NLO Event Generation for Chargino Production at the ILC

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W. Kilian, JR, T. Robens, EPJ **C 48** (2006), 389; hep-ph/0610425

Loopfest VI, Fermilab, April 18, 2007

Motivation: Precision Analysis

SUSY: stabilizes hierarchy, dark matter, radiative symmetry breaking

Charginos SUSY partners of Higgs and EW gauge bosons

Charginos/neutralinos at LHC only from gluino/squark decay chains

ILC allows for precision measurements at least at per cent-level

- Measuring their properties helps proving SUSY
- Important for determination of SUSY dark matter content
- . . . and for determination of SUSY-breaking mechanism SPA project

For the rest: always SPS1a' SUGRA-scenario with

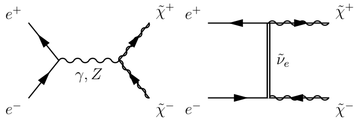
$m_0 = 70\,\mathrm{GeV} \quad m_{1/2} = 250\,\mathrm{GeV} \quad \tan\beta = 10 \quad \mathrm{sgn}\,\mu = 1 \quad A_0 = -300\,\mathrm{GeV}$

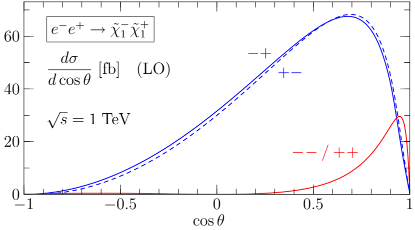
Chargino masses and widths:

	M	Γ	Γ/M
$\tilde\chi_1^+$	183.7 GeV	0.077 GeV	0.00042
$\tilde\chi_2^+$	415.4 GeV	3.1 GeV	0.0075

SPS1a'-preferred decay (2-step cascade): $\tilde\chi_1^+ \to \tilde\tau_1 \nu_\tau \to \tau^+ \tilde\chi_1^0 \nu_\tau$

The Born part





cos θ angle between e^- and $\tilde{\chi}_1^-$

- Born helicity amplitudes known analytically Choi et al., 9812236, 0002033
- Implemented in narrow width approximation in many programs PYTHIA, Herwig, Isajet, Comphep, Grace, . . .
- Full processes in Sherpa, SMadgraph, WHiZard

- No massless t-channel particles $\Rightarrow$ neglect m_e for phase space

- to clarify notation

$$\sigma_{\text{Born}}(s) = \int d\Gamma_2 \, |\mathcal{M}_{\text{Born}}(s, \cos\theta)|^2$$

Classification of NLO corrections

- Loop corrections to SUSY production and decay processes
- nonfactorizable, maximally resonant photon exchange between production and decay
- real radiation of photons
- off-shell kinematics for the signal process
- irreducible background from all other SUSY processes
- reducible, experimentally indistinguishable SM background processes

Multi-pole approximation, justified from EW SM processes Denner et al., 0006307, 0502063, 0604011.

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implemented in Sherpa, Smadgraph, WHiZard thoroughly checked

Hagiwara et al., 0512260; JR et al., 0512012

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Comparison of Automated Tools for Perturbative Interactions in SuperSymmetry

cf. e.g. `http://www.physics.carleton.ca/~reuter/susy_comparison.html`

$\tau^+\tau^- \to X$

Process	status	Madgraph/Helas 0.5 TeV	Madgraph/Helas 2 TeV	Whizard/O'Mega 0.5 TeV	Whizard/O'Mega 2 TeV	Sherpa/A'Megic 0.5 TeV	Sherpa/A'Megic 2 TeV
$\tilde\tau_1\tilde\tau_1^*$	●	257.57(7)	79.63(4)	257.32(1)	79.636(4)	257.30(1)	79.638(4)
$\tilde\tau_2\tilde\tau_2^*$	●	46.55(1)	66.86(2)	46.368(2)	66.862(3)	46.372(2)	66.862(3)
$\tilde\tau_1\tilde\tau_2^*$	●	95.50(3)	19.00(1)	94.637(3)	19.0015(8)	94.645(5)	19.000(1)
$\tilde\nu_\tau\tilde\nu_\tau^*$	●	502.26(7)	272.01(8)	502.27(2)	272.01(1)	502.30(3)	272.01(1)
$\tilde\chi_1^0\tilde\chi_1^0$	●	249.94(2)	26.431(1)	249.954(9)	26.431(1)	249.96(1)	26.431(1)
$\tilde\chi_1^0\tilde\chi_2^0$	●	69.967(3)	9.8940(3)	69.969(2)	9.8940(4)	69.968(3)	9.8937(5)
$\tilde\chi_1^0\tilde\chi_3^0$	●	17.0387(3)	0.7913(1)	17.0394(1)	0.79136(2)	17.040(1)	0.79137(5)
$\tilde\chi_1^0\tilde\chi_4^0$	●	7.01378(4)	1.50743(3)	7.01414(6)	1.5075(5)	7.0141(4)	1.50740(8)
$\tilde\chi_2^0\tilde\chi_2^0$	●	82.351(7)	18.887(1)	82.353(3)	18.8879(9)	82.357(4)	18.8896(1)
$\tilde\chi_2^0\tilde\chi_3^0$	●	—	1.7588(1)	—	1.75884(5)	—	1.7588(1)
$\tilde\chi_2^0\tilde\chi_4^0$	●	—	2.96384(7)	—	2.9640(1)	—	2.9639(1)
$\tilde\chi_3^0\tilde\chi_3^0$	●	—	0.046995(4)	—	0.0469966(9)	—	0.046999(2)
$\tilde\chi_3^0\tilde\chi_4^0$	●	—	8.5852(4)	—	8.55857(3)	—	8.5856(4)
$\tilde\chi_4^0\tilde\chi_4^0$	●	—	0.26438(2)	—	0.264389(5)	—	0.26437(1)
$\tilde\chi_1^+\tilde\chi_1^-$	●	185.09(3)	45.15(1)	185.093(6)	45.147(2)	185.10(1)	45.151(2)
$\tilde\chi_2^+\tilde\chi_2^-$	●	—	26.515(1)	—	26.5162(6)	—	26.515(1)
$\tilde\chi_1^+\tilde\chi_2^-$	●	—	4.2127(4)	—	4.21267(9)	—	4.2125(2)
h^0h^0	●	0.3533827(3)	0.0001242(2)	0.35339(2)	0.00012422(3)	0.35340(2)	0.000124218(6)
h^0H^0	●	—	0.005167(4)	—	0.0051669(3)	—	0.0051671(3)
H^0H^0	●	—	0.07931(3)	—	0.079301(6)	—	0.079311(4)
A^0A^0	●	—	0.07975(3)	—	0.079758(6)	—	0.079744(4)
Zh^0	●	59.591(3)	3.1803(8)	59.589(3)	3.1802(1)	59.602(3)	3.1829(2)
ZH^0	●	2.8316(3)	4.671(5)	2.83169(9)	4.6706(3)	2.8318(1)	4.6706(2)
ZA^0	●	2.9915(4)	4.682(5)	2.99162(9)	4.6821(3)	2.9917(2)	4.6817(2)
A^0h^0	●	—	0.005143(4)	—	0.0051434(3)	—	0.0051440(3)
A^0H^0	●	—	1.4880(2)	—	1.48793(9)	—	1.48802(8)
H^+H^-	●	—	5.2344(6)	—	5.2344(2)	—	5.2345(3)

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Charginos quite narrow

$\Rightarrow$ nonfactorizable corrections significantly suppressed

be careful: $\longrightarrow$ D. Rainwater's talk

Virtual Corrections

Virtual corrections from SUSY and SM particles: self energies, vertex corrections, box diagrams (as usual)

(Semi-)automatized calculation with `FeynArts/FormCalc`

Hahn et al., 9807565, 0012260, 0105349 ; Fritzsche, ; Fritzsche/Hollik, 0407095

Independent check of numerical results Öller/Eberl/Majerotto, 0504109

Regulators:

- Electron mass m_e for collinear photon radiation
- Fictitious photon mass λ for infrared divergencies

Interference of Born and virtual corrections

$$\sigma_{\text{virt}}(s, \lambda^2, m_e^2) = \int d\Gamma_2 \left[2\text{Re}\left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2)\right)\right]$$

Eliminate dependence on λ by

- neglecting power corrections in λ
- Adding real (1st order) photon radiation with $E_\gamma < \Delta E_\gamma$
- Correction (terms $\propto \log \Delta E_\gamma$) is shifted into soft-photon factor

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Soft-photon factor:

$$f_{\text{soft}} = -\frac{\alpha}{2\pi} \sum_{i,j = e^\pm, \widetilde{\chi}^\pm} \int_{|\mathbf{k}| \leq \mathbf{\Delta E}_\gamma} \frac{d^3k}{2\omega_k} \frac{(\pm) p_i p_j \, Q_i \, Q_j}{(p_i k)(p_j k)}$$

Real and Collinear Photons

"Virtual + Soft"

$$\sigma_{\text{v+s}}(s, \Delta E_\gamma, m_e^2) = \int d\Gamma_2 \left[f_{\text{soft}}(\tfrac{\Delta E_\gamma}{\lambda}) \, |\mathcal{M}_{\text{Born}}(s)|^2 + 2\text{Re}\left(\mathcal{M}_{\text{Born}}(s)^* \, \mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2)\right)\right]$$

for simulation choose $\Delta E_\gamma \leq \Delta E_\gamma^{\text{exp}}$

Real radiation (i.e. the process $e^- e^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^- \gamma$):

$$\sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2) = \int_{\Delta E_\gamma} d\Gamma_3 \, |\mathcal{M}_{2\to3}(s, m_e^2)|^2.$$

"Total" cross section (fixed order):

$$\sigma_{\text{tot}}(s, m_e^2) = \sigma_{\text{Born}}(s) + \sigma_{\text{v+s}}(s, \Delta E_\gamma, m_e^2) + \sigma_{2\,\to\,3}(s, \Delta E_\gamma, m_e^2)$$

should not depend on ΔE_γ, but power corrections only in $\sigma_{2\,\to\,3}$, not in $\sigma_{\text{v+s}}$

As usual, split $2 \to 3$ cross section:

$$\sigma_{2\,\to\,3}(s,\Delta E_\gamma,m_e^2) = \sigma_{\text{hard,non-coll}}(s,\Delta E_\gamma,\Delta\theta_\gamma) + \sigma_{\text{hard,coll}}(s,\Delta E_\gamma,\Delta\theta_\gamma,m_e^2)$$

$x = 1 - 2E_\gamma/\sqrt{s}$ electron energy fraction after radiation

Approximate collinear radiation by convoluting the Born cross section with a structure function

$$\sigma_{\text{hard,coll}}(s,\Delta E_\gamma,\Delta\theta_\gamma,m_e^2) = \int_{\Delta E_\gamma,\Delta\theta_\gamma} d\Gamma_3\,|\mathcal{M}_{2\to3}(s,m_e^2)|^2$$
$$= \int_0^{x_0} dx\, f(x;\Delta\theta_\gamma,\tfrac{m_e^2}{s}) \int d\Gamma_2\,|\mathcal{M}_{\text{Born}}(xs,m_e^2)|^2.$$

collinear structure functions (helicity conserving and helicity flip):

Böhm/Dittmaier, 1993

$$f^+(x) = \frac{\eta}{4}\frac{1+x^2}{1-x} \qquad f^-(x) = \frac{\alpha}{2\pi}(1-x) \qquad \eta := \frac{2\alpha}{\pi}\left[\log\left(\frac{s}{4m_e^2}(\Delta\theta_\gamma)^2\right) - 1\right]$$

Cutoff $\Delta E_\gamma \to x_0 = 1 - 2\Delta E_\gamma/\sqrt{s}$ (no power corrections in $\Delta\theta_\gamma$)

Simulation

Combining all parts:

$$\sigma_{\text{tot}}(s, m_e^2) = \int dx\, f_{\text{eff}}(x_1,\, x_2; \Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2\, |\mathcal{M}_{\text{eff}}(s, x_1,\, x_2; m_e^2)|^2 + \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3\, |\mathcal{M}_{2\to 3}(s)|^2,$$

with

$$\begin{aligned} f_{\text{eff}}(x_1,\, x_2; \Delta E_\gamma, \Delta\theta_\gamma, \frac{m_e^2}{s}) &= \delta(1-x_1)\,\delta(1-x_2) \\ &+ \delta(1-x_1)\, f(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s})\,\theta(x_0-x_2) \\ &+ f(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s})\,\delta(1-x_2)\,\theta(x_0-x_1) \end{aligned}$$

$$\begin{aligned} |\mathcal{M}_{\text{eff}}(s, x_1,\, x_2; m_e^2)|^2 &= \left[1 + f_{\text{soft}}(\Delta E_\gamma, \lambda^2)\,\theta(x_1, x_2))\right]\, |\mathcal{M}_{\text{Born}}(s)|^2 \\ &+ 2\text{Re}\left[\mathcal{M}_{\text{Born}}(s)\,\mathcal{M}_{\text{1-loop}}(s, \lambda^2, m_e^2)\right]\,\theta(x_1-x_0)\theta(x_2-x_0) \end{aligned}$$

All corrections defined as a generalized structure function
$\Rightarrow$ suitable for implementation in an event generator

The WHiZard/O'Mega Generator Generator

Kilian/Ohl/JR

Level of Complexity:

- $e^+e^- \to ZHH \to ZWWWW \to bbjjjjjjjj$ (12,000,000 diagrams)
- $pp \to \tilde{\chi}^0_1\tilde{\chi}^0_1 bbbb$ (32,000 diagrams, 22 color flows, $\sim 10,000$ PS channels)
- $pp \to VVjj \to jj\ell\ell\nu\nu$ incl. anomalous TGC/QGC

Current versions: WHiZard 1.51 / O'Mega 000.011beta

`http://theorie.physik.uni-wuerzburg.de/~ohl/omega/`

`http://www-ttp.physik.uni-karlsruhe.de/whizard/`

Available from my homepage:

`http://www.physics.carleton.ca/~reuter`

Major upgrade this spring/summer: WHiZard 2.0 / O'Mega 1.0

Implemented models:

- Test models: QED, QCD
- SM
- Littlest/Simplest Little Higgs
- MSSM, NMSSM, extended SUSY models
- Extra dimensions
- Noncommutative Standard Model

Technical Details and Failure of Approach

Generate Born + $2 \to 3$ by O'Mega, convolute Born with generalized structure function (“user-defined structure function” in WHiZard)

Sampling δ-functions:

- splitting sampling region $[0, x_0] \cup [x_0, 1]$
- map first region as exactly as possible
- set $x = 1$ in the 2nd region (δ-functions)
- reweighting according to

$$w(x > x_0) : w(x < x_0) = 1 : \int_0^{x_0} dx\, f(x; \Delta\theta_\gamma, \frac{m_e^2}{s})$$

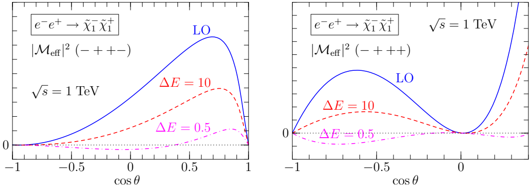
For fixed-order simulation avoid double-counting:
$f(x_1 < x_0, x_2 < x_0) \equiv 0$

Numerical agreement: WHiZard/O'Mega and fixed-order calculation

In the soft-photon region: negative event weights

- $2 \to 2$ and $2 \to 3$ runs separately
- Lowering the cutoff from $\Delta E_\gamma/\sqrt{s} < 10^{-2}$ to $\Delta E_\gamma/\sqrt{s} < 10^{-3}$: $2 \to 2$ NLO becomes negative, compensating the $2 \to 3$

Resumming photons



Experimental resolution drives one into negative weights region

Soft-collinear region: $E_\gamma < \Delta E_\gamma$, $\Delta\theta_\gamma < \theta_\gamma$: double logs $\frac{\alpha}{\pi} \log \frac{E_\gamma^2}{s} \log \theta_\gamma$ invalidate perturbative series

In that region resummation of all orders is possible

$$\sigma_{\text{Born+ISR}}(s, \Delta\theta_\gamma, m_e^2) = \int dx \, f_{\text{ISR}}(x; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int d\Gamma_2 \, |\mathcal{M}_{\text{Born}}(xs)|^2,$$

f_{ISR} includes all order soft-photon radiation (LLA), hard-collinear up to 3rd order Skrzypek/Jadach, 1991

For collinear photons cancellation of infrared divergencies built in, main source of negative weights removed

Matching with NLO

Combine ISR-resummed LO with NLO, avoid double-counting

Subtract contribution of one soft photon (already in soft-photon factor)

$$f_{\text{soft,ISR}}(\Delta E_\gamma, \Delta\theta_\gamma, m_e^2) = \frac{\eta}{4}\int_{x_0}^{1} dx \left(\frac{1+x^2}{1-x}\right)_+ = \frac{\eta}{4}\left(2\ln(1-x_0) + x_0 + \frac{1}{2}x_0^2\right).$$

After this subtraction we have

$$|\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2 = \left[1 + f_{\text{soft}}(\tfrac{\Delta E_\gamma}{\lambda}) - 2 f_{\text{soft,ISR}}(\Delta E_\gamma, \Delta\theta_\gamma, \tfrac{m_e^2}{s})\right] |\mathcal{M}_{\text{Born}}(\hat{s})|^2 + 2\text{Re}\left[\mathcal{M}_{\text{Born}}(\hat{s})\, \mathcal{M}_{\text{1-loop}}(\hat{s}, \lambda^2, m_e^2)\right],$$

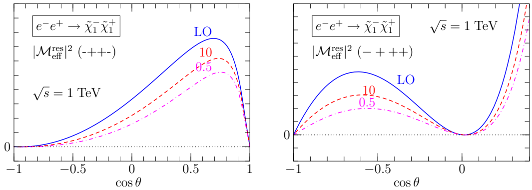
contains Born, virtual + soft contr. with LL part of virtual and soft-coll. removed

New "s+v" term (contains also soft/coll. corrections to Born/1-loop interference)

$$\sigma_{\text{v+s,ISR}}(s, \Delta E_\gamma, \Delta\theta_\gamma, m_e^2) = \int dx_1\, f_{\text{ISR}}(x_1; \Delta\theta_\gamma, \tfrac{m_e^2}{s}) \int dx_2\, f_{\text{ISR}}(x_2; \Delta\theta_\gamma, \tfrac{m_e^2}{s}) \int d\Gamma_2\, |\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2$$

Simulation

Resummation approach eliminates problem of negative weights:



Only source for negative weights: soft-noncollinear region, does not cause problems

Final improvement:

- convoluting $2 \to 3$ part with ISR structur function
- add $2 \to 4$ part

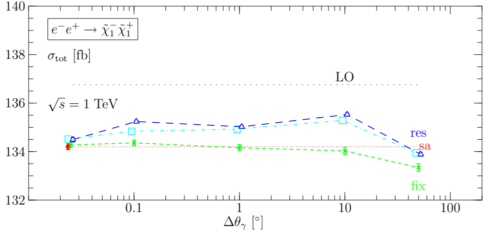
$$\sigma_{\text{tot,ISR+}}(s, m_e^2) = \int dx_1\, f_{\text{ISR}}(x_1; \Delta\theta_\gamma, \frac{m_e^2}{s}) \int dx_2\, f_{\text{ISR}}(x_2; \Delta\theta_\gamma, \frac{m_e^2}{s})$$
$$\times \left(\int d\Gamma_2\, |\widetilde{\mathcal{M}}_{\text{eff}}(\hat{s}; \Delta E_\gamma, \Delta\theta_\gamma, m_e^2)|^2 + \int_{\Delta E_\gamma, \Delta\theta_\gamma} d\Gamma_3\, |\mathcal{M}_{2\to 3}(\hat{s})|^2 \right)$$
$$+ \int_{\Delta E_{\gamma,i}, \Delta\theta_{\gamma,i}} d\Gamma_4\, |\mathcal{M}_{2\to 4}(s)|^2$$

Choosing Cutoffs

- **Collinear (angular) cutoff**

  Collinear approximation breaks down at $\theta_\gamma > 10°$

  Higher-order effects for emission angles below 0.1°



- **Energy cutoff**

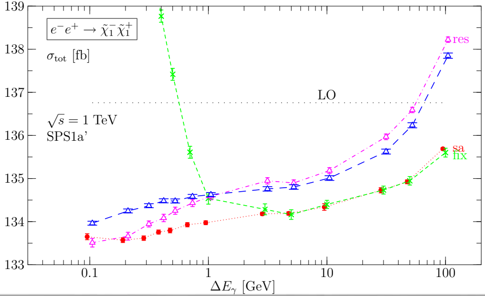
  Fixed order/semianalytic agree

  Small angles: interference term overshoots

  5 ‰correction from higher order γ radiation

  ILC statist. fluctuation: 2.5 ‰

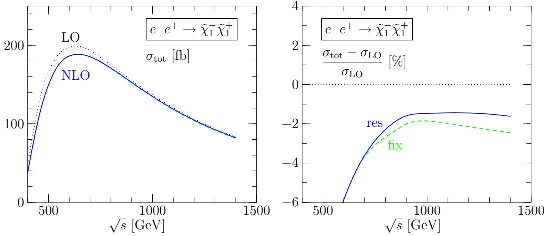
  $\Rightarrow \Delta E_\gamma \lesssim 0.5$ GeV



Results and Distributions

NLO corrections -5% (Xsec max.)

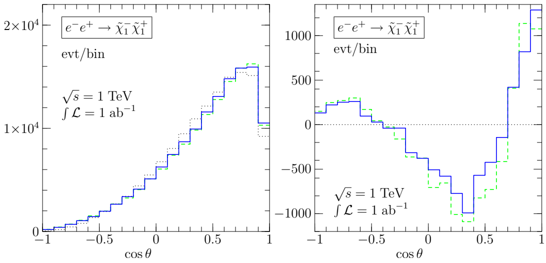
-2% (-1.5%) fixed-order (resummed) @ 1 TeV



Binned distribution of chargino scattering angle

Cutoffs: $\Delta\theta_\gamma = 1°$, $\Delta E_\gamma = 3$ GeV (fixed-order)

K-factor approach insufficient



Summary and Outlook

- Extended WHiZard/O'Mega: 1st NLO SUSY Monte Carlo Event Generator for the ILC
- All possible distributions available at NLO
- Matching of resummed soft-collinear photons and explicit NLO parts avoids negative weights
- Interface to `FeynArts/FormCalc`: all MSSM $2 \to 2$ processes for ILC available
- Maybe already part of new version WHiZard 2.0/O'Mega 1.0
- Open issues/Next step(s):
  - Include chargino decays (work in progress: **KKKRRR:** Kalinowski/Kilian/Kovaric/JR/Robens/Rolbiecki)
  - Resummation of Coulomb singularity: improved threshold behavior
  - Corresponding LHC processes:
    - More complicated parton shower
    - More complicated matching
    - Stay tuned!

Reshuffling contributions by changing cut-offs

- Shifting cutoffs changes the type of higher-order contributions
- lowering cutoffs is not necessarily improvement
- Focus on $O(\alpha^2)$ (real or virtual) correction,
- Resummation method, 3 different ways for real+virtual photons:
  - (a) Soft approximation (Denner, 1991) coll.+non-coll. soft γs; neglects contributions $\propto \frac{\Delta E_\gamma}{\sqrt{s}}$;
  - (b) ISR (Skrzypek/Jadach, 1990), coll. real+virtual γs; assumes $k^\top$-ordering of emitted γs, i.e. for $j > i$: $k_j^\top > k_i^\top$, and in n^{th} order: $\sum_{i=1}^n k_i^\top < k_{max}^\top$, where $k_{max}^\top$ is fixed
  - (c) exact (hard non-collinear) matrix element $\mathcal{M}_{2\to 3}$
- Radiation from the same leg:

$$\mathcal{O}(\alpha^2)_{\text{ISR}} - \mathcal{O}(\alpha)_{\text{ISR}}\,\mathcal{O}(\alpha)_{\text{ISR}}^{\text{soft}} + \mathcal{O}(\alpha)_{\text{ISR}}\,\mathcal{O}(\alpha)_{\text{soft}} \qquad \Delta_j = \mathcal{O}(\alpha)_{j,\text{soft}} - \mathcal{O}(\alpha)_{j,\text{ISR}}$$

- Radiation from different legs:

$$\mathcal{O}(\alpha)_{1,\text{ISR}}\mathcal{O}(\alpha)_{2,\text{ISR}} + \mathcal{O}(\alpha)_{1,\text{ISR}}\left(\mathcal{O}(\alpha)_{2,\text{soft}} - \mathcal{O}(\alpha)_{2,\text{ISR}}^{\text{soft}}\right) + \left(\mathcal{O}(\alpha)_{1,\text{soft}} - \mathcal{O}(\alpha)_{1,\text{ISR}}^{\text{soft}}\right)\mathcal{O}(\alpha)_{2,\text{ISR}}.$$

- $\Delta_1\mathcal{O}(\alpha)_{2,\text{ISR}} + \mathcal{O}(\alpha)_{1,\text{ISR}}\Delta_2 + \mathcal{O}(\alpha)_{1,\text{ISR}}\mathcal{O}(\alpha)_{2,\text{ISR}}$
- More details: T. Robens, PhD thesis, 2006.