Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Theoretische Teilchenphysik SS 08

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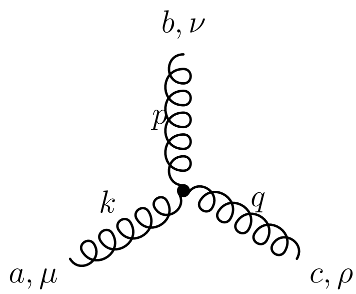
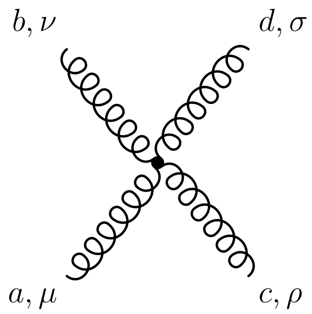
Blatt 3
05.05.08

Präsenz-Aufgabe 4 Non-Abelian Gauge Theories



$$= g f^{abc} \left[g^{\mu\nu} (k-p)^\rho + g^{\nu\rho} (p-q)^\mu + g^{\rho\mu} (q-k)^\nu \right]$$



$$= -ig^2 \left[f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \right]$$

Derive from the Yang-Mills Lagrangian the 3- and 4-gauge boson vertices to yield the above Feynman Rules (where p, k, q denote the incoming momenta). To correctly derive the symmetry factors rewrite the YM Lagrangian in the following form (this notation will be justified in the lecture):

$$\mathcal{L} = \mathcal{L}_{\text{quadratic}} + A^a_\mu A^b_\nu A^c_\rho \left(\frac{\delta^3}{\delta A^a_\mu \delta A^b_\nu \delta A^c_\rho} \mathcal{L} \Big|_{A=0} \right) + A^a_\mu A^b_\nu A^c_\rho A^d_\sigma \left(\frac{\delta^4}{\delta A^a_\mu \delta A^b_\nu \delta A^c_\rho \delta A^d_\sigma} \mathcal{L} \Big|_{A=0} \right) . \tag{1}$$

The coefficients of the cubic and quartic terms are the corresponding factors in the Feynman rules for the triple and quadruple gauge boson vertices.

Präsenz-Aufgabe 5 Casimir Operator

Show that the Casimir operator $\sum_a T^a T^a$ commutes with all generators T^b of the Lie group.

Hausaufgabe 4 Jacobi Identity

a) Prove the Jacobi identity

$$[T^a, [T^b, T^c]] + [T^b, [T^c, T^a]] + [T^c, [T^a, T^b]] = 0 \tag{2}$$

for constant matrices T^a.

b) Why does this hold for the covariant derivatives $\mathcal{D}_\mu, \mathcal{D}_\nu$ and $\mathcal{D}_\rho$ as well?

c) Show that this implies the so-called Bianchi identity

$$\epsilon^{\alpha\beta\gamma\delta}\mathcal{D}_\beta F_{\gamma\delta}. \tag{3}$$

d) With the definitions $F_{0i} = E_i$ and $F_{ij} = -\epsilon_{ijk}B_k$, what is the Bianchi identity for electrodynamics?

Hausaufgabe 5 Structure Constants, Adjoint Representation

a) Prove the Jacobi identity for the structure constants

$$f^{ade}f^{bcd} + f^{bde}f^{cad} + f^{cde}f^{abd} = 0. \tag{4}$$

b) Show that this is equivalent to the defining relation for the adjoint representation.