Unification without Doublet-Triplet Splitting — SUSY Exotics at the LHC

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W. Kilian, J. Reuter, PLB B642 (2006), 81, and work in progress (with F. Deppisch)

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The Standard Model (SM) – Theorist’s View

Renormalizable Quantum Field Theory (only with Higgs!) based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ non-simple gauge group

Reducible representation:

$$
q\left(3, 2\right)_{\frac{1}{3}} \oplus \ell\left(1, 2\right)_{-1} \\
\oplus u^c\left(3, 1\right)_{-\frac{4}{3}} \oplus d^c\left(3, 1\right)_{\frac{2}{3}} \\
\oplus e^c\left(1, 1\right) \oplus H\left(1, 2\right)_{1}
$$

Incompleteness

- Electroweak Symmetry Breaking
- Higgs boson
- Origin of neutrino masses
- Dark Matter: $m_{DM} \sim 100$ GeV

Theoretical Dissatisfaction

- 28 free parameters
- “strange” fractional $U(1)$ quantum numbers
- Hierarchy problem
Conventional and MSSM Unification

1971–74 Supersymmetry: consistent extrapolation to high scales

⇒ unification quantitatively testable
  (assuming a given spectrum)

⇒ two Higgs doublets $H^u, H^d$

⇒ superpartners for all SM particles,
  presumably in the TeV range

Bottom-Up Approach: just MSSM

1973 Unification of leptons and quarks by Pati/Salam:

$$G_{SM} \subset G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R \times Z_2$$

Each matter family in irreducible rep. (incl. $\nu_R$, 2nd Higgs doublet):

1974 Unification of gauge couplings by Georgi/Glashow:

$$G_{SM} \subset G_{GG} = SU(5)$$

Matter representation for $SU(5)$ is reducible (classically)
Simple-group unification: partial unification of leptons/quarks
The prime example: (SUSY) $SU(5)$

$SU(5) \rightarrow SU(3)_c \times SU(2)_w \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$

$SU(5)$ has $5^2 - 1 = 24$ generators:

$$24 \rightarrow (8, 1)_0 \oplus (1, 3)_0 \oplus (1, 1)_0 \oplus (3, 2)_{\frac{5}{3}} \oplus (\overline{3}, 2)_{-\frac{5}{3}}$$

$$G_\alpha^\beta \oplus W \oplus B \oplus X, Y \oplus \overline{X}, \overline{Y}$$

$$g A^a \frac{\lambda^a}{2} = \frac{g}{\sqrt{2}} \left( \sqrt{2} G^a \frac{\lambda^a_{GM}}{2} \begin{pmatrix} (\bar{X}, \bar{Y}) \\ (X, Y)^T \end{pmatrix} \sqrt{2} W^a \frac{\sigma}{2} \right) - \frac{g}{2\sqrt{15}} B \text{diag}(-2, -2, -2, 3, 3)$$

$SU(5)$ breaking: Higgs $\Sigma$ in adjoint 24 rep.

$$\langle \Sigma \rangle = w \times \text{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2}) \quad M_X = M_Y = \frac{5}{2\sqrt{2}} g w$$

other breaking mechanisms possible (e.g. orbifold)
Quantum numbers

- Hypercharge: \[ \frac{\lambda_{12}}{2} = \sqrt{\frac{3}{5}} \frac{Y}{2} \quad Y = \frac{1}{3} \text{diag}(-2, -2, 3, 3, 3) \]
  Quantized hypercharges are fixed by non-Abelian generator

- Weak Isospin: \[ T_{1,2,3} = \lambda_{9,10,11}/2 \]

- Electric Charge: \[ Q = T^3 + Y/2 = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0) \]

- Prediction for the weak mixing angle (with RGE running):
  \[ \alpha^{-1}(M_Z) = 128.91(2), \alpha_s(M_Z) = 0.1176(20), s_w^2(M_Z) = 0.2312(3) \]

  non-SUSY: \[ s_w^2(M_Z) = \frac{23}{134} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)} \frac{109}{201} \approx 0.207 \]

  SUSY: \[ s_w^2(M_Z) = \frac{15}{134} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)} \frac{109}{201} \approx 0.207 \]
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  \]

  \[
  \text{SUSY: } s_w^2(M_Z) = \frac{1}{5} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)} \frac{7}{15} \approx 0.231
  \]

New Gauge Bosons

Two colored EW doublets:
\( (X, Y), (\bar{X}, \bar{Y}) \) with charges \( \pm \frac{4}{3}, \pm \frac{1}{3} \)
Fermions (Matter Superfields)

The only possible way to group together the matter:

\[
\bar{5} = \begin{pmatrix} d^c \\ d^c \\ d^c \\ \ell \\ -\nu_\ell \end{pmatrix}, \quad 10 = \begin{array}{c} 1 \end{array} \begin{pmatrix} \begin{pmatrix} 0 & u^c & -u^c \\ -u^c & 0 & u^c \\ u^c & -u^c & 0 \end{pmatrix} & \begin{pmatrix} -u & -d \\ -u & -d \\ -u & -d \end{pmatrix} \\ \\ u & u & u \\ d & d & d \\ 0 & -e^c \\ e^c & 0 \end{pmatrix}
\]

\[
\bar{5} = (\bar{3}, 1)_{\frac{2}{3}} \oplus (1, 2)_{-1}, \quad 10 = (3, 2)_{\frac{1}{3}} \oplus (\bar{3}, 1)_{-\frac{4}{3}} \oplus (1, 1)_2
\]

Remarks

- \(2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad (5 \otimes 5)_a = 10, \quad (3 \otimes 3)_a = \bar{3}, \quad (\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 1 \end{pmatrix})_a = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\)
- Quarks and leptons in the same multiplet
- Fractional charges from tracelessness condition (color!)
- \(\bar{5}\) and 10 have equal and opposite anomalies
- \(\nu^c\) must be \(SU(5)\) singlet
The Doublet-Triplet Splitting Problem

MSSM Higgses included in $5_H \oplus \bar{5}_H$

$$5_H = (3, 1)_{-\frac{2}{3}} \oplus (1, 2)_1 : \begin{pmatrix} D \\ H_u \end{pmatrix} \quad \bar{5}_H = (\bar{3}, 1)_{\frac{2}{3}} \oplus (1, 2)_{-1} : \begin{pmatrix} D^c \\ \epsilon H_d \end{pmatrix}$$

$D, D^c$ colored triplet Higgses with charges $\pm \frac{1}{3}$ (EW singlet)

colored Dirac fermion $\tilde{D}$ with charge $-1/3$ (EW singlet)

Unification requires omitting colored part of SU(5) Higgs $5_H, \bar{5}_H$

- **Doublet-triplet splitting problem** \( (m_H \sim 100 \text{ GeV}, m_D \sim 10^{16} \text{ GeV}) \)

*Welcome*, since $SU(5)$-symmetric Higgs interactions would read

$$\bar{5} \ 10 \ 5_H = \ell H_d e^c + q \epsilon H_d d^c + q \epsilon \ell D^c + d^c u^c D^c$$

$$10 \ 5_H \ 10 = q \epsilon H_u u^c + D u^c e^c + D q \epsilon q$$

Generating SM masses $\Rightarrow$ leptoquark *and* diquark coupl. for $D, D^c$

$\Rightarrow$ triggers rapid proton decay
Interactions

Leptoquark couplings
(and SUSY vertices)

Diquark couplings
(and SUSY vertices)

Vector bosons induce e.g. decay $p \rightarrow e^+ \pi^0$
Doublet-Triplet Splitting

Possible scenarios:

1. Colored singlets are heavy (GUT scale) = doublet-triplet splitting
   - enables exact unification near $10^{16}$ GeV and excludes rapid proton decay
   - Proton decay may still be too fast (depending on the superpotential)
   - Doublet-triplet splitting is not trivially available

2. Colored singlets are light (TeV scale)
   - Simple unification no longer happens near $10^{16}$ GeV, nor elsewhere

   ▶ Proton-decay coupl. must be excluded: consistent with GUT symmetry?
Further MSSM Issues

Even if doublet-triplet splitting is accepted, the MSSM Higgs sector appears ad-hoc:

- **µ problem**
  - µ-term $\mu H_u H_d$ is supersymmetric, in principle not related to soft-SUSY-breaking Lagrangian:
  
  Why is it $O(100 \text{ GeV})$, not $O(10^{16} \text{ GeV})$?
  
  $\Rightarrow$ Possible extension as a solution: singlet Higgs $S$ with superpotential

  $$
  \lambda S H_u H_d \rightarrow \lambda \langle S \rangle H_u H_d = \mu H_u H_d
  $$

  (does not change the unification prediction)

  $\Rightarrow$ NMSSM, where $\langle S \rangle$ should be somehow related to soft-breaking Lagrangian
  
  How?
Radiative Symmetry Breaking

MSSM with cutoff $\Lambda \sim 10^{16}$ GeV: major contribution to Higgs potential comes through Coleman-Weinberg mechanism:

\[ \begin{array}{c}
\text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad = \quad \text{\textbullet} \quad \text{\textbullet} \\
\text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet}
\end{array} \]

$\Rightarrow$ Large top Yukawa coupl. drives effective $H_u$ mass squared negative:

\[
m_{\text{eff}}^2 = (m_{H,\text{soft}}^2 + \mu^2) + (\Lambda^2 \cdot 0) + m_{t,\text{soft}}^2 \frac{\lambda^2}{16\pi^2} \ln \frac{m_{t,\text{soft}}^2}{\Lambda^2}
\]

Such a mechanism may also be responsible for a $S$ vev in the NMSSM

- requires the existence of a vectorlike pair of chiral superfields
  - for instance, $D$ and $D^c$ (colored) with coupling $SDD^c$
  - ... as required by $SU(5)$, if $SH_uH_d$ is present
    ....... would simultaneously give a Dirac mass to $D$.

- Without tree-level quartic coupling, the CW mechanism implies $\langle S' \rangle \sim 4\pi m_{\text{soft}}$, so $\langle S' \rangle \gg \langle H \rangle$. 
Further MSSM Issues

Even if doublet-triplet splitting is accepted, the MSSM Higgs sector appears ad-hoc:

- Why is there only one family of Higgs matter? Neither $SU(5)$, nor $G_{PS}$ (nor $SO(10)$) does unify Higgs fields with SM matter...

“This does not necessarily mean that this is the Higgs mass!”
Higgs-Matter Unification

1976: Trinification: Treat all interactions equally

\[ G_{\text{Tri}} = SU(3)_c \times SU(3)_L \times SU(3)_R \times Z_3 \]

Multiplets:

\[
L(1, 3, \bar{3}) = \begin{pmatrix}
H_u^+ & H_d^0 & \nu_L \\
H_u^0 & H_d^- & e_L \\
e_R^c & v_R^c & S
\end{pmatrix}
\]

\[
Q_R(\bar{3}, 1, 3) = \begin{pmatrix}
\bar{u}_R^c & \bar{d}_R^c & \bar{D}_R^c
\end{pmatrix}
\]

1976: \( E_6 \) as superset of trinification (and \( SO(10) \))

with additional gauge bosons \( X(3, 3, 3) \) and \( \bar{X}(\bar{3}, \bar{3}, \bar{3}) \Rightarrow 78 \)

⇒ irreducible multiplet (27) unifies all matter, Higgs, colored and neutral singlets (within each family)

⇒ contains NMSSM, allows for radiative symmetry breaking in both singlet and doublet sectors
Higgs-Matter Unification

Complete $G_{\text{Tri}}$ or $E_6$ multiplet: no unification

Possible scenarios:

1. Omit one bi-triplet $D, D^c$ family $\Rightarrow$ doublet-triplet splitting
2. Add one extra MSSM Higgs family $\Rightarrow$ ESSM (S. King et al.)
3. Different unification pattern
Running With Triplets

Bottom-up approach: MSSM with one generation of triplets

$10^{15}$ GeV: crossing of $SU(2)_L$ and $U(1)_Y$
$\Rightarrow$ unification to LR symmetry $SU(2)_L \times SU(2)_R$, requires $\nu^c_R$
$SU(3)_c$ crosses at $10^{21}$ GeV: too high
Running With Triplets

Bottom-up approach: MSSM with one generation of triplets

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⇒ unification to LR symmetry $SU(2)_L \times SU(2)_R$, requires $\nu^c_R$

$SU(3)_C$ crosses at $10^{21}$ GeV: too high

⇒ extend to $SU(4)_C$: unification possible at $10^{18}$ GeV
Running With Triplets

Complete Model:

- Full SUSY $E_6/G_{Tri}$ matter spectrum above $10^3$ GeV, except $\nu_c$

- PS symmetry with $\nu_R$ above $10^{15}$ GeV

$$Q_L = (Q, L) = (4, 2, 1) \quad D = (D, D^c) = (6, 1, 1)$$

$$Q_R = ((u^c, d^c), (\nu^c, \ell^c)) = (\bar{4}, 1, 2) \quad S = (1, 1, 1)$$

$$H = (H_u, H_d) = (1, 2, 2)$$

- $E_6$ symmetry (and possibly extra fields) at $10^{18}$ GeV
Flavor Symmetry

Proton decay?

- Once triplets are included, a PS-symmetric superpotential contains leptoquark and diquark couplings simultaneously:
  \[ DQ_R Q_R = \epsilon_{\alpha\beta\gamma} \epsilon_{jk} D_\alpha (Q_R)_\beta (Q_R)_\gamma k \]

Possible solution: extra flavor symmetry \( SU(3)_F \) (or \( SO(3)_F \))

\[ \Rightarrow D \text{ diquark coupling with } SU(2)_R, SU(3)_c, SU(3)_F: \]

\[ DQ_R Q_R = \epsilon^{abc} \epsilon_{\alpha\beta\gamma} \epsilon_{jk} D^a_\alpha (Q_R)^b_\beta (Q_R)^c_\gamma k \]

Vanishes due to total antisymmetry \( \Rightarrow \) no proton decay

Analogous for \( \epsilon^{abc} \epsilon_{\alpha\beta\gamma} \epsilon_{jk} (D^c)^a_\alpha (Q_L)^b_\beta (Q_L)^c_\gamma k \)

- Leptoquark coupling of \( D \) not affected

Eff. superpotential from (spontan.) breaking of LR and/or flavor symm.:

- Exclude spurions \( \propto \epsilon_{\alpha\beta\gamma} \) (color space) \( \Rightarrow \) diquark couplings absent

- Integrating out heavy fields: baryon number as low-energy symmetry, flavor symmetry not
Sample Implementation

**Toy Model** (no dynamics!)

Extend $E_6 \times SU(3)_F$ to $E_8$

...by implementing $N = 2$ supersymmetry:

- We have: matter $27_3$ and gauge $78_1 + 1_8$.
- Add: mirror matter $\overline{27}_3$
- Supersymmetrize by adding *matter* $78_1 + 1_8$ and *gauge* $27_3 + \overline{27}_3$.

Decomposition of reps. in $E_8 \rightarrow E_6 \times SU(3)_F$:

$$248 = 27_3 \oplus \overline{27}_3 \oplus 78_1 \oplus 1_8$$

Result: matter 248 and gauge 248 (fundamental = adjoint)
Sample Implementation

Top-down

1. Somewhat below $M_{\text{Planck}}$
   - $N = 2 \rightarrow N = 1$ breaking removes mirror matter, leaving $E_6$ zero mode of chiral matter $\mathbf{27}_3$, maybe adjoint matter $\mathbf{78}_1$ and $\mathbf{1}_8$
   - Flavor $SU(3)$ on the zero modes (would be anomalous) is broken by colorless spurions, e.g., condensate $\langle \mathbf{1}_8 \rangle$.
   - $E_6$ is broken to $G_{\text{PS}}$ by colorless spurions, e.g., bilinear = Higgs ’$\mu$ term’ $\langle \bar{H}_u H_d \rangle$ in the $\mathbf{27}_3$ mirror representation
   - Additional allowed spurion = Singlet $\langle \mathbf{1}_1, \mathbf{1} \rangle = \langle \bar{S} \rangle$ (3. gen.)

Note: all spurions so far break flavor as well

Result:

- PS symmetry
- all MSSM superpotential terms allowed, but subject to PS symmetry and flavor constraints (no quark mixing)
- Flavor dynamics in higher-dim. superpotential due to $\mathbf{1}_8$ matter exchange
Sample Implementation

2. At $10^{15}$ GeV

Condensate in adjoint matter representation: $\langle 78_1 \rangle = \langle W_{23}^R \rangle$
+ higher-dimensional terms $\langle 27 78 27 \rangle^2$

$\langle W_{23}^R \rangle \times \langle W_{32}^R \rangle$

$\langle S \rangle \times \langle S \rangle$

$\Rightarrow \nu_R$ Majorana mass

$\Rightarrow$ PS symmetry broken to SM

$\Rightarrow$ Leptoquark couplings possible for $D, D^c$, but no diquark couplings
Sample Implementation

3. At $10^3$ GeV

Soft-breaking terms (hidden sector) induce radiative symmetry breaking $\langle S \rangle$ via $D/D^c$ loops

$\Rightarrow \mu_D$-term $D^c \langle S \rangle D$ (Dirac masses)

$\Rightarrow \mu_H$-term $H_u \langle S \rangle H_d$

$\Rightarrow Z'$ mass if the extra $U(1)$ broken by $\langle S \rangle$ was gauged

\[ \ldots \text{with flavor mixing} \]

4. At $10^2$ GeV

Soft-breaking + effective $\mu$-term induce radiative symmetry breaking $\langle H_u \rangle$ via $t/t^c$ loops

$\Rightarrow \langle H_d \rangle$ due to Higgs superpotential + soft-breaking terms

$\Rightarrow$ Dirac masses for all charged MSSM matter

$\Rightarrow$ Majorana masses (see-saw) for $\nu_L$

\[ \ldots \text{again, with flavor mixing} \]
Dark Matter

MSSM Higgses: $H_u^f, H_d^f$ with $f = 1, 2, 3$

* VEV selects single direction (taken as $f = 3$) in family space
  ⇒ 1 gen. MSSM Higgses, 2 gen. “unhiggses”

(2 bi-doublets = 8 charged and 8 neutral scalars + fermion superpartners)

In gauge interactions, unhiggses are pair-produced, thus suppressed in precision data, ... ... but also Yukawa interactions

1) FCNC  
2) resonant single production in $q\bar{q}$ or $e^+e^-$ annihilation

Unhiggses very heavy or artificially aligned or suppressed

⇒ (approximate?) $H$ parity: odd for unhiggses, even otherwise

And why not? Flavor symmetry removes the need for $R$ parity anyway.

If $H$ parity is exact:

▶ lightest unhiggs: $H$ parity protected dark matter
▶ Pair production of unhiggses/unhiggsinos, cascade decays

... and $R$ parity is exact:

▶ dark matter mix: interesting relic abundance
  (relaxes all neutralino bounds!)
A little bit of Pheno

Next step: Provide a viable low-energy spectrum

At LHC:

1) $1 - 3$ pairs of scalar leptoquarks $D_L, D_R$.
   - probably heavy $\gtrsim 1$ TeV (but hierarchy is possible)
   - pair-produced in $gg$ fusion at LHC
   - decay into $\ell u$ and $\nu d$:
     - generation-diagonal, or just third-generation: $\tau t$ and $\nu b$ or
     - generation-crossed (flavor symmetry!): $ec, eb, \mu d, te, t\mu \ldots$
     
     $gq \rightarrow D\ell$ production enhanced
     - or, if $R$-parity is violated, may mix with down-type squarks.

2) $1 - 3$ fermionic leptoquarkinos $\tilde{D}$
   - are probably heavy as well, but somewhat lighter than scalars
     (because $m^2 = \lambda \langle S \rangle^2 + m_{\text{soft}}^2$)
   - are also pair produced (maybe singly if $R$-parity is violated)
   - decay into $\tilde{\ell} j$, or $\ell \tilde{q}$, or $\nu \tilde{q}$
     - rich signatures!
     - spin measurement distinguishes from ordinary squarks
A little bit of Pheno

3) (non)"standard" MSSM Higgs
   ▶ Relaxed Higgs bounds (like in NMSSM)
   ▶ Possibly large invisible decay ratio ($\tilde{\chi}^0, a$)

4) 2 – 4 doublets of unhiggeses
   ▶ probably only pair-produced: Drell-Yan, maybe Higgs decays (singlets involved)
   ▶ missing-energy signatures, unique identification could be difficult: ILC?

5) 1 – 3 singlet scalars + pseudoscalars
   ▶ masses, properties?

6) and all associated neutralinos ($\leq 11$) and charginos ($\leq 4$)
   ▶ large and complicated chargino/neutralino mixing matrices. Decay chains at LHC become difficult to understand.

7) Either heavy $Z'$ (gauged NMSSM) or light pseudo-axion(s) $\eta$
   corresponding to extra $U(1)$

Conclusion: LHC phenomenology rich ... and confusing
Summary

3 independent building blocks for exotic SUSY phenomenology

Color-triplet 'leptoquark' scalars/fermions are present in the low-energy spectrum

- leads to a different unification pattern
- favoring PS symmetry above the R-neutrino mass scale

Flavor symmetry prohibits proton decay

- instead of (or in addition to) $R$ parity
- Superpotential terms are due to GUT- and flavor-breaking
- ...... therefore do not exhibit GUT relations

Higgs sector is flavored

- Unhiggses (1st and 2nd generation) carry conserved quantum number
- Unhiggses dark matter candidates
- Ordinary MSSM stuff might decay via $R$-parity violation

Confusing LHC pheno, but handle to GUT scale
Some Unification needs time