Physikalisches Institut, Albert-Ludwigs-Universität Freiburg

Übungen zur Vorlesung

Theoretische Teilchenphysik

SS 08

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Blatt 2
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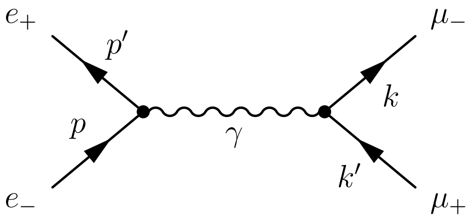
Präsenz-Aufgabe 3 QED: $e^+ \, e^- \; \to \; \mu^+ \, \mu^-$



Calculate the cross section for the above process at tree-level using the formula from the last tutorial

$$\frac{d\sigma}{d\Omega_{\text{CMS}}} = \frac{|\vec{k}_{\text{CMS}}|}{|\vec{p}_{\text{CMS}}|} \frac{1}{64\pi^2 s} |\mathcal{M}|^2 \tag{1}$$

and following the steps outlined below:

a) Derive the matrix element $\mathcal{M}$ using the Feynman rules given in the lecture.

b) The cross section is proportional to the absolut square of $\mathcal{M}$. Employ the following identities for the γ matrices

$$\gamma^{0\dagger} = \gamma^0, \; \gamma^{i\dagger} = -\gamma^i \text{ and } \{\gamma^\nu, \gamma^\mu\} = 2g^{\nu\mu} \tag{2}$$

in order to obtain

$$|\mathcal{M}|^2 = \frac{e^4}{q^4} \left(\bar{v}(p')\gamma^\mu u(p) \, \bar{u}(p)\gamma^\nu v(p')\right) \left(\bar{u}(k)\gamma_\mu v(k') \, \bar{v}(k')\gamma_\nu u(k)\right). \tag{3}$$

c) Average $|\mathcal{M}|^2$ over spins in the initial state and sum over spins in the final state using the following completeness relations:

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \; \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m. \tag{4}$$

What experimental setup does this correspond to?
The expression from eq.(3) should simplify to traces of products of γ matrices:

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4q^4} \text{tr}\left[(\not{p}' - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\right] \text{tr}\left[(\not{k} + m_\mu)\gamma_\mu(\not{k}' - m_\mu)\gamma_\nu\right]. \tag{5}$$

d) Evaluating the traces:
Assume the existance of a matrix γ^5 with the following properties:

$$(\gamma^5)^2 = 1 \text{ and } \{\gamma^\mu, \gamma^5\} = 0. \tag{6}$$

Show that the trace over an odd product of γ-matrices vanishes.
Furthermore, use eq.(2) to show that

$$\text{tr}[\gamma^\mu\gamma^\nu] = 4g^{\mu\nu}, \;\; \text{tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}). \tag{7}$$

Evaluate the electron and muon trace from eq.(5).

e) Now, neglect the electron mass and multiply the electron- with the muon-trace to obtain

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8e^4}{q^4} \left[(pk)(p'k') + (pk')(p'k) + m_\mu^2(pp')\right]. \tag{8}$$

f) To obtain a more explicit formula one has to choose a particular frame of reference - calculate $\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$ in the CMS with $\vec{p}, \vec{p}'$ along the z-axis and plug it into the formula for the differential cross section (eq.(1)):

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\sqrt{1 - \frac{4m_\mu^2}{s}}\left[\left(1 + \frac{4m_\mu^2}{s}\right) + \left(1 - \frac{4m_\mu^2}{s}\right)\cos^2\theta\right]. \tag{9}$$

g) Integrate over the solid angle to obtain the total cross section. Take the limit $m_\mu \to 0$. What is the problem with very high center of mass energies?

Hausaufgabe 3 QED: $e^- \mu^- \to e^- \mu^-$

Calculate the total cross section for the above process following the steps used in Präsenzübung 3 closely.