Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Theoretische Teilchenphysik

SS 08

Prof. Dr. J. Reuter
(Hochhaus/803), 8407

Blatt 10
30.06.08

Präsenz-Aufgabe 17 D-dim Gamma matrices

In order to calculate fermion loops using dimensional regularization, one has to generalize the Dirac matrices to arbitrary space-time dimensions. Let $g^{\mu\nu}$ be $D-$dim, e.g. $\mu, \nu = 0, 1, \ldots, D-1$ with

$$\delta^\mu_\mu \;=\; g^\mu_\mu \;=\; D \quad \Rightarrow \quad \#(\gamma^\mu) \;=\; D.$$

The defining Dirac algebra remains unchanged: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_4$.

Show that

$$\gamma^\mu\gamma_\mu \;=\; D\mathbb{1}_4, \quad \gamma^\mu \not{a} \gamma_\mu \;=\; (2-D)\not{a} \tag{1}$$

Präsenz-Aufgabe 18 Tensor Integrals

A general $N-$point tensor integral is given by

$$\begin{aligned}
&T^N_{\mu_1,\ldots,\mu_M}(p_1,\ldots,p_{N-1},m_0,\ldots,m_{N-1})\\
&= \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq \, \frac{q_{\mu_1}\cdots q_{\mu_M}}{[q^2-m_0^2+i\epsilon][(q+p_1)^2-m_1^2+i\epsilon]\ldots[(q+p_{N-1})^2-m_{N-1}^2+i\epsilon]}\\
&\equiv \langle \frac{q_{\mu_1}\cdots q_{\mu_M}}{[q^2-m_0^2+i\epsilon][(q+p_1)^2-m_1^2+i\epsilon]\ldots[(q+p_{N-1})^2-m_{N-1}^2+i\epsilon]}\rangle_q
\end{aligned}$$

By convention, one denotes $T^1 = A,\ T^2 = B,\ T^3 = C, \ldots$. Looking at their Lorenz-structure one can find the following tensor reduction

$$B^\mu \;=\; p_1^\mu B_1, \tag{2}$$
$$B^{\mu\nu} \;=\; g^{\mu\nu}B_{00} + p_1^\mu p_1^\nu B_{11} \tag{3}$$
$$C^\mu \;=\; p_1^\mu C_1 + p_2^\mu C_2, \tag{4}$$
$$C^{\mu\nu} \;=\; \ldots. \tag{5}$$

a) Contract the relation from eq.(2) with p^μ to express B_μ as linear combination of the **scalar master integrals** A_0, B_0.

b) Similarly, show that

$$B_{00} \;=\; \frac{1}{D-1}\left[A_0(m_1) + 2m_0^2 B_0 + (p^2 - m_1^2 + m_0^2)B_1\right], \tag{6}$$

$$B_{11} \;=\; \frac{1}{(D-1)p^2}\left[(D-2)A_0(m_1) - 2m_0^2 B_0 - D(p^2 - m_1^2 + m_0^2)B_1\right]. \tag{7}$$

Hausaufgabe 12 Scalar Master Integrals

With the expansion of the Γ-function around 0

$$\lim_{z\to 0}\Gamma(z) = \frac{1}{z} - \gamma_E + \mathcal{O}(z)$$

show that

a) the scalar 1-point function is given by

$$A_0(m) := \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\ (q^2 - m^2 + i\varepsilon)^{-1} = m^2\left[\Delta - \log\left(\frac{m^2}{\mu^2}\right) + 1\right] + \mathcal{O}(D-4), \tag{8}$$

where $\Delta = \frac{2}{4-D} - \gamma_E + \log 4\pi$,

b) the scalar 2-point function is given by

$$B_0(p, m_0, m_1) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\ \Big\{\underbrace{(q^2 - m_0^2 + i\varepsilon)}_{=:a}\underbrace{[(q+p)^2 - m_1^2 + i\varepsilon]}_{=:b}\Big\}^{-1} \tag{9}$$

$$= \Delta - \int_0^1 dx\ \log\left[\frac{x^2p^2 - x(p^2 - m_1^2 + m_0^2) + m_0^2 - i\varepsilon}{\mu^2}\right] + \mathcal{O}(D-4) \tag{10}$$

c) the following symmetry exists: $B_0(p^2, m_0, m_1) = B_0(p^2, m_1, m_0)$

d) the special relation

$$B_0(p^2, 0, m) = \Delta - \log\left(\frac{m^2}{\mu^2}\right) + 2 + \frac{m^2 - p^2}{p^2}\log\left(\frac{m^2 - p^2 - i\varepsilon}{m^2}\right), \tag{11}$$

holds.