\frac{\pi}{f}\right) \end{array}\right)$$

$$\xrightarrow[\text{gauge}]{\text{unitary}} \left(\begin{array}{c|cc} \mathbb{1}_{3\times3} & 0 & 0 \\ \hline 0 & \cos((v+h)/f) & \sin((v+h)/f) \\ & -\sin((v+h)/f) & \cos((v+h)/f) \end{array}\right)$$

Spinorial representation $\underline{4}$ of $SO(5)$

$$T^\alpha_{4L} = \frac{1}{2}\begin{pmatrix}\sigma^\alpha&0\\0&0\end{pmatrix},\quad T^\alpha_{4R}=\frac{1}{2}\begin{pmatrix}0&0\\0&\sigma^\alpha\end{pmatrix}\qquad \hat T^i_4 = \frac{1}{2\sqrt{2}}\begin{pmatrix}0&\bar\sigma^i\\ \bar\sigma_i^\dagger & 0\end{pmatrix}$$

$$\bar\sigma_i = (i\sigma_\alpha, \mathbb{1}_{2\times2})$$

Spinorial is the fundamental of $Sp(4) \simeq SO(5)$

$$\Omega \cdot T_4^A + (T_4^A)^T \cdot \Omega = 0, \quad \Omega = e^{i\pi[T_{4L}^2 - T_{4R}^2]} = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}$$

$$\mathrm{Tr}[T_4^A T_4^B] = \frac{1}{2}\delta^{AB}, \quad \sum_{A=1}^{10} (T_4^A)_I{}^J (T_4^A)_K{}^L = \frac{1}{4}\left(\delta_I{}^L \delta_K{}^J - \Omega_{IK}\Omega^{JL}\right)$$

$$U_4 = e^{i\frac{\sqrt{2}}{f}\pi_i(x)\hat{T}_4^i} = \begin{pmatrix} \cos\left(\frac{|H|}{\sqrt{2}f}\right)\mathbb{1}_{2\times 2} & i\sin\left(\frac{|H|}{\sqrt{2}f}\right)\frac{\Sigma}{|H|} \\ i\sin\left(\frac{|H|}{\sqrt{2}f}\right)\frac{\Sigma^+}{|H|} & \cos\left(\frac{|H|}{\sqrt{2}f}\right)\mathbb{1}_{2\times 2} \end{pmatrix} \qquad \Sigma = (H^c, H)$$

4x4 matrix

$$\xrightarrow[\text{gauge}]{\text{unitarity}} \begin{pmatrix} \cos\left(\frac{V+h}{2f}\right)\mathbb{1}_{2\times 2} & i\sin\left(\frac{V+h}{2f}\right)\mathbb{1}_{2\times 2} \\ i\sin\left(\frac{V+h}{2f}\right)\mathbb{1}_{2\times 2} & \cos\left(\frac{V+h}{2f}\right)\mathbb{1}_{2\times 2} \end{pmatrix}$$

$$A_{\mu,A} = \left\{A_{\mu,a} = \{A^L_{\mu,\alpha}, A^R_{\mu,\alpha}\},\ A_{\mu,i} = 0\right\}.$$

$$\longrightarrow \quad A^L_{\mu,\alpha} = \{gW_\mu^1, gW_\mu^2, gW_\mu^3\}$$

$$A^R_{\mu,\alpha} = \{0, 0, g'B_\mu\}.$$

$SO(4)$ covariant derivative: $\quad D_\mu \vec{\pi} = (\partial_\mu - iA^L_{\mu,\alpha}t_L^\alpha - iA^R_{\mu,\alpha}t_R^\alpha)\vec{\pi}$

$$\boxed{\begin{aligned} d_\mu^i &= \sqrt{2}\left(\frac{1}{\pi}\sin\left(\frac{\pi}{f}\right) - \frac{1}{f}\right)\frac{\vec{\pi}^T(D_\mu\vec{\pi})}{\pi^2}\pi^i - \frac{\sqrt{2}}{\pi}\sin\left(\frac{\pi}{f}\right)D_\mu\pi^i \\ e_\mu^{L\alpha} &= A^{L\alpha}_\mu - \frac{4}{\pi^2}\sin^2(\pi/2f)\,\vec{\pi}^T i t_L^\alpha D_\mu\vec{\pi} \\ e_\mu^{R\alpha} &= A^{R\alpha}_\mu - \frac{4}{\pi^2}\sin^2(\pi/2f)\,\vec{\pi}^T i t_R^\alpha D_\mu\vec{\pi} \end{aligned}}$$

$$\longrightarrow \quad E^{L\alpha}_{\mu\nu} = \partial_\mu e^{L\alpha}_\nu - \partial_\nu e^{L\alpha}_\mu + \varepsilon^{\alpha\beta\gamma}e^L_{\mu,\beta}e^L_{\nu,\gamma}$$

$$E^{R\alpha}_{\mu\nu} = \partial_\mu e^{R\alpha}_\nu - \partial_\nu e^{R\alpha}_\mu + \varepsilon^{\alpha\beta\gamma}e^R_{\mu,\beta}e^R_{\nu,\gamma}.$$

$$d_\mu^{(2)} = d_\mu^i\sigma_i = \left(\frac{1}{\sqrt{2}|H|}\sin\left(\frac{\sqrt{2}|H|}{f}\right) - \frac{1}{f}\right)\frac{\partial_\mu|H|^2}{|H|^2}\Sigma - \frac{\sqrt{2}}{|H|}\sin\left(\frac{\sqrt{2}|H|}{f}\right)D_\mu\Sigma$$

$$e^{(2)}_{L\mu} = e_\mu^{L\alpha}\frac{\sigma_\alpha}{2} = A^L_\mu + \frac{i}{2|H|^2}\sin^2\left(\frac{|H|}{\sqrt{2}f}\right)\cdot\left(\Sigma(D_\mu\Sigma)^+ - (D_\mu\Sigma)\Sigma^+\right)$$

$$e^{(2)}_{R\mu} = e_\mu^{R\alpha}\frac{\sigma_\alpha}{2} = A^R_\mu + \frac{i}{2|H|^2}\sin^2\left(\frac{|H|}{\sqrt{2}f}\right)\cdot\left(\Sigma^+(D_\mu\Sigma) - (D_\mu\Sigma)^+\Sigma\right)$$

with $D_\mu\Sigma = \partial_\mu\Sigma - \frac{i}{2}A^L_\mu\Sigma + \frac{i}{2}\Sigma A^R_\mu$

<u>Transformation properties:</u>

$$d_\mu^{(2)} \longrightarrow g_L \cdot d_\mu^{(2)} \cdot g_R^+$$

$$e^{(2)}_{L\mu} \longrightarrow g_L \cdot (e^{(2)}_{L\mu} + i\partial_\mu)\,g_L^+$$

$$e^{(2)}_{R\mu} \longrightarrow g_R \cdot (e^{(2)}_{R\mu} + i\partial_\mu)\,g_R^+$$

$$\mathcal{L}^{(2)} = \frac{f^2}{4}d_{\mu,i}d^{\mu,i} = \frac{f^2}{8}\mathrm{Tr}\left[(d_\mu^{(2)})^+ d^{(2),\mu}\right] =$$

$$\frac{f^2}{2|H|^2}\sin^2\left(\frac{\sqrt{2}|H|}{f}\right)D_\mu H^+ D^\mu H + \frac{f^2}{8|H|^4}\left(2\frac{|H|^2}{f^2} - \sin^2\left(\frac{\sqrt{2}|H|}{f}\right)\right)(\partial_\mu|H|^2)^2$$

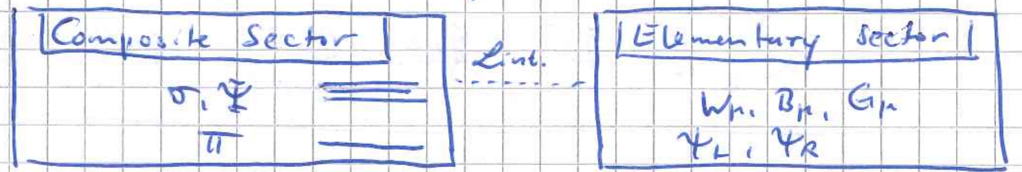
Unitar(it)y gauge: $\Sigma = \frac{v+h}{\sqrt{2}} \cdot \mathbb{1}_{2\times 2}$

$$d_\mu^{(2)} \xrightarrow{U.G.} -\frac{\sqrt{2}}{f}\,\partial_\mu h \,\mathbb{1}_{2\times2} + \sqrt{2}\, i \sin\left(\frac{v+h}{f}\right)\left(A_\mu^L - A_\mu^R\right)$$

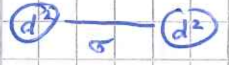
$$e_{L\mu}^{(2)} \xrightarrow{U.G.} A_\mu^L - \sin^2\left(\frac{v+h}{2f}\right)\left(A_\mu^L - A_\mu^R\right)$$

$$e_{R\mu}^{(2)} \xrightarrow{U.G.} A_\mu^R + \sin^2\left(\frac{v+h}{2f}\right)\left(A_\mu^L - A_\mu^R\right)$$

Up to now: non-linear sigma models.

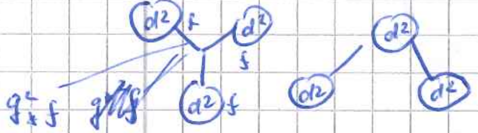


Power-Counting der Operatoren $\mathcal{L}_{EFT}^{(n)} = \frac{f^2}{m_*^{n-2}}\, \mathcal{O}^{(n)}$



$\left(\frac{f}{2}\sigma + \frac{1}{4}\sigma^2\right) d_{\mu,i}\, d^{\mu,i}$ integrate out the sigma field.

$$\longrightarrow \mathcal{L}_{EFT}^{(4)} = \frac{f^2}{8 m_*^2}\left(d^2\right)^2$$



$$\mathcal{L}^{(6)} \sim f^3 \frac{1}{m_*^6}\left(g_*^2 f\right)\left(d^2\right)^3$$

Assumption: $$\boxed{\mathcal{L}_{comp.} = \frac{m_*^4}{g_*^2}\,\hat{\mathcal{L}}\left[\frac{\partial}{m_*}, \frac{g_*\pi}{m_*}, \frac{g_*\sigma}{m_*}, \frac{g_*\Psi}{m_*^{3/2}}\right]} \quad (*)$$

m_* : typical resonance scale

g_* : typical resonance coupling.

$\Rightarrow$ $\boxed{\text{One Scale One Coupling (1S1C) models.}}$

With $f = \frac{m_*}{g_*}$, the linear σ-model is in that class.

Dim. Analysis: $[\mathcal{L}] = ET/L^4 = [\hbar]/L^4$

Canonically normalized fields: $[\pi] = [\sigma] = [\hbar]^{1/2}/L$, $[\Psi] = [\hbar]^{1/2}/L^{3/2}$

In $(*)$, we must then have $[m_*] = L^{-1}$, $[g_*] = [\hbar]^{-1/2}$ $\Longrightarrow$

m_* is a length scale, g_* a coupling $C = [\hbar]^{-1/2} = (ET)^{-1/2}$

Parameter	e	λ_H	G_F	$m_{W,Z}$	v	f_π
Dimension	C	C^2	C^2L^2	L^{-1}	$C^{-1}L^{-1}$	$C^{-1}L^{-1}$

$[f] = C^{-1}L^{-1}$

$$e\bar{\Psi}A_\mu\gamma^\mu\Psi \to [e]\,[\hbar]^{1/2}/L^{3/2}\,[\hbar]^{1/2}/L\,[\hbar]^{1/2}/L^{3/2}$$

$$= [e]\,[\hbar]^{3/2}/L^4 \Rightarrow [e] = [\hbar]^{-1/2}$$

Dimensional analysis then dictates the form $(*)$.

$$\mathcal{L}_{EFT} = \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}_{tree}\left[\frac{\partial}{m_*}, \frac{g_*\pi}{m_*}, \frac{g_*\sigma}{m_*}, \frac{g_*\Psi}{m_*^{3/2}}\right] +$$

$$+ \frac{g_*^2 \hbar}{16\pi^2} \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}_{1\text{-loop}}\left[\frac{\partial}{m_*}, \frac{g_*\pi}{m_*}, \frac{g_*\sigma}{m_*}, \frac{g_*\Psi}{m_*^{3/2}}\right] + \dots$$

Loop-wise expansion adds factors of $\hbar$ per loop.

Perturbativity bound $g_* \lesssim 4\pi/\sqrt{\hbar}$ from loop-expansion factor $\frac{g_*^2\hbar}{16\pi^2}$

$g \cdot A_\mu / m^*$ is the dimensionless combination appearing, $\lambda\Psi / m_*^{3/2}$

$$\Rightarrow \mathcal{L}_{EFT} = \frac{m_*^4}{g_*^2} \hat{\mathcal{L}}\left[\frac{\partial}{m_*}, \frac{g_*\pi}{m_*}, \frac{g_*\sigma}{m_*}, \frac{g_*\Psi}{m_*^{3/2}}, \frac{g \cdot A_\mu}{m_*}, \frac{\lambda \cdot \Psi}{m_*^{3/2}}\right]$$

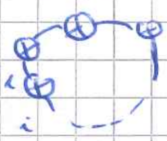
Power Counting in Strongly-Coupled Models (Large N)

Large N-limit $SU(N)$ with $N \to \infty$, but fixed coupling, hence 't Hooft coupling $g_s = g/\sqrt{N}$ is kept fixed.
In this limit, only planar diagrams survive (leading N approx.), as non-planar diagrams have less closed color loops.

Leading mesonic operator correlation functions then always scale with one closed color line:

$$\mathcal{O}^\Delta \equiv \langle 0 | M_1 \dots M_n | 0 \rangle \propto N.$$

$M \sim \bar{q}_i \Gamma q^i$



$\longrightarrow$ color factor N.

$$\boxed{\langle 0 | \mathcal{O}^\Delta | 0 \rangle \sim \frac{N}{16\pi^2} m_*^\Delta \, \mathcal{F}[p_i/m_*]}$$

by dimensional analysis. / LSZ formula: divide by wave function renormalizations

$$\langle 0 | M_i M_i | 0 \rangle \underset{p_i^2 \to m_i^2}{=} \frac{Z_i}{p_i^2 - m_i^2} \overset{!}{\sim} \frac{N}{16\pi^2} m_*^{\Delta_i} \mathcal{F}[p_i/m_*]$$

2-point (Källén-Lehmann) correlation function

$$\Rightarrow \sqrt{Z_i} \propto \frac{\sqrt{N}}{4\pi} m_*^{\frac{\Delta_i}{2}+1}$$

(wave function renormalization) ['t Hooft, 1974]

Powers of m_* combine to give the correct dimension of n-point Feynman amplitude: $[A_n] = E^{4-n}$

$$\Rightarrow \boxed{A_n \sim m_*^4 \frac{N}{16\pi^2} \left(\frac{4\pi}{\sqrt{N}}\right)^n \left(\frac{1}{m_*}\right)^n \mathcal{F}[p_i/m_*]}$$

3-point and higher amplitudes vanish in the strict large-N limit. Mesons become asymptotically free particles (!).
At finite N, they are weakly coupled with $g_* = 4\pi/\sqrt{N}$
$\rightarrow$
Each field insertion gives m_*^{-1} (bosons), $m_*^{-3/2}$ fermions and one power of g_* (justified by wave function renorm.)
Overall factor m_*^4/g_*^2 to reproduce the $m_*^4 - N$ scaling $\Rightarrow$ Scaling law for 1S1C models.

For $N=3$, power counting reduces to NDA ("Naive dimensional analysis") Georgi/Lentwyler, 197, Georgi, 1990 which is (mostly) confirmed by chiral perturbation calculations in meson physics.

Details of the Higgs potential

SM gauge and top Yukawa couplings break the Goldstone symmetries and generate the Higgs potential.

$$V_{g'^2} \propto c_+ \frac{g'^2}{2} \sin^2\frac{H}{f} + c_- g'^2 \cos\frac{H}{f}$$

$$V_{g^2} \propto \frac{3g^2}{2} \sin^2\frac{H}{f} - c_- 3g^2 \cos\frac{H}{f}$$

$$V_{\lambda^2}^{5\oplus5} \propto \left(\frac{c_L}{2}\lambda_{t_L}^2 - c_R\lambda_{t_R}^2\right)\sin^2\frac{H}{f} + \text{const.}$$ ← dominant, but allows only for VEVs 0 or $\pi f/2$.

Complete $O(\lambda^4)$ potential is:

$$V_{\lambda^4}^{5\oplus5} \propto \left(c_{LL}\lambda_{t_L}^4 + c_{RR}\lambda_{t_R}^4 + c_{LR}\lambda_{t_R}^2\lambda_{t_L}^2\right)\sin^2\frac{H}{f}$$
$$+\left(c'_{LL}\lambda_{t_L}^4 + c'_{RR}\lambda_{t_R}^4 + c'_{LR}\lambda_{t_R}^2\lambda_{t_L}^2\right)\sin^4\frac{H}{f}.$$

Now, realistically small ξ values can be obtained.

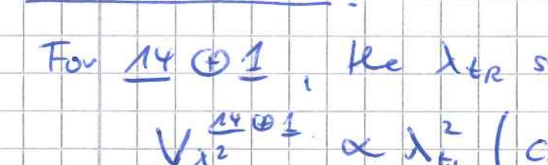
For $\underline{14}\oplus\underline{1}$, the λ_{t_R} singlet cannot generate a potential (singlet!)

$$V_{\lambda^2}^{14\oplus1} \propto \lambda_{t_L}^2\left(c_1\sin^2\frac{H}{f} + c_2\sin^4\frac{H}{f}\right)$$ ⟶ gives immediately reasonable VEVs.

Higgs potential is sum of 1PI (one-particle irreducible) diagrams with zero-momentum external Higgs lines.

- pure composite not allowed (Goldstone symmetry)
- pure elementary not possible (Higgs is composite)

⟹ mixed diagrams with at least one elementary lines, must go into a loop in order to be 1PI. Hence, V only generated at the radiative level.



$=$ composite sector lines, could be elementary or composite

$= \sim \langle \hat{O}\hat{O}\rangle$

$$V = \frac{N_c m_*^4}{16\pi^2}\left[\left(\frac{g_E}{g_*}\right)^2 V_{(2)}[H/f] + \left(\frac{g_E}{g_*}\right)^4 V_{(4)}[H/f] + \dots\right]$$

holds for both weakly-coupled and strongly coupled models.

Standard Coleman-Weinberg potential:

$$V_{gauge}[H] = \frac{3}{32\pi^2}\Lambda^2 \sum_i M_i^2[H] + \frac{3}{64\pi^2}\sum_i \left[M_i^4[H]\left(\log\left(\frac{M_i^2[H]}{\Lambda^2}\right) - \frac{1}{2}\right)\right]$$

$$V_{top}[H] = -\frac{N_c}{8\pi^2}\Lambda^2 M_t^2[H] - \frac{N_c}{16\pi^2} M_t^4[H]\left(\log\frac{M_t^2[H]}{\Lambda^2} - \frac{1}{2}\right)$$

Higgs as a (p)NGB does not generate a potential.

Quadratic terms are scheme-dependent and cancel after renormalization.

$$V^{IR}_{gauge}[H] = \frac{3f^4}{64\pi^2}\,\frac{3g^4 + 2g^2g'^2 + g'^4}{16}\,\sin^4\left(\frac{H}{f}\right)\left[\log\left[\frac{g^2}{g_*^2}\sin^2\frac{H}{f}\right] + \text{const.}\right]$$

$$V^{IR}_{top}[H] = -\frac{N_c}{16\pi^2}\,\frac{1}{16}\,\frac{y_t^4 f^4}{(1-\xi)^2}\,\sin^4\frac{2H}{f}\left(\log\left(\frac{y_t^2}{g_*^2}\sin^2\frac{2H}{f}\right) + \text{const.}\right)$$

using $\Lambda = m_* = g_* f$

Interpreted as RG-improved Higgs potential, logarithmically enhanced terms from running between m_* and $m_t/m_W/m_Z$.

Higgs VEV & Tuning

General form of Higgs potential: $\boxed{V[H] = -\alpha f^2 \sin^2\frac{H}{f} + \beta f^2 \sin^4\frac{H}{f}}$

α: contributions from $[g^2, g'^2]\ \lambda_t^2$ (gauge contributions ignored)

β: $\lambda_t^2\ (\underline{14}\oplus\underline{1})$, $\lambda_t^4\ (\underline{5}\oplus\underline{5})$

$$\left.\frac{\partial V}{\partial H}\right|_{\langle H\rangle} = -2\alpha\, \cancel{f^2/f}\ \cancel{\cos\frac{H}{f}\sin\frac{H}{f}} + 4\beta\, \cancel{f^2/f}\ \cancel{\cos\frac{H}{f}}\sin^3\frac{H}{f} \overset{!}{=} 0$$

assume $\langle H\rangle \neq 0 \implies -2\alpha + 4\beta \underbrace{\sin^2\frac{\langle H\rangle}{f}}_{\equiv \xi} \overset{!}{=} 0$

$\implies \boxed{\alpha = 2\beta\xi}$

$\sin^2\frac{\langle H\rangle}{f} \approx \frac{\langle H\rangle^2}{f^2} \equiv \xi.$ $\longrightarrow$

$$V = -2\beta\xi f^2 \sin^2\frac{H}{f} + \beta f^2 \sin^4\frac{H}{f} = \beta f^2\left(\sin^4\frac{H}{f} - 2\xi\sin^2\frac{H}{f}\right)$$
$$= \beta f^2 \sin^2\frac{H}{f}\left(\sin^2\frac{H}{f} - 2\xi\right)$$

$$0 \overset{!}{=} \left.\frac{\partial V}{\partial H}\right|_{\langle H\rangle} = 4\beta f\left(\sin^2\frac{\langle H\rangle}{f} - \xi\right)\sin\frac{\langle H\rangle}{f}\cos\frac{\langle H\rangle}{f} = 0$$

$$m_H^2 = \left.\frac{\partial^2 V}{\partial H^2}\right|_{\langle H\rangle} = 8\beta\sin^2\frac{\langle H\rangle}{f}\cos^2\frac{\langle H\rangle}{f} = 8\beta\,\xi(1-\xi)$$

$$\implies \boxed{V[H] = \frac{m_H^2 f^2}{8\xi(1-\xi)}\left(\sin^2\frac{H}{f} - \xi\right)^2 + \text{const.}}$$

Modifies the trilinear Higgs coupling: $\kappa_{3h} = \frac{1-2\xi}{\sqrt{1-\xi}}$

Measurement for ILC or CLIC.

$\frac{\alpha}{\beta} = 2\xi \ll 1$. Requires some tuning:

$$\alpha_{\underline{14}\oplus\underline{1}} = a \frac{N_c}{16\pi^2} \lambda_{t_L}^2 m_*^2 \,, \quad \beta_{\underline{14}\oplus\underline{1}} = b \frac{N_c}{16\pi^2} \lambda_{t_L}^2 m_*^2 .$$

Fine tuning: $\Delta_\xi^{14\oplus 1} = \frac{(\alpha/\beta)_{\text{expected}}}{(\alpha/\beta)_{\text{needed}}} \approx \frac{1}{2\xi}$.

$$\alpha_{\underline{5}\oplus\underline{5}} = a \frac{N_c}{16\pi^2} \lambda_t^2 m_*^2 \,, \quad \beta_{\underline{5}\oplus\underline{5}} = b \frac{N_c}{16\pi^2} \lambda_t^4 f^2$$

$$\Delta_\xi^{5\oplus 5} = \frac{(\alpha/\beta)_{\text{expected}}}{(\alpha/\beta)_{\text{needed}}} = \left(\frac{g_*}{\lambda_t}\right)^2 \cdot \frac{1}{2\xi} = \left(\frac{g_*}{\lambda_t}\right)^2 \cdot \Delta_\xi^{\underline{14}\oplus\underline{1}}$$

$\Longrightarrow$ $\underline{14}\oplus\underline{1}$ is favored over $\underline{5}\oplus\underline{5}$

COLLECTIVE SYMMETRY BREAKING

$$U(\pi) = \exp\left(i \frac{\sqrt{2}}{f} \pi_i \hat{T}^i\right) \qquad \underset{SO(5)}{T^A} = \{\underset{SO(4)}{T^a}, \hat{T}^i\} \quad \text{coset: } SO(5)/SO(4)$$

$$\mathcal{L}^\pi = \frac{f^2}{4} d_\mu^i d_i^\mu \quad \text{with} \quad d_\mu^i = i \,\mathrm{Tr}\left[(U^\dagger D_\mu U)\hat{T}^i\right]$$

$$D_\mu U = \partial_\mu U - i A_\mu U \,, \qquad A_\mu = g W^\alpha T_L^\alpha + g' B_\mu T_R^3 .$$

$$\mathcal{L}^g = -\frac{1}{4} \mathrm{Tr}[W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu] \,, \quad W_\mu = W_\mu^a T_L^a$$

Discuss spurions to systematically estimate calculability of Higgs potential

Concept of spurions is based on possibility of keeping track of consequences of a symmetry when it is broken by a small explicit perturbation.

Promote A_μ to a $SO(5)$ gauge field with two spurions:

$$A_\mu = \mathcal{G}_\alpha W_\mu^\alpha + \mathcal{G}' B_\mu = \mathcal{G}_{\alpha A} T^A W_\mu^\alpha + \mathcal{G}'_A T^A B_\mu$$

A: adjoint index of $SO(5)$. Physical values of the spurions:

$\mathcal{G}_\alpha = g T_L^\alpha$, $\mathcal{G}' = g' T_R^3$ breaking $SU(2)_L^0 \times SO(5) \xrightarrow{\mathcal{G}} SO(4) = SU(2)_L \times SU(2)_R$

$\xrightarrow{\mathcal{G}'} SU(2)_L \times U(1)_Y$

(elementary) symmetry of gauge-kinetic term.

Spurions that generate Higgs potential:

$$c_g f^4 (U^\dagger \mathcal{G}_\alpha \mathcal{G}_\alpha U)_{55} = \frac{3}{4} c_g f^4 g^2 \sin^2(H/f) \qquad (H \equiv \pi_4)$$

$$c_{g'} f^4 (U^\dagger \mathcal{G}' \mathcal{G}' U)_{55} = \frac{1}{4} c_g f^4 g'^2 \sin^2(H/f)$$

$SO(5)$ symmetry lowers degree of divergence from quartic to quadratic (Goldstone boson nature).

Enhance the (Goldstone) symmetries and make the Higgs charged under them to achieve even more protection in the Higgs ~~mechanism~~ potential generation.

$\Longrightarrow$ $\boxed{\text{Collective symmetry breaking}}$

Realized e.g. in N-site models and in Little Higgs models

Toy example: 2-Site Model

based on coset $SO(5)_L \times SO(5)_R / SO(5)_V$

pNGB matrix: $\mathcal{U}[\pi] = \exp\left(i \frac{\sqrt{2}}{f} \pi_A T^A\right)$, $\mathcal{U}[\pi] \to \mathcal{U}[\pi'] = g_L \mathcal{U}[\pi] g_R^T$

$SO(5)_L \times SO(5)_R \to SO(5)_V$ has 10 NGBs, 4 (π of $SO(4)_L$ are H), 6 are eaten up by gauged $SO(4)$ subgroup $SO(5)_R$.

EW bosons: gauging $SU(2)_L \times U(1)_Y$ of $SO(5)_L$.

By this construction, W/Z get admixtures of $\tilde{\rho}$ resonances (i.e. vectors of gauged $SO(4)_R$.)

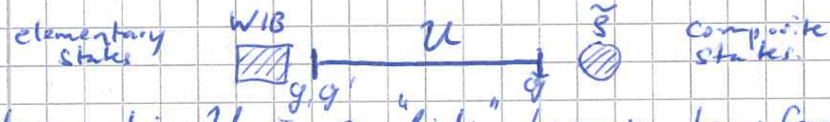
Structure of the model:

$$\mathcal{L}_0 = \mathcal{L}^\pi + \underbrace{\mathcal{L}^g_{comp.} + \mathcal{L}^g_{elem.}}$$

kinetic terms for composite and elementary gauge bosons.

$$\mathcal{L}^\pi = \frac{f^2}{4} \mathrm{Tr}\left[(D_\mu \mathcal{U})^T (D^\mu \mathcal{U})\right]$$

Link / Moose Notation (after antler of a moose)



Goldstone matrix $\mathcal{U}$ is a "link", because transforms from left with $SO(5)_L$ and from right with $SO(5)_R$.

Left side: "elementary group" $SU(2)_L^0 \times U(1)_Y^0$

Right side: $\widetilde{SO}(4)$ for (composite) gauge bosons $\tilde{\rho}$

$g, g', \tilde{g}$ spurions for the gauging.

$$D_\mu \mathcal{U} = \partial_\mu \mathcal{U} - i A_\mu \mathcal{U} + i \mathcal{U} \tilde{R}_\mu, \qquad A_\mu = g_0 W^\alpha_\mu T^\alpha_L + g_0' B_\mu T^3_R$$

$$\tilde{R}_\mu = \tilde{g}_\rho \tilde{\rho}^a_\mu T^a \quad \text{and then} \quad \mathcal{L}^g_{comp.} = -\frac{1}{4} \mathrm{Tr}\left[\tilde{\rho}_{\mu\nu} \tilde{\rho}^{\mu\nu}\right]$$

with $\tilde{\rho}_{\mu\nu} = \partial_\mu \tilde{\rho}_\nu - \partial_\nu \tilde{\rho}_\mu - i \tilde{g}_\rho [\tilde{\rho}_\mu, \tilde{\rho}_\nu]$

"Inverse Ohm law" for gauge couplings: $\frac{1}{g^2} = \frac{1}{g_0^2} + \frac{1}{\tilde{g}_\rho^2} \simeq \frac{1}{g_0^2}$,

$\frac{1}{g'^2} = \frac{1}{g_0'^2} + \frac{1}{\tilde{g}_\rho^2} \simeq \frac{1}{g_0'^2}$ (from the kinetic terms after rescaling $A_\mu \to A_\mu / g$.)

Higgs potential can only be generated if both symmetry $SO(5)_L$ and $SO(5)_R$ are broken because both $g_L = \mathcal{U}^T$ or $g_R = \mathcal{U}$ can rotate away a Higgs VEV.

Spurions: $A_\mu = \mathcal{G}_\alpha W_\mu^\alpha + \mathcal{G}' B_\mu = \mathcal{G}_{\alpha A_L} T^{A_L} W_\mu^\alpha + \mathcal{G}'_{A_L} T^{A_L} B_\mu$ with physical values $\mathcal{G}_\alpha = g_0 T_L^\alpha$, $\mathcal{G}' = g_0' T_R^3$,

$\tilde{R}_\mu = \tilde{\mathcal{G}}_a \tilde{\rho}_\mu^a = \tilde{\mathcal{G}}_{A_R a} T^{A_R} \tilde{\rho}_\mu^a$, physical value: $\tilde{\mathcal{G}}_a = \tilde{g}_\rho T^a$

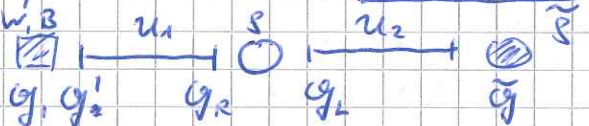
$\hookrightarrow$ breaks $SO(5)_R \times \widetilde{SO(4)} \longrightarrow SO(4)_{diag}$.

Higgs potential leading contributions from operators with 4 spurions.

$$\frac{c_g}{16\pi^2} f^4 \, \mathrm{Tr}\left[\mathcal{G}_\alpha \mathcal{G}_\alpha \mathcal{U} \tilde{\mathcal{G}}_a \tilde{\mathcal{G}}_a \mathcal{U}^T\right], \quad \frac{c_g'}{16\pi^2} f^4 \, \mathrm{Tr}\left[\mathcal{G}' \mathcal{G}' \mathcal{U} \tilde{\mathcal{G}}_a \tilde{\mathcal{G}}_a \mathcal{U}^T\right]$$

Degree of divergence reduced from quadratic to logarithmic!

Finite Higgs potential in the 3-site model:



Two copies of cosets $SO(5)_L \times SO(5)_R / SO(5)_V$

20 Goldstones, Π_1^A, Π_2^A

$$\mathcal{L}^\pi = \frac{f_1^2}{4} \mathrm{Tr}\left[(D_\mu \mathcal{U}_1)^T (D^\mu \mathcal{U}_1)\right] + \frac{f_2^2}{4} \mathrm{Tr}\left[(D_\mu \mathcal{U}_2)^T (D^\mu \mathcal{U}_2)\right]$$

$\underset{\substack{\downarrow \\ SU(2)_L^0 \times U(1)_Y^0}}{SO(5)_L^1} \times \underbrace{SO(5)_R^1, \; SO(5)_L^2}_{\text{vector contraction}} \times \underset{\substack{\searrow \\ \tilde{\rho}}}{SO(5)_R^2}$ broken by gauging.

broken by Gauging

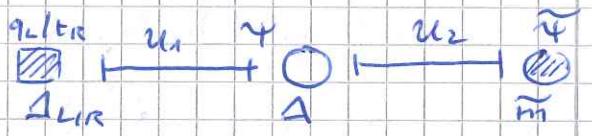
$$D_\mu \mathcal{U}_1 = \partial_\mu \mathcal{U}_1 - i A_\mu \mathcal{U}_1 + i \mathcal{U}_1 R_\mu \qquad R_\mu = L_\mu = g_\rho \rho_\mu^A T^A$$

$$D_\mu \mathcal{U}_2 = \partial_\mu \mathcal{U}_2 - i L_\mu \mathcal{U}_2 + i \mathcal{U}_2 \tilde{R}_\mu$$

Higgs decay constant linked: $\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2}$

Fermionic sector: 2-site model

3-site model



$\Rightarrow$ diverging contributions to Higgs potential only generated at 2-loop!

Collective symmetry breaking corresponds to "locality in theory space", i.e. only nearest-neighbor links in moose diagrams.

A wee bit on Collider Signatures of Top Partners and Vector Resonances.

Example: $SO(5)/SO(4)$ MCHM with a $\underline{5} = \underline{4} \oplus \underline{1}$

$$\mathcal{L}_{comp.} = i\bar{\Psi}_4 \not{D} \Psi_4 + i \bar{\Psi}_1 \not{D} \Psi_1 - m_4 \bar{\Psi}_4 \Psi_4 - m_1 \bar{\Psi}_1 \Psi_1$$
$$- \cancel{f}\left(i c_L \bar{\Psi}^i_{4L} \gamma^\mu d^i_\mu \Psi_{1L} + i c_R \bar{\Psi}^i_{4R} \gamma^\mu d^i_\mu \Psi_{1R} + h.c.\right)$$

$$D_\mu \Psi_4 = \left(\partial_\mu - \tfrac{2}{3} i g' B_\mu - i e_\mu - i g_s G_\mu\right)\Psi_4 .$$

$$D_\mu \Psi_1 = \left(\partial_\mu - \tfrac{2}{3} i g' B_\mu - i g_s G_\mu\right)\Psi_1$$

$$\Psi_4 = \frac{1}{\sqrt{2}}\begin{pmatrix} -iB + iX_{5/3} \\ -B - X_{5/3} \\ -iT - iX_{2/3} \\ T - X_{2/3}\end{pmatrix}$$

$\begin{pmatrix} T \\ B \end{pmatrix}$ $SU(2)_L$ doublet with $Y = \frac{1}{6}$

$\begin{pmatrix} X_{5/3} \\ X_{2/3} \end{pmatrix}$ " " " $Y = \frac{7}{6}$.

$$\mathcal{L}_{elem.} = i\bar{q}_L \not{D} q_L + i \bar{t}_R \not{D} t_R .$$

$$D_\mu q_L = \left(\partial_\mu - i g W^i_\mu \frac{\sigma^i}{2} - i \frac{1}{6} g' B_\mu - i g_s G_\mu\right) q_L$$

$$D_\mu t_R = \left(\partial_\mu - i \tfrac{2}{3} g' B_\mu - i g_s G_\mu\right) t_R$$

$$\mathcal{L}_{mix} = y_{L4} f \left(\bar{q}^5_L U\right)_i \Psi^i_4 + y_{L1} f \left(\bar{q}^5_L U\right)_5 \Psi_1 + h.c.$$
$$+ y_{R4} f \left(\bar{t}^5_R U\right)_i \Psi^i_4 + y_{R1} f \left(\bar{t}^5_R U\right)_5 \Psi_1 + h.c.$$

$$q^5_L = \frac{1}{\sqrt{2}}\begin{pmatrix} -ib_L \\ -b_L \\ -it_L \\ t_L \\ 0 \end{pmatrix}, \quad t^5_R = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}$$

embedding in incomplete $SO(5)$ multiplets.

8 free parameters: $y_{L4,1}, y_{R4,1}, m_4, m_1, c_{L/R}$

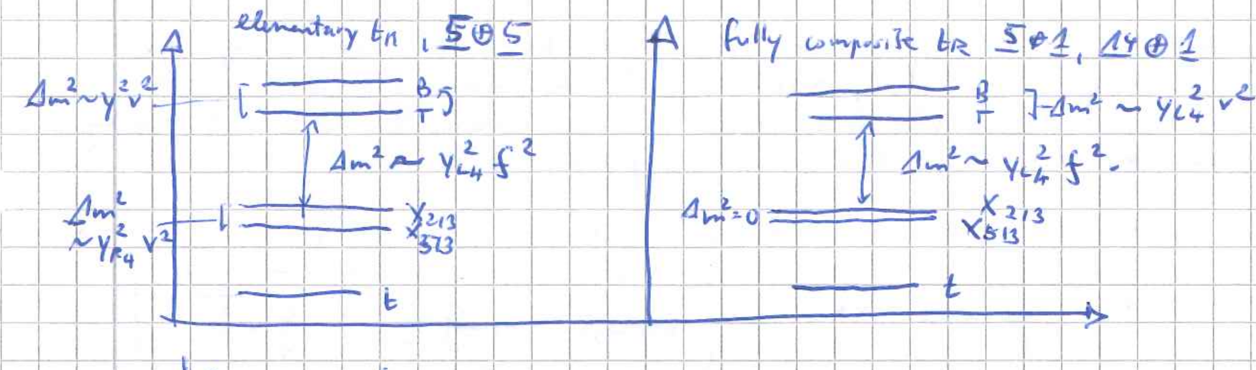
$$m_t^2 = \frac{\left(y_{L1} y_{R1} m_4 - y_{L4} y_{R4} m_1\right)^2 f^4}{\left(m_4^2 + y_{L4}^2 f^2\right)\left(m_1^2 + y_{R1}^2 f^2\right)} \frac{\xi}{2} + \mathcal{O}(f^2 \xi^2)$$

$$m_{X_{2/3}} = m_4 \left(1 + \frac{y_{R4}^2 f^2}{4 m_4^2} \xi + \ldots\right)$$

$$m_T = \sqrt{m_4^2 + y_{L4}^2 f^2}\left(1 - \frac{\left(y_{L4}^2 - y_{R4}^2\right) f^2}{4 m_4^2}\xi + \ldots\right)$$

$$m_{\tilde{T}} = \sqrt{m_1^2 + y_{R1}^2 f^2}\left(1 + \frac{\left(y_{L1}^2 - 2 y_{R1}^2\right) f^2}{4 m_1^2}\xi + \ldots\right)$$

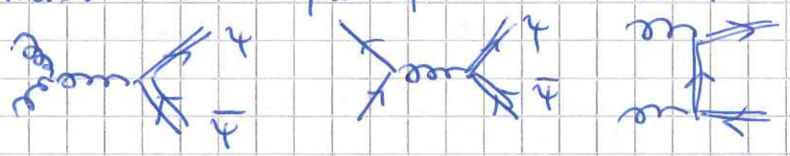
$$m_B = \sqrt{m_4^2 + y_{L4}^2 f^2} \qquad m_{X_{5/3}} = m_4$$



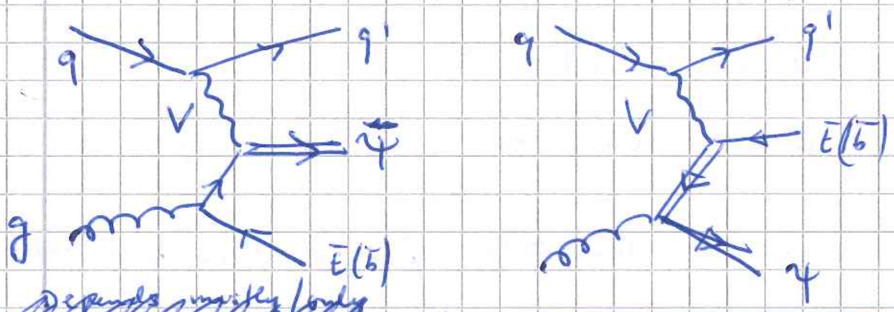
Mass spectra.

LHC production mechanisms.

Top partners: QCD pair production with universal Xsec. (function of m_ψ).

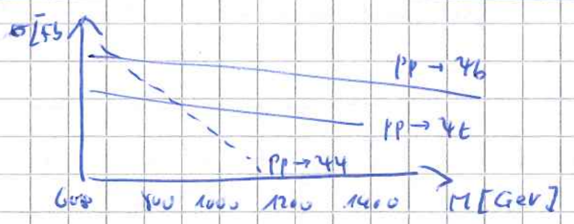


EW production:



Depends on the mixings and the details of the model.

~~Depends mostly/only on EW quantum numbers~~



For $m \sim 1$ TeV, single production becomes dominant over pair production.

Couplings of composite resonances with 3rd generation SM quarks mediated by W/Z/H

	Wt_R	Zt_R	ht_R	Wt_L	Zt_L	ht_L	Wb_L	Zb_L	hb_L
$X_{5/3}$	✓	—	—	ε	—	—	—	—	—
$X_{2/3}$	—	✓	✓	—	ε	ε	ε (not $\underline{5}+\underline{1}$)	—	—
T	—	✓	✓	—	ε	ε	ε	—	—
B	✓	—	—	ε	—	—	—	—	ε (only $\underline{14}+\underline{1}$)
$\tilde{T}$	—	ε	ε	—	✓	✓	✓	—	—
$Y_{5/3}$	ε	—	—	✓	—	—	—	—	—
$Y_{2/3}$	—	ε	ε	—	✓	✓	✓	—	—
$Z_{2/3}$	—	ε	ε	—	✓	✓	✓	—	—
$B^+_{-1/3}$	ε	—	—	✓	—	—	—	✓	—
$B^-_{-1/3}$	ε	—	—	✓	—	—	—	—	✓
$Z_{-4/3}$	—	—	—	—	—	—	✓	—	—

✓ leading order, ε subleading in v/f

In renormalizable models of vector-like quarks, the value of the $b/t\,(Z/W/h)\,X$ couplings is tightly related to the mixing between the SM fermions and the resonances and hence given by the SM gauge couplings and the EW quantum numbers of the resonances.

In composite Higgs models, these constraints do not exist.

Single production general rule: $\boxed{c_{single} \sim g \frac{v}{f} = g\sqrt{\xi}}$

$\Rightarrow$ $\xi \sim 0.1 \leftrightarrow c_{single} \sim 0.2$ for natural and viable scenarios.

---

(4) 4-plet $BR(X_{5/3} \to W^+ t) \sim 1.$

$\longrightarrow$ Same-sign (SS) lepton states as signal.

Same for B, only relevant if $\Delta m \ll 500$ GeV. dominant decay $B \to Wt$.

$BR(X_{2/3} \to Zt) \simeq BR(X_{2/3} \to ht) \simeq 1/2.$

$BR(T \to Zt) \simeq BR(T \to ht) \simeq 1/2.$

Chain decays possible:

$T \to Z X_{2/3}$
$T \to h X_{2/3}$
$T \to W X_{5/3}$

Full LHC program will push limits to 2-3 TeV range.

---

(1) $SU(2)_L$ singlet. $BR(\tilde{T} \to Wb) \simeq 1/2$, $BR(\tilde{T} \to Zt) \simeq BR(\tilde{T} \to ht) \simeq \frac{1}{4}$.

Strong single production in association with a bottom.

---

(14) 9-plet. $Z_{-4/3} \to W^- b$, $Z_{4/3}$ can be produced in association with a bottom.

$U_{8/3} \to W^+W^+t$ only has 3-body decays.

(either via off-shell $X_{5/3}$, $Y_{5/3}$ or via contact interaction)

$W^+W^+t \to$ high rate of SS leptons $\to$ enhances searches

already 8 TeV run $\Rightarrow$ $m_9 \gtrsim 990$ GeV limit.

Vector Resonances

Using effective parametrizations. Vectors can transform as $(\underline{3},\underline{1}), (\underline{1},\underline{3}), (\underline{1},\underline{1})$

$SU(2)_L$ triplet ρ^L_μ

$$\mathcal{L}_{\rho_L} = -\frac{1}{4g_{\rho_L}^2}\rho^{a_L}_{\mu\nu}\rho^{a_L,\mu\nu} + \frac{m_{\rho_L}^2}{2g_{\rho_L}^2}\left(\rho^{a_L}_\mu - e^{a_L}_\mu\right)^2 + \kappa\,\overline{\Psi}_4^i\gamma^\mu\left(\rho^{a_L}_\mu - e^{a_L}_\mu\right)(t^{a_L})_{ij}\Psi_4^j$$

$\rho^L_\mu - e^L_\mu$ with the CCWZ symbol transforms homogeneously under ~~SO(4)~~ SO(4).

$$\frac{1}{g^2} = \frac{1}{g_0^2} + \frac{1}{g_{\rho_L}^2}, \quad m_W^2 = \frac{g^2}{4} f^2 \xi, \quad m_Z^2 = \frac{g^2+g'^2}{4} f^2 \xi$$

$$M_{\rho_L^\pm}^2 = M_{\rho_L^0}^2 \simeq \frac{g_{\rho_L}^2}{g_{\rho_L}^2 - g^2} m_{\rho_L}^2 - \frac{\xi}{4} \frac{g^2}{g_{\rho_L}^2 - g^2} \left(2 m_{\rho_L}^2 - f^2 g^2\right)$$

$$M_{\rho_L}^2 - M_{\rho_L^\pm}^2 \simeq \frac{\xi^2 g'^2}{16\, g_{\rho_L}^2} m_{\rho_L}^2 \left(1 - \frac{f^2 g^2}{m_{\rho_L}^2}\right)$$

Mass splitting typically only 0.1% and can be neglected.

$SU(2)_R$ almost analogously. Here: $\frac{1}{g'^2} = \frac{1}{g_0'^2} + \frac{1}{g_{\rho_R}^2}$.

ρ_μ^X SO(4) singlets. $\rho_\mu^X \to \rho_\mu^X + \partial_\mu \alpha^X$ $U(1)_X$ group.

$$\mathcal{L}_{\rho_X} = -\frac{1}{4 g_{\rho_X}^2} \rho_{\mu\nu}^X \rho^{X\mu\nu} + \frac{m_{\rho_X}^2}{2 g_{\rho_X}^2} \left(\rho_\mu^X - g_0' B_\mu\right)^2 + \left(\rho_\mu^X - g_0' B_\mu\right) J^\mu_{\rho_X}$$

Heavy gluons $\mathcal{L}_{\rho_g} = -\frac{1}{4 g_{\rho_g}^2} \rho_{\mu\nu}^g \rho^{g\mu\nu} + \frac{m_{\rho_g}^2}{2 g_{\rho_g}^2} \left(\rho_\mu^g - g_{s0} G_\mu\right)^2 + \left(\rho_\mu^g - g_{s0} G_\mu\right) J^\mu_{\rho_g}$

$$M_{\rho_g}^2 = \frac{g_{\rho_g}^2}{g_{\rho_g}^2 - g_s^2} m_{\rho_g}^2 \qquad \frac{1}{g_s^2} = \frac{1}{g_{s0}^2} + \frac{1}{g_{\rho_g}^2}.$$

Goldstone boson equivalence theorem: at high energies, longitudinal modes of W and Z are well described by the Goldstone modes: $W^\pm_{\mu L} \sim \partial_\mu w^\pm$, $Z_L \sim \partial_\mu z$.

$$g_{\rho_{L/R}\phi\phi} \sim \underbrace{\left(\frac{m_{\rho_{L/R}}}{g_{\rho_{L/R}} f}\right)^2}_{1} g_{\rho_{L/R}} \sim g_{\rho_{L/R}}$$

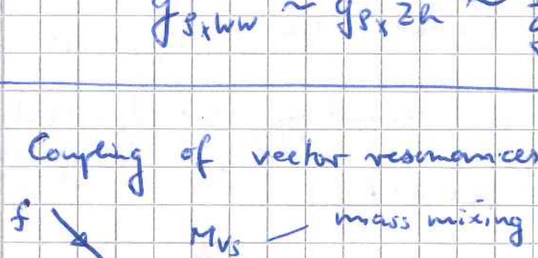
Sizable coupling, not surprising, is an all-composite resonances coupling

One Scale One Coupling Power Counting: $\frac{m_{\rho_{L/R}}}{g_{\rho_{L/R}}} \sim f$.

Singlets have very suppressed couplings (only after EWSB)

$$g_{\rho_X WW} \sim g_{\rho_X Zh} \sim \frac{g'^2}{g_{\rho_X}} \xi$$

Coupling of vector resonances to light SM fermions through mixing:



$SU(2)_L$ triplet: (mixing with W) $g_{\rho_L f_L f_L} \simeq c_L \frac{g^2}{g_{\rho_L}}$

$SU(2)_R$ triplet ρ_R, singlet ρ_X: (mixing with hypercharge boson) $g_{\rho_R^0 ff} \simeq c_Y \frac{g'^2}{g_{\rho_X}}$

Heavy gluons: $g_{\rho_g ff} \sim \frac{g_s^2}{g_{\rho_g}}$

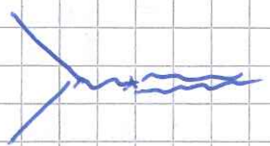
$$g_{\rho_X ff} \simeq c_Y \frac{g'^2}{g_{\rho_X}}$$

$$g_{\rho_R^\pm ff} \sim \frac{g^2}{g_{\rho_R}} \cdot \xi$$

(charged $\rho_R^\pm$ couplings only after EWSB)

Main LHC production processes.

Vector resonances are produced via Drell Yan (DY) and Vector-boson fusion (VBF)

additional α_{EW}^2  VBF  DY.

enhanced for large g_ρ suppressed for large g_ρ.

Composite vectors predominantly decay into heavy fermions, usually relatively broad.

Main decay channels of composite vector resonances

	Gauge	Light SM	3rd gen.	Heavy-light	Top partners
ρ_L^0	WW, Zh	$l^+l^-, u\bar{u}$	$t\bar{t}, b\bar{b}$	$T\bar{t}, B\bar{b}$	$X_{5/3}\bar{X}_{5/3}, X_{2/3}\bar{X}_{2/3}, (T\bar{T}, B\bar{B})$
$\rho_L^\pm$	WZ, Wh	$l\nu, u\bar{d}$	$t\bar{b}$	$B\bar{t}, T\bar{b}$	$X_{5/3}\bar{X}_{2/3}, T\bar{B}$
ρ_R^0	WW, Zh	$l^+l^-, u\bar{u}$	$t\bar{t}, b\bar{b}$	$T\bar{t}, B\bar{b}$	$X_{5/3}\bar{X}_{5/3}, X_{2/3}\bar{X}_{2/3}, (T\bar{T}, B\bar{B})$
$\rho_R^\pm$	WZ, Wh	—	—	$X_{5/3}\bar{t}, X_{2/3}\bar{b}$	$X_{5/3}\bar{T}, X_{2/3}\bar{B}$
ρ_X	—	$l\bar{l}, u\bar{u}$	$t\bar{t}, b\bar{b}$	$T\bar{t}, B\bar{b}, \tilde{T}\bar{t}$	$X_{5/3}\bar{X}_{5/3}, X_{2/3}\bar{X}_{2/3}, \tilde{T}\bar{\tilde{T}}, (T\bar{T}, B\bar{B})$
ρ_G	—	$u\bar{u}$	$t\bar{t}, b\bar{b}$	$T\bar{t}, B\bar{b}, \tilde{T}\bar{t}$	$X_{5/3}\bar{X}_{5/3}, X_{2/3}\bar{X}_{2/3}, \tilde{T}\bar{\tilde{T}}, (T\bar{T}, B\bar{B})$

~~$q\bar{q}$~~ $u\bar{u} \equiv u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c}$, $u\bar{d} \equiv u\bar{d} + c\bar{s}$

EW Precision Tests

Note: flavor constraints not at all discussed here!

EWPT: EW Precision Tests.

<u>Oblique Corrections</u>: Models are assumed to have the SM light fermions having interactions of the form (possibly after a field redefinition to account for mixing effects):

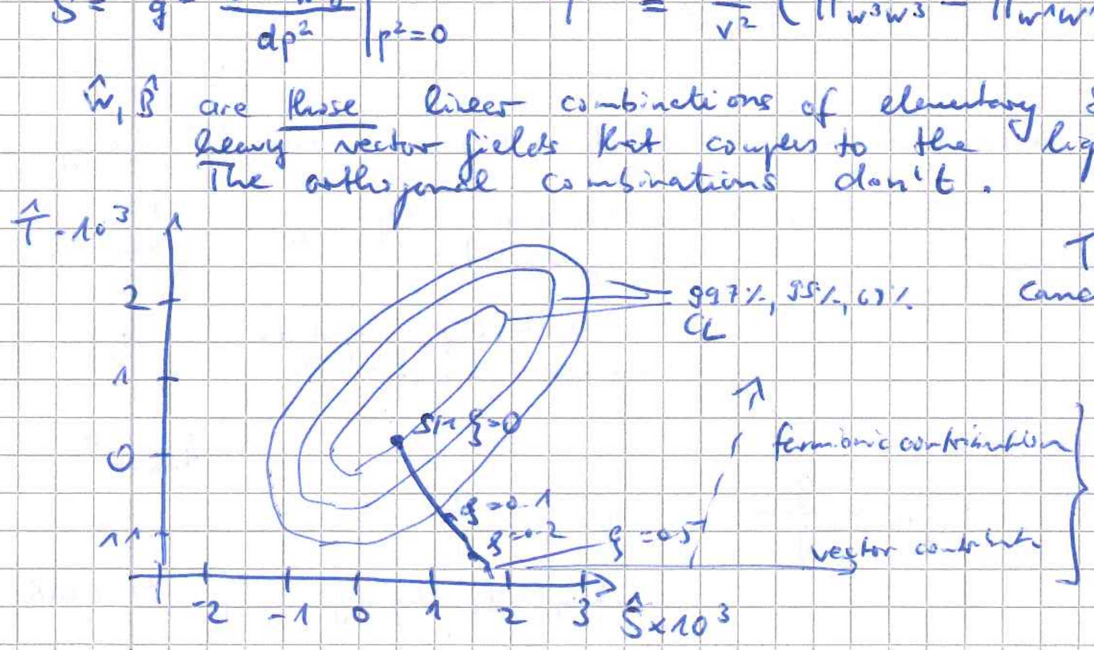
$$\mathcal{L}_{int} = \bar{\Psi}\gamma^\mu\left(T^\alpha \hat{W}_\mu^\alpha + Y\hat{B}_\mu\right)\Psi$$

Hence all the corrections are in the propagators (vacuum polarization diagrams) of $\{\hat{W}^\alpha, \hat{B}\}$.

$V \sim\!\!\otimes\!\!\sim V \quad \Pi_{VV}(p^2)$

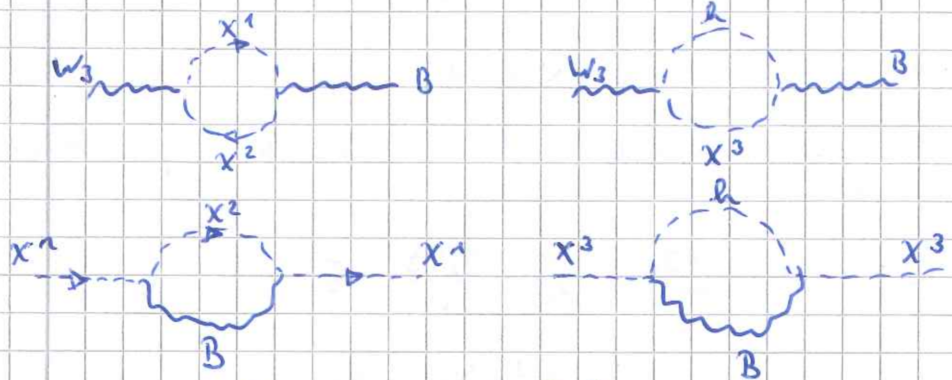
$$\hat{S} = g^2 \left.\frac{d\Pi_{W^3B}}{dp^2}\right|_{p^2=0} \qquad \hat{T} = \frac{4}{v^2}\left(\Pi_{W^3W^3} - \Pi_{W^1W^1}\right)\Big|_{p^2=0}$$

$\hat{W}, \hat{B}$ are <u>those</u> linear combinations of elementary $SU(2)_L \times U(1)_Y$ and heavy vector fields that couples to the light SM fermions. The orthogonal combinations don't.



There can be some cancellation between them.

Relevant contributions to $\hat{S}, \hat{T}$ from Higgs–Goldstone sector:



$\hat{S}$ (transversal fields)

$\hat{T}$ (longitudinal fields)

$$\Delta \hat{S} = \frac{g^2}{192\pi^2}\, \xi \log\left(\frac{m_\rho^2}{m_H^2}\right) \simeq 1.4 \cdot 10^{-3}\, \xi$$

$$\Delta \hat{T} = -\frac{3g'^2}{64\pi^2}\, \xi \log\left(\frac{m_\rho^2}{m_H^2}\right) \simeq -3.8 \cdot 10^{-3}\, \xi$$

Logarithmic divergence cancels in a renormalizable model like the SM.

$\Lambda_{UV}^{\text{cutoff}} \equiv m_\rho \longrightarrow 3\,\text{TeV}$ (numerically set for plots)

$\longrightarrow$ @ 2σ-level $\xi \lesssim 0.05 \mathrel{\hat{=}} f \gtrsim 1.1\,\text{TeV}$ from bounds on $\hat{T}$.

$SO(3)_c$ custodial symmetry forbids a quadratic divergence in $\hat{T}$.

↳ diagonal part of $SU(2)_L \times SU(2)_R$, hence respected by $SO(5)/SO(4)$ MCHM.

This is broken by $U(1)_Y$ gauging ($\propto g'^2$!) and mixing of elementary fermions. The necessity to insert two powers of g' reduces quadratic divergence to logarithmic one.

Vector resonance contributions to $\hat{T}$ lack IR enhancement logarithmic correction and hence are negligible.

$\Delta \hat{S}$:

$$\Delta \hat{S} \sim \frac{m_W^2}{m_\rho^2} \quad \Longrightarrow \quad m_\rho \gtrsim 2\,\text{TeV}$$

Radiative contributions to $\hat{S}$ from composite fermion loops:

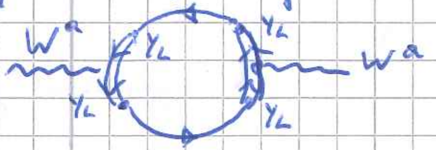
Spurions for custodial $SO(3)_c$ breaking / contributions to $\hat{T}$ parameter:

Field	Composite Operators ($SU(2)_L \times SU(2)_R$)	Spurion	$SO(5)$ reps.	$\hat{T}$ contribution
q_L	$(\underline{2}, \underline{1})$	$y_L \in (\underline{1}, \underline{1})$	$\underline{4}$	—
	$(\underline{2}, \underline{2})$	$y_L \in (\underline{1}, \underline{2})$	$\underline{5}, \underline{10}, \underline{14}$	4 insertions
t_R	$(\underline{1}, \underline{1})$	$y_R \in (\underline{1}, \underline{1})$	$\underline{5}, \underline{14}$	—
	$(\underline{1}, \underline{2})$	$y_R \in (\underline{1}, \underline{2})$	$\underline{4}$	4 insertions
	$(\underline{1}, \underline{3})$	$y_R \in (\underline{1}, \underline{3})$	$\underline{10}$	2 insertions
W_μ^a	$(\underline{3}, \underline{1})$	$g_0 \in (\underline{1}, \underline{1})$	$\underline{10}$	—
B_μ	$(\underline{1}, \underline{3})$	$g_0' \in (\underline{1}, \underline{3})$	$\underline{10}$	2 insertions

$\hat{T}$ parameter finite for 3, 4 or more insertions, logarithmically divergent for 2 insertions.

$\hat{T}$ is ~~...~~ in $(\underline{5}, \underline{1})$ rep. of $SU(2)_L \times SU(2)_R$

Schematic structure of radiative corrections to $\hat{T}$ parameter generated by a composite fermion loop:



$$\Delta \hat{T} \simeq \frac{N_c}{16\pi^2} \frac{y_L^4 f^2}{m^2} \xi$$

m: lightest top partner mass

Top Yukawa:

For $y_L \simeq y_R \simeq Y$, $\quad y_{top} \simeq Y^2 f / m \;\Rightarrow\; \Delta \hat{T} \simeq \frac{N_c}{16\pi^2} y_t^2 \xi \simeq 2 \cdot 10^{-2} \cdot \xi$

Contributions to the $\hat{T}$ parameter from the light composite fermions can (partially) cancel the IR contributions from the vectors.

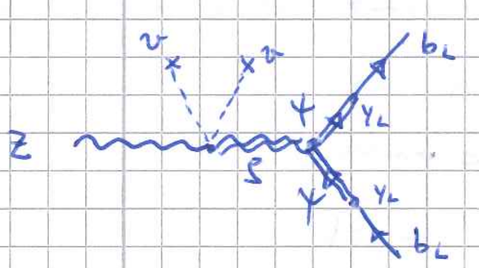
$\boxed{\text{Corrections to the } Zb\bar{b} \text{ coupling}}$

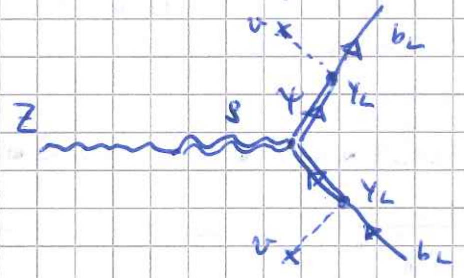
$$\mathcal{L}^Z = \frac{g}{\cos\theta_W} Z_\mu \bar{b} \gamma^\mu \left[(g_L^{SM} + \delta g_L) P_L + (g_R^{SM} + \delta g_R) P_R \right] b.$$

$g_{b_L}^{SM, tree} = -\frac{1}{2} + \frac{1}{3} \sin^2\theta_W$, $\quad g_{b_R}^{SM, tree} = \frac{1}{3} \sin^2\theta_W$

Loop corrections for limit $g \to 0$: $\quad g_{b_L}^{SM, loop} = \frac{m_t^2}{16\pi^2 v^2}$, $\quad g_{b_R}^{SM, loop} = 0$.

$|\delta g_{b_L}| \lesssim 3 \cdot 10^{-3}$ experimentally (LEP)





$$\delta g_{b_L} \sim \frac{y_L^2}{g_\psi} \xi \gtrsim \frac{y_t^2}{g_\psi^2} \xi$$

puts a stringent limit on ξ, especially for small values of $g_\psi = m_\psi / f \leq 2$, i.e. for light fermionic partners.

Bounds are much ~~...~~ relaxed for (additional) custodial symmetries.

P_{LR} parity exchanges $SU(2)_L$ and $SU(2)_R$ operators.

$P_{LR} = \text{diag}(-1, -1, -1, 1)^T$ is an $O(4)$, not $SO(4)$ element.

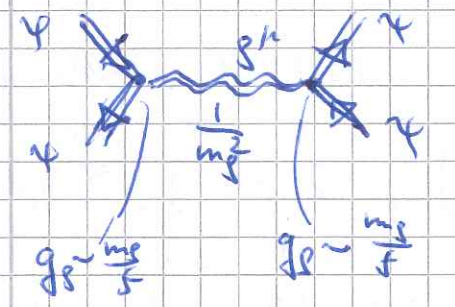
4th component of Higgs (VEV) is invariant under this $\Rightarrow$ Higgs VEV does <u>not</u> break P_{LR}.

P_{LR} invariance means: $\quad T_L = T_R$ and $T_L^3 = T_R^3$ for the ψ.

Typically, $\boxed{\delta g_{b_L} \simeq 2 \cdot 10^{-2} \frac{f}{m_\rho} \cdot \xi}$ tree-level contributions

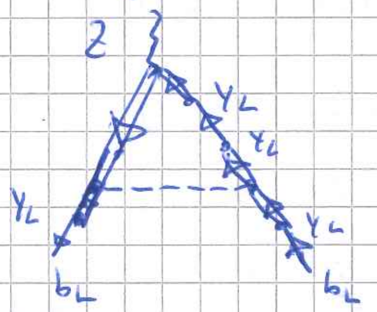
---

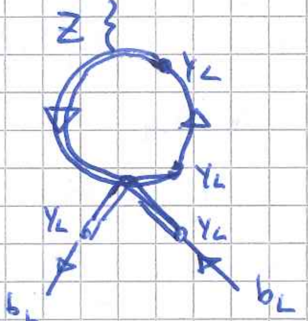
4-fermion- (dim 6) operators: $\mathcal{O}_{4\text{-ferm}} \sim \frac{1}{f^2} (\bar\Psi \gamma^\mu \Psi)(\bar\Psi \gamma_\mu \Psi)$



$\Longrightarrow \frac{1}{f^2}$

Loop contributions to the $Z b_L \bar b_L$ vertex:





$\leftarrow$ logarithmic divergencies can only appear in some of these diagrams.

$$\mathcal{O}_L^{4\text{-ferm}} = \frac{e_L}{f^2} (\bar B_L \gamma^\mu B_L)(\underbrace{\bar J_L \gamma_\mu J_L}_{\text{any } Q = +\frac{2}{3} \text{ states}}) \longrightarrow \text{protected and finite}$$

$$\mathcal{O}_R^{4\text{-ferm}} = \frac{e_R}{f^2} (\bar B_L \gamma^\mu B_L)(\bar J_R \gamma_\mu J_R)$$

$\hookrightarrow$ selection rule violated because J_R can mix via Y_L with q_L doublet.

$\Longrightarrow$ logarithmically divergent contribution

$$\Rightarrow \delta g_{b_L} \simeq \frac{Y_L^2}{16\pi^2} \xi \log\left(\frac{m_\rho^2}{m_\Psi^2}\right) \simeq 2 \cdot 10^{-2} \xi$$

for $m_\rho \simeq 3$ TeV, $m_\Psi \simeq 700$ GeV

In general, loop contributions a bit bigger than the tree-level ones.

In $\underline{5}+\underline{5}$ models: $\Delta \hat T = 3 \delta g_{b_L}$
(also in $\underline{5}+\underline{1}$ models with completely composite t_R)

$\boxed{\text{Anomalous top couplings}}$ $\mathcal{L}^Z = \frac{g}{\cos\theta_W} Z_\mu \bar t \gamma^\mu \left[(g_{t_L}^{SM} + \delta g_{t_L}) P_L + (g_{t_R}^{SM} + \delta g_{t_R}) P_R \right] t$

$g_{t_L}^{SM} = \frac{1}{2} - \frac{2}{3} \sin^2\theta_W$, $g_{t_R}^{SM} = -\frac{2}{3} \sin^2\theta_W$.

Deviations often parametrized as $V_{tb} = 1 - \delta V_{tb}$.

leading correction fulfills: $\boxed{\delta g_{t_L} = -\delta V_{tb}}$

Addendum: Custodial symmetries

$SO(3)_c$: forbids corrections to the $\hat{T}$ parameter (ρ parameter)

P_{LR}: discrete LR exchange parity.

$SO(4)$ global symmetries, Π^i: 4 plet, or a $(2, 2)$-$SU(2)_L \times SU(2)_R$ pseudo-real matrix:

$$\Sigma = \frac{1}{\sqrt{2}} \left(i\sigma_\alpha \Pi^\alpha + \mathbb{1}_{2\times 2} \Pi^4 \right), \quad \Sigma \overset{SU(2)_L \times SU(2)_R}{\longmapsto} g_L \Sigma g_R^\dagger$$

Composite Higgs with $SO(5)/SO(4)$ coset: $SO(4)$ unbroken subgroup
SM: accidental symmetry of Higgs sector

$SO(4)$ broken by gauge fields (gauging) and fermion couplings both in SM and Composite Higgs.

Higgs VEV $\langle \Pi^i \rangle = v\, \delta_{i4}$ $\quad SO(4) \xrightarrow{v} SO(3)$

$$\text{CUSTODIAL SYMMETRY} \Longrightarrow SO(3) \cong SO(3)_c$$

Either $SO(3)_c$ rotation on Goldstones Π^i, or vector symmetry of $SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$, by $g_L = g_R = g_V$

$$\langle \Sigma \rangle = \frac{v}{\sqrt{2}} \mathbb{1}_{2\times 2} \text{ is invariant under } SU(2)_V.$$

Assign W^α fields to $(3, 1)$, hence they preserve $SO(3)_c$

$\Longrightarrow \quad \mathcal{L}_{mass} = \frac{g^2 v^2}{8} W_\mu^\alpha W_\alpha^\mu$ contributes the same to

$\Pi_{W_3 W_3}$ and $\Pi_{W_+ W_-}$ $\Longrightarrow \Delta \hat{T}_{\mathcal{L}_{mass}} = 0$.

Hypercharge gauging corresponds to spurion $\mathcal{G}'$ in $(1, 3)$, namely $W_{R,\mu}^\alpha = \mathcal{G}'^\alpha B_\mu$

These generate (at loop-level) $\mathcal{L}_{mass} = \frac{v^2}{8} \left[(gW^1)^2 + (gW^2)^2 + (gW^3 - g' B)^2 \right]$

P_{LR}: assume $SU(2)_L$, $SU(2)_R$, $U(1)_X$ gauge fields:

$$W_{L\mu} = W_{L,\mu}^\alpha t_L^\alpha = g W_\mu^\alpha t_L^\alpha$$

$$W_{R,\mu} = W_{R,\mu}^\alpha t_R^\alpha = g' B_\mu t_R^3$$

$$X_\mu = \underbrace{g' B_\mu}_{\text{physical values}}.$$

$SU(2)_L \times SU(2)_R \times U(1)_X$ global group formally promoted to a local group.
Only the unbroken (linearly realized) $SO(3)_c \times U(1)_X$ is relevant.

$$\left.\begin{aligned} W_{L,\mu} &\longmapsto g_V \cdot (W_{L,\mu} + i\partial_\mu) \cdot g_V^\dagger \\ W_{R,\mu} &\longmapsto g_V \cdot (W_{R,\mu} + i\partial_\mu) \cdot g_V^\dagger \\ X_\mu &\longmapsto X_\mu + \partial_\mu \alpha_X \end{aligned}\right\} \text{identical transformation laws}$$

P_{LR} acts as $W^{\alpha}_{L,\mu} \leftrightarrow W^{\alpha}_{R,\mu}$, X_μ invariant

$P^{\mu}_{LR} = \mathrm{diag}(-1,-1,-1,+1) \Rightarrow P_{LR}$ enlarges $SO(3)_c \times U(1)_X$ to $O(3)_c \times U(1)_X$

$$c\,\underbrace{(W^3_{L,\mu} - W^3_{R,\mu})}_{P_{LR}\ \text{odd}}\ \underbrace{\bar{\Psi}\gamma^\mu\Psi}_{P_{LR}\ \text{eigenstate}}$$

$\Longrightarrow$ Flavour-violating Z interactions can only involve eigenstates with opposite parity.

$$\begin{matrix} \text{odd} \\ \text{odd} \\ \text{odd} \\ \text{even} \end{matrix} \begin{pmatrix} -ib_L \\ ib_L \\ -it_L \\ t_L \end{pmatrix}$$

embedding in $(2,2)$ fourplet.

$\{-ib_L, ib_L\} \to$ protected

$\{-it_L, t_L\} \to$ not protected.