The Non-Commutative Standard Model at a Photon Collider

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1 Non-Commutative SM 1

◦ Motivation ◦ Basics of NC (gauge) theories ◦ Seiberg-Witten Map/NCSM ◦ NC Standard Model (NCSM) ◦ Limits and Constraints

2 Pair production at the Photon Collider 6

◦ Pair Production ◦ Helicity amplitudes ◦ Angular Dependence/Z Resonance ◦ Plots - Spectrum ◦ Plots - Spectrum II

3 Conclusion 11

NOT mentioned: Hopf algebras, envelopping algebras, UV/IR mixing etc.

Old idea: Particle in a magnetic field Landau, (1930)

Particle coordinates transverse to $\vec{B}$ do not commute:

$$\{x_i, x_j\}_P = \frac{c}{e}\left(B^{-1}\right)_{ij}.$$

Motivation: String Theory

D branes with constant background field $B_{\mu\nu}$ ...,

Connes/Douglas/Schwartz (1998), Schomerus (1999) , Seiberg/Witten, (1999)

$$[\hat{x}_\mu, \hat{x}_\nu] \stackrel{\alpha'\to 0}{\longrightarrow} \mathrm{i}\left(B^{-1}\right)_{\mu\nu}$$

Non-Commutative space-time coordinates:

$$[\hat{x}^\mu, \hat{x}^\nu] = \mathrm{i}\theta^{\mu\nu}$$

Noncommutativity: antisymmetric tensor:

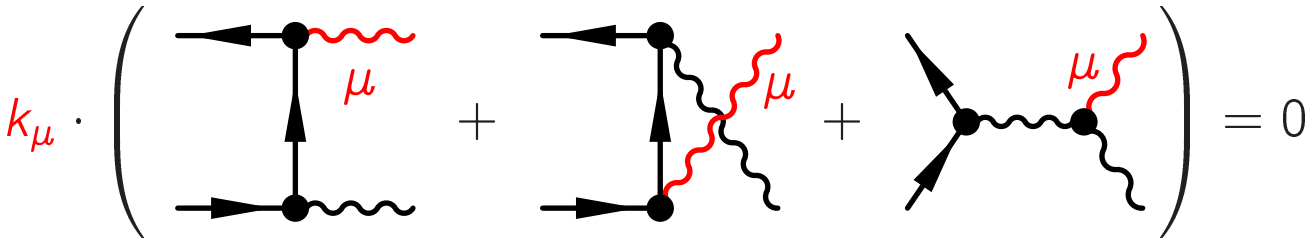
$$\theta^{\mu\nu} = \frac{1}{\Lambda_{NC}^2}\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Fields on noncommutative space-time $\Leftrightarrow$ Realisation on $\mathbb{R}^4$???

Moyal-Weyl product:

$$\hat{\mathbf{f}}(\hat{x})\cdot\hat{\mathbf{g}}(\hat{x}) \to f(x)\star g(x) := \exp\left[\partial_\mu^x \frac{i\theta^{\mu\nu}}{2}\partial_\nu^y\right] f(x)g(y)\Big|_{y\to x}$$
$$= f(x)\cdot g(x) + \frac{i}{2}\theta^{\mu\nu}\left(\partial_\mu f(x)\right)\cdot\left(\partial_\nu g(x)\right) + \mathcal{O}(\theta^2)$$

- **Demand: Gauge invariance on nc spaces**
- $\Rightarrow$ "Naive Approach": only certain gauge groups (basically $U(n)$)
  Problem: *Charge Quantization* , cf. Ward identity:



- **or:** Construction with infinitely many parameters
- *Map to finite number of DOF in comm. theories ?*

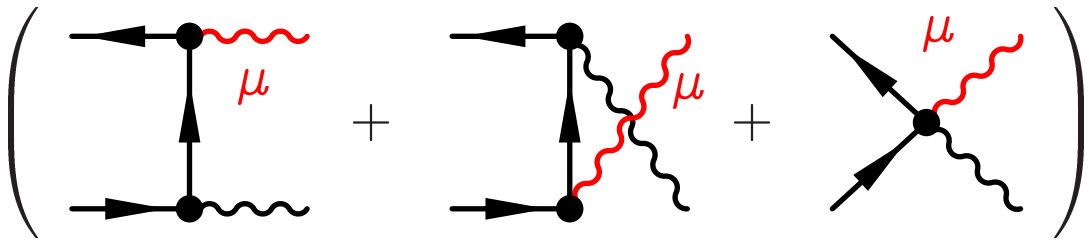
Seiberg-Witten Map

Seiberg/Witten, 1999; Wess, 2000

$$\hat{\Psi} = \Psi + \frac{1}{2}\theta^{\mu\nu} A_\nu \partial_\mu \Psi + \frac{\mathrm{i}}{8}\theta^{\mu\nu}[A_\mu, A_\nu]\Psi + \mathcal{O}(\theta^2)$$

$$\hat{\mathbf{A}}_\lambda = A_\lambda + \frac{1}{4}\theta^{\mu\nu}\{A_\nu, \partial_\mu A_\lambda\} + \frac{1}{4}\theta^{\mu\nu}\{F_{\mu\lambda}, A_\nu\} + \mathcal{O}(\theta^2)$$

No Charge Quantization , cf. Ward identity:

$k_\mu \cdot$ 

$= 0$

Non-Commutative Standard Model (NCSM)

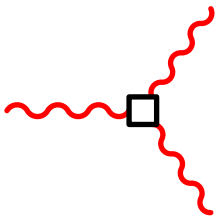
Wess *et al.*, 2001

SW map for SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$:

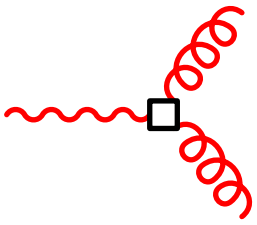
$$A_\mu = g' B_\mu Y + g \sum_{a=1}^{3} W_\mu^a T_L^a + g_s \sum_{a=1}^{8} G_\mu^a T_C^a$$

$$\mathcal{L}_{NCSM} = -\sum_i \frac{1}{2g_i^2} \mathrm{Tr}\left[\hat{\mathbf{F}}_{\mu\nu} \star \hat{\mathbf{F}}^{\mu\nu}\right] + \sum_k \overline{\hat{\Psi}}_k \star \mathrm{i}\hat{\not{D}} \star \hat{\Psi}_k + \ldots$$

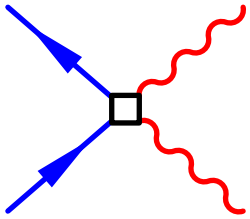
NCSM, additional Vertices:



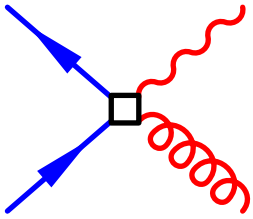
$\gamma\gamma\gamma,\ Z\gamma\gamma,$
$ZZ\gamma,\ ZZZ$

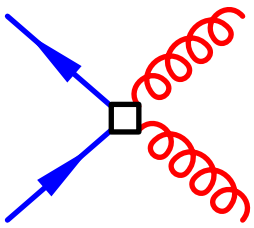


$Zgg,\ \gamma gg$



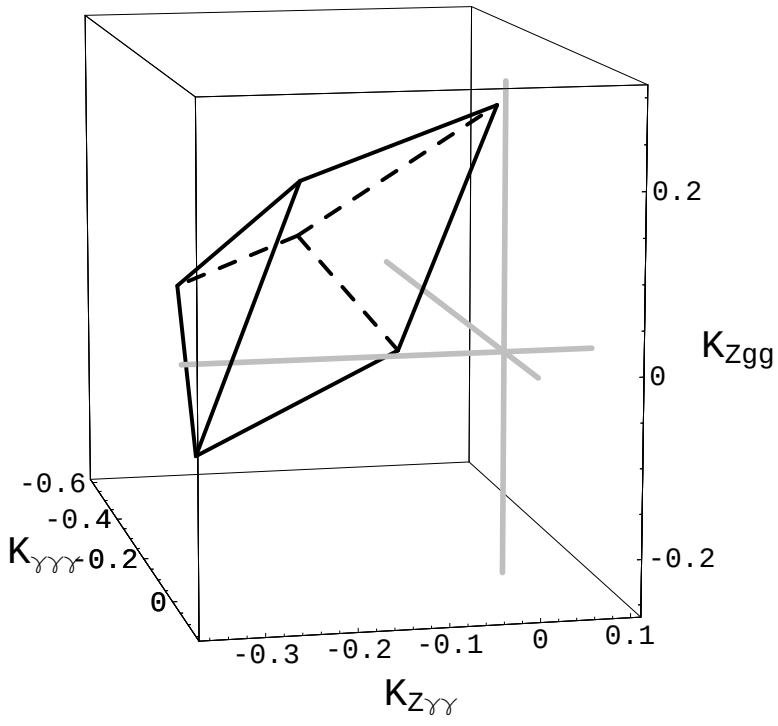
$f\bar{f}\gamma\gamma,\ f\bar{f}\gamma Z$
$f\bar{f}ZZ,\ f\bar{f}WW$

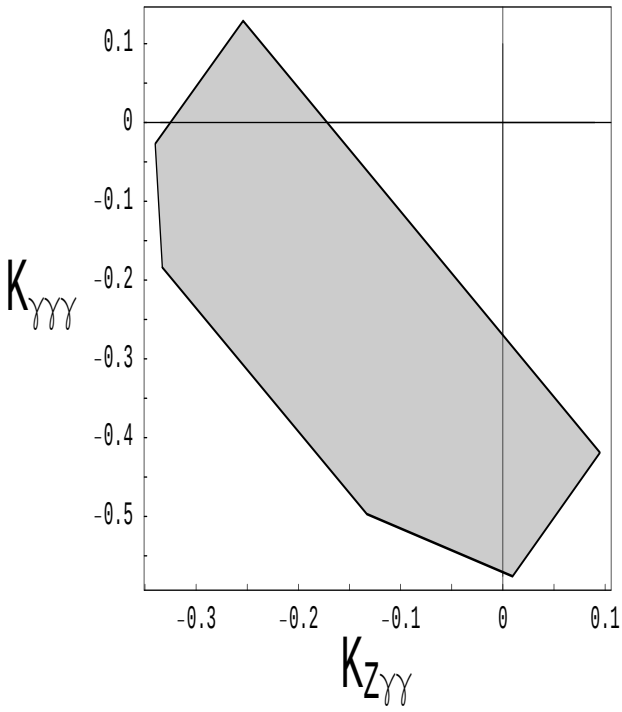




Constraints on Couplings from Matching to SM:

Duplančić/Schupp/Trampetić, 2003





SEARCHES and LIMITS:

- **NCQED:** ("Naive Approach") Hewett/Petriello/Rizzo, 2000
  several $2 \to 2$ processes @ LINAC $\Rightarrow \sim 1\,\text{TeV}$
- **"Plasmon Decay":** Raffelt *et al.*, 2001
  $\gamma_{\text{Pl.}} \to \bar{\nu}_R \nu_L$; Energy loss in star clusters
  $\Rightarrow \boxed{\Lambda_{NC} \gtrsim 80\,\text{GeV}}$

Yang–Landau Theorem Landau, 1948; Yang, 1950
Lorentz inv., Bose symm. $\Rightarrow$ Spin 1 particle $\not\to \gamma\gamma$ (gg)

- **Rare Z decays:** Behr *et al.*, 2001

$$\Gamma_{Z\to\gamma\gamma} \propto \alpha\, M_Z^5\, K_{Z\gamma\gamma}^2 \left[\frac{7}{3}\vec{E}^2 + \vec{B}^2\right]$$

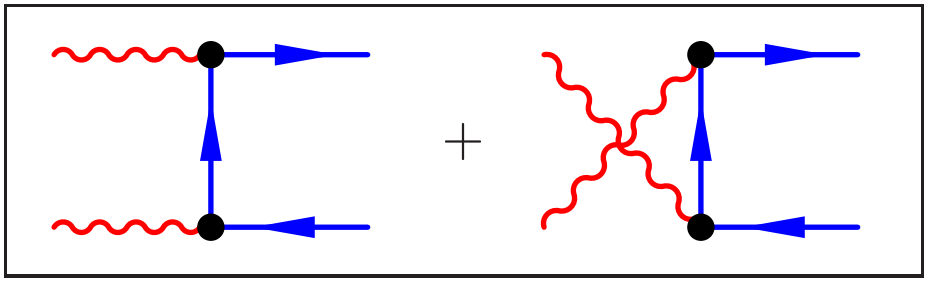
$$\Rightarrow\ \text{BR}(Z \to \gamma\gamma) \approx 4\times 10^{-8} \ \Rightarrow\ \boxed{\Lambda \gtrsim 100 - 200\,\text{GeV}}$$

- **Hadroproduction:** Alboteanu/Ohl, 2004 (in prep.)

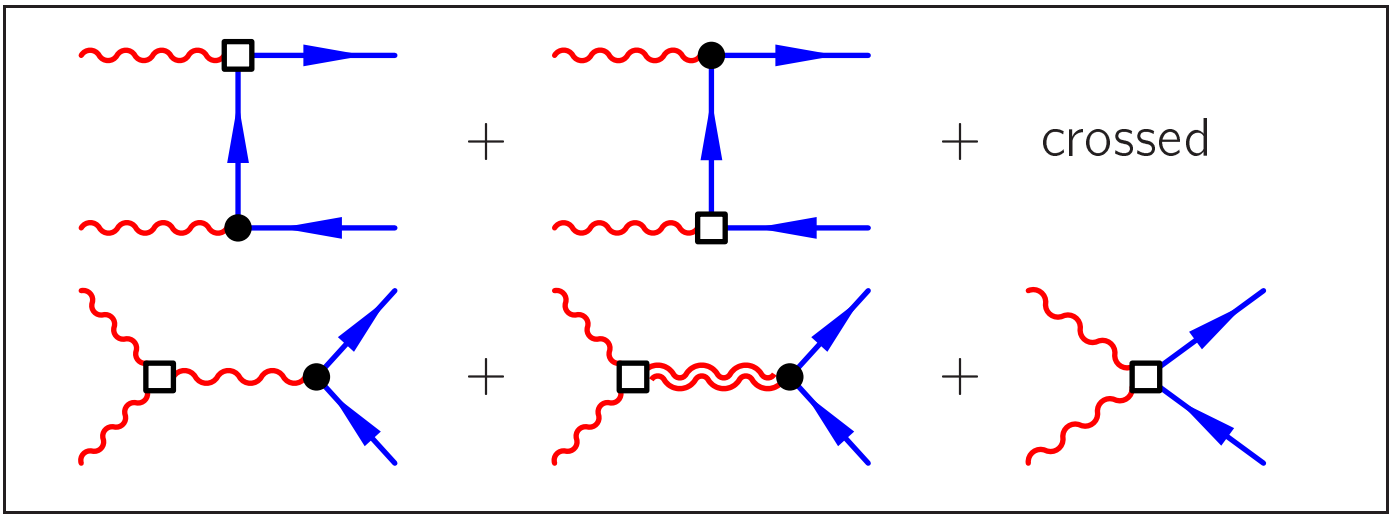
$$p\bar{p} \to \gamma Z, \quad \boxed{\Lambda_{NC} \gtrsim 200\,\text{GeV}}$$

NCSM Signal: Pair production at the Photon Collider

SM Amplitude:



NC Amplitude:



☹ Photons polarized ⇒ calculation complicated

☺ Photons polarized ⇒ more information

Numerically: O'Mega Ohl/JR/Schwinn, 2001-

Analytically: Only E_1, E_2 contribute

Analytical Calculation with helicity amplitudes

High energy limit $\rightarrow$ massless fermions $\rightarrow$ helicity states

van der Waerden/Weyl, 1930; Berends/Giele, 1981

Decomposition of lightlike vectors:

$$k^2 = 0 \rightarrow K_{\dot\alpha\beta} = k_{\dot\alpha} k_\beta \quad , \quad \langle kp \rangle = k^\alpha \epsilon_{\alpha\beta} p^\beta = k_1 p_2 - k_2 p_1$$

$$k_\alpha = \begin{pmatrix} (k^1 - \mathrm{i}k^2)/\sqrt{k^0 + k^3} \\ \sqrt{k^0 + k^3} \end{pmatrix}, \qquad 2(k \cdot p) = \langle kp \rangle \langle kp \rangle^*$$

Noncommutativity with Helicity amplitudes:

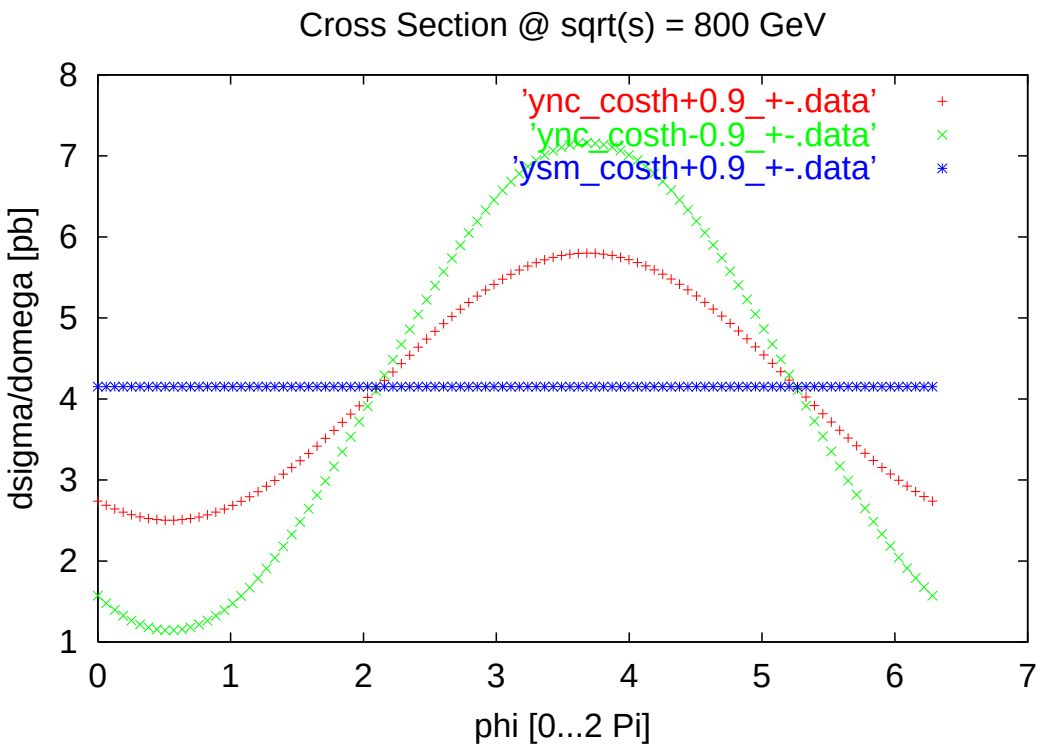
Spinor representation of antisymm. tensors: Penrose/Rindler

$$\boxed{\theta^{\mu\nu} \rightarrow \theta_{\dot\alpha\alpha,\dot\beta\beta} = \phi_{\alpha\beta}\, \epsilon_{\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\, \bar\phi_{\dot\alpha\dot\beta}}$$

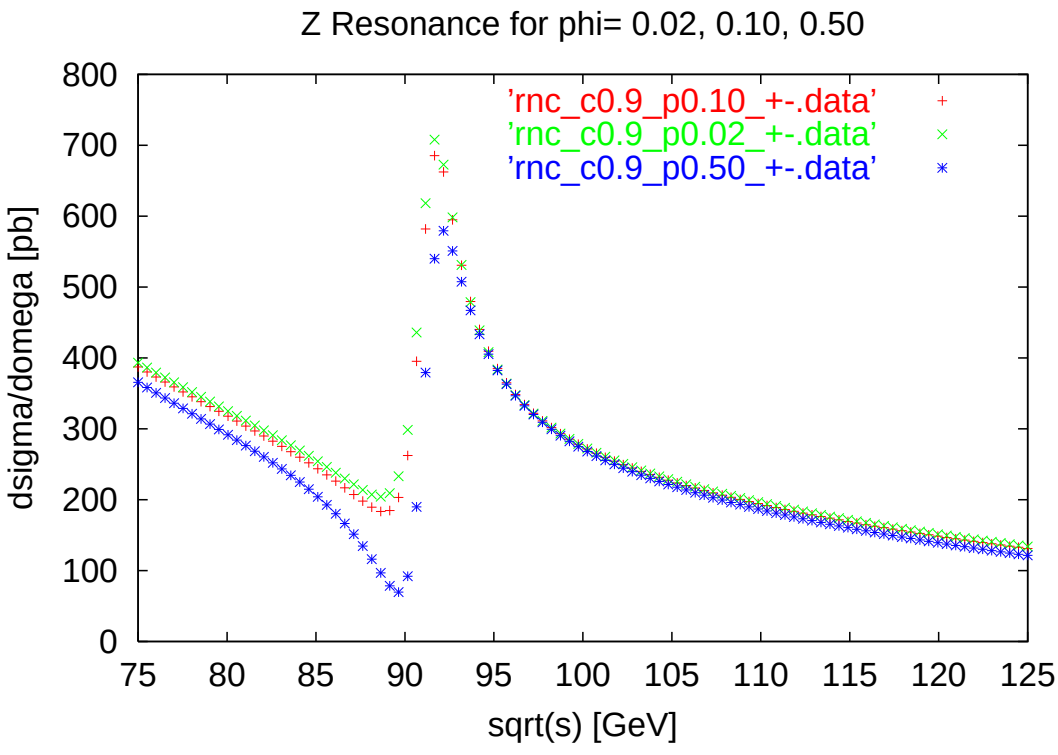
$$\mathbf{P}_i := E_i + \mathrm{i} B_i, \qquad \phi = \begin{pmatrix} -(\mathbf{P}_x - \mathrm{i}\mathbf{P}_y) & \mathbf{P}_z \\ \mathbf{P}_z & \mathbf{P}_x + \mathrm{i}\mathbf{P}_y \end{pmatrix}$$

Numerical Tests: Ward identity, Check with O'Mega,
Independence of the gauge spinor $\surd$

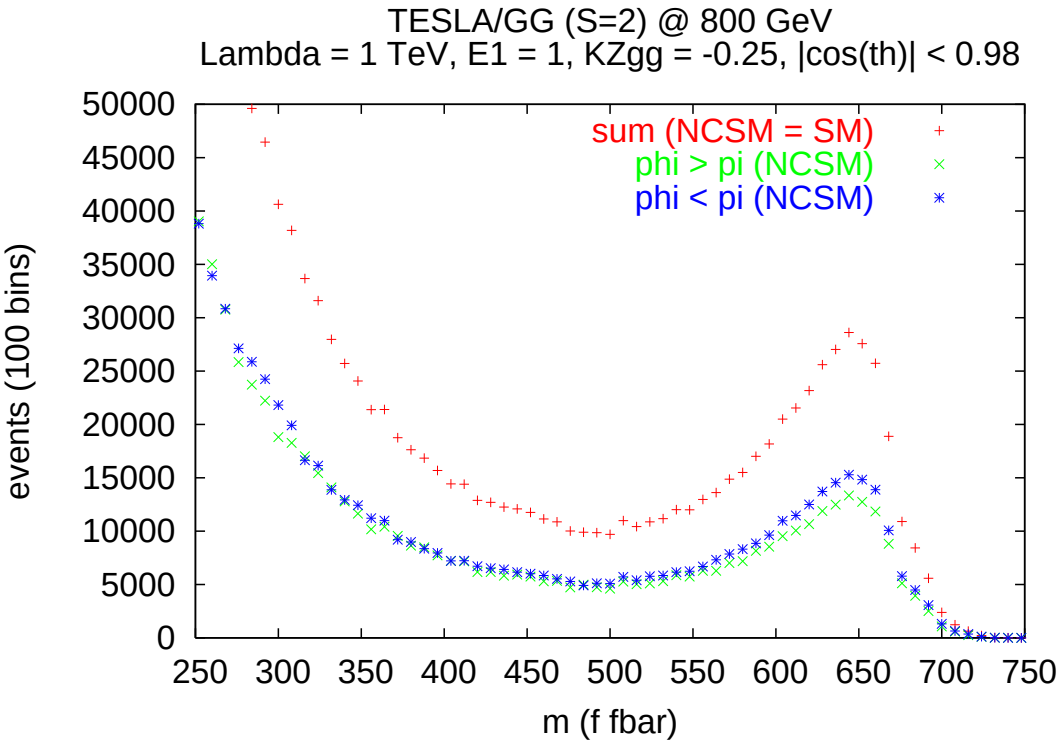
Azimuth dep. of CS @ $\sqrt{s} = 800$ GeV:

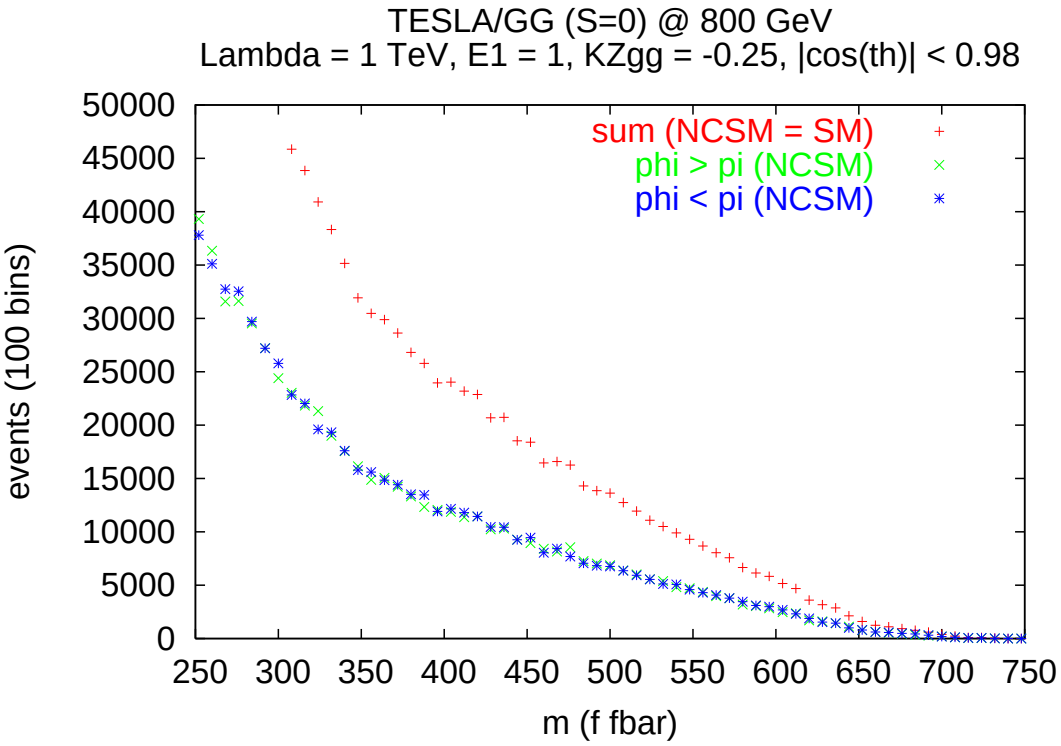


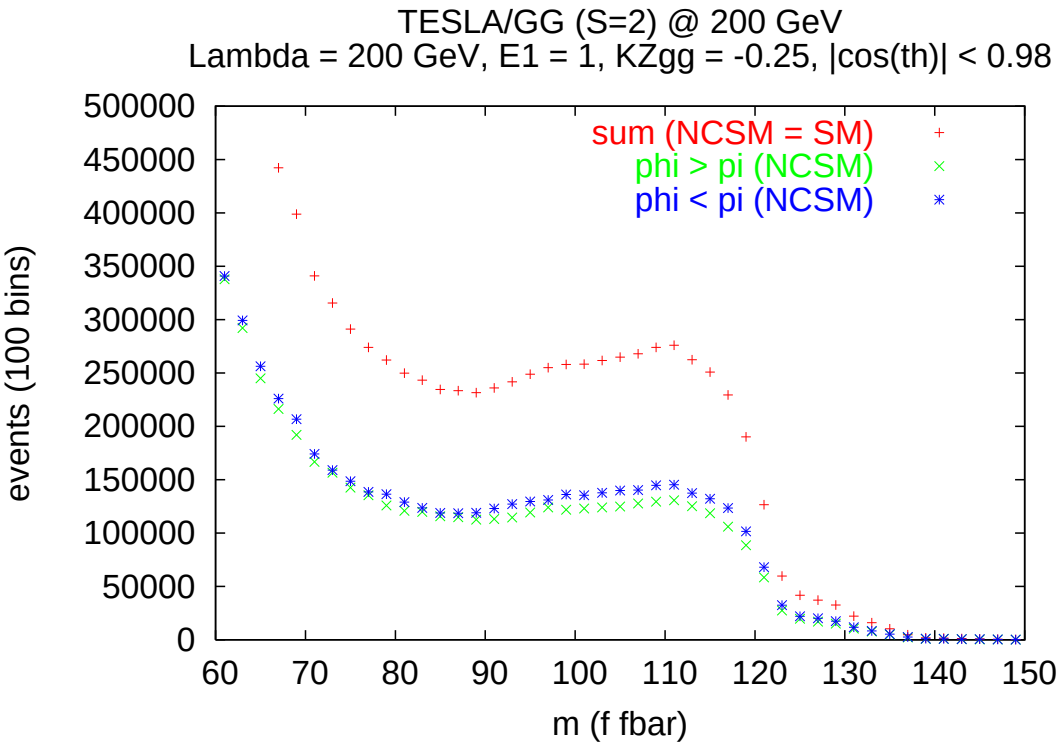
Z Resonance:

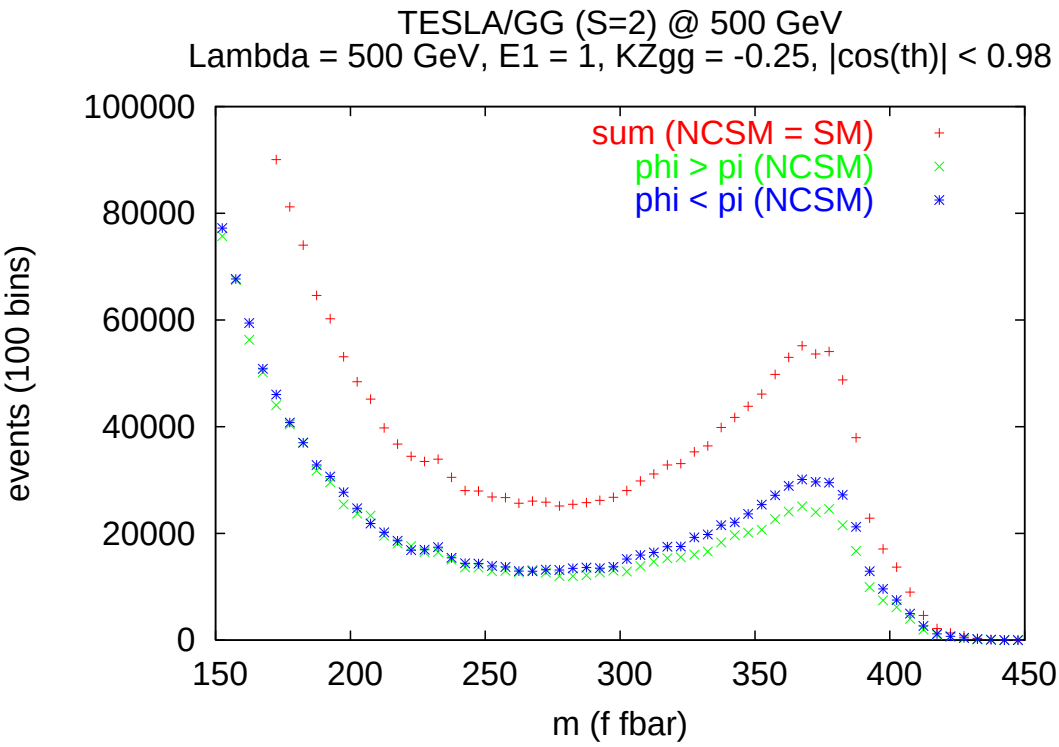


Photon spectrum: **Circe 2** Ohl, 2001






TESLA/GG (S=2) @ 200 GeV
Lambda = 200 GeV, E1 = 1, KZgg = -0.25, |cos(th)| < 0.98
sum (NCSM = SM)
phi > pi (NCSM)
phi < pi (NCSM)
events (100 bins)
m (f fbar)
500000
450000
400000
350000
300000
250000
200000
150000
100000
50000
0
60
70
80
90
100
110
120
130
140
150


TESLA/GG (S=2) @ 500 GeV
Lambda = 500 GeV, E1 = 1, KZgg = -0.25, |cos(th)| < 0.98
sum (NCSM = SM)
phi > pi (NCSM)
phi < pi (NCSM)
events (100 bins)
m (f fbar)
100000
80000
60000
40000
20000
0
150
200
250
300
350
400
450

Non-Commutative SM offers interesting new phenomena

- Generics: new – SM-forbidden – gauge-boson vertices, contact terms, etc.

Clear experimental signatures:
violation of angular momentum conservation, space-time dependence of the signal

Polarisation at the Photon Collider provides *crucial* information

Photon-Collider Mode: NC vs. Higgs Physics

Still missing: Seiberg-Witten @ $\mathcal{O}(\theta^2)$

Sensitivity up to $\sim 1\,\text{TeV}$ due to angular dependence