Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Theoretische Teilchenphysik

SS 08

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Blatt 6
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Präsenz-Aufgabe 10 Grassmann variables

Use the definition of Berezin for integration over anticommuting variables to show:

a)

$$\int d\bar{\alpha} d\alpha \exp\left[-\bar{\alpha} D\alpha\right] = \det D$$

$$\int d\bar{\alpha} d\alpha \exp\left[-\bar{\alpha} D\alpha + \bar{\xi}\alpha + \bar{\alpha}\xi\right] = \det D \exp\left[\bar{\xi} D^{-1}\xi\right],$$

b) as well as in the functional case:

$$\int d\bar{\theta}(x) d\theta(x) \exp\left[-(\bar{\theta}, D\theta)\right] = \det D$$

$$\int d\bar{\theta}(x) d\theta(x) \exp\left[-(\bar{\theta}, D\theta) + (\bar{\theta}, \xi) + (\bar{\xi}, \theta)\right] = \det D \exp\left[(\bar{\xi}, D^{-1}\xi)\right],$$

where

$$(\bar{\theta}, D\theta) = \int dx\, dy \bar{\theta}(x) D(x,y) \theta(y) \quad (1)$$

$$= \int dx \bar{\theta}(x) D(x) \theta(x), \text{ if } D(x,y) = D(x)\delta^{(4)}(x-y) \quad (2)$$

$$(\bar{\xi}, \theta) = \int dx \bar{\xi}(x) \theta(x). \quad (3)$$

Präsenz-Aufgabe 11 Gauge boson propagator

Prove that the propagator of a gauge boson in R_ξ-gauge is given by

$$D_F(k) = \frac{-i\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right)}{k^2 + i\epsilon} \quad (4)$$

Präsenz-Aufgabe 12 Bhabha scattering using computer algebra techniques

Calculate the cross section for $e^+e^- \to e^+e^-$, not neglecting the electron mass at first. Use the computer algebra program `FORM` by setting up a program `bhabha.frm` . The program and the manual can be found at `http://nikhef.nl/~form`.

Use the declarations:

```
Vectors p1,p2,k1,k2;
Indices m1,m2,m3,n1,n2,n3;
Symbols emass,s,t,u;
```

Then write the string of γ matrices as

```
Local  Msqt =
*      electron line in pure t-channel
       ((g_(1, k1) + emass) * g_(1,m1) * ( g_(1,p1) + emass) * g_(1,n1)
*      positron line in pure t-channel
       *(g_(2,k2) - emass) * g_(2,n1) * (g_(2,p2) - emass) * g_(2,m1))/4
       ;
```

for the spin-averaged and summed square of the t-channel. Run the program with `form bhabha.frm`.
Perform the traces, contractions and simplifications with

```
trace4,1;
  ...
contract;
id p1.p2 = s/2 - emass^2;
  ...
.sort
```

The output is produced shortly before the end statement:

```
print +s
.end;
```

You should obtain

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\Bigg\{ & \frac{8m_e^4 + 4m_e^2(s-t-u) + t^2 + u^2}{(s-m_e^2)^2} \\ & + \frac{8m_e^4 + 4m_e^2(t-s-u) + s^2 + u^2}{(t-m_e^2)^2} \\ & + \frac{8m_e^4 + 4m_e^2(t+s-3u) + u^2}{(s-m_e^2)(t-m_e^2)}\Bigg\}. \end{aligned} \tag{5}$$

Hausaufgabe 8 Bhabha scattering continued

Show that the Bhabha cross section in the limit of massless electrons is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left(u^2\left(\frac{1}{t}+\frac{1}{s}\right)^2 + \frac{t^2}{s^2} + \frac{s^2}{t^2}\right). \tag{6}$$

Insert back $\cos\theta$ and plot the differential cross section. What feature of the diagrams causes the differential cross section to diverge for $\theta \to 0$?

Derive the formula for $\frac{d\sigma}{dt}$.