High-Energy Vector Boson Scattering after the Higgs Discovery

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in collaboration with: W. Kilian, T. Ohl, M. Sekulla

Alboteanu/Kilian/JRR, **JHEP 0811** (2008) 010;
Beyer/Kilian/Krstonošić/Mönig/JRR/Schmitt/Schröder, **EPJC 48** (2006), 353;
JRR/Kilian/Sekulla, 1307.8170; Kilian/JRR/Ohl/Sekulla, 1408.6207 + in prep.

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Acknowledgments

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DESY

PHYSICS AT THE TERA SCALE Helmholtz Alliance

for providing the support for my research

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Disacknowledgments

for not providing me with internet and phone for 57 days

Motivation

- Light Higgs boson found
- SM-like (clear from EWPO)
- Mediator of EWSB found
- Mechanism of EWSB still poorly understood:
  - single Higgs field vs. Higgs sector
  - Higgs potential: stable vs. metastable vs. unstable !?
  - Higgs self-coupling vs. Higgs field scattering
  - Importance of longitudinal EW gauge bosons

Motivation

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  - Higgs potential: stable vs. metastable vs. unstable !?
  - Higgs self-coupling vs. Higgs field scattering
  - Importance of longitudinal EW gauge bosons
- Anomalous Triple Gauge Couplings: dibosons
- Anomalous Quartic Gauge Couplings: tribosons, VV scattering
- Higgs suppression makes VBS a prime candidate for BSM searches
- Hot topic: Snowmass BNL 04/13, SM@LHC Freiburg 04/13, LHCEWWG 04/13, Snowmass 07/13, Dresden 10/13, BNL workshop 10/14

Extensions of the SM

- Lagrangian of the EW SM (no fermions/QCD here):

$$\mathcal{L}_{EW} = -\frac{1}{2}\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

with building blocks:

$$D_\mu = \partial_\mu + \frac{i}{2}g\tau^I W^I_\mu + \frac{i}{2}g' B_\mu$$

$$W_{\mu\nu} = \frac{i}{2}g\tau^I(\partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\epsilon_{IJK}W^J_\mu W^K_\nu)$$

$$B_{\mu\nu} = \frac{i}{2}g'(\partial_\mu B_\nu - \partial_\nu B_\mu)$$

- Any EFT has higher-dimensional operators: Weinberg, 1979

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[\frac{a_i}{\Lambda}\mathcal{O}^{(5)}_i + \frac{c_i}{\Lambda^2}\mathcal{O}^{(6)}_i + \frac{e_i}{\Lambda^4}\mathcal{O}^{(8)}_i \cdots\right]$$

- without more fundamental theory $\Rightarrow$ no clue on the scale (neither on the coefficients)

Classification of Operators (I): Dim 6

(always v^2 subtracted)

- Dimension-6 operators (CP-conserving)

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_{W} = (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{B} = (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{\partial\Phi} = \partial_\mu \left(\Phi^\dagger \Phi\right) \partial^\mu \left(\Phi^\dagger \Phi\right)$$

$$\mathcal{O}_{\Phi W} = \left(\Phi^\dagger \Phi\right) \text{Tr}[W^{\mu\nu} W_{\mu\nu}]$$

$$\mathcal{O}_{\Phi B} = \left(\Phi^\dagger \Phi\right) B^{\mu\nu} B_{\mu\nu}$$

- Dimension-6 operators (CP-violating)

$$\mathcal{O}_{\widetilde{W}W} = \Phi^\dagger \widetilde{W}_{\mu\nu} W^{\mu\nu} \Phi$$

$$\mathcal{O}_{\tilde{B}B} = \Phi^\dagger \tilde{B}_{\mu\nu} B^{\mu\nu} \Phi$$

$$\mathcal{O}_{\widetilde{W}WW} = \text{Tr}[\widetilde{W}_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_{\widetilde{W}} = (D_\mu \Phi)^\dagger \widetilde{W}^{\mu\nu} (D_\nu \Phi)$$

	ZWW	AWW	HWW	HZZ	HZA	HAA	WWWW	ZZWW	ZAWW	AAWW
$\mathcal{O}_{WWW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{W}$	✓	✓	✓	✓	✓		✓	✓	✓	
$\mathcal{O}_{B}$	✓	✓		✓	✓					
$\mathcal{O}_{\Phi d}$			✓	✓						
$\mathcal{O}_{\Phi W}$			✓	✓	✓	✓				
$\mathcal{O}_{\Phi B}$				✓	✓	✓				
$\mathcal{O}_{\tilde{W}WW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{\tilde{W}}$	✓	✓	✓	✓	✓					
$\mathcal{O}_{\tilde{W}W}$			✓	✓	✓	✓				
$\mathcal{O}_{\tilde{B}B}$				✓	✓	✓				

Classification of Operators (II): Dim 8 (always v^2 subtracted)

- Dimension-8 operators (only $D_\mu \Phi$)

$$\mathcal{O}_{S,0} = \left[(D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[(D^\mu \Phi)^\dagger D^\nu \Phi \right] ,$$

$$\mathcal{O}_{S,1} = \left[(D_\mu \Phi)^\dagger D^\mu \Phi \right] \times \left[(D_\nu \Phi)^\dagger D^\nu \Phi \right] ,$$

- Dimension-8 operators (only field strength/mixed)

$$\mathcal{O}_{T,0} = \text{Tr}\left[W_{\mu\nu} W^{\mu\nu} \right] \cdot \text{Tr}\left[W_{\alpha\beta} W^{\alpha\beta} \right] ,$$
$$\mathcal{O}_{T,1} = \text{Tr}\left[W_{\alpha\nu} W^{\mu\beta} \right] \cdot \text{Tr}\left[W_{\mu\beta} W^{\alpha\nu} \right] ,$$
$$\mathcal{O}_{T,2} = \text{Tr}\left[W_{\alpha\mu} W^{\mu\beta} \right] \cdot \text{Tr}\left[W_{\beta\nu} W^{\nu\alpha} \right] ,$$
$$\mathcal{O}_{T,5} = \text{Tr}\left[W_{\mu\nu} W^{\mu\nu} \right] \cdot B_{\alpha\beta} B^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \text{Tr}\left[W_{\alpha\nu} W^{\mu\beta} \right] \cdot B_{\mu\beta} B^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \text{Tr}\left[W_{\alpha\mu} W^{\mu\beta} \right] \cdot B_{\beta\nu} B^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta}$$
$$\mathcal{O}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha} .$$

$$\mathcal{O}_{M,0} = \text{Tr}\left[W_{\mu\nu} W^{\mu\nu} \right] \cdot \left[(D_\beta \Phi)^\dagger D^\beta \Phi \right]$$
$$\mathcal{O}_{M,1} = \text{Tr}\left[W_{\mu\nu} W^{\nu\beta} \right] \cdot \left[(D_\beta \Phi)^\dagger D^\mu \Phi \right]$$
$$\mathcal{O}_{M,2} = \left[B_{\mu\nu} B^{\mu\nu} \right] \cdot \left[(D_\beta \Phi)^\dagger D^\beta \Phi \right] ,$$
$$\mathcal{O}_{M,3} = \left[B_{\mu\nu} B^{\nu\beta} \right] \cdot \left[(D_\beta \Phi)^\dagger D^\mu \Phi \right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} D^\mu \Phi \right] \cdot B^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} D^\nu \Phi \right] \cdot B^{\beta\mu} ,$$
$$\mathcal{O}_{M,6} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\nu} D^\mu \Phi \right] ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\mu} D^\nu \Phi \right] ,$$

Classification of Operators (III)

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0/1}$	✓	✓	✓						
$\mathcal{O}_{M,0/1/6/7}$	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{M,2/3/4/5}$		✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{T,0/1/2}$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,8/9}$			✓			✓	✓	✓	✓

- Dim. 8 operators generate aQGCs, but not aTGCs
- generate neutral quartics
- Redundancy of the operators:
  - Equations of motion: $D_\mu W^{\mu\nu} = \Phi^\dagger (D^\nu \Phi) - (D^\nu \Phi)^\dagger \Phi + \ldots$
  - Gauge symmetry structure: $[D_\mu, D_\nu] \, \Phi \propto W_{\mu\nu} \Phi$
  - Integration by parts (up to total derivatives)
  - Leads to relations like:

$$\begin{aligned} \mathcal{O}_B &= \mathcal{O}_{\tilde{W}} + \frac{1}{2}\mathcal{O}_{WW} - \frac{1}{2}\mathcal{O}_{BB} \\ \mathcal{O}_{BW} &= -2\,\mathcal{O}_W - \mathcal{O}_{WW} \\ \mathcal{O}_{\partial W} &= -4\,\mathcal{O}_{WWW} + \text{gauge-fermion operators} \end{aligned}$$

Classification of approaches

- Switch operator bases (vertex-dep.): Snowmass EW White Paper, 1310.6708

WWWW-Vertex:

$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{8}$$

$$\alpha_4 + 2\cdot\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{8}$$

WWZZ-Vertex:

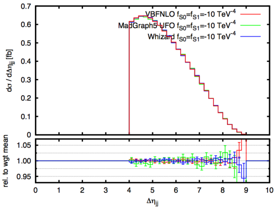
$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{16}$$

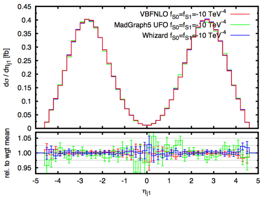
$$\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{16}$$

ZZZZ-Vertex:

$$\alpha_4 + \alpha_5 = \left(\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4}\right)\frac{v^4}{16}$$

- Full agreement among generators: VBF@NLO, WHIZARD, Madgraph





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- For the rest concentrate on:

$$\mathcal{L}_{HD} = F_{HD}\ \mathrm{tr}\left[\mathbf{H}^\dagger\mathbf{H} - \frac{v^2}{4}\right]\cdot\mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger(\mathbf{D}^\mu\mathbf{H})\right]$$

$$\mathcal{L}_{S,0} = F_{S,0}\ \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\mathbf{D}_\nu\mathbf{H}\right]\cdot\mathrm{tr}\left[(\mathbf{D}^\mu\mathbf{H})^\dagger\mathbf{D}^\nu\mathbf{H}\right]$$

$$\mathcal{L}_{S,1} = F_{S,1}\ \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\mathbf{D}^\mu\mathbf{H}\right]\cdot\mathrm{tr}\left[(\mathbf{D}_\nu\mathbf{H})^\dagger\mathbf{D}^\nu\mathbf{H}\right]$$

Unique way of operator assignment?

- Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- VVV and VBS to access the highest possible energies

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- Incoherent sum of channels at LHC prevent eliminating operators!

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- Incoherent sum of channels at LHC prevent eliminating operators!
- Similar to B physics: observables process [decay] specific

Simplified Models for VBS (and VVV)

- Rise of amplitude (6/8-dim. operator) may be Taylor expansion of a resonance
- *A priori*: No idea which resonances exist and wherefrom
- Including a resonance in the model, there still may be further sources for anomalous couplings (further resonances, $A_{\text{nonres}}(s)$, deviation from the Breit-Wigner shape, etc.)
- Beyond the resonance, the amplitude may eventually rise and need unitarization again.

Consequence:

- Resonances in all accessible spin/isospin channels
- Couplings to the Higgs and gauge sectors are unrelated and arbitrary
- Still include anomalous couplings
- Unitarization (later)

Resonances

Operator coefficients $\Rightarrow$ new physics scale Λ: $\alpha_i = v^k/\Lambda^k$

- Operator normalization is arbitrary
- Power counting can be intricate

New physics in electroweak sector:

- Narrow resonances $\Rightarrow$ particles
- Wide resonances $\Rightarrow$ continuum

$SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' \neq 0$ and fermion masses)

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 (Higgs ?)	ω^0 $(\gamma'/Z'\ ?)$	f^0 (Graviton ?)
$I = 1$	$\pi^\pm, \pi^0$ (2HDM ?)	$\rho^\pm, \rho^0$ $(W'/Z'\ ?)$	$a^\pm, a^0$
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

- $I = 0$: resonant in W^+W^- and ZZ scattering
- $I = 1$: resonant in W^+Z and W^-Z scattering
- $I = 2$: resonant in W^+W^+ and W^-W^- scattering

accounts for weakly and strongly interacting models

Example: a Scalar Resonance [Not counting ϕ with $M = 126$ GeV.]

- Mass M_σ.
- Coupling to the Higgs sector (Higgs and longitudinal W/Z):

$$g_L^\sigma (D_\mu \Phi)^\dagger (D^\mu \Phi)\, \sigma$$

- Coupling to the gauge sector (transversal W/Z):

$$g_T^\sigma \mathrm{tr}\left[\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right]\, \sigma$$

Possible Origin: 2HDM isosinglet (renormalizable)

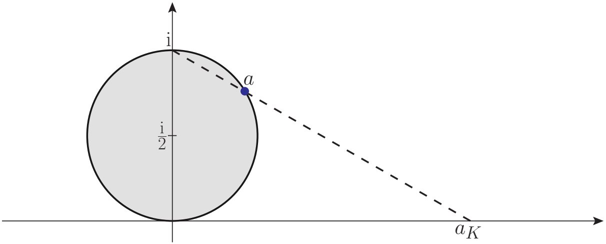
$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \qquad g_T^\sigma = O\left(\frac{1}{4\pi M_\sigma}\right) \quad \text{[loop]}$$

Possible Origin: new strong interactions

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \qquad g_T^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}$$

Unitarizing S matrices

- Cayley transform of S matrix: $S = \frac{\mathbf{1}+\mathrm{i}K/2}{\mathbf{1}-\mathrm{i}K/2}$ Heitler, 1941; Schwinger, 1948
- translates to transition operator: $T = \frac{K}{\mathbf{1}-\mathrm{i}K/2}$
- Works beyond perturbation theory, but allows perturbative expansion
- Diagonalize S matrix (partial waves):
  $\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1)\mathcal{A}_\ell(s)P_\ell(\cos\theta)$
- Complex eigenvalues: $t = 2a$ $k = 2a_K \Rightarrow a_K = \frac{a}{1+ia}$
- Corresponds to stereographic projection:



- Coulomb singularities Bloch/Nordsieck, 1937; Yennie/Frautschi/Suura, 1961

Unitarization Prescriptions

- **K-matrix unitarization prescription** Gupta, 1950; Berger/Chanowitz, 1991
  - Hermitian K-matrix interpreted as incompletely calculated approximation to true amplitude
  - $\Rightarrow$ Unitary S, T as a non-perturbativ completion of this approximation
  - Insert pert. expansion into expansion:

$$a = \frac{a_K}{1 - \mathrm{i} a_K} \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \mathrm{Re}\, a_0^{(2)} + \ldots}{1 - \mathrm{i}(a_0^{(1)} + \mathrm{Re}\, a_0^{(2)} + \ldots)}$$

  - Prescription does a partial resummation of perturbative series
  - Example Dyson resummation: $a_K^{(0)}(s) = \frac{\lambda}{s - m^2} \longrightarrow a^{(0)}(s) = \frac{\lambda}{s - m^2 - \mathrm{i}\lambda}$

- **Drawbacks of (original) K-matrix:**
  - Needs to construct self-adjoint K-matrix as intermediate step
  - Problem if S-matrix is not diagonal, or ...
    there are non-perturbative contributions

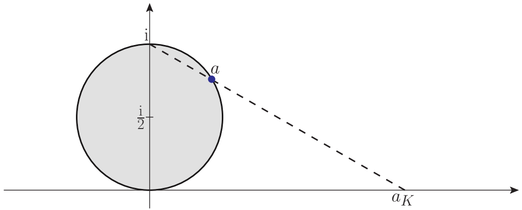
- **T-matrix unitarization**
  - a_0 complex approximation to eigenvalue of true T matrix
  - use again pseudo-stereographic projection (intersection of Argand circle with line $\overline{a_0\ i}$)
  - Results in: $a = \frac{\mathrm{Re}\, a_0}{1 - \mathrm{i} a_0^*} \quad \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \mathrm{Re}\, a_0^{(2)} + \ldots}{1 - \mathrm{i}(a_0^{(1)} + \mathrm{Re}\, a_0^{(2)} - \mathrm{i}\,\mathrm{Im}\, a_0^{(2)} + \ldots)}$

Alternative Unitarization Prescriptions

- **Comparison of T-matrix and (original) K-matrix:**
  - T-matrix does not rely on perturbation theory
  - Special treatment for non-normal T matrices (eigenvalues having imaginary parts larger than i; Riesz-Dunford operator calculus)

  1. T matrix description leads to point on the Argand circle
  2. For real a $\Rightarrow$ (original) K-matrix case
  3. a_0 on Argand circle $\Rightarrow$ left invariant

- **Thales circle construction:**



  - Defined via $\left|a - \frac{a_K}{2}\right| = \frac{a_K}{2} \;\Rightarrow\; a = \frac{1}{\mathrm{Re}\left(\frac{1}{a_0}\right) - \mathrm{i}}$
  - avoids non-normal matrices, but not single-valued around $a = 0$

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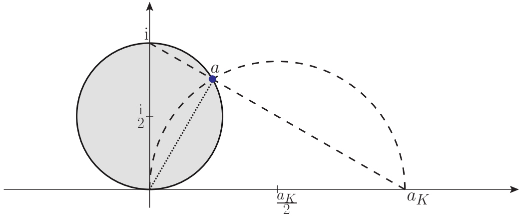

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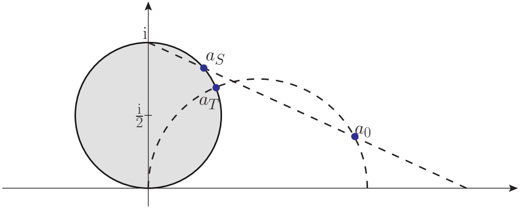

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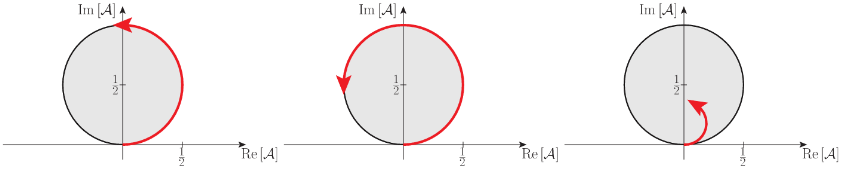
Unitarization Primer

Kilian/JRR/Ohl/Sekulla, 1408.6207

- Unitarization prescription not unique
- Padé (reordering pert. series) introduces artificial poles
- Form factors parameterize close-by new physics (additional parameters)
- minimal version (K or T matrix) $\Rightarrow$ just saturation no new parameters, does not rely on pert. expansion, stable against small perturbations
- Additional known features (resonances) should be implemented before unitarization

Unitary Description of EW interactions

- Five possible cases:
  - Amplitude perturbative, close to zero, small imag. part (SM)
  - Amplitude rises, gets imag. part, strongly interacting regime (presence of at least one dim. 8 operator)
  - Amplitude approaches maximum absolute value asymptotically
  - Turn over: new resonance
  - New inelastic channels open: eff. form factor, extra channels observable in multi-vector boson processes



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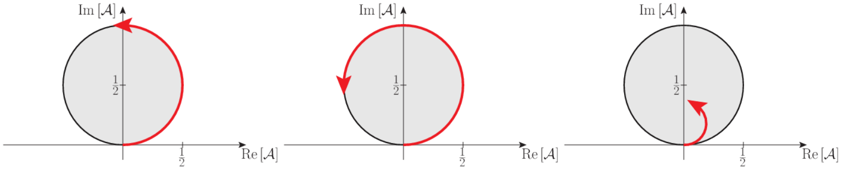


- Interpretation of EFT operator coefficients changes: formally still low-energy coefficients of Taylor expansion $\Rightarrow$ threshold parameters
- Complete description necessary (only) beyond threshold

Unitarity Bound for α_4 AQGC

Bounds for α_4

$$\ell = 0: \quad \sqrt{s} \leq \left(\frac{6\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.5\text{ TeV}}{\sqrt[4]{\alpha_4}}$$

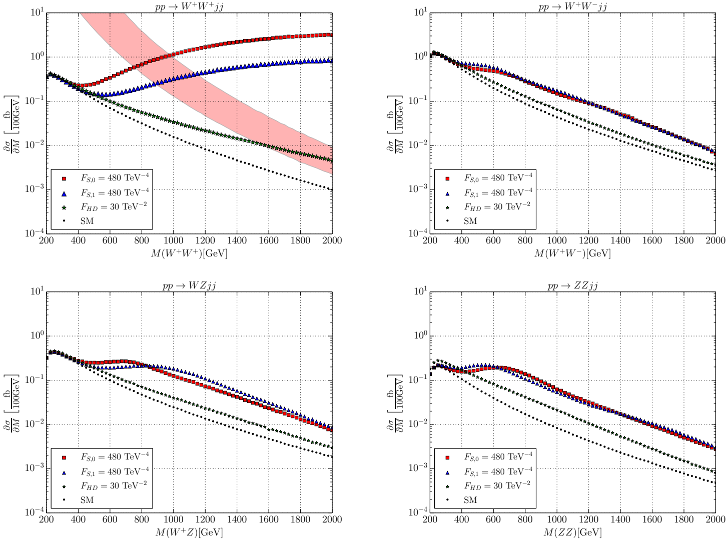
$$\ell = 2: \quad \sqrt{s} \leq \left(\frac{60\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.9\text{ TeV}}{\sqrt[4]{\alpha_4}}$$

α_4 AQGC contribution to $WW \to ZZ$

$$\mathcal{A}(s,t,u) = 4\alpha_4 \frac{t^2 + u^2}{v^4}$$

- Bound depends on coupling α_4
- Use strongest bound

Diboson invariant masses



General cuts: $M_{jj} > 500$ GeV; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$

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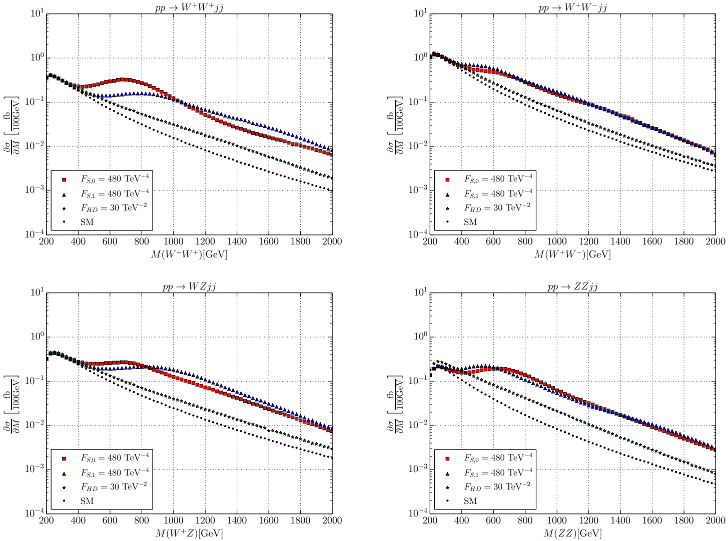

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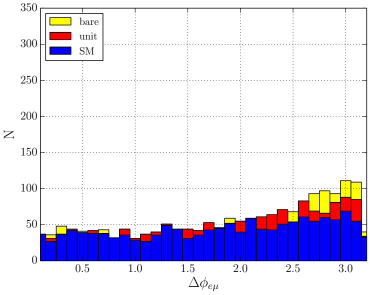
p_T and angular distributions

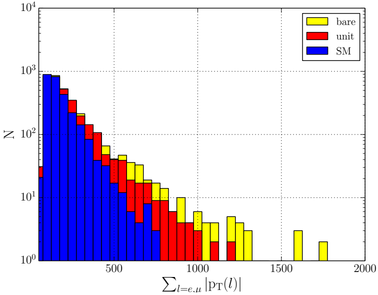
$pp \to e^+\mu^+\nu_e\nu_\mu jj$, $\sqrt{=}14$ TeV, $\mathcal{L} = 1000\,\text{fb}^{-1}$

Simulations with `WHIZARD` $\to$ JRR: Simulation

Not possible to use automated tool due to s-channel prescription

$F_{HD} = 30\,\text{TeV}^{-2}$





General cuts: $M_{jj} > 500$ GeV; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$, $p_T^\ell > 20$ GeV

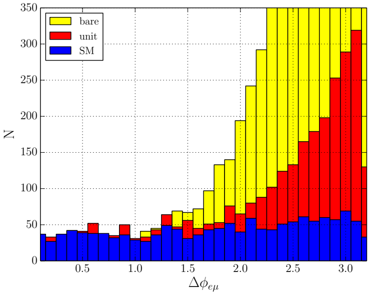
p_T and angular distributions

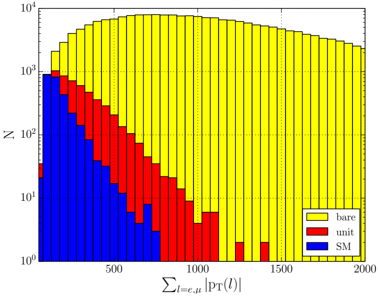
$pp \to e^+\mu^+\nu_e\nu_\mu jj$, $\sqrt{=}14$ TeV, $\mathcal{L} = 1000\,\text{fb}^{-1}$

Simulations with WHIZARD → JRR: Simulation

Not possible to use automated tool due to s-channel prescription

$F_{S,0} = 480\,\text{TeV}^{-4}$





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Not possible to use automated tool due to s-channel prescription

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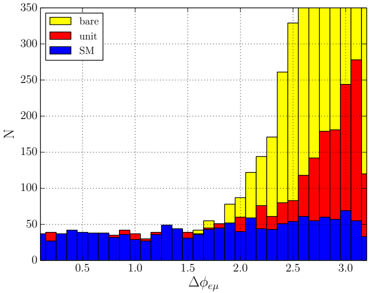
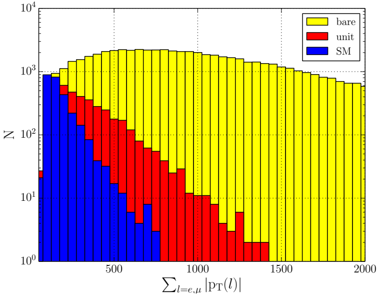
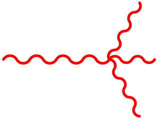
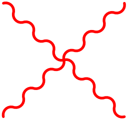




General cuts: $M_{jj} > 500$ GeV; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$, $p_T^\ell > 20$ GeV

And Triple Vector Boson Production?

relate  to  ??

Yes, the same Feynman graphs (in the SM), but. . .

Tribosons:

- one external $W/Z/\gamma$ is always far off-shell
- Unitarization has to proceed differently
- and a different set of (anomalous) couplings contributes
- particularly true for resonances

⇒ Important physics which should be treated independently w.r.t. VBS processes. Don't just combine the results!

Summary/Conclusions

- Triple/Quartic gauge couplings measured either
  - via diboson production
  - via triple boson production
  - via vector boson scattering
- Unify LHC and ILC/CLIC descriptions
- SM deviations in EW effective Lagrangian (SM + higher-dim. op.)
- Want to set model independent limits AQGC
- But: Energy range for testing AQGC is bound by Unitarity
- Simplified Models: minimally unitarized operators
- Unitarization scheme: no additional structure to the theory
- Unitarization introduces model dependence, but keeps model-dependence under control
- Sensitivity rises with number of intermediate states:
  - LHC sensitivity limited in pure EW sector: $\sim 1 - X$ TeV (???)
  - ILC1000 : $1.5 - 6$ TeV
  - (Tensor) Resonances very interesting Kilian/JRR/Sekulla, in preparation
  - Guess: 1.4 / 3 TeV e^+e^- [+ pol. ?] optimal choice

Always get the correct ellipses...

BACKUP SLIDES

Cut-Off Method (a.k.a. “Event Clipping”)

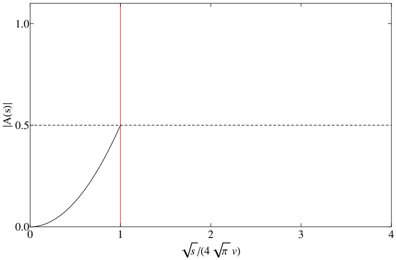
Cut-Off function

$$\Theta\left(\Lambda_C^2 - s\right)$$

- Naive prevention of Unitarity violation
- No continuous transition at Λ_C
- Ignore any interesting physics above Unitary bound
- Better: Use observables, which do not conflict unitarity condition

Cut-Off energy Λ_C

Λ_C equates unitarity bounds (often 0th partial wave)



Form Factor

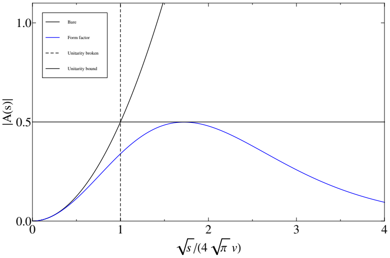
Form Factor

$$\frac{1}{\left(1+\frac{s}{\Lambda_{FF}^2}\right)^n}$$

Parameters

- n Chosen to prevent breaking of Unitarity
- Λ_{FF} Calculate highest possible value that satisfy real Unitarity bound ($0th$ partial wave)

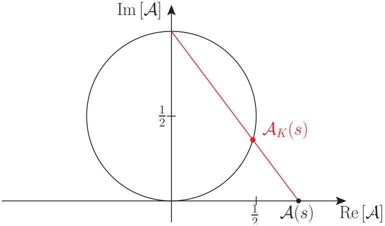
- Use Form Factor to suppress breaking of unitarity
- Can be generally used for arbitrary anomalous operator
- Need "Fine Tuning"



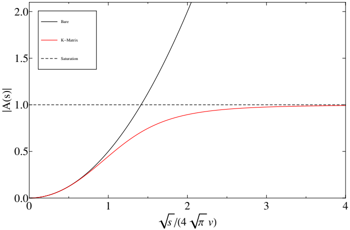
K-Matrix

K-Matrix Unitarisation

$$\mathcal{A}_K(s) = \frac{1}{\mathrm{Re}(\frac{1}{\mathcal{A}(s)}) - \mathrm{i}} = \frac{\mathcal{A}(s)}{1 - \mathrm{i}\mathcal{A}(s)} \quad \text{if } \mathcal{A}(s) \in \mathbb{R}$$



- Projection of elastic amplitudes onto Argand-Circle
- At high energies the amplitude saturises
- Is usable for complex amplitudes
- Not dependent on additional parameters



"Comparison"

- Which Unitarisation scheme provides the best description?

→ All of them:
Unitarisation schemes are an arbitrary way to guarantee Unitarity

Form Factor

- Suppression of amplitude to get below Unitarity bound

MC Generate less events than possible

K-Matrix

- Saturation of amplitude to achieve Unitarity

MC Generate maximal possible number of events

Vector Boson Scattering

Beyer et al.,hep-ph/0604048

1 TeV, $1\,\mathrm{ab}^{-1}$, full $6f$ final states, 80 % e^-_R, 60 % e^+_L polarization, binned likelihood

Contributing channels: $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$

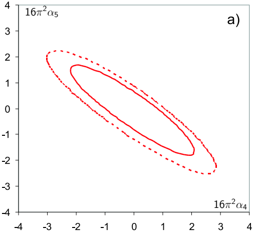
Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e\bar\nu_e q\bar q q\bar q$	$WW \to WW$	23.19
$e^+e^- \to \nu_e\bar\nu_e q\bar q q\bar q$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar\nu q\bar q q\bar q$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar q q\bar q$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar b X$	$e^+e^- \to t\bar t$	331.768
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar q$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar q$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar q$	1637.405

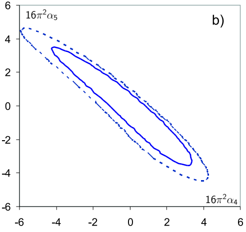
$SU(2)_c$ conserved case, all channels

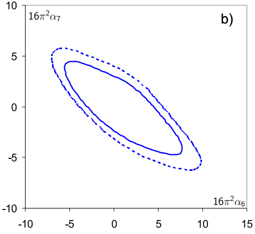
coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55







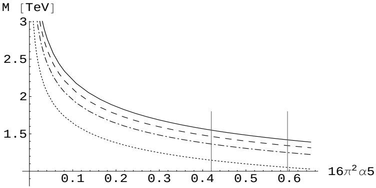
Interpretation as limits on resonances

Beyer et al.,hep-ph/0604048

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma / M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

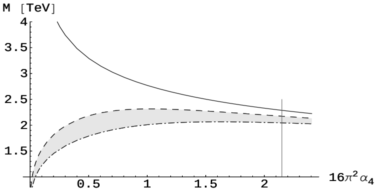


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2(\alpha_4^\lambda)^2/(2c_w^2)} \right)^{\frac{1}{4}}$$



upper/lower limit from λ_Z, grey area: magnetic moments

Final result:

Spin	$I=0$	$I=1$	$I=2$
0	1.55	–	1.95
1	–	2.49	–
2	3.29	–	4.30

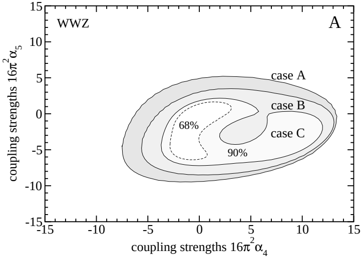
Spin	$I=0$	$I=1$	$I=2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

ILC Results: Triboson production

Beyer et al.,hep-ph/0604048

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\mathrm{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

	WWZ			ZZZ	best
$16\pi^2\times$	no pol.	e^- pol.	both pol.	no pol.	
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E^2_{\mathrm{mis}} + p^2_{\perp,\mathrm{mis}}$

No angular correlations yet

ILC Results: Triboson production

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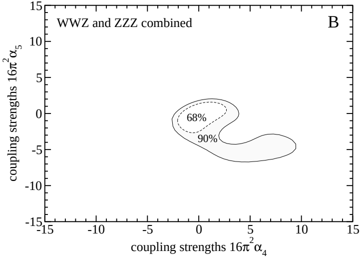

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	WWZ			ZZZ	best
$16\pi^2\times$	no pol.	e^- pol.	both pol.	no pol.	
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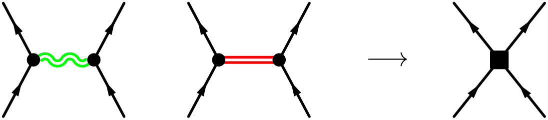
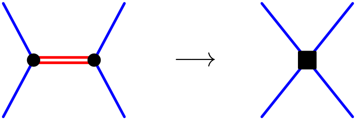
Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

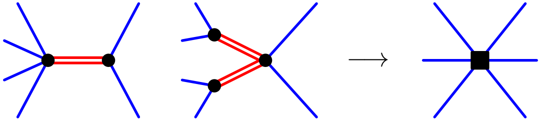


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{\Lambda^2} \mathrm{tr}\left[J^{(I)} \cdot J^{(I)}\right]$$

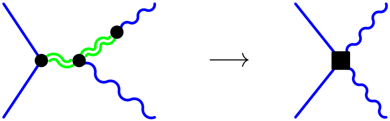


$$\mathcal{O}'_{h,1} = \frac{1}{F^2} \left((D\Phi)^\dagger \Phi\right) \cdot \left(h^\dagger (D^\Phi)\right) - \frac{v^2}{2} |D\Phi|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{\Lambda^2} (\Phi^\dagger \Phi - v^2/2)\, (D\Phi)^\dagger \cdot (D\Phi)$$



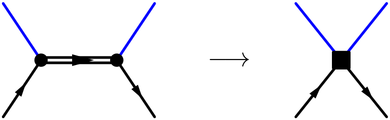
$$\mathcal{O}'_{h,3} = \frac{1}{\Lambda^2} \frac{1}{3} (\Phi^\dagger \Phi - v^2/2)^3$$



$$\mathcal{O}_{\Phi W} = -\frac{1}{\Lambda^2}\frac{1}{2}(\Phi^\dagger\Phi - v^2/2)\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

$$\mathcal{O}_{B} = \frac{1}{\Lambda^2}\frac{i}{2}(D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi)$$

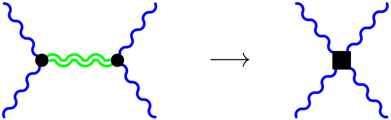
$$\mathcal{O}_{\Phi B} = -\frac{1}{\Lambda^2}\frac{1}{4}(\Phi^\dagger\Phi - v^2/2)B_{\mu\nu}B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{\Lambda^2}\overline{q}h(\not{D}h)q$$

Effective Dim. 8 Operators

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993



$$\mathcal{O}_\lambda = \frac{i}{\Lambda^4}\,\mathrm{tr}\left[W_{\mu\nu} \times W^{\nu\rho}(\Phi^\dagger \frac{\vec{\sigma}}{2}\left[D_\rho, D^\mu\right]\Phi)\right]$$
$$\mathcal{O}_\kappa = (D^\mu\Phi)^\dagger(D^\nu\Phi)(\Phi^\dagger\left[D_\mu, D_\nu\right]\Phi)$$

- operators linked through e.o.m.
- SM: 59 independent operators (1 fermion gen.) Buchmüller/Wyler, 1986; Grzadkowski/Iskrzynski/Misiak/Rosiek, 2010
- Renormalization mixes operators
- Beware of power counting

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = \mathrm{ie}\left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ A^{\rho\mu}\right] + \mathrm{ie}\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{QGC} = e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+ \right] + e^2 \frac{c_w}{s_w} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^- \right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+ \right] + e^2 \frac{c_w^2}{s_w^2} \left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+ \right] + \frac{e^2}{2 s_w^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW} \left(W^{-\mu} W_\mu^+ \right)^2 \right] + \frac{e^2}{4 s_w^2 c_w^4} h^{ZZ} (Z^\mu Z_\mu)^2$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 \qquad \Delta g_1^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) - g^2 (\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2 \left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10}) \right]$$

Parameters

$$\mathcal{L}_\sigma = -\frac{g_\sigma v}{2}\,\mathrm{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\sigma$$
$$\mathbf{V}_\mu = -\mathrm{i}g\mathbf{W}_\mu + \mathrm{i}g'\mathbf{B}_\mu$$
$$\mathbf{W}_\mu = W_\mu^a \frac{\tau^a}{2}$$
$$\mathbf{B}_\mu = W_\mu^a \frac{\tau^3}{2}$$

$$\mathcal{L}_\phi = \frac{g_\phi v}{4}\,\mathrm{Tr}\left[\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6}\,\mathrm{Tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right)\phi\right]$$
$$\phi = \sqrt{2}\left(\phi^{++}\tau^{++} + \phi^{+}\tau^{+} + \phi^{0}\tau^{0} + \phi^{-}\tau^{-} + \phi^{--}\tau^{--}\right)$$
$$\tau^{++} = \tau^{+} \otimes \tau^{+}$$
$$\tau^{+} = \frac{1}{2}\left(\tau^{+} \otimes \tau^{3} + \tau^{3} + \tau^{+}\right)$$
$$\tau^{0} = \frac{1}{\sqrt{6}}\left(\tau^{3} \otimes \tau^{3} - \tau^{+} \otimes \tau^{-} - \tau^{-} + \tau^{+}\right)$$

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SM Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{min}} = &-\frac{1}{2}\operatorname{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\operatorname{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] && W^{\pm},\ Z \\ &+ (\partial_\mu \phi)^\dagger \partial^\mu \phi - V(\phi) && h \\ &+ \frac{v^2}{4}\operatorname{tr}\left[(\mathbf{D}_\mu \Sigma)^\dagger (\mathbf{D}^\mu \Sigma)\right] && w^{\pm},\ z \\ &- \frac{g_h v}{2}\operatorname{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] h\end{aligned}$$

Vector Bosons

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + \mathrm{i}g\left[\mathbf{W}_\mu, \mathbf{W}_\nu\right]$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$$

$$\mathbf{W}_\mu = W_\mu^a \frac{\tau^a}{2} \qquad \mathbf{B}_\mu = B_\mu \frac{\tau^3}{2}$$

$$\mathbf{D}_\mu = \partial_\mu + \mathrm{i}g\mathbf{W}_\mu - \mathrm{i}g'\mathbf{B}_\mu$$

Higgs Sector

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$\Sigma = \exp\left[-\frac{\mathrm{i}}{v} w^a \tau^a\right]$$

$$\mathbf{V}_\mu = \Sigma\,(\mathbf{D}_\mu \Sigma)$$

Unitary Gauge

- Goldstone bosons are absorbed by vector bosons as longitudinal degrees of freedom
- $w^a \equiv 0 \to \Sigma \equiv 1$
- $\mathbf{D}_\mu = \partial_\mu - \mathbf{V}_\mu = \partial_\mu + \frac{\mathrm{i}g}{2}\left(\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_w} Z\tau^3\right)$

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \underbrace{(\partial_\mu\phi)^\dagger\,\partial^\mu\phi - \frac{v^2}{4}\text{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] - \frac{g_h v}{2}\text{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h}_{\widehat{=}_{g_h=1}(\mathbf{D}_\mu\phi)^\dagger\mathbf{D}^\mu\phi} - V(\phi)$$

- Coincides with known SM parametrisation

Isospin decomposition

- Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + \alpha_4 \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right] + \alpha_5\left(\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right)^2$$

- Leads to the following amplitudes: $s = (p_1 + p2)^2 \quad t = (p1 - p3)^2 \quad u = (p1 - p4)^2$

$$\boxed{\mathcal{A}(s,t,u) =:} \quad \mathcal{A}(w^+w^- \to zz) = \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(w^+z \to w^+z) = \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2+u^2}{v^4}$$

$$\mathcal{A}(w^+w^- \to w^+w^-) = -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5)\frac{s^2+t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4}$$

$$\mathcal{A}(w^+w^+ \to w^+w^+) = -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5)\frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(zz \to zz) = 8(\alpha_4 + \alpha_5)\frac{s^2+t^2+u^2}{v^4}$$

- (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\mathcal{A}(I=0) = 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$

$$\mathcal{A}(I=1) = \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t)$$

$$\mathcal{A}(I=2) = \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$