Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Theoretische Teilchenphysik

SS 08

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Blatt 6
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Präsenz-Aufgabe 10 Nilpotence of the BRS transform

Take the BRS transformations (with the Grassmann-valued parameter λ stripped off):

$$\begin{aligned} sA^a_\mu &= (D_\mu c)^a \\ s\psi &= igc^aT^a\psi \\ sc^a &= -\frac{g}{2}f^{abc}c^bc^c \\ s\bar{c}^a &= B^a. \end{aligned}$$

Show $s^2\phi = 0\ \forall\phi$.

Präsenz-Aufgabe 11 Color algebra

For the color algebra, use:

$$\mathrm{tr}[T^aT^b] = T_F\delta^{ab},$$

with Dynkin index $T_F = 1/2$ for the fundamental representation.

Prove

$$T^bT^aT^b = (C_F - \frac{1}{2}C_A)T^a$$

with the quadratic Casimir operators $C_F = 4/3$, $C_A = 3$ for $SU(3)$.
What gives $\mathrm{tr}[T^aT^bT^aT^b]$ then?

Hausaufgabe 8 QCD amplitudes

With the QED results from the previous exercise sheets at hand, calculate the cross sections for the corresponding QCD processes. Average and sum over initial and final color states, respectively. Employing the above relations as well as crossing symmetry, show (you may use a computer algebra program like `FORM` or `FeynCALC` for `Mathematica`):

a)

$$\begin{aligned} \frac{d\sigma}{dt}(ud\to ud) &= \frac{4\pi\alpha_s^2}{9s^2}\left(\frac{s^2+t^2}{t^2}\right) \\ \frac{d\sigma}{dt}(u\bar{u}\to d\bar{d}) &= \frac{4\pi\alpha_s^2}{9s^2}\left(\frac{u^2+t^2}{s^2}\right) \\ \frac{d\sigma}{dt}(u\bar{u}\to u\bar{u}) &= \frac{4\pi\alpha_s^2}{9s^2}\left(\frac{u^2+t^2}{s^2}+\frac{u^2+s^2}{t^2}-\frac{2}{3}\frac{u^2}{st}\right) \\ \frac{d\sigma}{dt}(uu\to uu) &= ? \end{aligned}$$

b)

$$\frac{d\sigma}{dt}(q\bar{q}\to gg) = \frac{32\pi\alpha_s^2}{27s^2}\left(\frac{u}{t}+\frac{t}{u}-\frac{9}{4}\left(\frac{t^2+u^2}{s^2}\right)\right).$$