SUSY Lectures and Exercises

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Maria Laach, 09/2007

Literature

- Kalka/Soff: Supersymmetrie, Teubner, 1997 (quite elementary; in German)
- Wess/Bagger, Supersymmetry and Supergravity, Princeton Univ. Press, 1992
- S.P. Martin, SUSY Primer, hep-ph/9709356
- S. Weinberg, The Quantum Theory of Fields, Vol. III, 1998 (beware of notation!)
- M. Drees/R. Godbole/Roy, Theory and Pheno of Sparticles (World Scientific, 2004)
- H. Baer/X. Tata, Weak Scale Supersymmetry, Cambridge Univ. Press, 2006
- ...and many more

Julius Wess lecturing at the SUSY07 conference on July 25th, 2007 in Karlsruhe

It is with great sadness that we learned about the sudden death of Julius Wess on August, 8th, 2007. Only two weeks ago Julius still gave a special colloquium at the University of Karlsruhe on how he and Bruno Zumino introduced Supersymmetry into particle physics in 1973. Julius was then professor in Karlsruhe and Bruno held a position at CERN. This special colloquium, entitled "From Symmetry to Supersymmetry" was part of the PreSUSY07 school of the 15th International conference on Supersymmetry and the Unification of Fundamental Interactions, which was held in Karlsruhe from 26.7.2007-1.8.2007. Julius gave also the opening lecture at this SUSY07 conference and participated actively in his charming manner.

Julius Wess was one of the most renowned and successful theoretical particle physicists as exemplified by important awards: he was recipient of the Max Planck medal, the Wigner medal , the Gottfried Wilhelm Leibniz Prize. He also received an honorary Ph.D. from the Humboldt Universität in Berlin.

Wess was born in 1934 in Austria and received his Ph.D. in 1957 in Vienna where he was a student of Hans Thirring. His Ph.D. examiner was the famous quantum mechanics physicist, Erwin Schrödinger. His scientific career brought him first to CERN. In 1966 he became associate professor at the Courant Institute of the Univerisity of New York, in 1968 full professor at the Universität Karlsruhe (TH). In 1990 he left Karlsruhe to become director of the Max-Planck-Institut für Physik (Werner-Heisenberg-Institut) in Munich and professor at the Ludwig-Maximilians-Universität. After his retirement he worked at DESY in Hamburg.

Outline

Motivation for SUSY

The Standard Model (SM) cannot be the ultimate theory

– The SM does not contain gravity

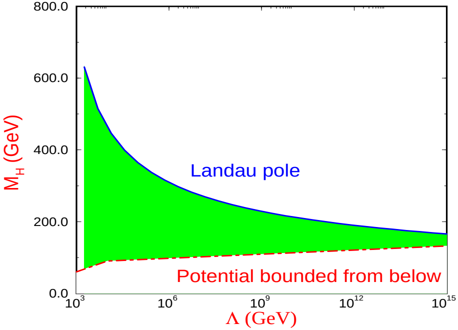
– Further problems: Hierarchy problem

Up to which energy scale Λ can the SM be valid?

– $\Lambda < M_{\mathrm{Pl}}$: inclusion of gravity effects necessary

– stability of Higgs potential: $\Rightarrow$

– Hierarchy problem : Higgs mass unstable w.r.t. quantum corrections $\delta M_H^2 \sim \Lambda^2$ (but what does this mean?)



Ideas for New Physics since 1970

(1) **New Ingredients**

– Technicolour: Higgs a bound state of strongly-interacting particles

(2) **Symmetries for cancellation of quantum corrections**

– Supersymmetry: Spin-statistics $\Rightarrow$ corrections from bosons and fermions cancel each other
– Little Higgs models: Global symmetries $\Rightarrow$ corrections from like-statistics particles cancel each other

(3) **Nontrivial Space-time structure eliminates hierarchy**

– Additional space dimensions: gravity appears only week
– Noncommutative space-time: coarse-grained space-time

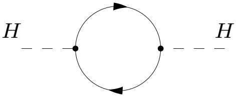
(4) **Ignoring the Hierarchy**

– Anthropic Principle: parameters have their values, *because* we (can) measure them

Mass is what determines the properties of the free propagation of a particle

Free propagation: 

inverse propag.: $i(p^2 - M_H^2)$

Loop corrections: 

inverse propag.: $i(p^2 - M_H^2 + \Sigma_H^f)$

QM: integration over all possible loop momenta k

dimensional analysis:

$$\Sigma_H^f \quad \sim \quad N_f \; \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

$$\text{for } \Lambda \to \infty : \qquad \Sigma_H^f \quad \sim \quad N_f \; \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_{\mathrm{f}}^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_{\rm Pl}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\rm Pl}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30}\, M_H^2$$

(for $M_H \lesssim 1$ TeV)

– no additional symmetry for $M_H = 0$

– no protection against large corrections

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections

in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\rm GUT}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

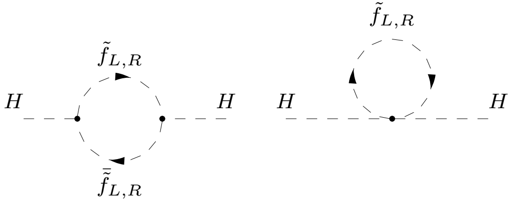
Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}} \, \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}} \, \lambda_{\tilde{f}}^2 \, \Lambda^2$

$\Rightarrow$ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda^2_{\tilde{f}} = \lambda^2_f$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \lambda^2_{\tilde{f}} = \lambda^2_f$

$$\Rightarrow \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda^2_f \, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\,\text{TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Outline

SUSY Field Theory – Symmetry and Algebra

Symmetry: a group of transformations that leaves the Lagrangian invariant

Generators of the group fulfill certain algebra

Examples:

0. Angular rotation: $\Psi \to \Psi e^{i\theta^a L_a}$
   theory is invariant under rotation
   generators: L_a, algebra: $[L_a, L_b] = i\varepsilon_{abc} L^c$
   Quantum numbers: (max. spin)^2, spin $[l(l+1), m = +l \ldots - l]$
1. Internal symmetry: $SU(3) \times SU(2) \times U(1)$
   gauge symmetry for description of strong and electroweak force
   generators: T_a, algebra: $[T_a, T_b] = i f_{abc} T^c$
   Quantum numbers: color, weak isospin, hyper charge
2. Poincaré symmetry (includes rotation)
   space–time symmetries:
   Lorentz transformations: $\Lambda^{\mu\nu}$, translations: P^ρ
   Quantum numbers: mass, spin → Exercise #1

Ex. 1: Spinor Representations of the Lorentz Group

The Lorentz group ($SO(1,3)$) has the following algebra (generalization of angular momentum commutation relations):

$$[J^{\mu\nu}, J^{\rho\sigma}] = -i(g^{\nu\sigma}J^{\mu\rho} + g^{\mu\rho}J^{\nu\sigma} - g^{\nu\rho}J^{\mu\sigma} - g^{\mu\sigma}J^{\nu\rho})$$

a) Define the generators for rotations and boosts as:

$$L^i = \frac{1}{2}\epsilon^{ijk}J^{jk} \qquad K^i = J^{0i},$$

$i, j, k = 1, 2, 3$. An infinitesimal Lorentz transformation can be written as

$$\Phi \rightarrow (1 - i\vec{\theta}\vec{L} - i\vec{\beta}\vec{K})\Phi.$$

Find the commutation relations for the operators L^i, K^j. Show that the sets

$$\vec{J}_+ = \frac{1}{2}\left(\vec{L} + i\vec{K}\right) \qquad \vec{J}_- = \frac{1}{2}\left(\vec{L} - i\vec{K}\right)$$

commute with each other and are each a $SU(2)$ angular momentum algebra.

b) Hence, representations of angular momentum (multiples of $1/2$) imply representations of the Lorentz algebra as (j_+, j_-). Use $\vec{J} = \vec{\sigma}/2$ to find the transformation law for the two fundamental representations $(\frac{1}{2}, 0)$ (undotted or left-handed spinors $(\psi_L)_\alpha$) and $(0, \frac{1}{2})$ (dotted or right-handed spinors). Proof that $\epsilon\psi_L^*$ with $\epsilon = i\sigma^2$ transforms like ψ_R and $-\epsilon\psi_R^*$ like ψ_L. (Dotting by complex conjugation and raising by ϵ).

Lorentz group: Representations of Lorentz group are labelled by two 'spins', $j_1, j_2 = 0$, where $j_1, j_2 = 0, \frac{1}{2}, 1, \ldots$

Basic representations $M_\alpha{}^\beta$ act on:

$(\frac{1}{2}, 0)$: LEFT-handed 2-component Weyl spinor, ψ_α

$(0, \frac{1}{2})$: RIGHT-handed 2-component Weyl spinor, $\bar{\psi}^{\dot{\alpha}}$

they transform under Lorentz transformations as follows:

$$\begin{aligned} \psi'_\alpha &= M_\alpha{}^\beta \psi_\beta; & \bar{\psi}'_{\dot{\alpha}} &= (M^*)_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}} \\ \psi'^\alpha &= (M^{-1})_\beta{}^\alpha \psi^\beta; & \bar{\psi}'^{\dot{\alpha}} &= (M^{*-1})_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} \end{aligned}$$

$M = \exp(i\frac{\vec{\sigma}}{2}(\vec{\vartheta} - i\vec{\varphi}))$, $\vec{\vartheta}$ and $\vec{\varphi}$ are the three rotation angles and boost parameters, respectively

$\Rightarrow$ spinors with undotted indices (first two components of Dirac spinor) transform according to $(\frac{1}{2}, 0)$-representation of Lorentz group, spinors with dotted indices (last two components of Dirac spinor) transform according to $(0, \frac{1}{2})$-representation

Our world (the SM) is described by:

- internal symmetry: T_a
- Poincaré symmetry: $\Lambda^{\mu\nu}$, P^ρ

internal symmetry is a trivial extension of the Poincaré symmetry:

$$[\Lambda^{\mu\nu}, T^a] = 0, \qquad [P^\rho, T^a] = 0$$

$\Rightarrow$ direct product: (Poincaré group) $\otimes$ (internal symmetry group)
Particle states characterized by maximal set of commuting observables:

$$|\underbrace{m, s; \vec{p}, s_3;}_{\text{space–time}} \underbrace{Q, I, I_3, Y, \ldots}_{\text{internal}}\rangle$$

quantum numbers

Wanted: extension of the SM

Theorem # 1: No-go theorem [*Coleman, Mandula '67*]

Any Lie-group containing Poincaré group P and internal symmetry group $\tilde{G}$ must be direct product $P \otimes \tilde{G}$

$$\Big|\underbrace{m, s; \vec{p}, s_3}_{\text{space–time}};\ \underbrace{\tilde{g}, \ldots}_{\text{internal}}\Big\rangle$$

quantum numbers

New group $\tilde{G}$ with generators Q^α and

$$[\Lambda^{\mu\nu}, Q^\alpha] \neq 0, \qquad [P^\rho, Q^\alpha] \neq 0$$

impossible

Direct product $\Rightarrow$ no irreducible multiplets can contain particles with different mass or different spin

$\Rightarrow$ new symmetry must predict new particles with the same mass and spin as in the SM

$\Rightarrow$ experimentally excluded, no such symmetry possible ☹

Theorem # 2: How-To-Avoid-the-No-go theorem

[Gol'fand, Likhtman '71] [Volkov, Akulov '72] [Wess, Zumino '74]

No go theorem can be evaded if instead of Lie-group (generators fulfill commutator relations):

$$[\ldots,\ldots] \to \{\ldots,\ldots\}$$

Anticommutator: $\{A, B\} = AB + BA$

$\Rightarrow$ Generator Q^α is fermionic (i.e. it has spin $\frac{1}{2}$)

$\Rightarrow$ Particles with different spin in one multiplet possible

$$Q|\text{boson}\rangle = |\text{fermion}\rangle, \quad Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Q changes spin (behavior under spatial rotations) by $\frac{1}{2}$

E.g.:

spin 2	$\to$	spin $\frac{3}{2}$	$\to$	spin 1
graviton		gravitino		photon

Simplest case: only one fermionic generator Q_α (and conjugate $\bar{Q}_{\dot\beta}$)

$\Rightarrow N = 1$ SUSY algebra:

$$
\begin{aligned}
[Q_\alpha, P_\mu] = \left[\bar{Q}_{\dot\beta}, P_\mu\right] &= 0 \\
[Q_\alpha, M^{\mu\nu}] &= i(\sigma^{\mu\nu})_\alpha^{\ \beta} Q_\beta \qquad (\star) \\
\{Q_\alpha, Q_\beta\} = \left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} &= 0 \\
\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} &= 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu \\
[M^{\mu\nu}, M^{\rho\sigma}] &= -i(g^{\nu\sigma} M^{\mu\rho} + g^{\mu\rho} M^{\nu\sigma} - g^{\nu\rho} M^{\mu\sigma} - g^{\mu\sigma} M^{\nu\rho})
\end{aligned}
$$

$\text{Energy} = H = P_0, \Rightarrow [Q_\alpha, H] = 0 \Rightarrow$ conserved charge

$(\star) \Rightarrow Q_\alpha$ ($\bar{Q}_{\dot\beta}$) is a left-handed (right-handed) spinor

$\Rightarrow$ SUSY: symmetry that relates bosons to fermions
unique extension of Poincaré group of $D = 4$ relativistic QFT

$\longrightarrow$ Exercise #2

Ex. 2: SUSY Basics: Harmonic Oscillator

a) Consider a (bosonic) harmonic oscillator. For simplicity assume: $\hbar = c = \omega = \cdots = 1$. There are the usual relations

$$[q, p] = i, \quad a = \frac{1}{\sqrt{2}}(q + ip), \quad a^\dagger = \frac{1}{\sqrt{2}}(q - ip), \quad \left[a, a^\dagger\right] = 1.$$

For the eigenstates $|n\rangle$ we have: $a\,|n\rangle = \sqrt{n}\,|n-1\rangle\,, \quad a^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$. Up to now everything is bosonic. Number and Hamilton operator:

$$N_B = a^\dagger a, \quad H_B = \frac{1}{2}(p^2 + q^2) = ?$$

What do we get for $[N_B, a]$, $\left[N_B, a^\dagger\right]$, $N_B\,|n\rangle$ and $H_B\,|n\rangle$?

b) Add a 2-state system (analogous to Spin-1/2 states $|\vec{S}^{\,2}, S_z\rangle$):

$$|\frac{1}{2}, +\frac{1}{2}\rangle = |+\rangle\,, \quad |\frac{1}{2}, -\frac{1}{2}\rangle = |-\rangle\,.$$

What is the algebra of the S_i? Use $S_\pm = S_x \pm iS_y$ to define fermionic annihilation and creation operators:

$$d^\dagger := S_+, \quad d := S_-.$$

What are the anticommuation relations of $d^\dagger, d$?
Analogous to a spin in a magnetic field define fermionic number and Hamiltonian operator:

$$N_F = d^\dagger d, \quad H_F = S_z = ?$$

How do $d^\dagger, d, N_F$ act on the states $|+\rangle\,, |-\rangle$?

c) Now couple the fermionic and bosonic systems:

$$H := H_B + H_F = ?$$

The coupled states are:

$$|n,+\rangle = |n\rangle \otimes |+\rangle\,, \quad |n,-\rangle = |n\rangle \otimes |-\rangle\,.$$

What does the spectrum of H look like? What about degeneration?

d) Now we derive the SUSY generators. They should do the following:

$$Q\,|1,+\rangle = |2,-\rangle \qquad (\text{allg.: } Q\,|n,+\rangle \to \sqrt{n+1}\,|n+1,-\rangle)$$
$$Q^\dagger\,|2,-\rangle = |1,+\rangle \qquad (\text{allg.: } Q^\dagger\,|n,-\rangle \to \sqrt{n}\,|n-1,+\rangle).$$

What follows for $Q, Q^\dagger$? Calculate $\left[N_{B,F}, Q^{(\dagger)}\right]$. What do you get for

$$\left\{Q^{(\dagger)}, Q^{(\dagger)}\right\}, \quad \left[H, Q^{(\dagger)}\right]?$$

What are the energy eigenvalues?

Can SUSY be an exact symmetry?

Consider fermionic state $|f\rangle$ with mass m
$\Rightarrow$ there is a bosonic state $|b\rangle = Q_\alpha |f\rangle$

$$P^2 |f\rangle = m^2 |f\rangle$$

$$\Rightarrow \quad P^2 |b\rangle = P^2 Q_\alpha |f\rangle = Q_\alpha P^2 |f\rangle = Q_\alpha m^2 |f\rangle = m^2 |b\rangle$$

$\Rightarrow$ for each fermionic state there is a bosonic state with the same mass

$\Rightarrow$ states are paired bosonic $\leftrightarrow$ fermionic

$\Rightarrow$ (still) experimentally excluded

$\Rightarrow$ SUSY must be broken

More on spinors:

The components of the spinors are Grassmann variables ,
i.e. anticommuting c-numbers

Raising and lowering of indices through the totally antisymmetric ϵ-tensor:

$$\epsilon^{\alpha\beta} = \epsilon^{\dot\alpha\dot\beta} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \; \epsilon_{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta; \;\; \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta; \;\; \bar\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\psi^{\dot\beta}; \;\; \bar\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta};$$

In particular

$$\epsilon^\alpha{}_\beta = \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^\alpha_\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of two Grassmann spinors is defined through

$$\theta\zeta \equiv \theta^\alpha\zeta_\alpha = \theta^\alpha\epsilon_{\alpha\beta}\zeta^\beta = -\epsilon_{\alpha\beta}\zeta^\beta\theta^\alpha = \zeta^\beta\epsilon_{\beta\alpha}\theta^\alpha = \zeta^\beta\theta_\beta = \zeta\theta$$

$$\bar\theta\bar\zeta \equiv \bar\theta_{\dot\alpha}\bar\zeta^{\dot\alpha} = -\bar\zeta^{\dot\alpha}\bar\theta_{\dot\alpha} = \bar\zeta\bar\theta$$

$\rightarrow$ Exercise #3

Exercise 3: Grassmann numbers

A Grassmann number, θ, is an anticommuting entity: $\{\theta, \theta\} = 0$.

a) What follows for the Taylor series of the function $\phi(\theta)$?

b) If we impose translational invariance for the integrals

$$\int_{-\infty}^{\infty} \mathrm{d}x \; \phi(x) = \int_{-\infty}^{\infty} \mathrm{d}x \; \phi(x+c)$$

for Grassmann variables, what integration rules do we get?

$$\int \mathrm{d}\theta, \quad \int \theta \mathrm{d}\theta, \quad \frac{\partial}{\partial\theta} \qquad ?$$

Use the most simple normalization for $\int \theta \mathrm{d}\theta$!

c) What are the changes in the rules if there is a set of Grassmann variables, $\Theta = \{\theta_1, \theta_2, \theta_3, \ldots\}$?

The γ-matrices are defined by

$$\gamma^\mu \equiv \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot\alpha} \\ (\bar\sigma^\mu)^{\dot\alpha\alpha} & 0 \end{pmatrix}; \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

where $(\sigma^\mu)_{\alpha\dot\alpha} \equiv \{1, \sigma^1, \sigma^2, \sigma^3\}_{\alpha\dot\alpha}$; $(\bar\sigma^\mu)^{\dot\alpha\alpha} \equiv \{1, -\sigma^1, -\sigma^2, -\sigma^3\}^{\dot\alpha\alpha}$; $(\sigma_\mu)_{\alpha\dot\alpha} = g_{\mu\nu}(\sigma^\nu)_{\alpha\dot\alpha}$; $(\bar\sigma_\mu)^{\dot\alpha\alpha} = g_{\mu\nu}(\bar\sigma^\nu)^{\dot\alpha\alpha}$.

The convention for the metric is: $g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$.

The γ-matrices have the usual commutation relation, $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.

This follows from

$$(\sigma^\mu)_{\alpha\dot\alpha}(\bar\sigma^\nu)^{\dot\alpha\alpha} = \mathrm{Tr}(\sigma^\mu\bar\sigma^\nu) = 2g^{\mu\nu}.$$

Furthermore,

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{4}\left(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu\right), \\ \bar\sigma^{\mu\nu} &= \frac{1}{4}\left(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu\right). \end{aligned}$$

The Lorentz-covariant expressions involving 4-component Dirac spinors can be written in two-component notation using

$$\Phi = \begin{pmatrix} \lambda_\alpha \\ \bar{\phi}^{\dot{\alpha}} \end{pmatrix}; \quad \overline{\Psi} = \begin{pmatrix} \chi^\alpha & \bar{\psi}_{\dot{\alpha}} \end{pmatrix}$$

$$\begin{aligned}
\overline{\Psi}\Phi &= \chi\lambda + \bar{\psi}\bar{\phi} = \chi^\alpha\lambda_\alpha + \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^5\Phi &= \bar{\psi}\bar{\phi} - \chi\lambda = \bar{\psi}_{\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \chi^\alpha\lambda_\alpha \\
\overline{\Psi}\gamma^\mu\Phi &= \chi\sigma^\mu\bar{\phi} - \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} - \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^5\Phi &= \chi\sigma^\mu\bar{\phi} + \lambda\sigma^\mu\bar{\psi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\phi}^{\dot{\alpha}} + \lambda^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} \\
\overline{\Psi}\gamma^\mu\gamma^\nu\Phi &= \chi\sigma^\mu\bar{\sigma}^\nu\lambda + \bar{\psi}\bar{\sigma}^\mu\sigma^\nu\bar{\phi} = \chi^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma})^{\dot{\alpha}\beta}\lambda_\beta + \bar{\psi}_{\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\beta}(\sigma^\nu)_{\beta\dot{\beta}}\bar{\phi}^{\dot{\beta}}
\end{aligned}$$

$$P_L = \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix}.$$

$\Rightarrow P_L\Phi = \lambda_\alpha$ left-handed Weyl spinor (LHWS)

$\Rightarrow P_R\Phi = \bar{\phi}^{\dot{\alpha}}$ right-handed Weyl spinor (RHWS)

$\longrightarrow$ Exercise #4

Exercise 4: Majorana Spinors

A Majorana spinor (real 4-spinor) has the property

$$\Psi = \mathcal{C}\overline{\Psi}^T \qquad \text{mit} \mathcal{C} = i\gamma^2\gamma^0 \; .$$

a) Show $\mathcal{C}^{-1} = \mathcal{C}^T = -\mathcal{C}$, as well as:

$$\Gamma^T = \begin{cases} +\mathcal{C}\Gamma\mathcal{C}^{-1} & \Gamma = \mathbb{I}, \gamma^5\gamma^\mu, \gamma^5 \\ -\mathcal{C}\Gamma\mathcal{C}^{-1} & \Gamma = \gamma^\mu, [\gamma^\mu, \gamma^\nu] \end{cases} \qquad (1)$$

b) With the help of a) proof that

$$\Gamma^T = \begin{cases} +\mathcal{C}\Gamma\mathcal{C}^{-1} & \Gamma = \mathbb{I}, \gamma^5\gamma^\mu, \gamma^5 \\ -\mathcal{C}\Gamma\mathcal{C}^{-1} & \Gamma = \gamma^\mu, [\gamma^\mu, \gamma^\nu] \end{cases} \qquad (2)$$

c) Express $\mathcal{C}$ with the help of the epsilon tensors.

d) What has to hold for the left- and righthanded components of a 4-spinor being Majorana?

e) Show for 2-spinors: $\theta^\alpha\theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}(\theta\theta)$

Further consequences of the SUSY algebra

$$\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\Rightarrow \left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\} \bar\sigma_\nu^{\dot\beta\alpha} = 2\,\underbrace{\sigma^\mu_{\alpha\dot\beta}\bar\sigma_\nu^{\dot\beta\alpha}}_{2g^\mu{}_\nu} P_\mu = 4P_\nu$$

$$\nu = 0 \Rightarrow H = P_0 = \frac{1}{4}\left\{Q_\alpha, \bar{Q}_{\dot\beta}\right\}\bar\sigma_0^{\dot\beta\alpha} = \frac{1}{4}\left(\left\{Q_1, Q_1^\dagger\right\} + \left\{Q_2, Q_2^\dagger\right\}\right)$$

where $\bar{Q}_{\dot\alpha} = (Q_\alpha)^\dagger$

$\left\{Q_i, Q_i^\dagger\right\} = Q_i Q_i^\dagger + Q_i^\dagger Q_i$: hermitian operator, eigenvalues ≥ 0

$\Rightarrow$ for any state $|\alpha\rangle$: $\langle\alpha|H|\alpha\rangle \geq 0$

spectrum of H is bounded from below, ≥ 0

$\Rightarrow$ no negative eigenvalues

State with lowest energy: vacuum state $|0\rangle$

if vacuum state is symmetric, i.e. $Q|0\rangle = 0$, $Q^\dagger|0\rangle = 0$ for all Q

$\Rightarrow$ vacuum has zero energy, $\langle 0|H|0\rangle = E_{\text{vac}} = 0$

For spontaneous symmetry breaking: vacuum state is not invariant

$\Rightarrow$ If (global) SUSY is spontaneously broken, i.e. $Q_\alpha|0\rangle \neq 0$,
then $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ non-vanishing vacuum energy

Further consequences for SUSY multiplets:

$\left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} = 0 \Rightarrow \bar{Q}_{\dot{\alpha}}^2 = 0$ (and $Q_\alpha^2 = 0$)

Consider multiplet, start with state of lowest helicity λ_0

application of $\bar{Q}_{\dot{\alpha}} \Rightarrow$ one additional state with helicity $\lambda_0 + \frac{1}{2}$

further application of $\bar{Q}_{\dot{\alpha}} \Rightarrow$ 0, no further state

$\Rightarrow$ one fermionic + one bosonic state

(N SUSY generators $\Rightarrow 2^{N-1}$ bosonic and 2^{N-1} fermionic states)
SUSY multiplet contains equal number of bosonic and fermionic state

Most relevant multiplets (possess also CPT conjugate ‘mirrors’):

- chiral supermultiplet: $-\frac{1}{2}, 0$

  Weyl fermion (quark, lepton, . . .) $+$ complex scalar (squark, slepton)
- vector supermultiplet: $-1, -\frac{1}{2}$

  Gauge boson (massless vector) $+$ Weyl fermion (gaugino)
- graviton supermultiplet: $-2, -\frac{3}{2}$

  graviton $+$ gravitino

SUSY Theory – Superfields and Superspace

Translation transformation: P_μ , parameter: x^μ

SUSY transformation: $Q_\alpha, \bar{Q}_{\dot\alpha}$, parameter: $\theta_\alpha, \bar\theta_{\dot\alpha}$
$\rightarrow$ anticommuting c-numbers ("Grassmann variables")

$\Rightarrow$ Extension of 4-dim. space–time by coordinates θ^α, $\bar\theta^{\dot\alpha}$: superspace

Point in superspace: $X = (x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$, Superfield: $\phi(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$

Taylor expansion in Grassmann variable: $\theta^\alpha\theta^\beta\theta^\gamma = 0$ $(\alpha, \beta, \gamma = 1, 2)$

$\Rightarrow$ Taylor expansion terminates after 2nd term, i.e. $\phi(\theta) = a + \theta\psi + \theta\theta\, f$

Integration: $\int d\theta = 0$, $\int d\theta\, \theta = 1$

$$\Rightarrow \quad \int d^2\theta\, \phi(\theta) = \int d^2\theta (a + \theta\psi + \theta\theta\, f) = f$$

With Grassmann variables:

SUSY algebra can be written in terms of commutators only

$$\begin{aligned} [\theta Q, \bar{\theta}\bar{Q}] &= 2\theta\sigma^\mu\bar{\theta}P_\mu \\ [\theta Q, \theta Q] = [\bar{\theta}\bar{Q}, \bar{\theta}\bar{Q}] &= 0 \\ [P^\mu, \theta Q] = [P^\mu, \bar{\theta}\bar{Q}] &= 0 \end{aligned}$$

$\Rightarrow$ can be treated like Lie-group with anticommuting parameters

SUSY transformations:

(Lagrangian should be invariant!)

Group element of finite SUSY transformation:

$$S(y, \xi, \bar{\xi}) = \exp\left[i\left(\xi Q + \bar{\xi}\bar{Q} - y^\mu P_\mu\right)\right]$$

in analogy to group elements for Lie-groups

ξ, $\bar{\xi}$ are independent of y^μ: global SUSY transformation

Hausdorff's formula ($e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$ for $[A,[A,B]] = [A,[A,B]] = 0$): and SUSY algebra $[(\xi Q),(\bar{\xi}\bar{Q})] = 2(\xi\sigma^\mu\bar{\xi})P_\mu$

Group Multiplication law:

$$S(y,\xi,\bar{\xi})S(x,\zeta,\bar{\zeta}) = S(x+y-(\xi\sigma\bar{\zeta})+(\zeta\sigma\bar{\xi}),\xi+\zeta,\bar{\xi}+\bar{\zeta})$$

representations of generators from infinitesimal superfield transformation

$$\Rightarrow \quad P_\mu = i\partial_\mu, \quad Q_\alpha = \partial_\alpha - i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu, \quad \bar{Q}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu$$

with $\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}$, $\bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}$ $\rightarrow$ Exercise #5

Ex. 5: Superspace Representation, Chiral Superfields

Use the following definitions for supercharges and covariant derivatives:

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu \qquad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu$$

$$\bar Q_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \qquad \bar D_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu$$

a) Calculate all anticommutators.

b) Proof

$$\epsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}(\theta\theta) = 4.$$

c) The left-handed superfield Φ is defined by the irreducibility condition $\bar D_{\dot\alpha}\Phi = 0$. (Why is it invariant under SUSY trafos?) Transform supercharges and covariant derivatives to the variables: θ, $\bar\theta$, $y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta$. What is that good for?

d) Show that the chiral multiplett can be written as:

$$\phi_L(y,\theta) = \varphi(y) + \sqrt{2}\,\theta^\alpha\,\psi_\alpha(y) + \theta\theta\,F(y)\,.$$

(Historical normalization!!!) Which fields are in with how many components? If you like, expand back to the old variables.

e) Proof that the SUSY trafos in that representation are (ε infinitesimal):

$$\begin{aligned} \theta^\alpha &\quad\longrightarrow\quad \theta^\alpha + \varepsilon^\alpha \\ x^\mu &\quad\longrightarrow\quad x^\mu + 2\,i\,\theta\sigma^\mu\bar{\varepsilon}\ . \end{aligned}$$

Derive the SUSY transformations of φ, ψ and F?

Total derivatives in integrals can be rewritten as (vanishing) “surface terms”. Which term can therefore be used for the construction of an action invariant under SUSY transformations, $S = \int \mathrm{d}^4x\mathcal{L}$?

f) Let θ^α be a Grassmann spinor. Let’s define:

$$\mathrm{d}^2\theta := -\frac{1}{4}\varepsilon_{\alpha\beta}\ \mathrm{d}\theta^\alpha\ \mathrm{d}\theta^\beta\ .$$

With $\phi(\theta) = a + \theta\psi + \theta\theta f$, calculate $\int \mathrm{d}^2\theta\ \phi(\theta)$. How can we now write the SUSY-invariant term?

g) What about the products of chiral superfields?
Which terms built from products of two or three left-chiral superfields can therefore be used for the construction of an invariant action? Interpret these terms.

Definition of covariant derivatives:

$$D_\alpha = -i\,\partial_\alpha - (\sigma^\mu\bar\theta)_\alpha\partial_\mu, \quad \bar D_{\dot\alpha} = i\,\bar\partial_{\dot\alpha} + (\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$$

Anticommutation relations of Q_α, D_α:

$$\{Q_\alpha, D_\beta\} = \{\bar Q_{\dot\alpha}, \bar D_{\dot\beta}\} = \{Q_\alpha, \bar D_{\dot\beta}\} = 0$$

$$\{Q_\alpha, Q_\beta\} = \{\bar Q_{\dot\alpha}, \bar Q_{\dot\beta}\} = 0$$

$$\{D_\alpha, D_\beta\} = \{\bar D_{\dot\alpha}, \bar D_{\dot\beta}\} = 0$$

$$\{Q_\alpha, \bar Q_{\dot\alpha}\} = 2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = 2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu$$

$$\{D_\alpha, \bar D_{\dot\alpha}\} = -2i(\sigma^\mu)_{\alpha\dot\alpha}\partial_\mu = -2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu$$

$\Rightarrow$ D_α, $\bar D_{\dot\alpha}$ anticommute with SUSY generators

$\Rightarrow$ are invariant under SUSY transformations

$$[(\xi Q + \bar\xi\bar Q), D_\alpha] = 0, \quad [(\xi Q + \bar\xi\bar Q), \bar D_{\dot\alpha}] = 0$$

General superfield in component form

Most general form of field depending on x, θ, $\bar{\theta}$:

$$\begin{aligned}\Phi(x,\theta,\bar{\theta}) \quad = \quad & \varphi(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta F(x) + \bar{\theta}\bar{\theta}H(x) + \theta\sigma^\mu\bar{\theta}A_\mu(x) \\ & + (\theta\theta)\bar{\theta}\bar{\lambda}(x) + (\bar{\theta}\bar{\theta})\theta\xi(x) + (\theta\theta)(\bar{\theta}\bar{\theta})D(x)\end{aligned}$$

Further terms vanish because of $\theta\theta\theta = \bar{\theta}\bar{\theta}\bar{\theta} = 0$

Components (can be complex):

φ, F, H, D: scalar fields

A_μ: vector field

ψ, $\bar{\chi}$, $\bar{\lambda}$, ξ: Weyl-spinor fields

$\Rightarrow$ Too many components in 4-dim. for irreducible representation of SUSY with spin ≤ 1 (chiral or vector multiplet)

$\Rightarrow$ representation is reducible
(not all component fields mix with each other under SUSY transf.)

⇒ Irreducible superfields (smallest building blocks) from imposing conditions on general superfield
conditions need to be invariant under SUSY transformations:

$\bar{D}_{\dot{\alpha}}\Phi = 0$: left-handed chiral superfield (LχSF)

$D_{\alpha}\Phi = 0$: right-handed chiral superfield (RχSF)

$\Phi = \Phi^{\dagger}$: vector superfield

⇒ chiral superfields describe left- or right-handed component of SM fermion $+$ scalar partner

LχSF in components:

$$\begin{aligned}\phi(x,\theta,\bar{\theta}) \quad &= \quad \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta}) \\ &\quad - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^{\mu}\partial_{\mu}\varphi(x) - (\theta\theta)F(x)\end{aligned}$$

φ, F: scalar fields , ψ: Weyl-spinor field

LχSF: Transf. of component fields with infinitesimal SUSY param. $\xi, \bar{\xi}$:

Comparison with $$\delta\phi(x,\theta,\bar\theta) = i(\xi Q + \bar\xi\bar Q)\phi(x,\theta,\bar\theta)$$

$$\begin{aligned}\delta\phi(x,\theta,\bar\theta) \quad &= \quad \delta\varphi(x) + \sqrt{2}\theta\delta\psi(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\delta\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\delta\psi(x)\sigma^\mu\bar\theta) \\ &\quad - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\delta\varphi(x) - (\theta\theta)\delta F(x)\end{aligned}$$

$\Rightarrow$ determination of $\delta\varphi$, $\delta\psi$, δF:

$$\delta\varphi = \sqrt{2}\xi\psi \qquad \text{boson} \to \text{fermion}$$

$$\delta\psi_\alpha = \sqrt{2}F\xi_\alpha + i\sqrt{2}(\sigma^\mu\bar\xi)_\alpha\partial_\mu\varphi \qquad \text{fermion} \to \text{boson}$$

$$\delta F = \partial_\mu(i\sqrt{2}\bar\xi\bar\sigma^\mu\psi) \qquad F \to \text{total derivative}$$

RχSF: analogously

SUSY Theory – Supersymmetric Lagrangians

Aim: construct an action that is invariant under SUSY transformations:

$$\delta \int d^4x \mathcal{L}(x) = 0$$

Satisfied if $\mathcal{L} \longrightarrow \mathcal{L} +$ total derivative

F and D terms (the terms with the largest number of θ and $\bar{\theta}$ factors) of chiral and vector superfields transform into a total derivative under SUSY transformations

⇒ Use F-terms (LχSF, RχSF) and D-terms (Vector SF) to construct an invariant action:

$$S = \int d^4x \left(\int d^2\theta \mathcal{L}_F + \int d^2\theta d^2\bar{\theta} \mathcal{L}_D \right)$$

If Φ is a LχSF ⇒ Φ^n is also a LχSF (since $\bar{D}_{\dot{\alpha}}\Phi^n = 0$ for $\bar{D}_{\dot{\alpha}}\Phi = 0$)

⇒ products of chiral superfields are chiral superfields, products of vector superfields are vector superfields

F-term Lagrangian:

$$\mathcal{L}_F = \int d^2\theta \sum_{ijk} \left(a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k \right) + \text{ h.c.}$$

Terms of higher order in Φ_i lead to non-renormalizable Lagrangians

$\Rightarrow$ F-term Lagrangian contains mass terms, scalar–fermion interactions ($\rightarrow$ superpotential), but no kinetic terms

D-term Lagrangian:

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} V$$

$\Rightarrow$ D-term Lagrangian contains kinetic terms

$\rightarrow$ Exercise #6

Exercise 6: Wess-Zumino model

The only other SUSY-invariant combination of chiral superfields is the product of left- and right-chiral superfields. This product is a vector superfield whose highest component again transforms into a total derivative. It has the form

$$(\phi_L^\dagger \, \phi_L)\Big|_{\theta\theta\bar\theta\bar\theta} = \int \mathrm{d}^2\theta \, \mathrm{d}^2\bar\theta \, (\phi_L^\dagger \, \phi_L) = F \, F^* \; - \; \varphi \, \partial_\mu \partial^\mu \, \varphi^* \; - \; i\bar\psi \, \sigma_\mu \, \partial^\mu \, \psi \; .$$

For hard guys: Take the expanded form for chiral fields:

$$\begin{aligned} \phi(x,\theta,\bar\theta) \quad &= \quad \varphi(x) + \sqrt{2}\theta\psi(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\partial_\mu\psi(x)\sigma^\mu\bar\theta) \\ &\quad - \frac{1}{4}(\theta\theta)(\bar\theta\bar\theta)\partial^\mu\partial_\mu\varphi(x) - (\theta\theta)F(x) \end{aligned}$$

and check it. At least, make it plausible to you...

Take the following ansatz for the Lagrangian density

$$\mathcal{L} \; = \; (\phi_L^\dagger \, \phi_L)\Big|_{(\theta\theta)(\bar\theta\bar\theta)} - (\frac{m}{2}\phi_L^2 + \frac{g}{3}\phi_L^3)\Big|_{(\theta\theta)} - (\frac{m}{2}\phi_L^{\dagger\,2} + \frac{g}{3}\phi_L^{\dagger\,3})\Big|_{(\bar\theta\bar\theta)} \; ,$$

m mass parameter, g coupling constant

Interpret the first term of $\mathcal{L}$, especially with respect to the number of physical degrees of freedom.

Write $\mathcal{L}$ with component fields. What's the equation of motion for F?

Use the result to eliminate F from $\mathcal{L}$. What follows for the masses and couplings of the fields φ and ψ?

Example: the Wess–Zumino Lagrangian

Construction of Lagrangian from chiral superfields Φ_i

$\Rightarrow$ Φ_i, $\Phi_i \Phi_j$, $\Phi_i \Phi_j \Phi_k$

$\Phi_i^\dagger \Phi_i$: vector superfield, $(\Phi_i^\dagger \Phi_i)^\dagger = \Phi_i^\dagger \Phi_i$

$$\left[\Phi_i^\dagger \Phi_i\right]_{\theta\theta\bar\theta\bar\theta} = F^\dagger F + (\partial_\mu \varphi^*)(\partial^\mu \varphi) + \frac{i}{2}(\psi \sigma^\mu \partial_\mu \bar\psi - \partial_\mu \psi \sigma^\mu \bar\psi) + \partial_\mu(\ldots)$$

Auxiliary field F can be eliminated via equations of motion

$$\Rightarrow \quad \mathcal{L}_D = \frac{i}{2}(\psi_i \sigma^\mu \partial_\mu \bar\psi_i - (\partial_\mu \psi_i)\sigma^\mu \bar\psi_i) - \frac{1}{2} m_{ij}(\psi_i \psi_j + \bar\psi_i \bar\psi_j)$$
$$+ (\partial_\mu \varphi_i^*)(\partial^\mu \varphi_i) - \sum_i \left| a_i + \frac{1}{2} m_{ij} \varphi_j + \frac{1}{3} \lambda_{ijk} \varphi_j \varphi_k \right|^2$$
$$- \lambda_{ijk} \varphi_i \psi_j \psi_k - \lambda_{ijk}^\dagger \varphi_i^* \bar\psi_j \bar\psi_k$$

$\mathcal{L}$ can be rewritten as kinetic part + contribution of superpotential $\mathcal{W}$:

$$\mathcal{W}(\Phi_i) = a_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \lambda_{ijk} \Phi_i \Phi_j \Phi_k$$

Mass and interaction terms can be gained by derivatives of the superpotential as a function of the scalar components only:

$$\mathcal{W}(\varphi_i) = a_i \varphi_i + \frac{1}{2} m_{ij} \varphi_i \varphi_j + \frac{1}{3} \lambda_{ijk} \varphi_i \varphi_j \varphi_k$$

$$\begin{aligned} \Rightarrow \quad \mathcal{L} \quad = \quad & \frac{i}{2} (\psi_i \sigma^\mu \partial_\mu \bar{\psi}_i - (\partial_\mu \psi_i) \sigma^\mu \bar{\psi}_i) + (\partial_\mu \varphi_i^*)(\partial^\mu \varphi_i) \\ & - \sum_i \left| \frac{\partial \mathcal{W}}{\partial \varphi_i} \right|^2 - \frac{1}{2} \frac{\partial^2 \mathcal{W}}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 \mathcal{W}^*}{\partial \varphi_i^* \partial \varphi_j^*} \bar{\psi}_i \bar{\psi}_j \end{aligned}$$

$\mathcal{W}$ determines all interactions and mass terms

Special case $a_i = 0$: Wess–Zumino model

SUSY Gauge Theories

- Gauge trafo has local scalar phase: $\phi(x) \to e^{-iv(x)}\phi(x)$
- Mismatch between trafo of $\partial\phi$ and $(\partial\phi)^\dagger$ is cured by $A_\mu(x) \to A_\mu(x) - \frac{1}{g}\partial_\mu v(x)$
- We have SUSY! (Theoretically!) $\Rightarrow$ Trafo must contain a superfield Λ, not only a scalar

$$\phi \to \phi' = e^{-i\Lambda(x)}\phi \qquad \text{demand} \quad \bar{D}_{\dot\alpha}\phi' = 0 \qquad \text{Hence, } \bar{D}_{\dot\alpha}\Lambda = 0$$

- But now not SUSY invariant any more:

$$\mathcal{L}_{\text{kin.}} = \Phi^\dagger\Phi\Big|_{(\theta\theta)(\bar\theta\bar\theta)} \to \Phi^\dagger e^{i(\Lambda^\dagger - \Lambda)}\Phi\Big|_{(\theta\theta)(\bar\theta\bar\theta)}$$

- Introduce vector superfield

$$\mathcal{L}_{\text{kin.}} = \Phi^\dagger e^{2gV}\Phi\Big|_{(\theta\theta)(\bar\theta\bar\theta)} \quad \text{invariant, if} \qquad V \to V' = V + \frac{i}{2g}(\Lambda - \Lambda^\dagger)$$

$\longrightarrow$ Exercise #7

Exercise 7: A little bit of SUSY gauge theories

a) Use the series expansion of the general real superfield

$$\begin{aligned} V(x,\theta,\bar\theta) &= c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta A_\mu(x) \\ &+ \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(\bar\theta\bar\theta)(M(x)-iN(x)) \\ &+ i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\ &+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right) \end{aligned}$$

and the SUSY gauge transformation $V \to V + \frac{i}{2g}(\Phi - \Phi^\dagger)$

to derive the trafos for the components. Show that one can choose the Wess-Zumino gauge:
$V(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta A_\mu(x) + i(\theta\theta)\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\theta\lambda(x) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x)$

Calculate e^V.

c) For the kinetic terms of the gauge fields we need again chiral superfields:

$$\mathcal{L} = \frac{1}{4}W^\alpha W_\alpha\Big|_{(\theta\theta)} + \frac{1}{4}\bar W_{\dot\alpha}\bar W^{\dot\alpha}\Big|_{(\bar\theta\bar\theta)}$$

with $W_\alpha = -\frac{1}{4}\bar D\bar D D_\alpha V$ and $\bar W_{\dot\alpha} = -\frac{1}{4}DD\bar D_{\dot\alpha}V$. Are these fields really chiral?

d) For people addicted to calculations: consult the literature for non-Abelian gauge theories. There is always a more complicated calculation!

Vector superfield in components:

$$\begin{aligned} V(x,\theta,\bar\theta) &= c(x) + i\theta\chi(x) - i\bar\theta\bar\chi(x) + \theta\sigma^\mu\bar\theta\, A_\mu(x) \\ &+ \frac{i}{2}(\theta\theta)(M(x)+iN(x)) - \frac{i}{2}(\bar\theta\bar\theta)(M(x)-iN(x)) \\ &+ i(\theta\theta)\bar\theta\left(\bar\lambda(x) + \frac{i}{2}\partial_\mu\chi(x)\sigma^\mu\right) - i(\bar\theta\bar\theta)\theta\left(\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\bar\chi(x)\right) \\ &+ \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)\left(D(x) - \frac{1}{2}\partial^\mu\partial_\mu c(x)\right) \end{aligned}$$

Number of components can be reduced by SUSY gauge transformation:

Wess-Zumino gauge: $c(x) = \chi(x) = \bar\chi(x) = M(x) = N(x) \equiv 0$

Vector SF:

$$V(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta\, A_\mu(x) + i(\theta\theta)\bar\theta\bar\lambda(x) - i(\bar\theta\bar\theta)\theta\lambda(x) + \tfrac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x)$$

$$\begin{aligned} \delta D &= -\xi\sigma^\mu\partial_\mu\bar\lambda(x) - \partial_\mu\lambda(x)\sigma^\mu\bar\xi \\ &= \partial_\mu\left[-\xi\sigma^\mu\bar\lambda(x) - \lambda(x)\sigma^\mu\bar\xi\right] \end{aligned}$$

$D \to$ total derivative

Auxiliary fields are eliminated via equations of motions:

$$\text{abelian}: \quad F = m\varphi^* + g{\varphi^*}^2$$

$$\text{non-abelian, gauge group } G: \quad D^G = \ldots \sum_a g_G \left(\varphi_i^\dagger (T_G)^a \varphi_i\right)$$

(internal indices of T_G, φ_i suppressed)

$$\Rightarrow \quad \mathcal{L}_D = F\,F^* + \frac{1}{2}\sum_G D^G D^G + \ldots$$

Lagrangian for scalar fields φ_i and spinor fields ψ_i with the same mass m_{ii} contains couplings of type $hf\bar{f}$ and $\tilde{h}\tilde{f}\bar{f}$ with the same strength

$\Rightarrow$ SUSY implies relations between masses and couplings

Combined SUSY and Gauge Transformations

Non-Abelian groups (like QCD): $[T^a, T^b] = if^{abc}T^c \qquad \phi_i \to \left(e^{igT^a v^a(x)}\phi\right)_i$

Gauge covariant derivatives

$$\begin{aligned} D_\mu \phi_i &= \partial_\mu \phi_i + igA^a_\mu (T^a\phi)_i \\ D_\mu \phi^{*\,i} &= \partial_\mu \phi^{*\,i} - igA^a_\mu (\phi^* T^a \phi)^i \\ D_\mu \psi_i &= \partial_\mu \psi_i + igA^a_\mu (T^a \psi)_i \\ D_\mu \lambda^a &= \partial_\mu \lambda^a + gf^{abc}A^b_\mu \lambda^c \end{aligned}$$

Chiral Superfields

$$\begin{aligned} \delta\varphi_i &= \sqrt{2}\epsilon\psi_i \\ \delta\psi_i &= i\sqrt{2}(\sigma^\mu\bar\epsilon)D_\mu\varphi_i + \sqrt{2}F_i\epsilon \\ \delta F_i &= i\sqrt{2}\bar\epsilon\bar\sigma^\mu(D_\mu\psi)_i + g(T^a\phi)_i(\bar\epsilon\bar\lambda^a) \end{aligned}$$

Vector Superfields

$$\begin{aligned} \delta A^a_\mu &= \frac{1}{2}((\bar\epsilon\bar\sigma_\mu\lambda^a) + (\bar\lambda^a\bar\sigma_\mu\epsilon)) \\ \delta\lambda^a &= \frac{i}{2}(\sigma^\mu\bar\sigma^\nu\epsilon)F^a_{\mu\nu} + \epsilon D^a \\ \delta D^a &= \frac{i}{2}\left(\bar\epsilon\bar\sigma^\mu(D_\mu\lambda)^a + (D_\mu\bar\lambda^a)\bar\sigma^\mu\epsilon\right) \end{aligned}$$

Summary of Lagrangian terms

- Kinetic terms for matter (chiral) superfields:

$$\int d^2\theta d^2\bar{\theta}\Phi^\dagger e^{2gV}\Phi = |D_\mu\varphi|^2 + i(\overline{\Psi}\not{D}\Psi) + |F|^2 + \sqrt{2}g(\overline{\Psi}T^a\lambda^a)\varphi + g(\varphi^\dagger T^a\varphi)D^a$$

kinetic terms of SM fermions + sfermions incl. their gauge couplings + F terms + "SUSY gauge Yukawas" + "D term couplings"

- Kinetic terms for vector (gauge) superfields

$$\int d^2\theta W^a W^a + \int d^2\bar{\theta}\bar{W}^a\bar{W}^a = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \frac{i}{2}\overline{\lambda^a}(\not{D}\lambda)^a + \frac{1}{2}D^a D^a$$

- Superpotential

$$\int d^2\theta\mathcal{W}(\Phi) + \text{h.c.} = -F\frac{\partial\mathcal{W}(\varphi)}{\partial\varphi} - F^\dagger\frac{\partial\bar{\mathcal{W}}(\varphi^\dagger)}{\partial\varphi^\dagger} - \frac{\partial^2\mathcal{W}(\varphi)}{\partial\varphi^2}\overline{\Psi^c_R}\Psi_R - \frac{\partial^2\bar{\mathcal{W}}(\varphi^\dagger)}{\partial\varphi^{\dagger\,2}}\overline{\Psi^c_L}\Psi_L$$

"Matter Yukawas" + scalar potential

- Eliminate auxiliary fields by E.O.M.:

$$D^a = -g(\varphi^\dagger T^a\varphi) \qquad F = -\frac{\partial\bar{\mathcal{W}}(\varphi^\dagger)}{\partial\varphi^\dagger} \qquad F^\dagger = -\frac{\partial\mathcal{W}(\varphi)}{\partial\varphi}$$

Supersymmetry breaking

Wanted: spontaneous SUSY breaking

Remember:

$$\{Q, \bar{Q}\} \sim P_\mu \quad \Rightarrow \quad Q\bar{Q} \sim P_\mu$$
$$P_0 = H \quad \Rightarrow \quad Q\bar{Q} \sim H$$

$\{Q, \bar{Q}\} \sim Q\bar{Q} + \bar{Q}Q$ (with $\bar{Q} = Q^\dagger$): hermitian Operator $\Rightarrow$ Eigenvalues ≥ 0
$\Rightarrow$ For any state $|\alpha\rangle$: $\langle\alpha| H |\alpha\rangle \geq 0$
spectrum bounded from below

Vacuum: $|0\rangle$

No spontaneous symmetry breaking: vacuum symm.: $Q|0\rangle = 0$, $\bar{Q}|0\rangle = 0$

$$\Rightarrow \langle 0| H |0\rangle = E_{\text{vac}} = 0$$

Spontaneous symmetry breaking: vacuum is not invariant: $Q|0\rangle \neq 0$

$$\Rightarrow \langle 0| H |0\rangle = E_{\text{vac}} > 0$$

$\Rightarrow$ non-vanishing vacuum energy

There are basically three different mechanisms for SUSY breaking

- D-term breaking for $U(1)$ gauge groups (Fayet/Iliopoulos, 1974)

$$\mathcal{L}_{\mathsf{FI}} = -\kappa D \qquad \Rightarrow V = \kappa D - \frac{1}{2} D^2 - gD \sum_i q_i |\phi_i|^2$$

is gauge invariant and supersymmetric

$$\Rightarrow \quad V = m_i^2 |\phi_i|^2 + \frac{1}{2} \left(\kappa - g \sum_i q_i |\phi_i|^2 \right)^2$$

$\Rightarrow$ Fermion masses: m_i^2 Scalar masses: $m_i^2 - gq\kappa$

- F-term breaking (O'Raifeartaigh, 1975)

$\rightarrow$ Exercise #8

- Dynamical Symmetry Breaking (Witten, 1981)
  Strongly coupled superfields condense and break SUSY

Exercise 8: O'Raifeartaigh model

- How to pronounce Lochlainn O'Raifeartaigh?
- F-term breaking is only possible with at least three chiral superfields. Consider the superpotential

$$\mathcal{W} = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2$$

  Calculate the scalar potential.
- Minimize the potential unter the assumption $m^2 > yk$. Why has supersymmetry to be broken?
- Evaluate the masses of the scalars and fermions. What about the spectrum?

SUSY breaking

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ in a realistic model: SUSY must be broken

Most satisfactory way for model of SUSY breaking:

spontaneous SUSY breaking

Specific SUSY-breaking schemes (see below) in general yield effective Lagrangian at low energies, which is supersymmetric except for explicit soft SUSY-breaking terms

Soft SUSY-breaking terms: do not alter dimensionless couplings

(i.e. dimension of coupling constants of soft SUSY-breaking terms > 0)
otherwise: re-introduction of the hierarchy problem

$\Rightarrow$ no quadratic divergences (in all orders of perturbation theory)
scale of SUSY-breaking terms: $M_{\mathrm{SUSY}} \lesssim 1$ TeV

Classification of possible soft breaking terms:

[*L. Girardello, M. Grisaru '82*]

- scalar mass terms: $m^2_{\phi_i} \left|\phi_i\right|^2$
- trilinear scalar interactions: $A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$
- gaugino mass terms: $\frac{1}{2} m \bar{\lambda}\lambda$
- bilinear terms: $B_{ij}\phi_i\phi_j + \text{h.c.}$
- linear terms: $C_i\phi_i$

$\Rightarrow$ relations between dimensionless couplings unchanged
no additional mass terms for chiral fermions

$N > 1$ SUSY

So far: $N = 1$ SUSY, simplest case, only one fermionic generator and its hermitian adjoint: Q_α, $\bar{Q}^{\dot{\beta}}$

$\Rightarrow$ one superpartner for photon: photino

N-extended SUSY: N generators Q_α^A, $\bar{Q}_B^{\dot{\beta}}$, $A = 1, \ldots, N$

$\Rightarrow$ N superpartners for the photon, ...

Generalization of anticommutator relation:

$$\{Q_\alpha^A, Q_\beta^B\} = \epsilon_{\alpha\beta} X^{AB}$$

$X^{AB} = -X^{BA}$: "central charges"

Problem:

helicity $+\frac{1}{2}$ and helicity $-\frac{1}{2}$ fermions are in same supermultiplet (e.g.: hypermultiplet for $N = 2$ SUSY)

$\Rightarrow$ helicity $+\frac{1}{2}$ and $-\frac{1}{2}$ fermions need to transform in the same way under gauge transformations

not possible for chiral fermions of electroweak theory

$\Rightarrow$ $N > 1$ SUSY theories are 'non-chiral'

$\Rightarrow$ $N = 1$ SUSY theories are the best candidates for a realistic low-energy theory (extension of the SM)

However: $N > 1$ SUSY have interesting properties

e.g.: $N = 4$ SUSY field theory (flat space) is finite

Seiberg–Witten solution in $N = 2$ SUSY, . . .

Outline

Minimal Supersymmetric Standard Model (MSSM)

MSSM: superpartners for SM fields

SM matter fermions have different quantum numbers than
SM gauge bosons

$\Rightarrow$ need to be placed in different superfields

$\Rightarrow$ no SM fermion is a gaugino

no Higgs is a sfermion (e.g. scalar neutrino)

agnostic about how SUSY breaking is achieved

no particular SUSY breaking mechanism assumed
parameterization of possible soft SUSY-breaking terms

$\Rightarrow$ most general case: 105 new parameters: masses, mixing angles, phases

1. Fermions, sfermions:

left-handed chiral superfields give SM fermions/sfermions
($\Rightarrow$ the conjugates of right-handed ones appear)

LχSF Q: quark, squark SU(2) doublets

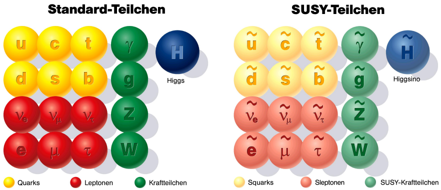
LχSF U^c: up-type quark, squark singlets

LχSF D^c: down-type quark, squark singlets

LχSF L: lepton, slepton SU(2) doublets

LχSF E^c: lepton, slepton singlets

$\Rightarrow$ one generation of SM fermions and their superpartners described by five LχSFs



2. Gauge bosons, gauginos:

Vector superfields:

– gluons g	and	gluinos $\tilde{g}$
– W bosons $W^\pm$, W^0	and	winos $\tilde{W}^\pm$, $\tilde{W}^0$
– B boson B^0	and	bino $\tilde{B}^0$

3. Higgs bosons, higgsinos:

LχSF

In MSSM: two Higgs doublets needed $\Rightarrow$ two LχSFs

Supersymmetrization of SM Yukawas

$$\mathcal{L}_{\rm SM} = \underbrace{y_d \overline{Q}_L H d_R}_{\text{d-quark mass}} + \underbrace{y_u \overline{Q}_L \hat{H} u_R}_{\text{u-quark mass}}$$

d-quark mass $m_d = y_d v$ u-quark mass $m_u = y_u v$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L , \quad \hat{H} = i\sigma_2 H^\dagger, \quad H \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \hat{H} \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: not both terms simultaneously allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on Φ_i, not on $\Phi_i^\dagger$

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow H_d$ and H_u needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Chiral supermultiplets of the MSSM

		spin 0	spin $\frac{1}{2}$	(SU(3)$_c$, SU(2), U(1)$_Y$)
squarks and quarks	Q	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(3, 2, \frac{1}{6})$
	U^c	$\tilde{u}_R^*$	u_R^c	$(\bar{3}, 1, -\frac{2}{3})$
	D^c	$\tilde{d}_R^*$	d_R^c	$(\bar{3}, 1, \frac{1}{3})$
sleptons and leptons	L	$(\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(1, 2, -\frac{1}{2})$
	E^c	$\tilde{e}_R^*$	e_R^c	$(1, 1, 1)$
higgs and higgsinos	H_u	(h_u^+, h_u^0)	$(\tilde{h}_u^+, \tilde{h}_u^0)$	$(1, 2, \frac{1}{2})$
	H_d	(h_d^0, h_d^-)	$(\tilde{h}_d^0, \tilde{h}_d^-)$	$(1, 2, -\frac{1}{2})$

$\longrightarrow$ Exercise #9

Exercise 9: The superpotential of the MSSM

	$SU(2)$ Singlett			$SU(2)$ Doublett			
chiral SF	U^c	D^c	E^c	Q	L	H_u	H_d
Hypercharge Y	-2/3	1/3	1	1/6	-1/2	1/2	-1/2

a) Construct the most general superpotential from these chiral superfields, subject to the following constraints:

- Renormalizability
- $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance

What follows from these constraints for the general structure of the allowed terms? Which interaction terms appear?

b) Which terms appear to be strange? What can you do against them? Consider the combination

$$P_R = (-1)^{3(B-L)}$$

with the baryon number B and lepton number L. This matter parity is often rephrased as R parity acting on the component fields, $(-1)^{3(B-L)+2s}$, where s is the spin. What does this for the interactions?

Vector supermultiplets

	spin $\frac{1}{2}$	spin 1	(SU(3)$_c$, SU(2), U(1)$_Y$)
gluinos and gluons	$\tilde{g}$	g	$(8,1,0)$
winos and W-bosons	$\widetilde{W}^{\pm}, \widetilde{W}^0$	$W^{\pm}, W^0$	$(1,3,0)$
bino and B-boson	$\widetilde{B}$	B	$(1,1,0)$

$\Rightarrow$ MSSM has further symmetry: “R parity” (see below)

Restricts the superpotential of the MSSM:

$$\mathcal{W}_{\text{MSSM}} = U^c \mathbf{Y_u} Q H_u - D^c \mathbf{Y_d} Q H_d - E^c \mathbf{Y_e} L H_d + \mu H_u H_d$$

Get used to signs, matrix and group invariant structure!

Particle content of the MSSM

Superpartners for Standard Model particles:

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad W^\pm, H^\pm \quad \gamma, Z, H_1^0, H_2^0 \qquad \text{Spin } 1 \text{ / Spin } 0$$

$$\underbrace{\tilde{g}} \quad \underbrace{\tilde{W}^\pm, \tilde{H}^\pm} \quad \underbrace{\tilde{\gamma}, \tilde{Z}, \tilde{H}_1^0, \tilde{H}_2^0}$$

$$\tilde{g} \quad \tilde{\chi}_{1,2}^\pm \quad \tilde{\chi}_{1,2,3,4}^0 \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:

Two Higgs doublets, physical states: $h^0, H^0, A^0, H^\pm$

as usual: Breaking of SU(2) $\times$ U(1)$_Y$ (electroweak symmetry breaking)

$\Rightarrow$ fields with the same SU(3)$_c$, U(1)$_{\text{em}}$ quantum numbers mix after EWSB

R parity

Most general gauge-invariant and renormalizable superpotential with chiral superfields of the MSSM:

$$\mathcal{W} = \mathcal{W}_{\rm MSSM} + \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j E_k + \lambda'^{ijk} L_i Q_j D_k + \mu'^i L_i H_u}_{\text{violate lepton number}} + \underbrace{\frac{1}{2}\lambda''^{ijk} U_i D_j D_k}_{\text{violates baryon number}}$$

If both lepton and baryon number are violated

⇒ rapid proton decay

Minimal choice (MSSM) contains only terms in the Lagrangian with even number of SUSY particles

⇒ additional symmetry: "R parity"

⇒ all SM particles have even R parity, all SUSY particles have odd R parity

The Lightest SUSY Particle (LSP)

MSSM has further symmetry: "R-parity"

all SM-particles and Higgs bosons: even R-parity, $P_R = +1$

all superpartners: odd R-parity, $P_R = -1$

$\Rightarrow$ SUSY particles appear only in pairs, e.g. $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$

$\Rightarrow$ lightest SUSY particle (LSP) is stable
(usually the lightest neutralino)

good candidate for Cold Dark Matter

$\Rightarrow M_{\text{SUSY}} \lesssim 1$ TeV

LSP neutral, uncolored $\Rightarrow$ leaves no traces in collider detectors

$\Rightarrow$ Typical SUSY signatures: "missing energy"

Spontaneous breaking of global SUSY

If global SUSY is spontaneously broken $\Rightarrow$ $\langle 0|H|0\rangle = E_{\text{vac}} > 0$

$\Rightarrow$ fields need vacuum expectation value

$\Rightarrow$ either $\langle F\rangle > 0$ (F-term breaking) or $\langle D\rangle > 0$ (D-term breaking)

$\Rightarrow$ requires that $F_i = 0$, $D^a = 0$ cannot be simultaneously satisfied for any values of the fields

F-term breaking:

need linear term in superpotential $\mathcal{W}(\phi_i) = a_i\phi_i + \frac{1}{2}m_{ij}\phi_i\phi_j + \frac{1}{3}\lambda_{ijk}\phi_i\phi_j\phi_k$

$\Rightarrow$ requires a chiral superfield that is a singlet under all gauge groups

$\Rightarrow$ not possible within the MSSM

D-term breaking:

Does not work in the MSSM (leads to charge and color-breaking minima)

Problems can be overcome if SUSY breaking happens in a 'hidden sector', i.e. by fields which have only very small couplings to ordinary matter

SUSY breaking in the hidden sector:
– tree-level (like F- and D-term breaking)
– dynamical breaking (similar to chiral symmetry breaking in QCD), . . .

SUSY-breaking terms in the MSSM arise radiatively via interaction that communicates SUSY breaking rather than through tree-level couplings to SUSY breaking v.e.v.s

$\Rightarrow$ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

If mediating interactions are $\approx$ flavor-diagonal
$\Rightarrow$ universal soft-breaking terms

"Hidden sector": SUSY breaking	$\longrightarrow$	Visible sector: MSSM

"Gravity-mediated": mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
"Gaugino-mediated"
...

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB, Gaugino-mediation: SUSY breaking happens on a different brane in a higher-dimensional theory

(more details later)

Soft breaking terms

$$\begin{aligned}\mathcal{L}_{\rm soft} \;=\; & -\frac{1}{2}\Big(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}\widetilde{W} + M_3 \tilde{g}\tilde{g}\Big) + \text{h.c.} \\ & - m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - (b H_u H_d + \text{h.c.}) \\ & - \Big(\tilde{u}_R \mathbf{A_u} \widetilde{Q} H_u - \tilde{d}_R \mathbf{A_d} \widetilde{Q} H_d - \tilde{e}_R \mathbf{A_e} \widetilde{L} H_d\Big) + \text{h.c.} \\ & - \widetilde{Q}^\dagger \mathbf{m_Q^2} \widetilde{Q} - \widetilde{L}^\dagger \mathbf{m_L^2} \widetilde{L} - \tilde{u}_R \mathbf{m_u^2} \tilde{u}_R^* - \tilde{d}_R \mathbf{m_d^2} \tilde{d}_R^* - \tilde{e}_R \mathbf{m_e^2} \tilde{e}_R^*\end{aligned}$$

Most general parameterization of SUSY-breaking terms that keep relations between dimensionless couplings unchanged
$\Rightarrow$ no quadratic divergences

$\mathbf{m_i^2}, \mathbf{A_j}$: 3×3 matrices in family space

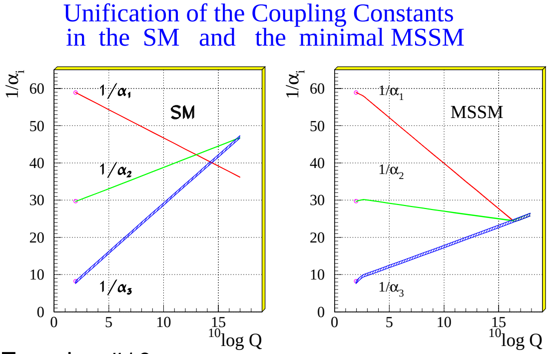
$\Rightarrow$ many new parameters

Gauge Coupling Unification

[RGE: equations that connect parameters at different energy scales]

$\longrightarrow$ use RGE's to evolve gauge coupling constants from electroweak scale to the GUT scale

$$\alpha_i(Q_{\text{electroweak}}) \longrightarrow \alpha_i(Q_{\text{GUT}})$$



gauge couplings do not meet in the SM

they unify in the MSSM although it was not designed for it!

$\Rightarrow M_{\text{SUSY}} \approx 1$ TeV

$\longrightarrow$ Exercise #10

Exercise 10: SUSY GUTs and $\sin^2\theta_W$

The renormalization group equations for the coupling constants $g_i, i = 1, 2, 3$ are given by:

$$\frac{\partial \alpha_i}{\partial \ln \mu} = b_i \frac{\alpha_i^2}{2\pi}. \tag{1}$$

In the SM, the coefficients b_i for the gauge groups SU(i) are:

$$\begin{aligned} b_3 &= -11 + \frac{4}{3} N_{\rm g}, \\ b_2 &= -\frac{22}{3} + \frac{4}{3} N_{\rm g} + \frac{1}{6} N_{\rm H}, \\ b_1 &= \frac{20}{9} N_{\rm g} + \frac{1}{6} N_{\rm H}, \end{aligned}$$

where $N_{\rm g}$ is the number of generations and $N_{\rm H}$ the number of Higgs doublets.

a) Why has b_1 a different form as b_2 and b_3? Show that the solution of (1) is:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu_0)} - \frac{b_i}{4\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right) \tag{2}$$

b) In a Grand Unified Theory (GUT) we should have the following relation at the GUT M_X :

$$\sqrt{5/3}\, g_1(M_X) = g_2(M_X) = g_3(M_X) = g_{\rm GUT}. \tag{3}$$

In a GUT like $SU(5) \stackrel{M_X}{\to} SU(3)_c \times SU(2)_L \times U(1)_Y$, d^c, L are together in a multiplet ($\bar{\mathbf{5}}$), as well as u^c, e^c, Q ($\mathbf{10}$). Demand that $\mathrm{tr}\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$ to find the normalization of hypercharge.

Furthermore, we have:

$$\alpha_1 = \frac{\alpha(\mu)}{c_{\rm W}^2(\mu)} \qquad\qquad \alpha_2 = \frac{\alpha(\mu)}{s_{\rm W}^2(\mu)} \tag{4}$$

c) For the scale $\mu_0 = M_X$ take the the GUT scale. Express $s_{\rm W}^2(\mu)$ as a function of $\alpha(\mu)$ ans $\ln(M_{\rm GUT}/\mu)$. Replace the logarithm by a corresponding relation containing $\alpha(\mu)$ and $\alpha_3(\mu) \equiv \alpha_s(\mu)$. Keep the b_i explicite.

c) Wo do you get with $\alpha(M_Z) \approx 1/128$ and $\alpha_s(M_Z) \approx 0.12$ for $s_{\rm W}^2(M_Z)$, M_X and $\alpha_{\rm GUT}$? Experimentally, the weak mixing angle has the value $s_{\rm W}^2 = 0.2312(3)$.

d) Due to the additional particle content of the MSSM the coefficients of the renormalization group equations for the coupling constants are changed with respect to the SM. In the MSSM they are:

$$\begin{aligned} b_3 &= -9 + 2N_{\rm g}, \\ b_2 &= -6 + 2N_{\rm g} + \frac{1}{2}N_{\rm H}, \\ b_1 &= \frac{10}{3}N_{\rm g} - \frac{1}{2}N_{\rm H}, \end{aligned}$$

Repeat the above calculations for the MSSM. What changes?

SUSY relations among parameters

Symmetry properties of MSSM Lagrangian (SUSY, gauge invariance) give rise to coupling and mass relations

Soft SUSY breaking does not affect SUSY relations between dimensionless couplings

E.g.:

gauge boson–fermion coupling

=

gaugino–fermion–sfermion coupling

for U(1), SU(2), SU(3) gauge groups

In SM: all masses are free input parameters (except M_W–M_Z relation)

MSSM:

- Relations between neutralino and chargino masses
- Sfermion mass relations, e.g. $m^2_{\tilde{e}_L} = m^2_{\tilde{\nu}_L} - M_W^2 \cos(2\beta)$

All relations receive corrections from loop effects

⇔ effects of soft SUSY breaking, electroweak symmetry breaking

The MSSM Higgs sector

Higgs mechanism needed to give masses to W and Z bosons:

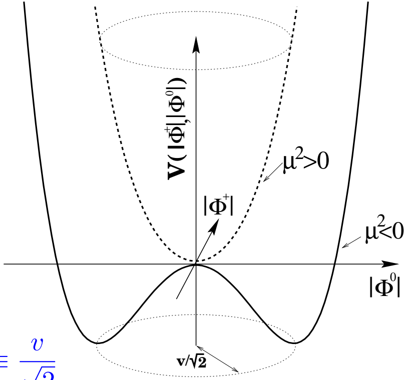
SM: Scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2_{\text{SM}} \left|\Phi^\dagger\Phi\right| + \lambda \left|\Phi^\dagger\Phi\right|^2, \quad \lambda > 0$$

$\mu^2_{\text{SM}} < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle\Phi_0\rangle| = \sqrt{\frac{-\mu^2_{\text{SM}}}{2\,\lambda}} \equiv \frac{v}{\sqrt{2}}$



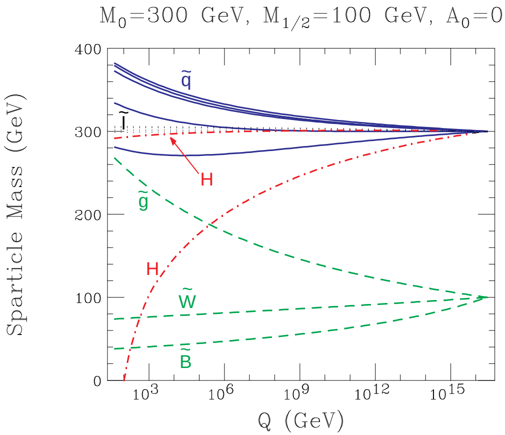
SM: sign of μ_{SM} has to be set by hand

MSSM: negative sign of μ comes for free provided that . . .

– assume GUT scale (as motivated by coupling constant unification)

– take universal input parameters at the GUT scale (see mSUGRA below)

– run down to the electroweak scale with RGEs



Exactly one parameter turns negative: the "μ" in the Higgs potential

But this only works if

$m_t = 150 \ldots 200$ GeV

and $M_{\rm SUSY} \approx 1$ TeV

Exercise 11: The MSSM Higgs potential

a) Derive the scalar potential for the Higgs fields in the MSSM. There are two contributions from the superpotential, three soft-breaking terms and two D terms.

b) Show that performing $SU(2)_L$ rotations allows to rotate the VEVs to the neutral components in the potential

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - b(H_u^0 H_d^0 + \text{h.c.}) + \frac{g'^2 + g^2}{8}(|H_u^0|^2 - |H_d^0|^2)^2$$

c) Although the quartic term is positive, there is a D-flat direction $|H_u^0| = |H_d^0|$. Show that the requirement for the potential to be bounded from below for large values of the fields along the D-flat directions leads to

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d^2} \ . \qquad (1)$$

d) Show that the requirement for one linear combination of H_U^0 and H_d^0 has a negative mass squared implies

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) \qquad (2)$$

e) Next, we set the VEVs equal to $\langle H_u^0 \rangle =: v_u$, $\langle H_d^0 \rangle =: v_d$ and introduce $\tan\beta := v_u/v_d$. Then, $v^2 = v_u^2 + v_d^2 \approx (246\,\text{GeV}/\sqrt{2})^2$. Show that both VEVs being positive implies $0 < \beta < \pi/2$. Minimize the potential with respect to H_u^0 and H_d^0 and proof that this leads to

$$m_{H_u}^2 + |\mu|^2 - b\cot\beta - (m_Z^2/2)\cos(2\beta) = 0 \qquad (3a)$$

$$m_{H_d}^2 + |\mu|^2 - b\tan\beta + (m_Z^2/2)\cos(2\beta) = 0 \qquad (3b)$$

Check that these equations satisfy the conditions (1) and (2).

f) Taking $|\mu|^2$, b, $m^2_{H_u}$, $m^2_{H_d}$ as input parameters, show that the solution of (3a) is given by

$$\sin(2\beta) = \frac{2b}{m_{H_u^2} + m^2_{H_d} + 2|\mu|^2} \tag{4a}$$

$$m_Z^2 = \frac{|m_{H_u^2} + m^2_{H_d}|}{\sqrt{1-\sin^2(2\beta)}} - (m_{H_u^2} + m^2_{H_d} + 2|\mu|^2) \tag{4b}$$

g) Use the following rotations

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} H_u^+ \\ (H_d^-)^* \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}$$

to find the masses for the states h^0, H^0, A^0, $H^\pm$, $G^\pm$ and G^0. Express $\sin(2\alpha)/\sin(2\beta)$ and $\tan(2\alpha)/\tan(2\beta)$ as functions of the masses of the Higgs and gauge bosons.

h) Derive the bound for the lightest Higgs boson

$$m_{h^0} < m_Z|\cos(2\beta)|. \tag{6}$$

Why is this not already ruled out?

The MSSM Higgs sector

Two Higgs doublets:

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = \begin{pmatrix} v_d + (\phi_d^0 + i\chi_d^0)/\sqrt{2} \\ \phi_d^- \end{pmatrix}$$

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} = \begin{pmatrix} \phi_u^+ \\ v_u + (\phi_u^0 + i\chi_u^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = (m_{H_d}^2 + |\mu|^2)|H_d|^2 + (m_{H_u}^2 + |\mu|^2)|H_u|^2 - b(\epsilon_{ab} H_u^a H_d^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}} (|H_d|^2 - |H_u|^2)^2 + \underbrace{\frac{g^2}{2}} |H_d^\dagger H_u|^2$$

gauge couplings, in contrast to SM

Mixing of $\mathcal{CP}$-even, $\mathcal{CP}$-odd, charged fields:

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_d^0 \\ \phi_u^0 \end{pmatrix}$$

$$\begin{pmatrix} A^0 \\ G^0 \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \chi_d^0 \\ \chi_u^0 \end{pmatrix}, \quad \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ -\cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

$$\tan(2\alpha) = \tan(2\beta)\,\frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}$$

Three Goldstone bosons (as in SM): G^0, $G^\pm$

$\longrightarrow$ longitudinal components of $W^\pm$, Z

$\Rightarrow$ Five physical states: $h^0, H^0, A^0, H^\pm$

h, H: neutral, $\mathcal{CP}$-even, A^0: neutral, $\mathcal{CP}$-odd, $H^\pm$: charged

Gauge-boson masses:

$$M_W^2 = \frac{1}{2} g'^2 (v_d^2 + v_u^2), \quad M_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_d^2 + v_u^2), \quad M_\gamma = 0$$

Problem:

MSSM contains term $\mu H_u H_d$ in superpotential

μ: dimensionful parameter

For ew symmetry breaking required: $\mu \sim$ electroweak scale

But: no a priori reason for $\mu \neq 0$, $\mu \ll M_{\mathrm{P}}$
(problem mainly in GMSB scenario, easier to overcome in mSUGRA)

Possible solution:

μ related to v.e.v. of additional field

$\Rightarrow$ Introduction of extra singlet field S, v.e.v. s $\Rightarrow$ "NMSSM"

Superpotential: $\mathcal{V} = \lambda H_u H_d S + \frac{1}{3}\kappa S^3 + \dots$

Physical states in NMSSM Higgs-sector:

S_1, S_2, S_3 (CP-even), P_1, P_2 (CP-odd), $H^{\pm}$

Parameters in MSSM Higgs potential V (besides g, g'):

$$v_d,\ v_u,\ m_{H_d},\ m_{H_d},\ |\mu|,\ b$$

relation for M_W^2, M_Z^2 $\Rightarrow$ 1 condition

minimization of V w.r.t. neutral Higgs fields H_d^0, H_u^0 $\Rightarrow$ 2 conditions

$\Rightarrow$ only two free parameters remain in V, conventionally chosen as

$$\tan\beta = \frac{v_u}{v_d}, \qquad M_A^2 = b(\tan\beta + \cot\beta)$$

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{\mathrm{H}^\pm}$: no free parameters, can be predicted

In lowest order:

$$m_{\mathrm{H}^\pm}^2 = M_A^2 + M_W^2$$

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_d - \phi_u$ basis:

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} m^2_{\phi_d} & m^2_{\phi_d\phi_u} \\ m^2_{\phi_d\phi_u} & m^2_{\phi_u} \end{pmatrix} =$$

$$\begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\,\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\,\cos\beta & M_A^2 \cos^2\beta + M_Z^2 \sin^2\beta \end{pmatrix}$$

$$\Downarrow \quad \leftarrow \text{Diagonalization}, \ \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

Tree-level result for m_h, m_H:

$$m_{H,h}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_h, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$g_{hVV} = \sin(\beta - \alpha)\, g^{\rm SM}_{HVV}, \quad V = W^\pm, Z$$

$$g_{HVV} = \cos(\beta - \alpha)\, g^{\rm SM}_{HVV}$$

$$g_{hAZ} = \cos(\beta - \alpha)\, \frac{g'}{2\cos\theta_W}$$

$$g_{hb\bar{b}}, g_{h\tau^+\tau^-} = -\frac{\sin\alpha}{\cos\beta}\, g^{\rm SM}_{Hb\bar{b},H\tau^+\tau^-}$$

$$g_{ht\bar{t}} = \frac{\cos\alpha}{\sin\beta}\, g^{\rm SM}_{Ht\bar{t}}$$

$$g_{Ab\bar{b}}, g_{A\tau^+\tau^-} = \gamma_5 \tan\beta\, g^{\rm SM}_{Hb\bar{b}}$$

$\Rightarrow$ $g_{hVV} \leq g^{\rm SM}_{HVV}$, $\quad g_{hVV}, g_{HVV}, g_{hAZ}$ cannot all be small

$g_{hb\bar{b}}, g_{h\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling

$g_{hb\bar{b}}, g_{h\tau^+\tau^-}$: possible

Higgs mass bounds in SUSY theories

MSSM predicts upper bound on m_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

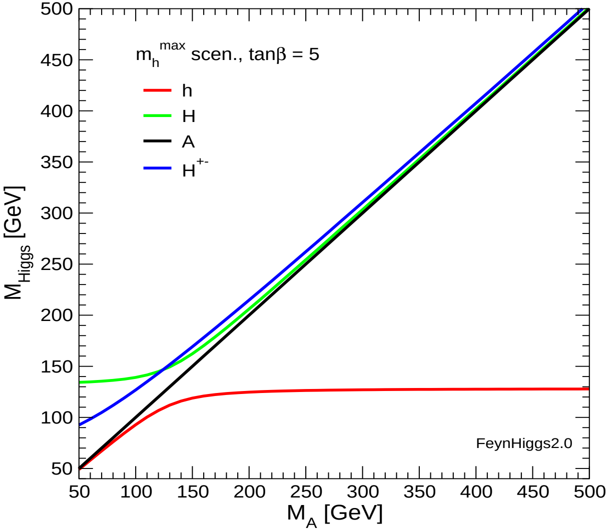
Yukawa couplings: $\frac{\mathrm{e}\, m_t}{2 M_W s_\mathrm{W}}, \frac{\mathrm{e}\, m_t^2}{M_W s_\mathrm{W}}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

Present status of m_h prediction in the MSSM:

Complete one-loop and ‘almost complete’ two-loop result available

Upper bound on m_h saturated for large M_A , large $\tan\beta$,
significant mixing in $\tilde{t}$ sector



Upper bound $m_h \lesssim 135$ GeV reduced by

$\approx 7, 12, 11$ GeV in mSUGRA , GMSB , AMSB scenarios

[*S. Ambrosanio et al. '01*]

Upper bound on m_h in extensions of MSSM: $m_h \lesssim 200$ GeV
(no new gauge groups!)

[*G. Kane, C. Kolda, J. Wells '93*] [*J. Espinosa, M. Quirós '93, '98*]

Remaining theoretical uncertainties in prediction for m_h in the MSSM:

[*G. Degrassi et al. '02*]

- From unknown higher-order corrections:
  $\Rightarrow \Delta m_h \approx 3$ GeV

- From uncertainties in input parameters
  $m_t, \ldots, M_A, \tan\beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \ldots$
  $\Delta m_t \approx 2$ GeV $\Rightarrow \Delta m_h \approx 2$ GeV

The Sfermions

Scalar potential terms for the sfermions:

- Soft-breaking mass terms: $m^2_{\tilde{f}_L}|\tilde{f}_L|^2 + m^2_{\tilde{f}_R}|\tilde{f}_R|^2$
- F terms from the superpotential:
$$Y_f^2|H_{u/d}\tilde{f}_L|^2 + Y_f^2|H_{u/d}\tilde{f}_R|^2 + |\mu|^2(H_{u/d}\tilde{f}_L\tilde{f}_R^* + \text{h.c.})$$
- Soft-breaking A terms: $A_f H_{u/d}\tilde{f}_L\tilde{f}_R^* + \text{h.c.}$
- D-term contributions: $\propto g^2|\tilde{f}_L|^2|H_{u/d}|^2 + g'^2|\tilde{f}|^2|H_{u/d}|^2$

$\Rightarrow$ Characteristic mass matrix structure for up/down-type sfermions:

$X_{f_\uparrow} = A_{f_\uparrow} - \mu/\tan\beta \qquad X_{f_\downarrow} = A_{f_\downarrow} - \mu\tan\beta$

$$\mathcal{M}^2_{\tilde{f}_\uparrow} = \begin{pmatrix} M^2_{\tilde{f}_L} + m^2_{f_\uparrow} + \Delta D_{\tilde{f}_\uparrow,L} & m_{f_\uparrow}X_{f_\uparrow} \\ m_{f_\uparrow}X_{f_\uparrow} & M^2_{\tilde{f}_R} + m^2_{f_\uparrow} + \Delta D_{\tilde{f}_\uparrow,R} \end{pmatrix} \xrightarrow{\theta_{\tilde{f}_\uparrow}} \begin{pmatrix} m^2_{\tilde{f}_\uparrow,1} & 0 \\ 0 & m^2_{\tilde{f}_\uparrow,2} \end{pmatrix}$$

$$\mathcal{M}^2_{\tilde{f}_\downarrow} = \begin{pmatrix} M^2_{\tilde{f}_L} + m^2_{f_\downarrow} + \Delta D_{\tilde{f}_\downarrow,L} & m_{f_\downarrow}X_{f_\downarrow} \\ m_{f_\downarrow}X_{f_\downarrow} & M^2_{\tilde{f}_{\downarrow,R}} + m^2_{f_\downarrow} + \Delta D_{\tilde{f}_\downarrow,R} \end{pmatrix} \xrightarrow{\theta_{\tilde{f}_\downarrow}} \begin{pmatrix} m^2_{\tilde{f}_\downarrow,1} & 0 \\ 0 & m^2_{\tilde{f}_\downarrow,2} \end{pmatrix}$$

$$\Delta D_{\tilde{f}} = (T_{3,\tilde{f}} - Q_{\tilde{f}}\sin^2\theta_W)\cos(2\beta)m_Z^2$$

Sfermion Systematics

- Off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

  $\Rightarrow$ mixing important in stop sector (sbottom sector for large $\tan\beta$)

  Taken into account also for stau sector
- Mixing makes (often) $\tilde\tau_1$ lightest slepton (NLSP), $\tilde t_1$ lightest squark
- gauge invariance $\Rightarrow$ relation between $m_{\tilde t_1}, m_{\tilde t_2}, \theta_{\tilde t}, m_{\tilde b_1}, m_{\tilde b_2}, \theta_{\tilde b}$
- No right-handed sneutrinos (GUT-scale ?)
- Characteristics from renormalization group equations:

$$
\begin{array}{llllllllllll}
m^2_{\tilde d_L} & = m_0^2 & + & K_3 & + & K_2 & + & \frac{1}{36} K_1 & + & \Delta D_{\tilde d_L} & & \\
m^2_{\tilde u_L} & = m_0^2 & + & K_3 & + & K_2 & + & \frac{1}{36} K_1 & + & \Delta D_{\tilde u_L} & & K_1 \approx 0.15\, m^2_{1/2} \\
m^2_{\tilde u_R} & = m_0^2 & & & + & K_3 & + & \frac{4}{9} K_1 & + & \Delta D_{\tilde u_R} & & \\
m^2_{\tilde d_R} & = m_0^2 & + & K_3 & & & + & \frac{1}{9} K_1 & + & \Delta D_{\tilde d_R} & & K_2 \approx 0.5\, m^2_{1/2} \\
m^2_{\tilde e_L} & = m_0^2 & & & + & K_2 & + & \frac{1}{4} K_1 & + & \Delta D_{\tilde e_L} & & \\
m^2_{\tilde \nu} & = m_0^2 & & & + & K_2 & + & \frac{1}{4} K_1 & + & \Delta D_{\tilde \nu} & & K_3 \approx (4.5 - 6.5)\, m^2_{1/2} \\
m^2_{\tilde e_L} & = m_0^2 & & & & & + & K_1 & + & \Delta D_{\tilde e_R} & &
\end{array}
$$

Neutralinos

In gauge-eigenbasis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$, the neutralino mass matrix is:

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

Origin of terms:

- M_1 and M_2 soft-breaking gaugino masses
- $-\mu$ entries: SUSY Higgs(-ino) mass term
- g, g' terms: Higgs-higgsino-gaugino “SUSY gauge Yukawas”

Rotate this to the mass eigenbasis:

$$\tilde{N}_i = \mathbf{N}_{ij} \psi_j^0, \qquad \mathbf{N}^* \mathbf{M}_{\tilde{N}} \mathbf{N}^{-1} = \begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix}$$

if EWSB negligible: a “bino-like" $\tilde{\chi}_1^0 \approx \tilde{B}$; a “wino-like" $\tilde{\chi}_2^0 \approx \tilde{W}^0$; and “higgsino-like" $\tilde{\chi}_3^0, \tilde{\chi}_4^0 \approx (\tilde{H}_u^0 \pm \tilde{H}_d^0)/\sqrt{2}$

Neutralinos

In gauge-eigenbasis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0)$, the neutralino mass matrix is:

$$\mathbf{M}_{\tilde{N}} = \begin{pmatrix} M_1 & 0 & -\cos\beta\ s_W\, m_Z & \sin\beta\ s_W\, m_Z \\ 0 & M_2 & \cos\beta\ c_W\, m_Z & -\sin\beta\ c_W\, m_Z \\ -\cos\beta\ s_W m_Z & \cos\beta\ c_W\, m_Z & 0 & -\mu \\ \sin\beta\ s_W\, m_Z & -\sin\beta\ c_W\, m_Z & -\mu & 0 \end{pmatrix}$$

Origin of terms:

- M_1 and M_2 soft-breaking gaugino masses
- $-\mu$ entries: SUSY Higgs(-ino) mass term
- g, g' terms: Higgs-higgsino-gaugino “SUSY gauge Yukawas”

Rotate this to the mass eigenbasis:

$$\tilde{N}_i = \mathbf{N}_{ij}\psi_j^0, \qquad \mathbf{N}^*\mathbf{M}_{\tilde{N}}\mathbf{N}^{-1} = \begin{pmatrix} m_{\tilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\tilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\tilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\tilde{N}_4} \end{pmatrix}$$

if EWSB negligible: a “bino-like" $\tilde{\chi}_1^0 \approx \tilde{B}$; a “wino-like" $\tilde{\chi}_2^0 \approx \tilde{W}^0$; and “higgsino-like" $\tilde{\chi}_3^0, \tilde{\chi}_4^0 \approx (\tilde{H}_u^0 \pm \tilde{H}_d^0)/\sqrt{2}$

Charginos

In gauge eigenbasis $\psi^\pm = (\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-)$:

$$\mathbf{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix},$$

with

$$\mathbf{X} = \begin{pmatrix} M_2 & g v_u \\ g v_d & \mu \end{pmatrix} = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta\, m_W \\ \sqrt{2}\cos\beta\, m_W & \mu \end{pmatrix}.$$

Rotate into mass eigenstates by two unitary matrices:

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix}, \qquad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix}. \qquad \mathbf{U}^* \mathbf{X} \mathbf{V}^{-1} = \begin{pmatrix} m_{\tilde{\chi}_1} & 0 \\ 0 & m_{\tilde{\chi}_2} \end{pmatrix},$$

Often: $\tilde{\chi}_2^0$ degenerate with $\tilde{\chi}_1^\pm$

The gluino

Color octet fermion $\Rightarrow$ cannot mix with any other MSSM state

Pure QCD interactions

GUT-inspired models imply at any RG scale (up to tiny 2-loop corrections):

$$M_3 = \frac{\alpha_s}{\alpha} \sin^2\theta_W \, M_2 = \frac{3}{5}\frac{\alpha_s}{\alpha} \cos^2\theta_W \, M_1$$

Translates to mass relations near TeV scale:

$$\boxed{M_3 : M_2 : M_1 \approx 6 : 2 : 1}$$

Crucial for SUSY detection:

- Proof its fermion nature
- Proof its Majorana nature
- Proof its octet nature

Exercise 12: Structure of MSSM Feynman rules

Consider the structure of the Feynman rules of the MSSM. First, recall what general Lorentz structures are possible.

- Gauge self-couplings
- Fermion gauge interactions
- Sfermion gauge interactions
- all sorts of Yukawa couplings
- Triple and quartic scalar couplings
- Never try to count!

Models for SUSY-breaking communication

Remember:

$\Rightarrow$ phenomenology depends mainly on mechanism for communicating SUSY breaking rather than on SUSY-breaking mechanism itself

"Hidden sector":	$\longrightarrow$	Visible sector:
SUSY breaking		MSSM

"Gravity-mediated": CMSSM, mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
...

SUGRA: mediating interactions are gravitational

GMSB: mediating interactions are ordinary electroweak and QCD gauge interactions

AMSB: SUSY breaking on a different brane in a higher-dimensional theory

All constrained models are special versions of the MSSM !!!

Gravity-mediated SUSY breaking (aka mSUGRA)

$\Rightarrow$ Quantum field theory of supergravity: graviton and gravitino

QFT with spin 2 and spin $\frac{3}{2}$ field is not renormalizable

$\Rightarrow$ cannot be extended to arbitrarily high energies

$\Rightarrow$ QFT of supergravity has to be interpreted as effective theory

contains non-renormalizable terms prop. to inverse powers of M_{Pl}

Best candidate for fundamental theory: string theory

SUSY breaking in hidden sector:

$\Rightarrow$ supergravity Lagrangian contains non-renormalizable terms that communicate between hidden and visible sector $\sim 1/M_{\text{Pl}}^n$

Dimensional analysis:

SUSY breaking in hidden sector by v.e.v. $\langle F \rangle$ (dim $\langle F \rangle = \mathrm{mass}^2$)
coupling $\sim 1/M_{\mathrm{Pl}}$

require $m_{\mathrm{soft}} \to 0$ for $\langle F \rangle \to 0$ (no SUSY breaking) and for $M_{\mathrm{Pl}} \to \infty$ (vanishing gravitational interaction)

$$\Rightarrow \quad m_{\mathrm{soft}} \approx \frac{\langle F \rangle}{M_{\mathrm{Pl}}}$$

Wanted: $m_{\mathrm{soft}} \lesssim 1$ TeV (hierarchy problem)

$\Rightarrow \sqrt{\langle F \rangle} \approx 10^{11}$ GeV : scale of SUSY breaking in hidden sector

In general: $m_{\mathrm{gravitino}} = m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\mathrm{Pl}}}$

$\Rightarrow$ $m_{\frac{3}{2}} \approx m_{\mathrm{soft}}$, gravitational interactions

$\Rightarrow$ gravitino not important for collider phenomenology

Non-renormalizable terms in supergravity Lagrangian:

$$\mathcal{L}_{\rm NR} = -\frac{1}{M_{\rm Pl}} F_X \sum_a \frac{1}{2} f_a \lambda^a \lambda^a + \text{h.c.} - \frac{1}{M_{\rm Pl}^2} F_X F_X^* \, k^i_j \varphi_i \varphi^{*j}$$
$$- \frac{1}{M_{\rm Pl}} F_X \left(\frac{1}{6} y'^{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{2} \mu'^{ij} \varphi_i \varphi_j\right) + \text{h.c.}$$

F_X : (auxiliary) field for a chiral supermultiplet X in the hidden sector

φ_i, λ^a: scalar and gaugino fields in the MSSM

If $\sqrt{\langle F_X \rangle} \sim 10^{10}$–$10^{11}$ GeV

$\Rightarrow$ soft SUSY-breaking terms of MSSM with $m_{\rm soft} \approx 10^2$–$10^3$ GeV

Assumption of a “minimal” form of the supergravity Lagrangian

$\Rightarrow$ soft-breaking terms which obey “universality” and “proportionality”

Results in exactly the known MSSM Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\mathrm{soft}} \quad = \quad & -\frac{1}{2}\left(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}\widetilde{W} + M_3\tilde{g}\tilde{g}\right) + \mathrm{h.c.} \\
& - (m^2_{H_u} + |\mu|^2)H_u^+H_u - (m^2_{H_d} + |\mu|^2)H_d^+H_d - (bH_uH_d + \mathrm{h.c.}) \\
& - \left(\tilde{u}_R\mathbf{a_u}\widetilde{Q}H_u - \tilde{d}_R\mathbf{a_d}\widetilde{Q}H_d - \tilde{e}_R\mathbf{a_e}\widetilde{L}H_d\right) + \mathrm{h.c.} \\
& - \widetilde{Q}^+\mathbf{m_Q^2}\widetilde{Q} - \widetilde{L}^+\mathbf{m_L^2}\widetilde{L} - \tilde{u}_R\mathbf{m_u^2}\tilde{u}_R^* - \tilde{d}_R\mathbf{m_d^2}\tilde{d}_R^* - \tilde{e}_R\mathbf{m_e^2}\tilde{e}_R^*
\end{aligned}
$$

with 5 independent parameters at the GUT scale:

$$
\begin{aligned}
M_1 = M_2 = M_3 \quad &= \quad m_{1/2} \\
m^2_{H_u} = m^2_{H_d} = \mathbf{m_Q^2} = \mathbf{m_L^2} = \mathbf{m_u^2} = \mathbf{m_d^2} = \mathbf{m_e^2} \quad &= \quad m_0 \\
\mathbf{a_u} = \mathbf{a_d} = \mathbf{a_e} \quad &= \quad A_0 \\
& \quad\quad b \\
& \quad\quad |\mu|^2
\end{aligned}
$$

Still to do: parameter(Q_{GUT}) $\rightarrow$ parameter($Q_{\mathrm{electroweak}}$)

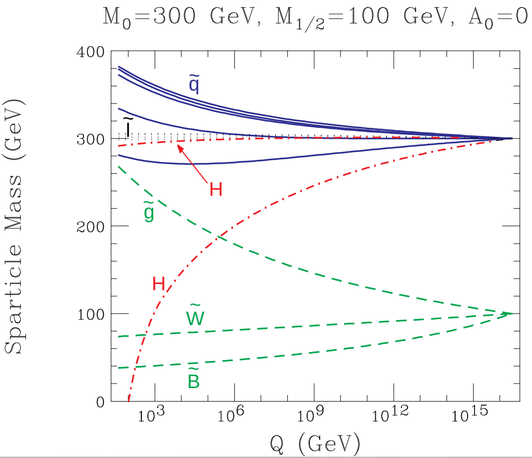
Low-energy parameters (at the electroweak (EW) scale) via "Renormalization group equations" (RGEs)

[RGE: equations that connect parameters at different energy scales]

$\Rightarrow M_1, M_2, M_3, m^2_{H_u}, m^2_{H_d}, \mathbf{m^2_Q}, \mathbf{m^2_L}, \mathbf{m^2_u}, \mathbf{m^2_d}, \mathbf{m^2_e}, \mathbf{a_u}, \mathbf{a_d}, \mathbf{a_e}, b, |\mu|^2$

at the EW scale

Example:



Five new parameters, if possible phases are ignored:

$$m_0^2,\ m_{1/2},\ A_0,\ b,\ \mu$$

Final "trick": require radiative electroweak symmetry breaking:
Require correct value of M_Z at the EW scale:

$$\begin{aligned} |\mu|^2 + m_{H_d}^2 &= b\tan\beta - M_Z^2/2\cos 2\beta \\ |\mu|^2 + m_{H_u}^2 &= b\cot\beta + M_Z^2/2\cos 2\beta \end{aligned}$$

$\Rightarrow$ $|\mu|$, b given in terms of $\tan\beta$, $\mathrm{sign}\,\mu$

$\Rightarrow$ Scenario characterized by

$$m_0^2,\ m_{1/2},\ A_0,\ \tan\beta,\ \mathrm{sign}\,\mu$$

Usually called 'CMSSM' (constrained MSSM) or 'mSUGRA'

In agreement with all phenomenological constraints (see below)

Summary: "supergravity inspired scenario", "mSUGRA"

characterized by five parameters:

$$m_0^2, \ m_{1/2}, \ A_0, \ \tan\beta, \ \text{sign}\,\mu$$

m_0: universal scalar mass parameter

$m_{1/2}$: universal gaugino mass parameter

A_0: universal trilinear coupling

$\tan\beta$: ratio of Higgs vacuum expectation values

sign(μ): sign of supersymmetric Higgs parameter

$m_0, m_{1/2}, A_0$: GUT scale parameters

$\Rightarrow$ particle spectra from renormalization group running to weak scale

Lightest SUSY particle (LSP) is usually lightest neutralino

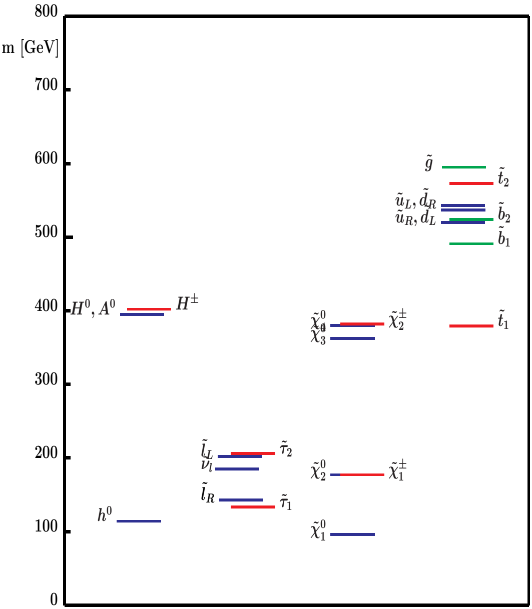
gaugino masses run in same way as gauge couplings

$\Rightarrow$ gluino heavier than charginos, neutralinos

“Typical” mSUGRA scenario (SPS 1a benchmark scenario):

SPS home page:

`www.ippp.dur.ac.uk/~georg/sps`



Gauge mediated SUSY breaking: GMSB

New chiral supermultiplets, "messengers" , couple to SUSY breaking in hidden sector

Couple indirectly to MSSM fields via gauge interactions

$\Rightarrow$ mediation of SUSY breaking via electroweak and QCD gauge interactions

$\Rightarrow$ $\approx$ flavor-diagonal

SUSY breaking already in messenger spectrum

$\Rightarrow$ SUSY particle masses from loop diagrams with messenger particles, gauge-interaction strength

$$\Rightarrow \qquad m_{\text{soft}} \approx \frac{\alpha_i}{4\pi} \frac{\langle F \rangle}{M_{\text{mess}}}, \qquad M_{\text{mess}} \sim \sqrt{\langle F \rangle}$$

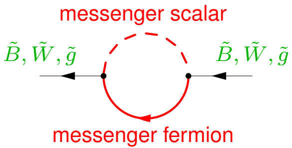
For $m_{\text{soft}} \lesssim 1$ TeV $\Rightarrow \sqrt{\langle F \rangle} \approx 10^4$–$10^5$ GeV

$\Rightarrow$ scale of SUSY breaking in hidden sector much lower than in SUGRA

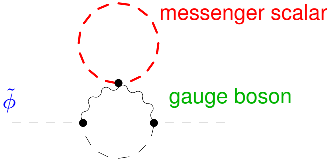
Gravitino mass: $m_{\frac{3}{2}} \approx \frac{\langle F \rangle}{M_{\rm Pl}} \approx 10^{-9}$ GeV

$\Rightarrow$ Gravitino is always the lightest SUSY particle (LSP)

Gaugino masses generated at one-loop order, $m_\lambda \approx \frac{\alpha_i}{4\pi}$



Scalar masses generated at two-loop order, $m_\varphi^2 \approx \left(\frac{\alpha_i}{4\pi}\right)^2$



$\Rightarrow$ Typical mass hierarchy in GMSB scenario between strongly interacting and weakly interacting particles $\sim \alpha_3/\alpha_2/\alpha_1$

GMSB scenario characterized by

$$M_{\text{mess}}, N_{\text{mess}}, \Lambda, \tan\beta, \text{sign}(\mu)$$

M_{mess}: messenger mass scale

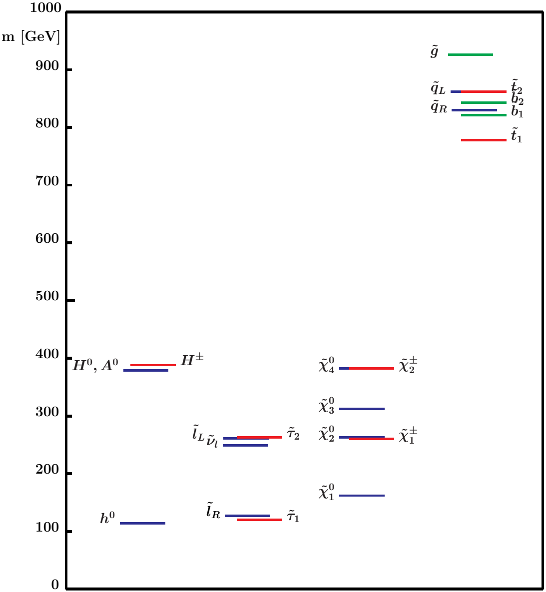
N_{mess}: messenger index (number of messenger multiplets)

Λ: universal soft SUSY breaking mass scale felt by low-energy sector

LSP is always the gravitino
next-to-lightest SUSY particle (NLSP): $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$

can decay into LSP inside or outside the detector

GMSB scenario with $\tilde{\tau}$ NLSP
(SPS 7 benchmark scenario):



Anomaly mediated SUSY breaking: AMSB

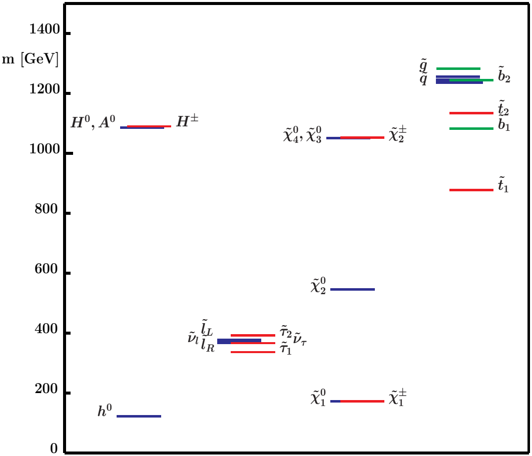
mAMSB scenario characterized by

$$m_{\rm aux}, m_0, \tan\beta, \text{sign}(\mu)$$

$m_{\rm aux}$: overall scale of SUSY particle masses

m_0: phenomenological parameter: universal scalar mass term

AMSB spectrum (SPS 9):

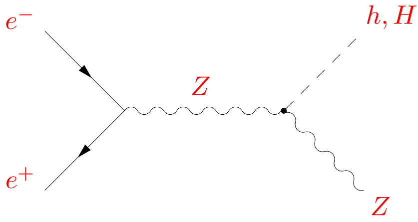


typical feature: very small neutralino–chargino mass difference

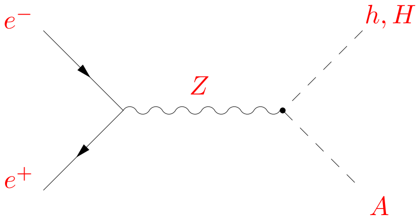
Outline

Search for neutral MSSM Higgs bosons at LEP:

$e^+e^- \to Zh, ZH$



$$\sigma_{hZ} \propto \sin^2(\beta - \alpha_{\rm eff})\sigma_{hZ}^{\rm SM}$$

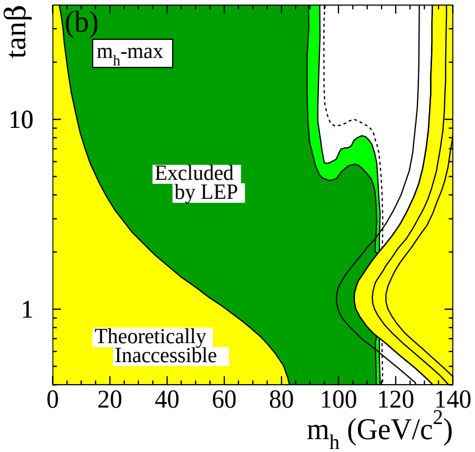


$$\sigma_{hZ} \propto \cos^2(\beta - \alpha_{\rm eff})\sigma_{AZ}^{\rm SM}$$

Constraints from the Higgs search at LEP [*LEP Higgs Working Group '06*]

Experimental search vs. upper m_h-bound

$m_h^{\max}$-scenario ($m_t = 174.3$ GeV, $M_{\text{SUSY}} = 1$ TeV):



$m_h > 92.8$ GeV
(expected: 94.9 GeV)
95% C.L.

$M_A > 93.4$ GeV
(expected: 95.2 GeV)

Parameter region where experimental lower bound on m_h is significantly lower than SM bound, $M_H > 114.4$ GeV, corresponds to $\sin^2(\beta - \alpha_{\text{eff}}) \ll 1$

"Excluded" $\tan\beta$ region:

$$0.7 < \tan\beta < 2.0$$

Note: this exclusion bound assumes

m_t, M_{SUSY} fixed, $m_t = 174.3$ GeV, $M_{\text{SUSY}} = 1$ TeV

no theoretical uncertainties included

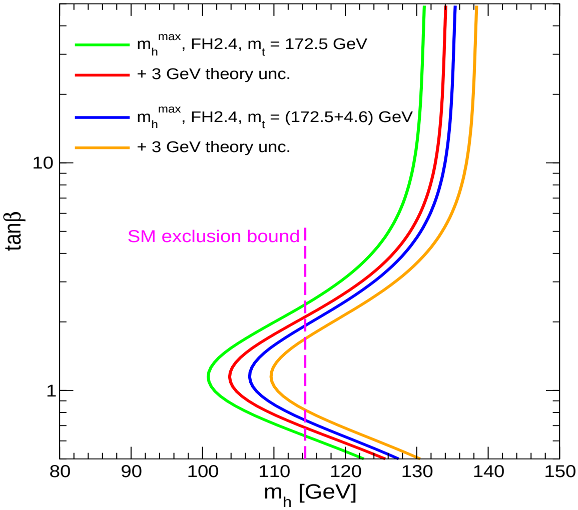
Note: new m_t value: $m_t = 171.4 \pm 2.1$ GeV [*Tevatron EWWG '06*]

Note: new m_t value: $m_t = 170.9 \pm 1.8$ GeV [*Tevatron EWWG '06*]

parametric uncertainty: $\delta m_h^{\text{para}} \approx \delta m_t$

Effect of new corrections and $m_t \to m_t + 2\,\sigma_{m_t}$

[*Heinemeyer et al. '05*]



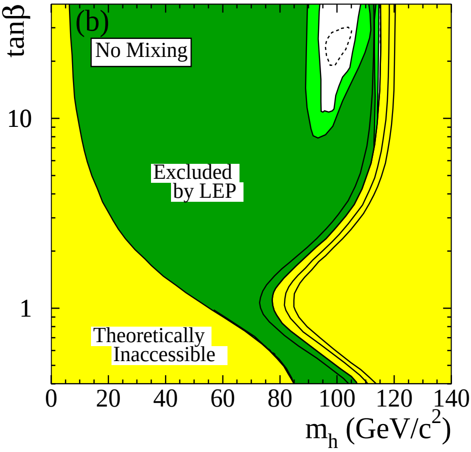
$\Rightarrow$ precise knowledge of m_t important!

$\Rightarrow$ Low $\tan\beta$ LEP exclusion region can vary strongly

Constraints from the Higgs search at LEP [*LEP Higgs Working Group '06*]

Experimental search vs. upper m_h-bound

no-mixing scenario ($m_t = 174.3$ GeV, $M_{\rm SUSY} = 1$ TeV):



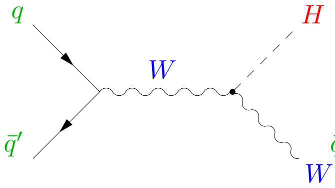
$m_h > 93.6$ GeV (expected: 96.0 GeV) 95% C.L.

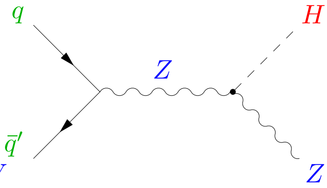
$M_A > 93.6$ GeV (expected: 96.4 GeV)

Current Higgs search: Tevatron (Fermilab)

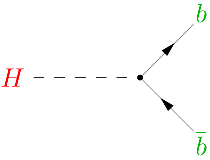
Tevatron: $p\bar{p}$ accelerator:

Dominant production processes:





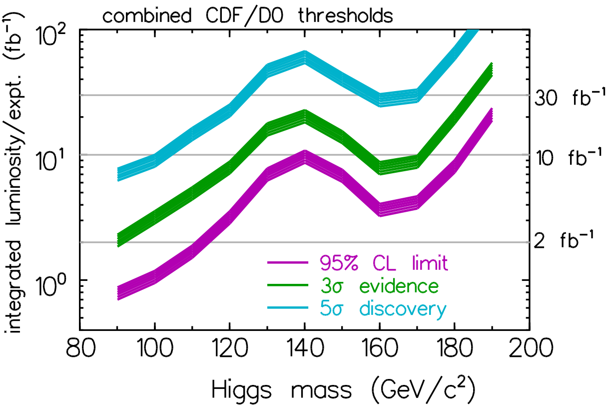
Dominant decays: $H \to b\bar{b}$



Search for SUSY Higgs at the Tevatron:

$\Rightarrow$ very similar to SM Higgs search

Expectations for Higgs discovery at the Tevatron:



Unfortunately: luminosity problems in the start of RunII
$\Rightarrow$ progress slower than anticipated

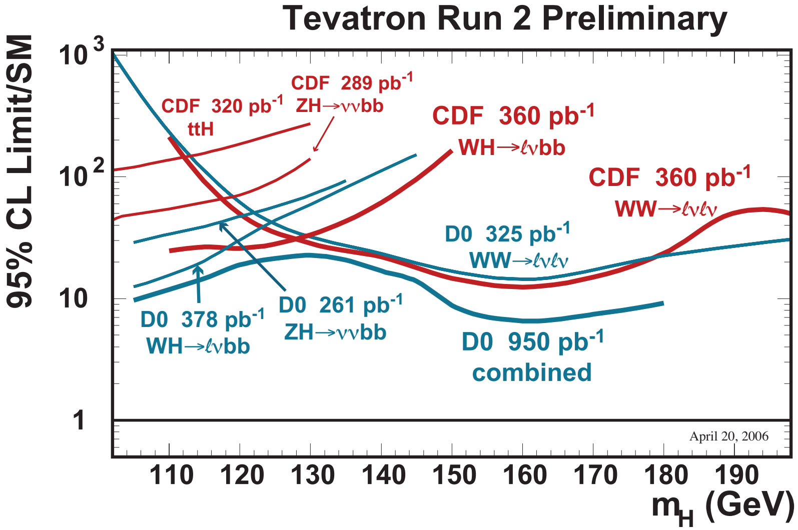
For SM Higgs boson with $M_H \sim 120$ GeV:

$\approx 2007/08$: sensitivity for 95% C.L. exclusion

≈ 2009: sensitivity for 3σ evidence

Current status of SM Higgs search:

[*CDF, DØ '06*]



Can they close the gap?

Tevatron search for a “SM-like” light Higgs:

Prediction in “simplified” versions of the MSSM:
($m_t^{\rm exp} = 172.5$ GeV, $\delta m_t^{\rm exp} = 2.3$ GeV, $M_{\rm SUSY} \lesssim \text{few}$ TeV)
[*A. Dedes et al. '03*] [*Heinemeyer et al. '04,'05*]

	max. m_h [GeV]	$\delta m_h / \delta m_t$	for $m_t^{\rm exp} + 2\delta m_t$
mSUGRA/CMSSM	125.5	0.65	128.5
mGMSB	119.9	0.70	123.1
mAMSB	121.4	0.58	124.1

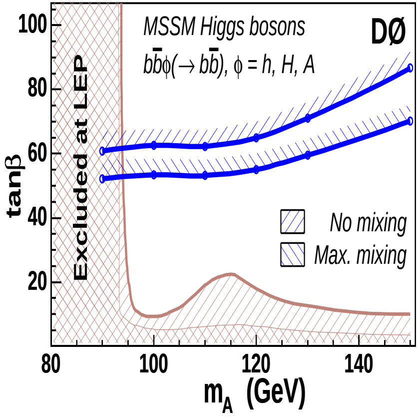
Exclusion potential of the Tevatron: $M_H^{\rm SM} \lesssim 130$ GeV

mSUGRA/CMSSM, mGMSB, mAMSB: no suppression of hVV coupling
$\Rightarrow$ SM bound applies

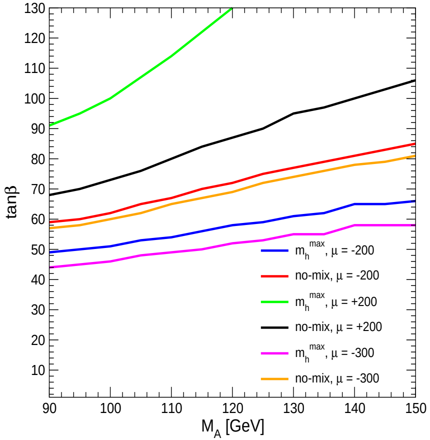
$\Rightarrow$ Tevatron can exclude mSUGRA/CMSSM, mGMSB, mAMSB, . . .

$\Rightarrow$ potentially huge impact on search strategies at LHC

Existing Tevatron data allows bounds on SUSY parameter space:



[*DØ '05*]



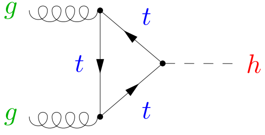
[*Theory prediction*]

Future search: LHC (CERN)

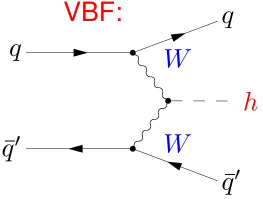
LHC: pp accelerator: start: summer 2008

Important production channel at the LHC:

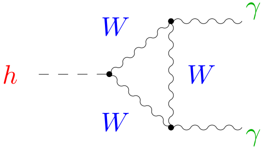
Gluon Fusion:

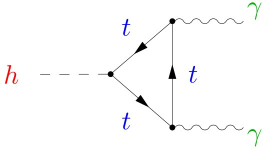


VBF:



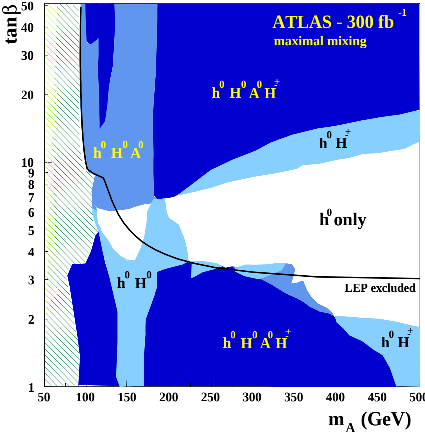
Important decay for Higgs mass measurement:

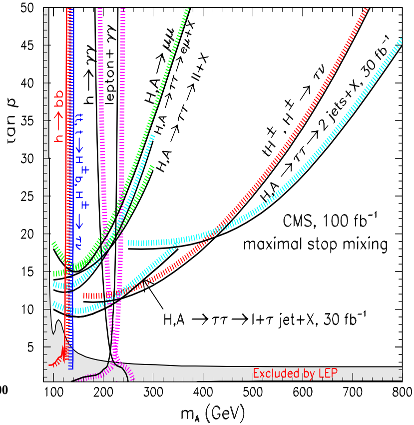




SM Higgs search at the LHC: $\Rightarrow$ full parameter accessible

MSSM Higgs discovery contours in M_A–$\tan\beta$ plane ($m_h^{\max}$ benchmark scenario): [ATLAS '99] [CMS '03]

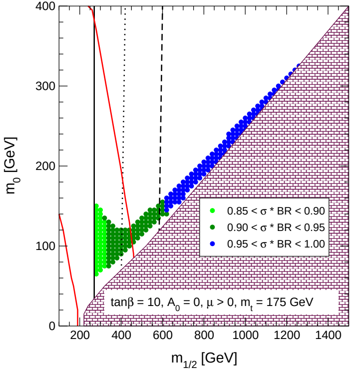


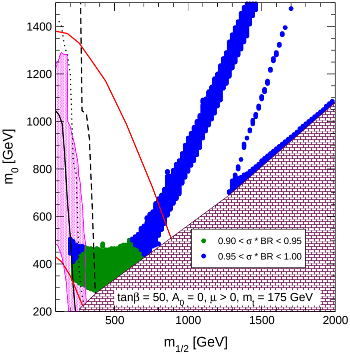


Higgs searches at the Tevatron and the LHC

mSUGRA scenario with CDM constraints $\mu > 0, \tan\beta = 10, 50$:

$$\Big[\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)\Big]_{\mathrm{CMSSM}} \Big/ \Big[\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)\Big]_{\mathrm{SM}}:$$





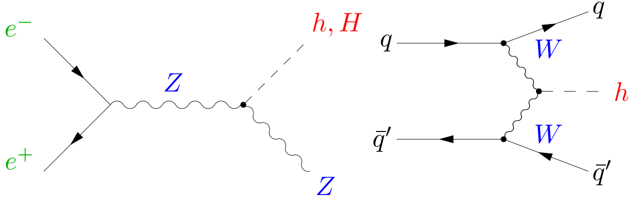
$\Rightarrow$ no significant suppression of $\sigma(gg \to h) \times \mathrm{BR}(h \to \gamma\gamma)$ compared to SM

$\Rightarrow$ Discovery of lightest Higgs boson within about one year at LHC possible

Future search: ILC

ILC: e^+e^- accelerator: possible start: 2020-25

Higgs-Strahlung: Weak boson fusion:



$\Rightarrow$ Measurement of masses, couplings, ... in per cent/per mille

$\Rightarrow$ Theory prediction have to be at the same level of accuracy!

$\Rightarrow$ Strong sensitivity to high scales of the (and beyond the) MSSM

Precision physics in the MSSM Higgs sector

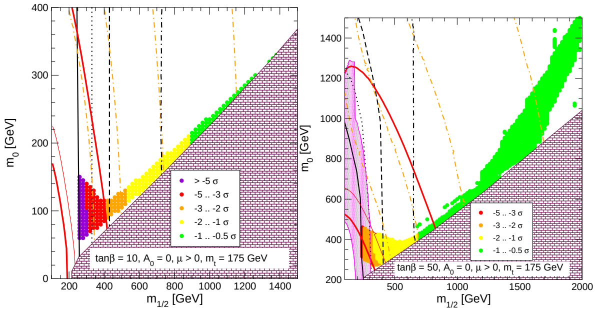
Precise measurement of Higgs branching ratios

$\Rightarrow$ Sensitivity to deviations SM / MSSM

E.g.: Prediction for $\sigma(e^+e^- \to Zh) \times \mathrm{BR}(h \to WW^*)$ in parameter region allowed by cosmology: comparison mSUGRA – SM:

[*J. Ellis et al. '02*]

$\mu > 0, \tan\beta = 10, 50$:



$\Rightarrow$ In allowed parameter space: sizable deviations from SM predictions for precision observables in the Higgs sector possible

SUSY at colliders – Sparticle searches

In order to establish SUSY experimentally:

Need to demonstrate that:

– every particle has superpartner
– their spins differ by $1/2$
– their gauge quantum numbers are the same
– their couplings are identical
– mass relations hold

...

$\Rightarrow$ Precise measurements of masses, branching ratios, cross sections, angular distributions, ... mandatory for

- establishing SUSY experimentally
- disentangling patterns of SUSY breaking

$\Rightarrow$ We need both: hadron colliders (Tevatron/LHC) and high luminosity ILC

SUSY searches at the Tevatron, Run II:

compared to Run I: $\approx 100\times$ higher luminosity, slightly increased energy (1.8 $\to$ 1.96 TeV)

Limited mass window in which discovery of SUSY particles above Run I is possible

Best prospects for:

- 'Trilepton signal': $\tilde{\chi}_2^0 \tilde{\chi}_1^+ \to \ell^+ \ell^- \tilde{\chi}_1^0 \ell^+ \nu \tilde{\chi}_1^0$
- $\tilde{t}$, $\tilde{b}$, $\tilde{g}$ searches
- SUSY Higgs searches in region of large $\tan\beta$

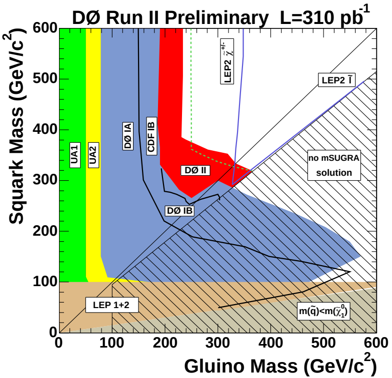
SUSY searches at the LHC:

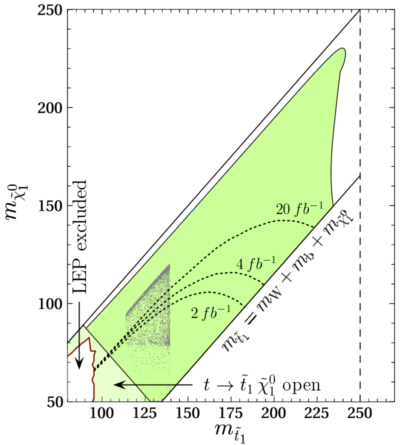
Dominated by production of colored particles: gluino, squarks

Very large mass range in the searches for jets $+$ missing energy

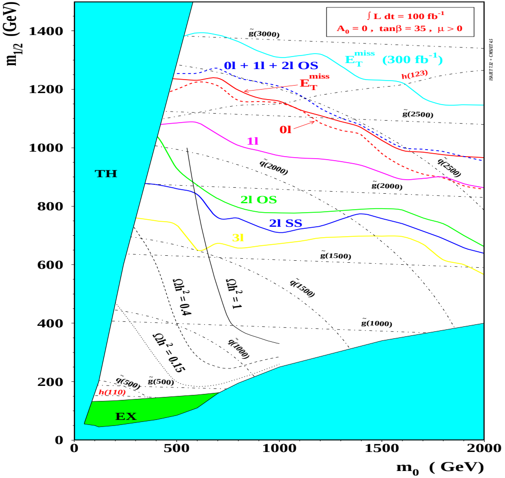
$\Rightarrow$ gluino, squarks accessible up to 2–3 TeV

Search for gluinos, squarks and stops :





Discovery reach contours in m_0–$m_{1/2}$ plane (mSUGRA scenario) for various final states with 100 fb^{-1}: [*CMS '99*]



⇒ discovery of SUSY particles expected if low-energy SUSY is realized

Production of SUSY particles at the LHC will in general result in complicated final states, e.g.

$$\tilde{g} \to \bar{q}\tilde{q} \to \bar{q}q\tilde{\chi}_2^0 \to \bar{q}q\tilde{\tau}\tau \to \bar{q}q\tau\tau\tilde{\chi}_1^0$$

Production of uncolored particles via cascade decays often dominates over direct production

Many states are produced at once

$\Rightarrow$ **Main background for SUSY is SUSY itself!**

Searches for MSSM Higgs bosons:

good prospects for detecting light Higgs h

H/A discovery possible in significant part of parameter space

In order to establish SUSY experimentally:

Need to demonstrate that:

- every particle has superpartner
- their spins differ by $1/2$
- their gauge quantum numbers are the same
- their couplings are identical
- mass relations hold

...

⇒ Precise measurements of masses, branching ratios, cross sections, angular distributions, ... mandatory for

- establishing SUSY experimentally
- disentangling patterns of SUSY breaking

Requires clean experimental environment, high luminosity, beam polarization, . . .

$\Rightarrow$ High luminosity ILC necessary, complementary to hadron machines

SUSY searches at the ILC:

Clean signatures, small backgrounds

Thresholds for pair production of SUSY particles

$\Rightarrow$ precise determination of mass and spin of SUSY particles, mixing angles, complex phases, . . .

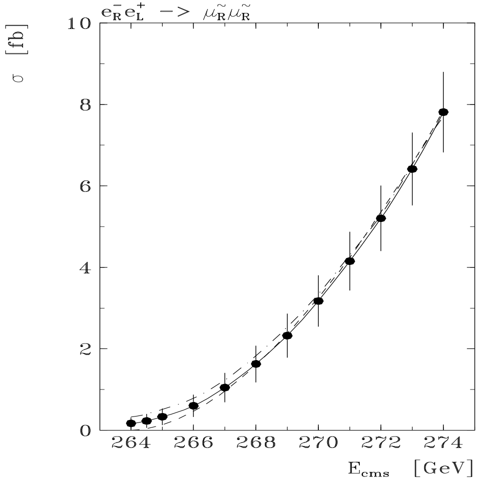
Limited by kinematic reach

Good prospects for production of uncolored particles

$\Rightarrow$ LHC / ILC complementarity

Example for SUSY physics at the ILC (I):

Determination of mass and spin of $\tilde{\mu}_R$ from production at threshold: [*TESLA TDR '01*]



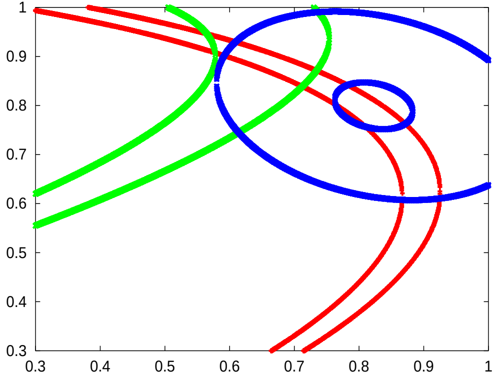
$$\Rightarrow \quad \frac{\Delta m_{\tilde{\mu}_R}}{m_{\tilde{\mu}_R}} < 1 \times 10^{-3}$$

$\Rightarrow$ test of $J = 0$ hypothesis

Example for SUSY physics at the ILC (II):

Determination of ϕ_R, ϕ_L in neutralino sector from measurement of $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ and $P_{e^-} = \pm 80\%, P_{e^+} = \pm 60\%$ $\mathcal{L} = 500$ fb^{-1}:

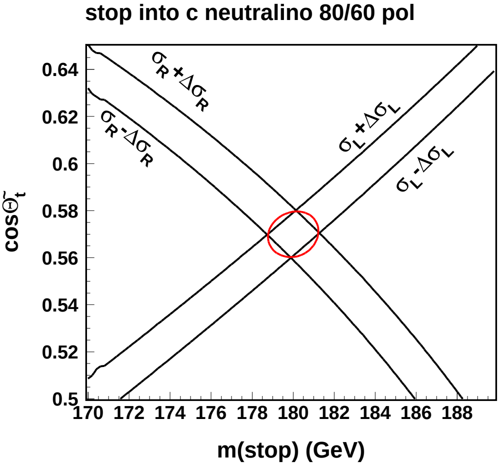
[*K. Desch, J. Kalinowski, G. Moortgat-Pick, M. Nojiri, G. Polesello '03*]



$\Rightarrow \cos 2\phi_L = [0.62, 0.72], \cos 2\phi_R = [0.87, 0.91]$

Example for SUSY physics at the ILC (III):

Determination of $m_{\tilde{t}_1}$, $\theta_{\tilde{t}}$ from $\sigma(e^+e^- \to \tilde{t}_1\tilde{t}_1)$ with polarized beams:
[*R. Keränen, H. Nowak, A. Sopczak '00*]



$$\Rightarrow \frac{\Delta m_{\tilde{t}_1}}{m_{\tilde{t}_1}} \approx 0.5\%,$$

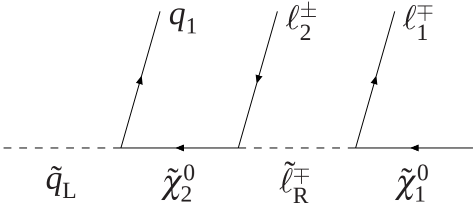
$$\frac{\Delta \cos\theta_{\tilde{t}}}{\cos\theta_{\tilde{t}}} \approx 1.5\%$$

Example (I): SUSY parameters at LHC and ILC

Reconstruction of sparticle masses at the LHC

[*B. Gjelsten, E. Lytken, D. Miller, P. Osland, G. Polesello, M. Chiorboli, A. Tricomi*]

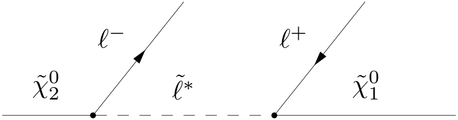
Complicated decay chains for squarks and gluinos



Examples worked out for SPS1a from ATLAS and CMS
main tool: dilepton "edge" from $\tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0$
$\longrightarrow$ Exercise #13

Exercise 13: Dilepton edges

Consider the following decay chain:



a) Show that in the rest frame of the particle A, the momentum of the particles B and C in the decay $A \to B + C$ is given by

$$|\vec{p}_B| = |\vec{p}_C| = \frac{1}{2M_A}\sqrt{\lambda(M_A^2, m_b^2, m_c^2)}$$

with

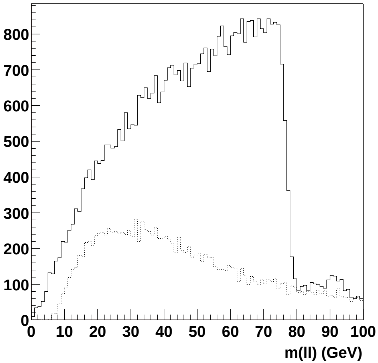
$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \; .$$

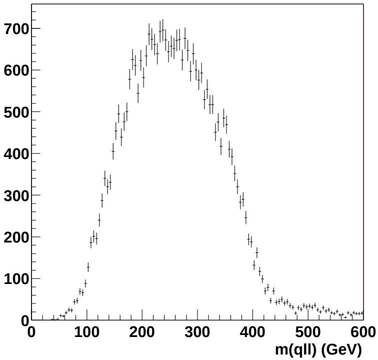
b) Calculate the dilepton edge, i.e. the maximum invariant mass the lepton pair can have. Neglect the lepton masses.
The result should be

$$(m_{\ell\ell}^{\text{max.}})^2 = \frac{(m^2_{\tilde{\chi}_2^0} - m^2_{\tilde{\ell}})(m^2_{\tilde{\ell}} - m^2_{\tilde{\chi}_1^0})}{m_{\tilde{\ell}^2}}$$

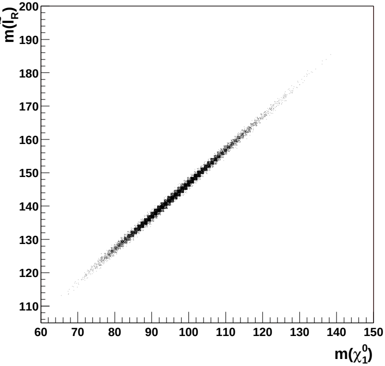
Sbottom/squark and gluino reconstruction:

Edge in same flavor-opposite sign lepton distribution (left), invariant mass distributions with kinematical endpoints (right)





Strong correlation between slepton mass and LSP mass, LSP mass can be constrained at LHC at the 10% level only:

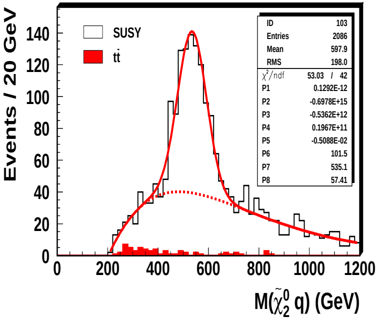


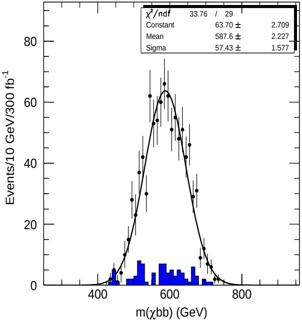
$\Rightarrow$ Take LSP mass as input from ILC

$\Rightarrow$ feed LSP mass from ILC in the LHC analysis

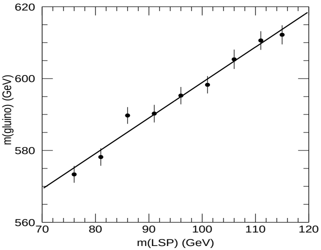
$\Rightarrow$ Get sbottom/squark mass if LSP mass is known

Squark peak (left) and gluino reconstruction from $(\chi_2^0 bb)$ invariant mass distribution (right):





$m_{\tilde{g}}$ as function of the LSP mass:



$\Rightarrow \Delta m_{\tilde{g}} \approx \Delta m_{\mathrm{LSP}}$

Accuracies for the case of the LHC alone (left) and with the ILC measurement of the LSP mass with $\delta m_{\rm LSP} = \pm 0.05$ GeV:

[*LHC/ILC Study Group report '04*]

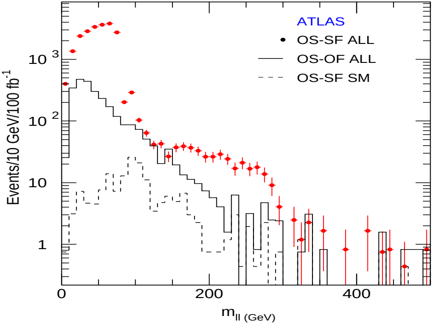
	LHC [GeV]	LHC + ILC [GeV]
$\Delta m_{\tilde{\chi}_1^0}$	4.8	0.05
$\Delta m_{\tilde{\chi}_2^0}$	4.2	0.08
$\Delta m_{\tilde{l}_L}$	4.8	0.05
$\Delta m_{\tilde{b}_1}$	7.1	5.7
$\Delta m_{\tilde{q}_L}$	8.7	4.9
$\Delta m_{\tilde{q}_R}$	7-12	5-11
$\Delta m_{\tilde{g}}$	8.0	6.5

$\Rightarrow$ ILC input improves accuracy significantly

One step further:

Determination of the mass of the heaviest neutralino at the LHC using ILC input from the neutralino/chargino sector:

[*J. Kalinowski, G. Moortgat-Pick, M. Nojiri, G. Polesello '03*]



⇒ Need besides LSP mass also masses of sleptons and charginos from ILC in order to correctly identify $\tilde{\chi}_4^0$

⇒ Feeding $m(\tilde{\chi}_4^0)$ back into ILC analysis improves accuracy of parameter determination at the ILC

Even further:

Full reconstruction of stop/sbottom parameters with LHC $\otimes$ ILC

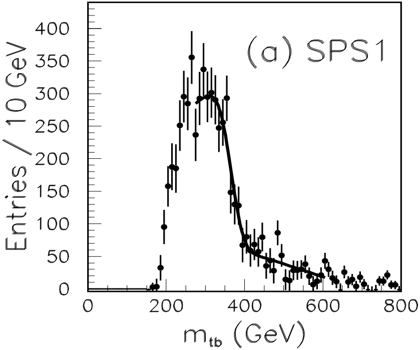
[*J. Hisano, K. Kawagoe, M. Nojiri '03*]

Complete set of electroweak SUSY parameters (from ILC) and branching ratios used to exploit LHC rate measurements

$\tilde{t}/\tilde{b}$ sector determined by 5 parameters, e.g. $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$, $m_{\tilde{t}_1}$, $\theta_{\tilde{b}}$, $\theta_{\tilde{t}}$

- Take $m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$ from previous study
- $\Rightarrow$ need three more observables:
  - tb invariant mass distribution
  - rate of "edge-events" in m_{tb} distribution (chargino chain)
  - rate of events in $\ell\ell b$ distribution ($\tilde{\chi}_2^0$ chain)

m_{tb} distribution for SPS1a:



Involved couplings are stop–sbottom–W, top–sbottom–chargino and stop–bottom–chargino

$\Rightarrow$ If chargino couplings + BRs are known then observed rates are sensitive to sbottom/stop mixing parameters

Necessary improvements in accuracy of theoretical prediction in order to match experimental precision at LHC, $\delta m_h^{\rm exp} \approx 0.2$ GeV:

- Uncertainty from experimental errors of input parameters:

  $\Rightarrow$ Complementarity example:

  In order to match

  experimental precision at LHC, $\delta m_h^{\rm exp} \approx 0.2$ GeV

  need

  ILC precision on m_t, $\delta m_t^{\rm exp} \lesssim 0.2$ GeV

- Uncertainty from unknown higher-order corrections:

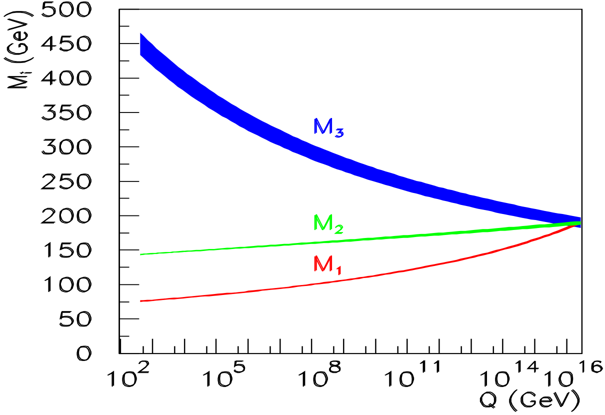
  $\Rightarrow$ Need improvement by more than a factor 10!

LHC/ILC synergy example (III):

If all low-energy parameters are known: Extrapolation to high scales

from combination of LHC and ILC results, precise measurement of masses of SUSY particles, couplings

E.g.: Test of gaugino mass unification
[*G. Blair, W. Porod, P. Zerwas '01*]



SUSY Simulations

Need precise predictions for cross sections and arbitrary distributions

Off-Shell and Interference effects important: $\mathcal{O}(10 - 30\%)$

[*Hagiwara et al., '05; Berdine et al., '07*]

Monte-Carlo event generators for multi-particle productions

- `WHIZARD` [*Kilian/Ohl/Reuter*]
  `http://whizard.event-generator.org`
- `Sherpa` [*Krauss/Schumann et al.*]
  `http://www.mc-sherpa.de`
- `Madgraph` [*Maltoni/Stelzer et al.*]
  `http://www.uiuc.edu/~madgraph`

Validated by

- Unitarity checks
- Gauge and SUSY invariance checks
- Direct comparison (see below)

Read in input values via SUSY Les Houches Accord (SLHA) file

Check your own codes against reference list:

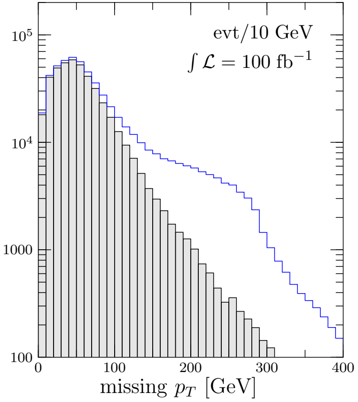
`http://whizard.event-generator.org/susy_comparison.html`

Sample LHC and ILC SUSY Simulations

LHC: simulation with WHIZARD

Sbottom production:
$pp \to b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0$ including full matrix elements

Bumps in missing p_T

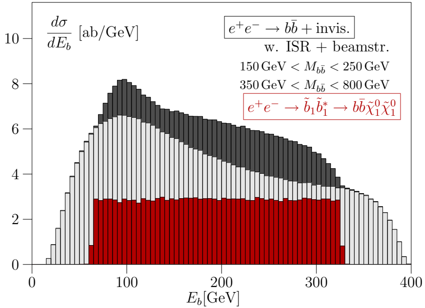


ILC: Sbottom production

Exclusive final state:
$e^+e^- \to b\bar{b}\tilde{\chi}_1^0\tilde{\chi}_1^0$

Intermediate states:
$\tilde{\chi}_i^0\tilde{\chi}_j^0, H^0A^0, Zh^0, ZH^0$

In red: Breit-Wigner approximation, grey: plus SM/SUSY bkgds



Electroweak precision observables

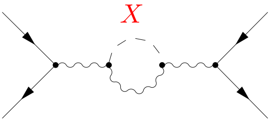
Precision Observables (POs):

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\mathrm{eff}}, a_\mu$	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections



$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

Electroweak precision tests: SM vs. MSSM

Electroweak precision measurements:

M_Z [GeV]	$=$	91.1875 ± 0.0021	0.002%
G_μ [GeV^{-2}]	$=$	$1.16637(1)\,10^{-5}$	0.0009%
m_t [GeV]	$=$	172.7 ± 2.9	1.7%
M_W [GeV]	$=$	80.426 ± 0.034	0.04%
$\sin^2\theta_{\text{eff}}^{\text{lept}}$	$=$	0.23148 ± 0.00016	0.07%
Γ_Z [GeV]	$=$	2.4952 ± 0.0023	0.09%

Quantum effects of the theory: loop corrections: $\sim \mathcal{O}(1\%)$

SM: M_H is free parameter

precise measurement of $M_W, \sin^2\theta_{\text{eff}}, \ldots \Rightarrow$ constraints on M_H

MSSM: m_h is predicted

precise meas. of $M_W, \sin^2\theta_{\text{eff}}, m_h, \ldots \Rightarrow$ constr. on $m_{\tilde t}, \theta_{\tilde t}, m_{\tilde b}, \theta_{\tilde b}, \ldots$

Indirect determination of m_t from precision data:

$$m_t = 178.9^{+12.0}_{-9.0} \text{ GeV}$$

Direct measurement:

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

Leading corrections to precision observables:

$$\sim m_t^2$$
$$\sim \ln M_H$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

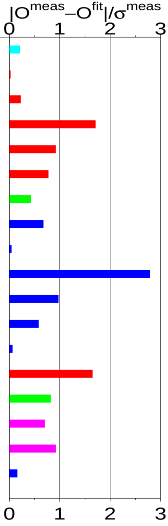
– experimental error of input parameters: m_t, $\Delta\alpha_{\text{had}}$, ...

Global fit of the SM to all data:

Basic assumption: SM provides correct description of experimental data

Comparison of SM prediction with the data: [*LEPEWWG '06*]

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02766
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01640
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479
R_b	0.21629 ± 0.00066	0.21585
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0741
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1479
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.392 ± 0.029	80.371
Γ_W [GeV]	2.147 ± 0.060	2.091
m_t [GeV]	171.4 ± 2.1	171.7



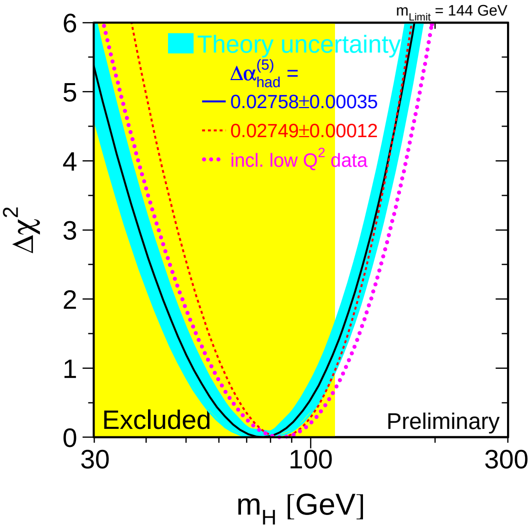
Global fit to all SM data:
[*LEPEWWG '06*]

$\Rightarrow M_H = 85^{+39}_{-28}$ GeV

$M_H < 166$ GeV, 95% C.L.

Assumption for the fit:
SM incl. Higgs boson

$\Rightarrow$ no confirmation of Higgs mechanism



$\Rightarrow$ Higgs boson seems to be light, $M_H \lesssim 200$ GeV

Powerful precision observables for SUSY:

M_W, $\sin^2\theta_{\rm eff}$, m_h, $(g-2)_\mu$, b physics, ...

Comparison of current experimental errors with anticipated precision at

Run II of the Tevatron ($p\bar{p}$ collider, $E_{\rm CM} \approx 2$ TeV; ≥ 2001),

LHC (pp collider, $E_{\rm CM} \approx 14$ TeV; $\gtrsim 2008$,

ILC (e^+e^- collider, $E_{\rm CM} \approx 500$–1000 GeV, $\gtrsim 201x$?)

with and without low-energy running mode (GigaZ)

	now	Tev. Run II	LHC	ILC	GigaZ
$\delta\sin^2\theta_{\rm eff}(\times 10^5)$	16	78	14–20	—	1.3
δM_W [MeV]	29	20	15	10	7
δm_t [GeV]	2.1	1.2	1.0	0.1	0.1
δm_h [MeV]	—	$\mathcal{O}(2000)$	200	50	—

M_W, $\sin^2\theta_{\text{eff}}$, m_h, $(g-2)_\mu$, b physics, ...

A) Theoretical prediction for M_W in terms

of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu} \left(\frac{1}{1 - \Delta r}\right)$$

$\Updownarrow$

loop corrections

B) Effective mixing angle:

$$\sin^2\theta_{\text{eff}} = \frac{1}{4\,|Q_f|} \left(1 - \frac{\text{Re}\, g_V^f}{\text{Re}\, g_A^f}\right)$$

Higher order contributions:

$$g_V^f \to g_V^f + \Delta g_V^f, \quad g_A^f \to g_A^f + \Delta g_A^f$$

Corrections to M_W, $\sin^2\theta_{\rm eff}$

→ can be approximated with the ρ**-parameter:**

ρ measures the relative strength between
neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta\rho} \qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2} \frac{c_{\rm w}^2}{c_{\rm w}^2 - s_{\rm w}^2} \Delta\rho, \qquad \Delta\sin^2\theta_W^{\rm eff} \approx -\frac{c_{\rm w}^2 s_{\rm w}^2}{c_{\rm w}^2 - s_{\rm w}^2} \Delta\rho$$

→ Exercise #8

Additional sources for sizable radiative corrections in the MSSM:

- Mass and couplings of light $\mathcal{CP}$-even Higgs: Large Yukawa corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right), \ldots$
- t / $\tilde{t}$ loops, b / $\tilde{b}$ loops (for large $\tan\beta$)
- Corr. to relation between bottom mass and bottom Yukawa coupling:

$$y_b = \frac{\sqrt{2}}{v \cos\beta} \frac{m_b}{1 + \Delta_b},$$

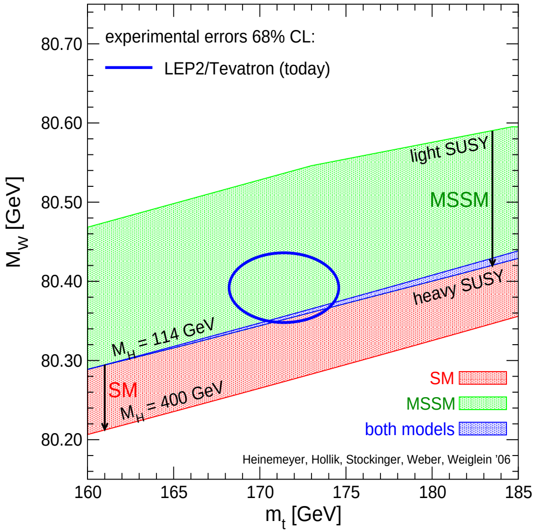
$$\Delta_b = \mu \tan\beta \left(\alpha_s I(\ldots) + \alpha_t I(\ldots)\right)$$

$\Rightarrow$ Coupling non-perturbative for $\Delta_b \to -1$

- Loop contributions from light SUSY particles
- ...

MSSM Example (I): Prediction for M_W in the SM and the MSSM :

[*A.M. Weber et al. '06*]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of M_H^{SM}

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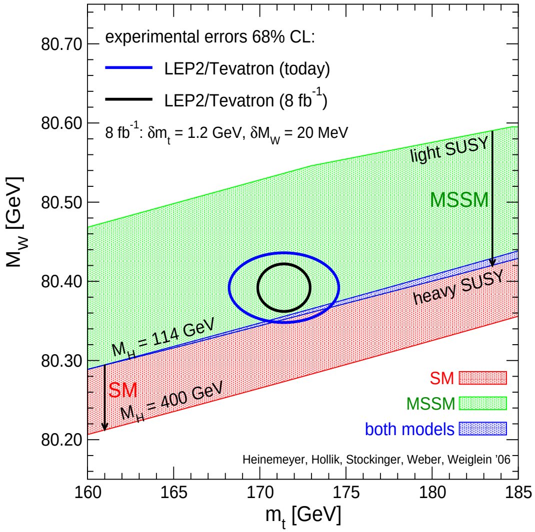
[*A.M. Weber et al. '06*]



MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of $M_H^{\rm SM}$

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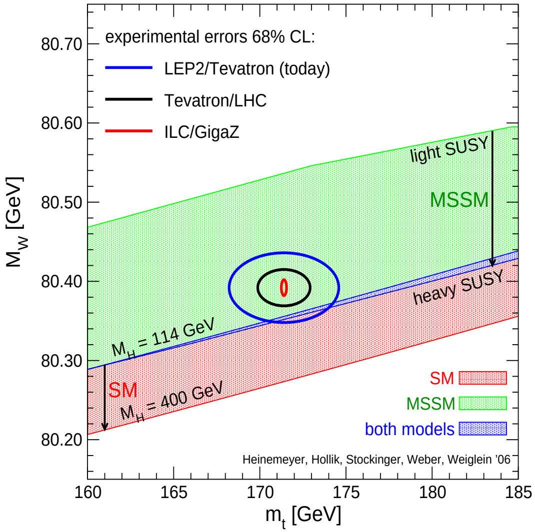


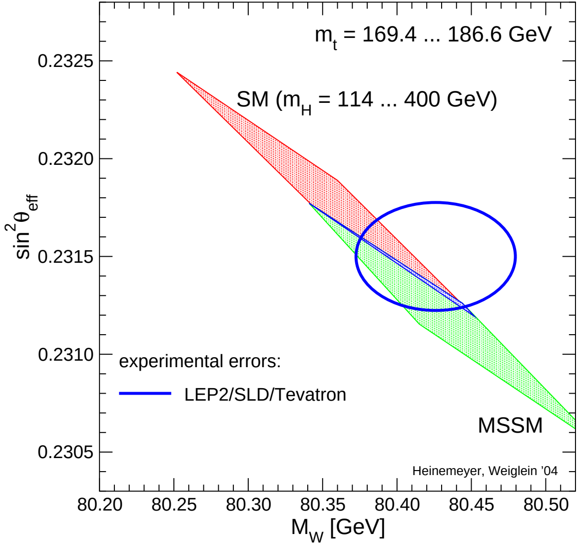
MSSM band:
scan over
SUSY masses

overlap:
SM is MSSM-like
MSSM is SM-like

SM band:
variation of $M_H^{\rm SM}$

MSSM Example (II):

Prediction for M_W and $\sin^2\theta_{\rm eff}$ in the SM and the MSSM :

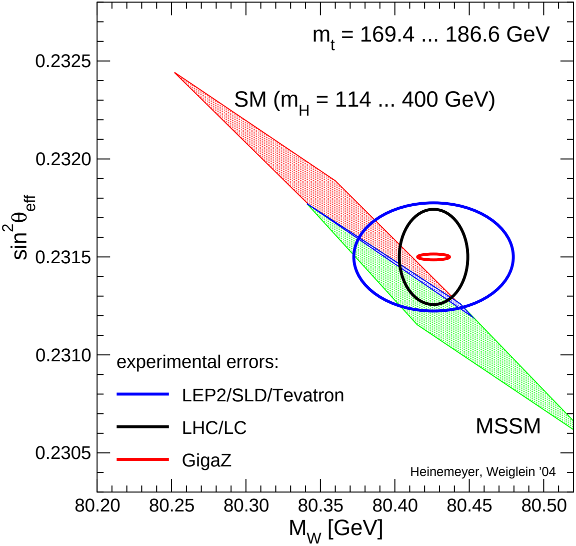


MSSM uncertainty:
unknown masses of SUSY particles

SM uncertainty:
unknown Higgs mass

MSSM Example (II):

Prediction for M_W and $\sin^2\theta_{\text{eff}}$ in the SM and the MSSM :



MSSM uncertainty:
unknown masses
of SUSY particles

SM uncertainty:
unknown Higgs mass

C) Theoretical prediction of the lightest MSSM Higgs boson mass: m_h

Contrary to the SM: m_h is not a free parameter

MSSM tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:

$$\Delta m_h^2 \sim G_\mu m_t^4 \ln \left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$$

> The MSSM Higgs sector is connected to all other sector via loop corrections
> (especially to the scalar top sector)

Measurement of m_h, Higgs couplings $\Rightarrow$ test of the theory

LHC: $\Delta m_h \approx 0.2$ GeV
ILC: $\Delta m_h \approx 0.05$ GeV

$\Rightarrow$ m_h will be (the best?) electroweak precision observable

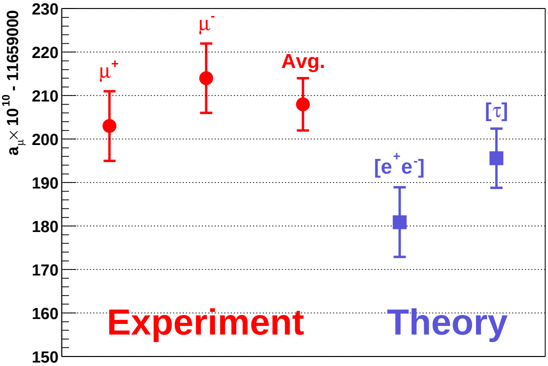
Muon anomalous magnetic moment: $(g-2)_\mu$

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \qquad F_2(0) = (g-2)_\mu$$

Overview about the current experimental and SM (theory) result:

[*g-2 Collaboration, hep-ex/0401008*]



Overview of the SM theory evaluation:

Source	contr. to $a_\mu [10^{-10}]$	
LO hadr.	$\sim 695 \pm 7\ (e^+e^-)$	[Davier et al, Hagiwara et al. '03]
		[Ghozzi, Jegerlehner '03]
	$711.0 \pm 6\ (\tau)$	[Davier, Eidelman, Höcker, Zhang '03]
LBL	8 ± 4	[Knecht, Nyffeler '02]
	13.6 ± 2.5 tbc	[Melnikov, Vainshtein '03]
EW 1L	19	
EW 2L	-4	[Czarnecki, Krause, Marciano '98]
exp. res.	6	[BNL E821 '04]

$\rightarrow$ "Isospin breaking effects" in τ data problematic [Ghozzi, Jegerlehner '03]

e^+e^- data: good agreement between new SND, CMD2, KLOE data

$\Rightarrow$ general agreement at ICHEP'06 Moscow: discard τ data

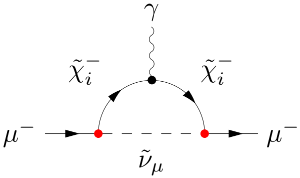
$$a_\mu^{\text{exp}} - a_\mu^{\text{theo,SM}} \approx (27.5 \pm 8.4) \times 10^{-10}$$

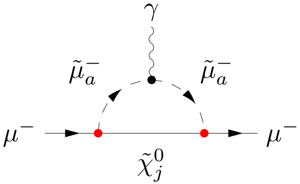
MSSM contributions to $(g-2)_\mu$:

Coupling of muon to magnetic field : $\mu - \mu - \gamma$ coupling

$$\bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i}{2m_\mu} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p) A_\mu \qquad F_2(0) = (g-2)_\mu$$

Feynman diagrams for MSSM 1L corrections:





Enhancement factor as compared to SM:

$$\mu - \tilde{\chi}_i^\pm - \tilde{\nu}_\mu \quad : \quad \sim m_\mu \; \tan\beta$$
$$\mu - \tilde{\chi}_j^0 - \tilde{\mu}_a \quad : \quad \sim m_\mu \; \tan\beta$$

Precision Observables in the CMSSM: collider implications

What do we know about the SUSY mass scale?

1. Coupling constant unification $\Rightarrow M_{\text{SUSY}} \approx 1$ TeV
2. LSP should be cold dark matter $\Rightarrow M_{\text{SUSY}} \lesssim 1$ TeV
3. Indirect hints from existing data?

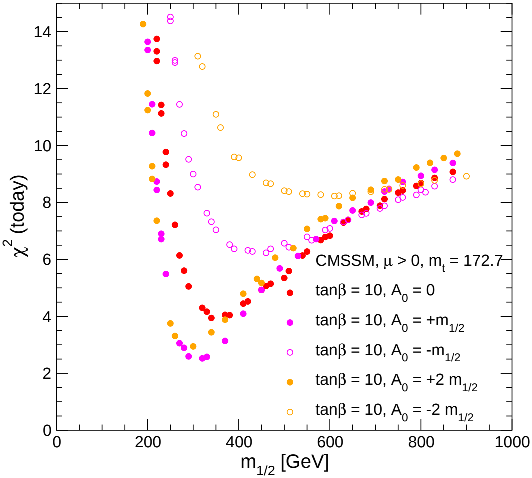
   [*J. Ellis, S.H., K. Olive, G. Weiglein '04*]

   – Focus on mSUGRA/CMSSM
   free parameters: $m_{1/2}, m_0, A_0, \tan\beta$

   – hard constraint: LSP gives right amount of cold dark matter
   only thin strips allowed in the $m_{1/2}$–m_0 plane
   fix $\tan\beta = 10, 50$ (lower/upper edge in CMSSM) and $\mu > 0$

   – Use existing data of M_W, $\sin^2\theta_{\text{eff}}$, $\text{BR}(b \to s\gamma)$, $(g-2)_\mu$
   $\Rightarrow \chi^2$ fit with these observables
   determine best fit values of $m_{1/2}, m_0, A_0$
   $\Rightarrow$ best fit values for masses, couplings, ...

χ^2 fit result for $m_{1/2}$: ($m_t = 172.7$ GeV, $\tan\beta = 10$, $A_0/m_{1/2}$ varied)



Very good fit!

Best fit obtained for
$m_{1/2} \approx 320$ GeV
$A_0 \lesssim 320$ GeV
$m_0 \approx 90$ GeV

$\Rightarrow$ SUSY particles relatively light

$\Rightarrow$ very good prospects for the LHC/ILC

similar for $\tan\beta = 50$

MSSM Dark matter

The LSP as a dark matter candidate

Astrophysical data (cosmic microwave background, . . .) $\Rightarrow$ existence of non-baryonic cold dark matter in the Universe

SUSY with R parity conservation $\Rightarrow$ LSP relic density falls naturally in favored range if $m_{\mathrm{LSP}} \lesssim 1$ TeV

WMAP results (2006 update: no significant change)
[*C. Bennet et al. '03*] , [*D. Spergel et al. '03*]

$\Rightarrow$ cold dark matter density:

$$\Omega_{\mathrm{CDM}} h^2 = 0.112 \pm 0.018 \qquad \text{at 95\% C.L.}$$

(h: normalized Hubble constant)

$\Rightarrow$ Constraints on the SUSY parameter space

Combination of all existing astrophysical data:

$\Rightarrow$ It all fits together

$$\Omega_{\rm tot} \approx 1$$

$$\Omega_M h^2 = 0.135^{+0.008}_{-0.009}$$

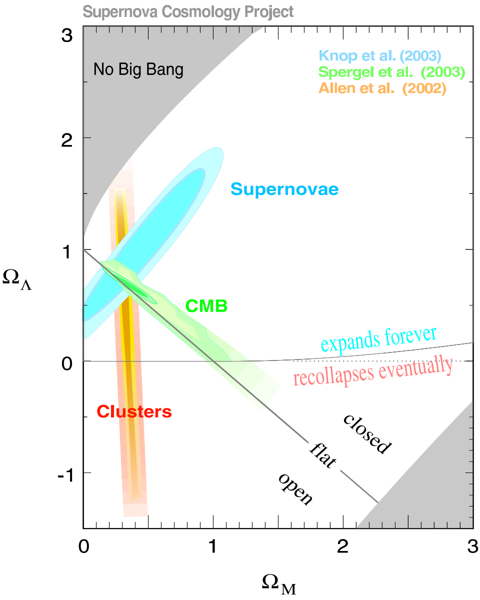
$$\Omega_B h^2 = 0.0224 \pm 0.0009$$

$$\Omega_\chi h^2 = 0.112 \pm 0.018$$

$$\Omega_\Lambda \approx 0.73$$

$\Omega_\chi \Rightarrow$ dark matter

$\Omega_\Lambda \Rightarrow$ dark energy …



Dark matter in the CMSSM parameter space:

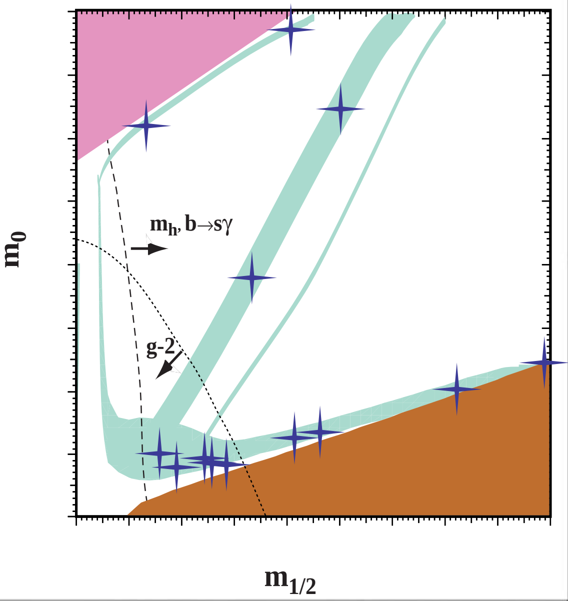
schematic representation
$(0.1 \leq \Omega_\chi h^2 \leq 0.3)$
[*K. Olive et al. '02*]

Despite its simplicity:
CMSSM fulfills all experimental bounds

Four mechanisms for "good" $\langle \sigma v \rangle$:

- Bulk
- stau coannihilation
- Higgs pole annihilation
- focus point

Crosses: benchmark points



Direct search for dark matter

Several methods:

(A) Search with earth-based detectors
dark matter scatters with matter in the detector
[*CDMS, CRESST, DAMA, EDELWEISS, GERDA, ZEPLIN, ...*]

(B) Search for photons from DM annihilation
[*ARGO, CELESTE, EGRET, GLAST, HEGRA, HESS, MAGIC, VERITAS, ...*]

(C) Search for neutrinos from DM annihilation
[*AMANDA, ANTARES, IceCube, ...*]

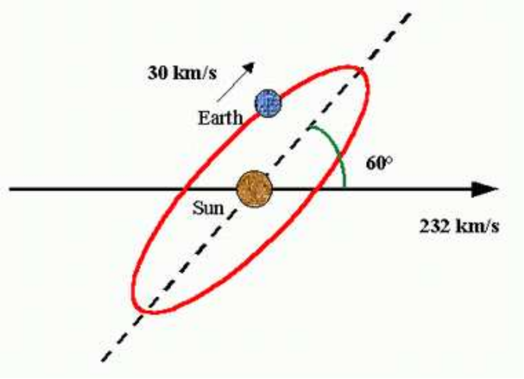
(D) Search for antimatter from DM annihilation
[*AMD, HEAT, ...*]

A) Search with earth-based detectors

⇒ Calculation of cross section for DM matter interaction
⇒ Comparison of calculation with measurement

Successful measurement with DAMA?

Idea: change of DM wind within the year



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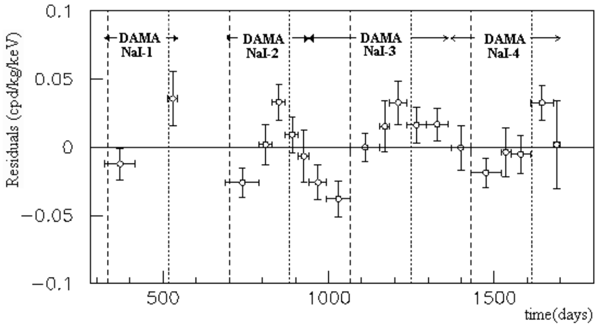
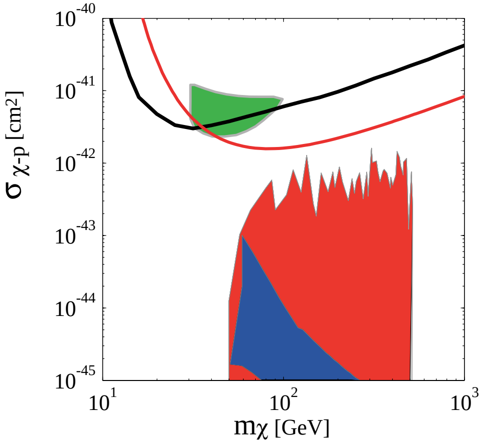


⇒ Change of number of events with a one-year period

Comparison of DAMA with CDMS'03 and EDELWEISS'03:



[*K. Olive et al. '03*] (situation in 2006 later)

⇒ DAMA measurement in conflict with other experiments

⇒ CMSSM/SUSY prediction in reach

B) Search for photons from DM annihilation: EGRET

DM annihilates (with each other) $\Rightarrow$ excess of photons

Excess of photons within our galaxy?
Two contradicting data analyses:

- [*de Boer et al. '04*]
  Photon excess is fitted with $\tilde{\chi}$ annihilation

  $\Rightarrow m_{\tilde{\chi}} \approx 65$ GeV
  Background under control?
  (extragalactic background ok)
- [*D. Elsässer, K. Mannheim '05*]
  Photon excess is fitted with $\tilde{\chi}$ annihilation
  $\Rightarrow m_{\tilde{\chi}} \approx 520$ GeV
  Background under control?
  (Excess only in extragalactic background)

Systematics?

Outline

Outlook

- Only two candidates can explain hierarchies:
  - Low-energy supersymmetry (focus here on SUSY!)
  - Dynamical symmetry breaking

- Plea for data:

  We need experimental information from Tevatron, LHC, ILC, ν experiments, dark matter searches, low-energy experiments, ... to verify / falsify models of new physics understanding

- The experiments in the next years will bring a decisive test of our ideas about low-energy SUSY

$\Rightarrow$ Very exciting prospects for the coming years

May we live in interesting times!

One Ring to Find Them, One Ring to Rule Them Out?