Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Theoretische Teilchenphysik

SS 08

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Präsenz-Aufgabe 16 Pion decay

Consider the current-current interactions for weak decays proposed by Fermi,

$$\mathcal{L} = -i\frac{G_F}{\sqrt{2}}\mathcal{J}_\mu^{(\pi)}(x)\mathcal{J}^{(l),\mu}(x), \tag{1}$$

with the pion decay matrix element from the lecture:

$$\langle 0|\mathcal{J}_\mu^{(\pi)}(x)|\pi(p)\rangle = iF_\pi \quad p_\mu^{(\pi)}e^{-ip^{(\pi)}x}. \tag{2}$$

a) For the leptonic current coupled to a pion take Fermi's ansatz:

$$\mathcal{J}_\mu^{(i)}(x) = \left(\bar{l}_i\gamma_\mu\left(C_V + C_A\gamma_5\right)\nu_i\right), \tag{3}$$

where i denotes the lepton species. Calculate the decay width of $\pi^- \to e^-\bar{\nu}_e$ and $\pi^- \to \mu^-\bar{\nu}_\mu$.
Does the ratio of the two quantities depend on the values of C_V and C_A? Determine the rates for the physical values of the so-called $(V-A)$-coupling, $C_V = 1$, $C_A = -1$. Use

$$\begin{aligned}
\text{tr}\left[\gamma^5\gamma^{\mu_1}...\gamma^{\mu_{\text{odd}}}\right] &= 0 \\
\text{tr}\left[\gamma^5\right] = 0, \quad \text{tr}\left[\gamma^5\gamma^\mu\gamma^\nu\right] &= 0 \\
\text{tr}\left[\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right] &= -4i\epsilon^{\mu\nu\rho\sigma}.
\end{aligned} \tag{4}$$

b) Assume a scalar current with

$$\mathcal{J}^{(i)}(x) = \left(\bar{l}_i\left(C_V + C_A\gamma_5\right)\nu_i\right). \tag{5}$$

How would the pion decay matrix element look like in this case?
Calculate the decay rate. What changes with respect to the vector current?

c) Discuss the conservation of angular momentum for the pion decay. Use the fact that the anti-neutrino in the standard model is (almost exclusively) right-handed, in order to explain the suppression of the electron decay.

Hausaufgabe 11 Preliminaries for evaluating loop integrals

a) **Feynman parameters**

Show:

$$\frac{1}{ab} = \int_0^1 dx \, \frac{1}{[ax + b(1-x)]^2} = \int_0^1 dx \, dy \, \delta(1-x-y) \frac{1}{[ax+by]^2} \tag{6}$$

$$\frac{1}{ab^n} = \int_0^1 dx \, dy \, \delta(1-x-y) \frac{ny^{n-1}}{[ax+by]^{n+1}} \tag{7}$$

$$\frac{1}{a_1 \dots a_n} = \int_0^1 \left(\prod_{i=1}^n dx_i \right) \, \delta(1 - \sum_{i=1}^n x_i) \frac{(n-1)!}{[a_1 x_1 + \dots + a_n x_n]^n} \tag{8}$$

$$\frac{1}{a_1^{m_1} \dots a_n^{m_n}} = \int_0^1 \left(\prod_{i=1}^n dx_i \right) \, \delta(1 - \sum_{i=1}^n x_i) \frac{\prod_{i=1}^n x_i^{m_i - 1}}{[\sum_{i=1}^n x_i a_i]^{\Sigma m_i}} \frac{\Gamma(m_1 + \dots + m_n)}{\Gamma(m_1) \dots \Gamma(m_n)} \tag{9}$$

hint: Use induction over n in order to prove relation (8).

b) **Beta funciton**

Given the integral representation of the Gamma and Beta function,

$$\Gamma(x) = \int_0^\infty dt \; t^{x-1} e^{-t}, \qquad B(x,y) = \int_0^1 dt \; t^{x-1}(1-t)^{y-1}, \tag{10}$$

show that they obey the following relation:

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}. \tag{11}$$