The Low-Energy Structure of Little Higgs Models

Jürgen Reuter
— University of Karlsruhe —
⟨reuter@particle.uni-karlsruhe.de⟩

collaboration with: Wolfgang Kilian, DESY

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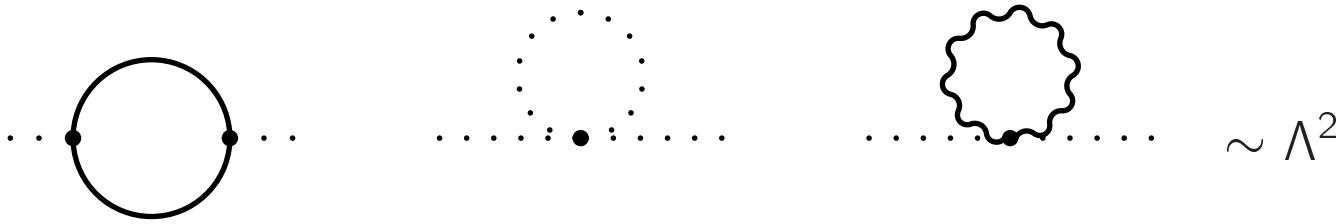
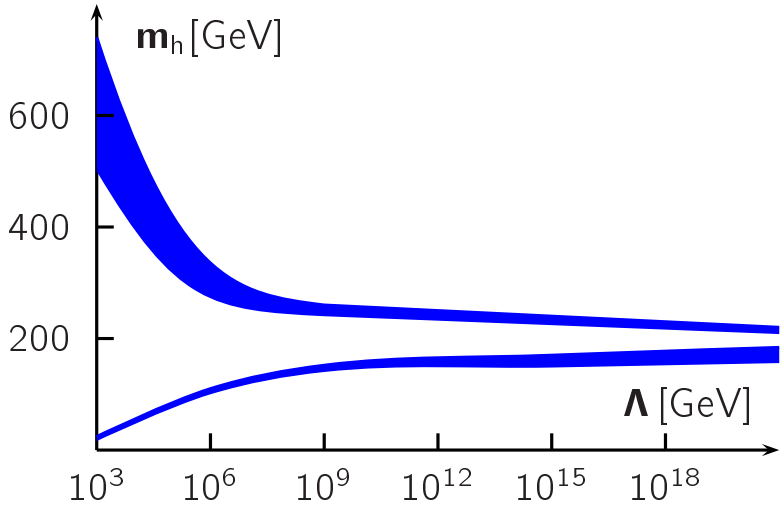
Motivation: Hierarchy Problem

Effective theories below a scale Λ $\Rightarrow$ Loop integration cut off at order $\sim \Lambda$:





◊ *Light* Higgs favoured by electroweak precision observables ($m_h < 0.5\,\text{TeV}$)

Problem: Natural Higgs mass is $\mathcal{O}(\Lambda^2)$:

$$m_h^2 = m_0^2 + \Lambda^2 \times (\text{loop factors})$$

$m_h \ll \Lambda \quad \Longleftrightarrow \quad$ Fine-Tuning !?

Solution: Mechanism for elimination of loop contributions

Invent (partial) symmetry to protect particle mass

Traditional (SUSY):
Spin-Statistics $\Longrightarrow$ Loops of bosons and fermions

Little Higgs:
Gauge group structure $\Longrightarrow$ Loops of particles of *like statistics*

Old Idea: Georgi/Pais, 1974; Georgi/Dimopoulos/Kaplan, 1984

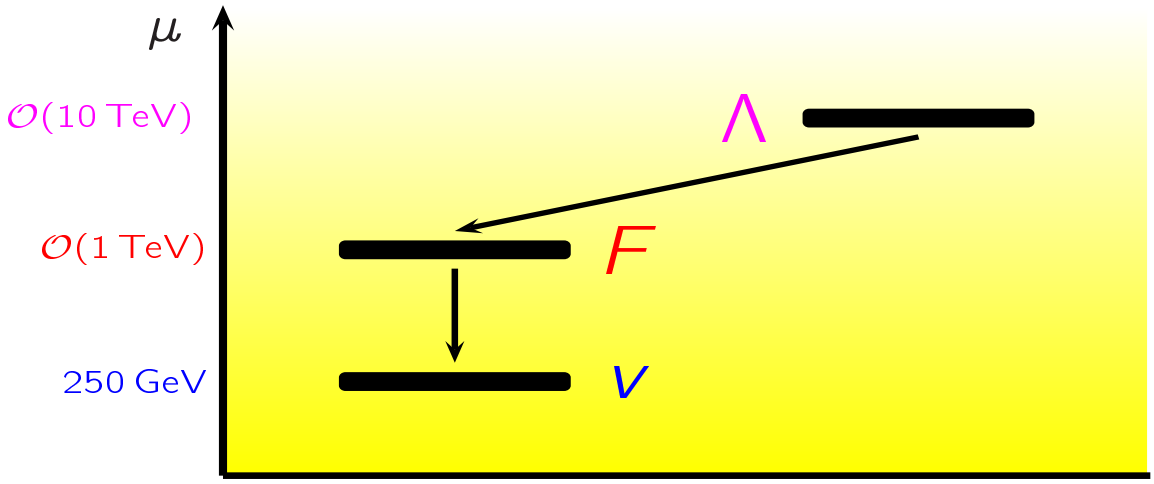
Light Higgs as Pseudo-Goldstone boson $\Leftrightarrow$ spontaneously broken (approximate) *global* symmetry; non-linear sigma model

- w/o Fine-Tuning: $v \sim \Lambda/4\pi$

New Ingredience: Arkani-Hamed/Cohen/Georgi/..., 2001

Gauge Group Structure eliminates quadratic divergences @ 1-loop level $\Longrightarrow$ 3-scale model

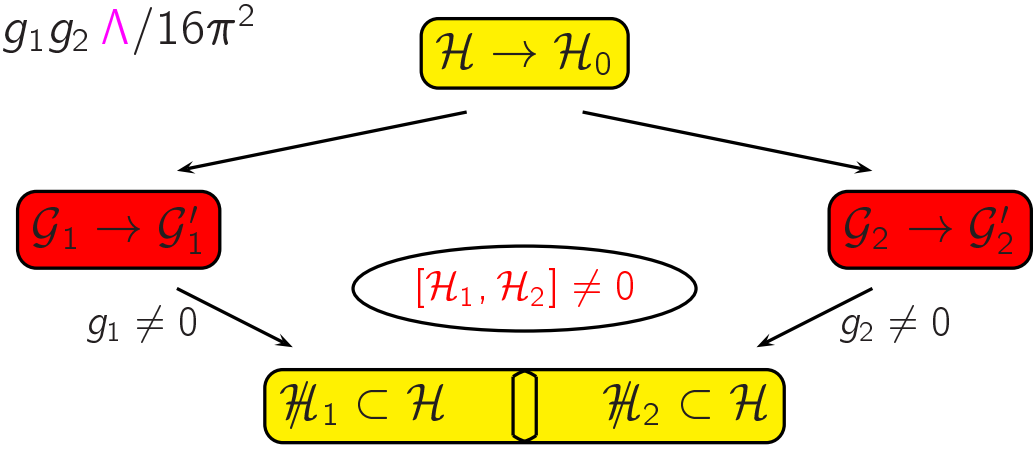
- Scale Λ: global SB, new dynamics, UV embedding
- Scale F: Pseudo-Goldstone bosons, new vector bosons and fermions
- Scale v: Higgs, $W^\pm$, Z, $\ell^\pm$, ...



How does Little Higgs actually work?

Boson masses radiative (Coleman-Weinberg), **but:** Higgs protected by symmetries *against quadratic corrections @ 1-loop level*

$M_H \sim g_1 g_2 \Lambda/16\pi^2$



- Extended scalar (Higgs-) sector
- Extended global symmetry
- **Specific form of the potential:**

$$\mathcal{V}(h,\Phi) = C_1 F^2 \left|\Phi + \frac{\mathrm{i}}{F} h \otimes h\right|^2 + C_2 F^2 \left|\Phi - \frac{\mathrm{i}}{F} h \otimes h\right|^2$$

h massless @ 1-loop level, $\langle h \rangle = v$, $M_h \sim v$

$M_\Phi \sim F$, $\mathrm{i} \cdot \langle \Phi \rangle \sim v^2/F$

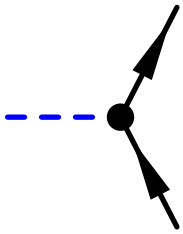
h and Φ mix

- **Extended Gauge Sector:**

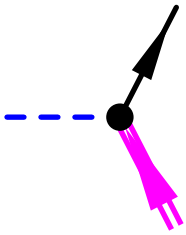
$\mathrm{SM}' \times \mathcal{G} \to \mathrm{SM} \implies$ Duplication of electroweak gauge bosons: $B', Z', W'^{\pm}$ (Hypercharge singlets & triplets)

- Extended top sector: new heavy quarks, t, t' loops $\Rightarrow$ *negative* M_h^2 $\Rightarrow$ EWSB

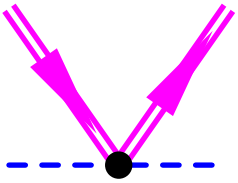
Couplings in the sector top, Higgs, T:



$$\frac{i\lambda_t}{\sqrt{2}}$$

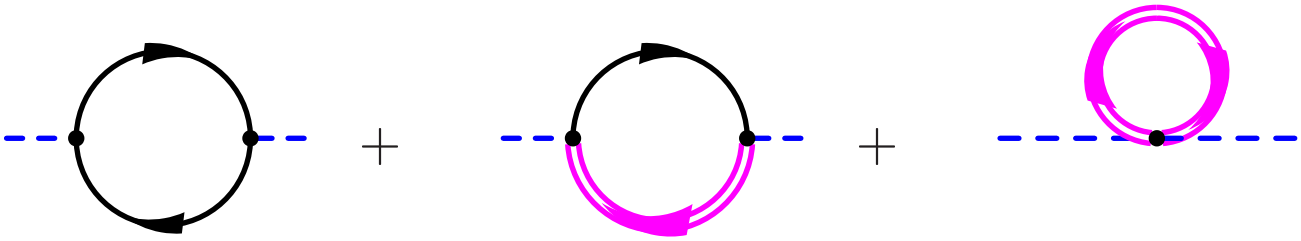


$$\frac{i\lambda_T}{\sqrt{2}}$$



$$\frac{i\lambda_T}{2F}$$

1-Loop Contributions to m_H^2



$$\propto \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k^2-m_T^2)} \left\{ \lambda_t^2(k^2-m_T^2) + k^2\lambda_T^2 - \frac{m_T}{F}\lambda_T k^2 \right\}$$

Little Higgs global symmetry imposes relation

$$\boxed{\frac{m_T}{F} = \frac{\lambda_t^2+\lambda_T^2}{\lambda_T}}$$

$$\Longrightarrow \quad \boxed{\textbf{Quadratic divergence cancels}}$$

Littlest Higgs: Arkani-Hamed/Cohen/Katz/Nelson, 2002

Symmetry breaking:

$$\begin{aligned} SU(5) &\to SO(5) && \text{(global)} \\ [SU(2)\times U(1)]^2 &\to SU(2)_L \times U(1)_Y && \text{(local)} \end{aligned}$$

1 heavy triplet X_μ, 1 heavy singlet Y_μ

The unbroken Lagrangian:

$$\mathcal{L} = \mathcal{L}_0^{(3)} + \mathcal{L}_0^{(1)} + \mathcal{L}_0^{G}.$$

$$\mathcal{L}_0^{(3)} = -\frac{1}{2g_1^2}\,\mathrm{Tr}\,\mathbf{A}_{1\mu\nu}\mathbf{A}_1{}^{\mu\nu} - \frac{1}{2g_2^2}\,\mathrm{Tr}\,\mathbf{A}_{2\mu\nu}\mathbf{A}_2{}^{\mu\nu} - 2\,\mathrm{tr}\,A_1{}^{\mu}\,J^{(3)}{}_{\mu}$$

$$\mathcal{L}_0^{(1)} = -\frac{1}{2g_1'^2}\,\mathrm{Tr}\,\mathbf{B}_{1\mu\nu}\mathbf{B}_1{}^{\mu\nu} - \frac{1}{2g_2'^2}\,\mathrm{Tr}\,\mathbf{B}_{2\mu\nu}\mathbf{B}_2{}^{\mu\nu} - B_1{}^{\mu}\,J^{(1)}{}_{\mu}.$$

Gauge group generators:

$$T_1^a = \frac{1}{2}\begin{pmatrix} \tau^a & & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad T_2^a = \frac{1}{2}\begin{pmatrix} 0 & & \\ & 0 & \\ & & -\tau^{a*} \end{pmatrix}$$

$$\begin{aligned} Y_1 &= \mathrm{diag}(3,3,-2,-2,-2)/10, \\ Y_2 &= \mathrm{diag}(2,2,2,-3,-3)/10, \end{aligned}$$

Triplet current: $J^{(3)}{}_\mu = J^{(3),a}_\mu \frac{\tau^a}{2}$

Singlet current: $J^{(1)}{}_\mu$ *Couplings not unique*, but:
(Anomaly cancellation!)

Goldstone Lagrangian

Spontaneous Symmetry Breaking (SSB) @ scale $F \sim$ TeV

$$\mathcal{L}_0^G = \frac{F^2}{8}\,\mathrm{Tr}(D_\mu \Xi)(D^\mu \Xi)^*, \qquad \Xi = \left(\exp \frac{2i}{F}\Pi\right)\Xi_0,$$

$$\Xi_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & h & \phi \\ h^\dagger & 0 & h^T \\ \phi^\dagger & h^* & 0 \end{pmatrix}.$$

Covariant derivative

$$D^\mu \Xi = \partial^\mu \Xi + i \sum_{k=1,2} \left[(\mathbf{A}_k{}^\mu \Xi + \Xi (\mathbf{A}_k{}^\mu)^T) + (\mathbf{B}_k{}^\mu \Xi + \Xi (\mathbf{B}_k{}^\mu)^T) \right]$$

Building blocks:

$$h = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + H + iz) \end{pmatrix}, \qquad \phi = \begin{pmatrix} \sqrt{2}\,\Phi^{++} & \Phi^+ \\ \Phi^+ & \Phi_0 + i\Phi_1 \end{pmatrix}.$$

3rd generation *distinguished* $\Leftrightarrow$ new vector-like top quark mixes with top

- Heavy top renders Higgs (mass)^2 negative
- Interaction with heavy quark *cancels quadratic cut-off dependence* in top-sector ($\rightarrow$ example)

Chiral fields:

$$\mathbf{Q}_R:\ b_R,\ t_R,\ T_R, \quad \text{and} \quad \mathbf{Q}_L:\ q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix},\ T_L,$$

$$\mathcal{L}_t = \sum_Q \bar{\mathbf{Q}} i \not{D} \mathbf{Q} + \mathcal{L}_Y - \lambda_2 F (\bar{T}_L T_R + \text{h.c.}).$$

Use a global $SU(3)$ subsymmetry

$$\chi_L = \begin{pmatrix} i\tau^2 T_L & i q_L & 0 \\ -i q_L^T & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$\boxed{\mathcal{L}_Y = \lambda_1 F\, \bar{t}_R\, \text{Tr}\left[\Xi^*(iT_2^2)\Xi^* \hat{\chi}_L\right] + \text{h.c.}}$$

∗ *Naturalness does not require cancellation mechanism for light fermions*

> Lepton-number violating interactions can generate **neutrino masses** (due to presence of triplet scalars)

Lagrangian invariant under **full gauge symmetry**

$$\boxed{\mathcal{L}_N = -g_N F (\bar{L}^c)^T \Xi L}, \quad \text{with} \quad L = (\mathrm{i}\tau^2 \ell_L, 0, 0)^T$$

EWSB: Generation of neutrino masses $\boxed{m_\nu \sim g_N v^2/F}$

Caveat: m_ν too large compared to observations

$\Rightarrow$ g_N small, e.g. F/Λ', where Λ' : scale of lepton number breaking

Mixing of the gauge fields:

$$A_1{}^\mu = W^\mu + g_X c^2 X^\mu, \qquad B_1{}^\mu = B^\mu + g_Y c'^2 Y^\mu,$$
$$A_2{}^\mu = W^\mu - g_X s^2 X^\mu, \qquad B_2{}^\mu = B^\mu - g_Y s'^2 Y^\mu,$$

$$\mathcal{L}_0^{(1)} + \mathcal{L}_0^{(3)} = \mathcal{L}_{g,gf}^{\text{EW}} - 2 g_X c^2 \operatorname{tr} X^\mu J_\mu - \frac{1}{2}\operatorname{tr} X_{\mu\nu} X^{\mu\nu} - g_Y c^2 Y^\mu J_\mu^{(1)} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu}$$

Expand the Goldstone Lagrangian

$$\mathcal{L}_0^G = M_X^2 \operatorname{tr} X_\mu X^\mu + g_X \frac{c^2 - s^2}{2} \operatorname{tr}[X^\mu V^{(3)}{}_\mu] + \frac{1}{2} M_Y^2 Y_\mu Y^\mu + g_Y \frac{c'^2 - s'^2}{4} Y^\mu V^{(1)}{}_\mu + \frac{1}{2}\operatorname{tr}(D_\mu \phi)^\dagger (D^\mu \phi) + (D_\mu h)^\dagger (D^\mu h) - \frac{1}{6F^2}\operatorname{tr}\left[V^{(3)}{}_\mu V^{(3),\mu}\right] + \ldots,$$

Heavy vector masses:

$$\boxed{M_X = g_X F/2}, \qquad \boxed{M_Y = g_Y F/(2\sqrt{5})},$$

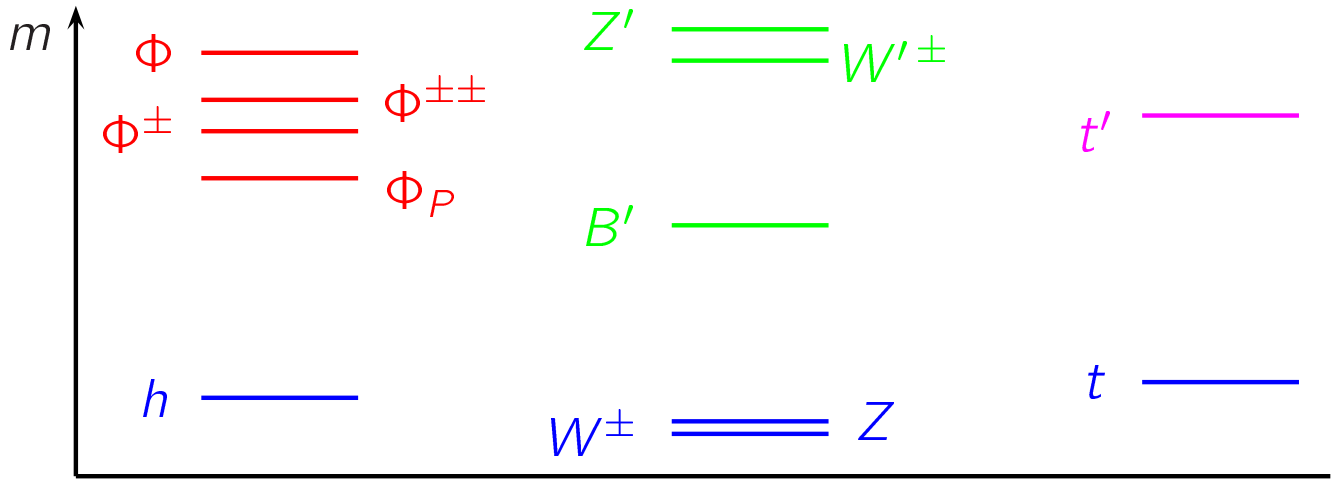
Higgs current

$$\boxed{V_\mu = i\left[h(D_\mu h)^\dagger - (D_\mu h) h^\dagger\right]}$$

$$\mathbf{1}_0: \quad V^{(1)}{}_\mu = \operatorname{tr} V_\mu, \qquad \mathbf{3}_0: \quad V^{(3)}{}_\mu = V_\mu - \frac{1}{2}\operatorname{tr} V_\mu.$$

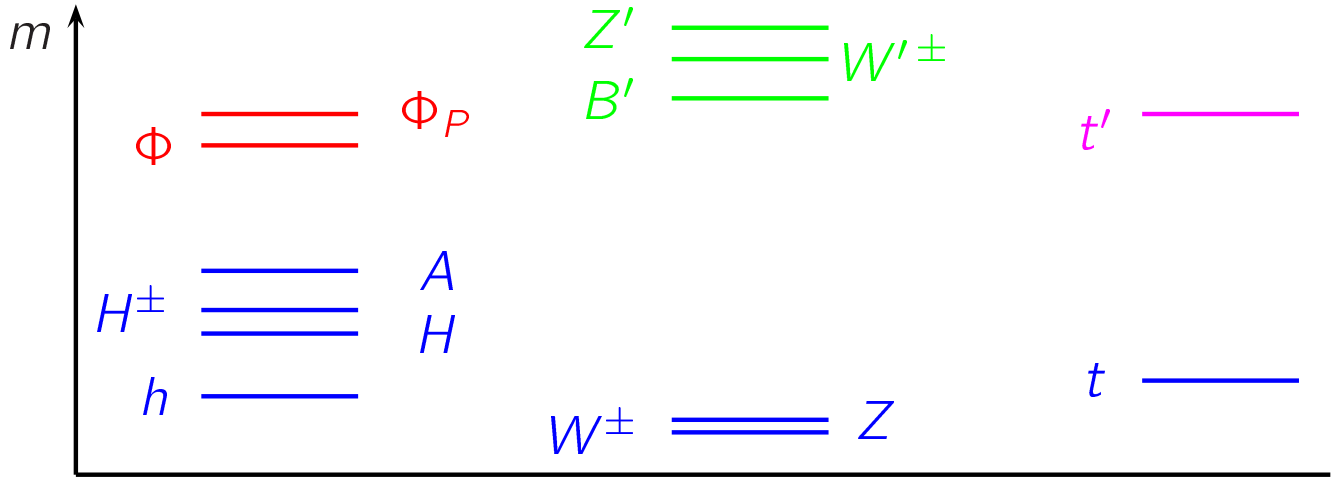
- $\boxed{SU(5) \to SO(5)}$ Arkani-Hamed/Cohen/Katz/Nelson, 2002

  $\mathcal{G} = SU(2) \otimes U(1)$, heavy complex Higgs triplet



- $\boxed{SO(6) \to Sp(6)}$ Low/Skiba/Smith, 2002

  $\mathcal{G} = SU(2) \otimes U(1)$, heavy complex singlet



- $\boxed{[SU(4)]^4 \to [SU(3)]^4}$ Kaplan/Schmaltz, 2003

  2HDM, $h_{1/2}$, $\Phi'_{1,2,3}$, $\Phi'_{P\,1,2,3}$, $Z'_{1,\ldots,8}$, $W'^{\pm}_{1,2}$, q', ℓ'

How to *clearly* separate effects of heavy degrees of freedom?

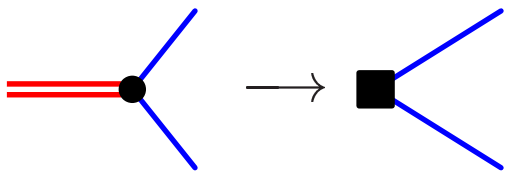
Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j,J] = \int \mathcal{D}[\Phi]\,\mathcal{D}[\varphi]\ \exp\left[i \int dx \left(\tfrac{1}{2}(\partial\varphi)^2 + \tfrac{1}{2}(\partial\Phi)^2 - \tfrac{1}{2}M^2\Phi^2 - \lambda\varphi^2\Phi - \ldots + J\Phi + j\varphi \right) \right],$$

Low-energy effective theory $\Rightarrow$ integrating out heavy degrees of freedom (DOF) in path integrals, set up Power Counting

Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2}\left(1 + \frac{\partial^2}{M^2}\right)^{-1}\varphi^2 :$$



$$\frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\varphi^2\Phi = - \frac{1}{2}\Phi'(M^2+\partial^2)\Phi' + \frac{\lambda^2}{2M^2}\varphi^2\left(1+\frac{\partial^2}{M^2}\right)^{-1}\varphi^2,$$

- ◇ Accounts for *all tree-level effects*
- ◇ Φ VEV, mixing, etc. **easily incorporated**
- ◇ UV-divergent corrections determined by matching conditions

- Experimental precision @ ‰ level consistent with truncation of expansion at order $1/M^2$, if $M \sim$ TeV
- **EFT for Little Higgs:** SM + EW Dim. 6 Operators
- Generically $v^2/M^2 \sim 1/16\pi^2$ (Naturalness)
- Loop corrections of heavy fields *suppressed by further powers of* v^2/M^2.
- Quadrat. div.: Coleman-Weinberg potential
- restrict calculation to the coefficients of dimension-six operators at tree-level

Integrating out the X and Y vector fields $\Rightarrow$

$$\begin{aligned}\mathcal{L}^{(1)} + \mathcal{L}^{(3)} = \mathcal{L}^{\text{EW}}_{g,gf} &+ f^{(3)}_{JJ}\,\text{tr}[J^{(3),\mu} J^{(3)}_\mu] + f^{(3)}_{VJ}\,\text{tr}[V^{(3),\mu} J^{(3)}_\mu] \\ &+ f^{(1)}_{VV}\,\text{tr}[V^{(3),\mu} V^{(3)}{}_\mu] + f^{(1)}_{JJ}\,J^{(1),\mu} J^{(1)}_\mu \\ &+ f^{(1)}_{VJ} V^{(1),\mu} J^{(1)}_\mu + f^{(1)}_{VV} V^{(1),\mu} V^{(1)}{}_\mu,\end{aligned}$$

In the Littlest Higgs e.g.

$$\boxed{f^{(3)}_{VV} = -\frac{1}{6F^2}\left(1 + \frac{3}{2}(c^2 - s^2)^2\right)}$$

Coleman-Weinberg potential of the scalar fields @ 1-loop:

$$\begin{aligned}\mathcal{L}^{CW}_0 = &-\frac{1}{2}M^2_\phi\,\text{tr}[\phi\phi^\dagger] + \mu^2(h^\dagger h) - \lambda_4(h^\dagger h)^2 \\ &- \lambda_{2\phi}\text{i}\left(h^\dagger\phi h^* - h^T\phi^\dagger h\right) - \lambda_{2\phi\phi}\,\text{tr}[(\phi\phi^\dagger)(hh^\dagger)] \\ &- \lambda_{4\phi}\text{i}(h^\dagger h)\left(h^\dagger\phi h^* - h^T\phi^\dagger h\right) - \lambda_6(h^\dagger h)^3\end{aligned}$$

Sensitive to UV completion, dimensionless parameters k and k'. EWSB $\Rightarrow$ Constraints on k, k'

Higgs mass parameter μ^2 in leading-logarithmic 1-loop order:

$$\mu^2 = +\frac{3\lambda_t^2}{8\pi^2}M_T^2 \log\frac{\Lambda^2}{M_T^2} - \ldots$$

Integrate out the heavy scalar (Power counting!)

$$\phi' = \phi - \frac{2i\lambda_{2\phi}}{M_\phi^2}\left(1 + \frac{D^2}{M_\phi^2} + \frac{2\lambda_{2\phi\phi}}{M_\phi^2} hh^\dagger\right)^{-1} \times \left(1 + \frac{\lambda_{4\phi}}{\lambda_{2\phi}} h^\dagger h\right) hh^T$$

Higgs mass up to order v^4/F^2

$$m_H^2 = 2\lambda_4^{\text{eff}} v^2$$

$$= -2v^2 \left(\frac{e^2}{s_w^2 c^2} + \frac{e^2}{c_w^2 c'^2}\right) k \cdot \frac{\left(\frac{e^2}{s_w^2 s^2} + \frac{e^2}{c_w^2 s'^2}\right) k + \frac{\lambda_t^2}{c_t^2} k'}{\left(\frac{e^2}{s_w^2 s^2 c^2} + \frac{e^2}{c_w^2 s'^2 c'^2}\right) k + \frac{\lambda_t^2}{c_t^2} k'}$$

(Remember $\mu^2 = m_H^2/2$)

EWSB

$$\lambda_4^{\text{eff}} > 0 \quad \Rightarrow \quad \boxed{\frac{\lambda_{2\phi}^2}{M_\phi^4} < \frac{1}{8F^2}}$$

- Couplings $V - J$ between Higgs and fermion currents induce after SSB anomalous couplings of the W and Z bosons to fermions
- SM gauge couplings def. by IA w. fermions convenient to eliminate the corrections by appropriate field redefinitions, i.e., by applying EOM
- Separate triplet/singlet $SU(2)_c$ conserving/violating

Custodial-$SU(2)$ Conserving Terms

EOM eliminates $V - J$ coupling

$$0 = \frac{1}{2} f_{VJ}^{(3)} \operatorname{tr}[V^{(3)}{}_\mu \frac{\delta \mathcal{L}}{\delta W_\mu}] = -\frac{1}{g^2} f_{VJ}^{(3)} \operatorname{tr}[V^{(3)}{}_\mu D_\nu W^{\mu\nu}] + \frac{1}{2} f_{VJ}^{(3)} \operatorname{tr}[V^{(3),\mu} V^{(3)}{}_\mu] - f_{VJ}^{(3)} \operatorname{tr}[V^{(3),\mu} J_\mu^{(3)}]$$

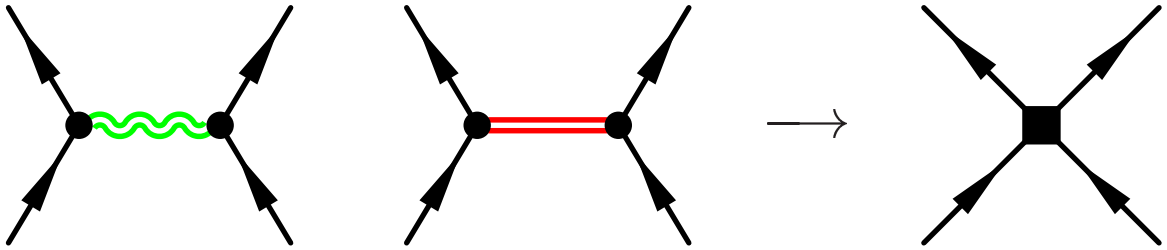
$$\boxed{\mathcal{L}^{(3)} = \mathcal{L}_{g,gf}^{\mathrm{EW}} + f_{JJ}^{(3)} \mathcal{O}_{JJ}^{(3)} + f_{VW} \mathcal{O}_{VW} + f_{VV}^{(3)} \mathcal{O}_{VV}^{(3)}}$$

$$\mathcal{O}_{VW} = \operatorname{tr} V^{(3)}{}_{\mu\nu} W^{\mu\nu}, \qquad \mathcal{O}_{JJ}^{(3)} = \operatorname{tr} J^{(3),\mu} J_\mu^{(3)}$$

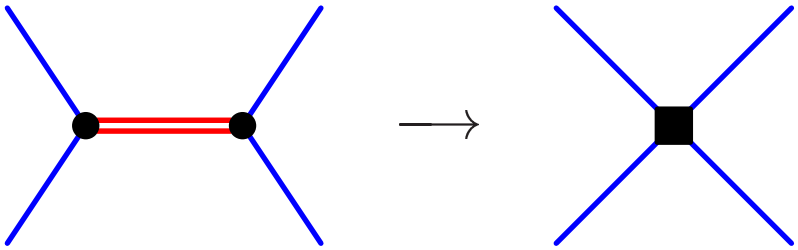
$$\mathcal{O}_{VV}^{(3)} = \operatorname{tr} V^{(3),\mu} V^{(3)}{}_\mu$$

Custodial-$SU(2)$ Violating Terms

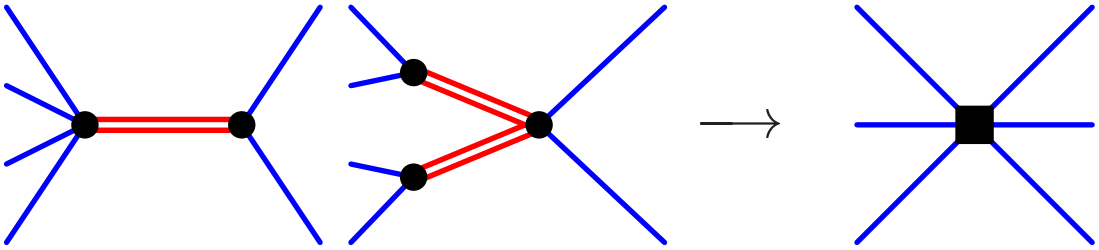
$$\boxed{\mathcal{L}^{(1)} = \mathcal{L}_{g,gf}^{\mathrm{EW}} + f_{JJ}^{(1)} \mathcal{O}_{JJ}^{(1)} + f_{VB} \mathcal{O}_{VB} + f_{VV}^{(1)} \mathcal{O}_{VV}^{(1)}}$$



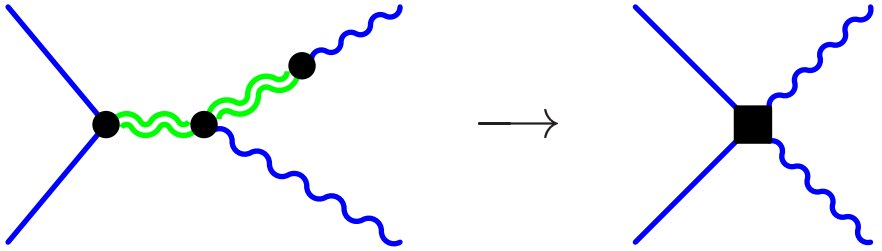
$$\frac{1}{F^2}\mathcal{O}_{JJ}^{(I)}$$



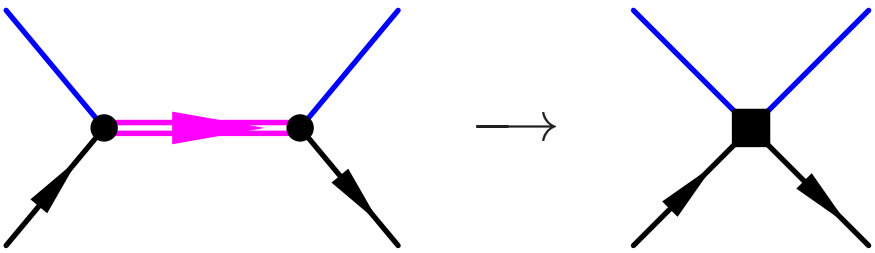
$$\frac{1}{F^2}\mathcal{O}_{VV}^{(I)}$$



$$\frac{1}{F^2}\mathcal{O}_{VV}^{(I)}$$



$$\frac{1}{F^2}\mathcal{O}_{VW},\ \frac{1}{F^2}\mathcal{O}_{VB}$$



$$\frac{1}{F^2}\mathcal{O}_{Vq}$$

- ◇ All low-energy effects order v^2/F^2
- ◊ Low-energy observables parameterized by ΔS, ΔT, 2 parameters for contact interactions (no ΔU op. here)

$\mathcal{O}_{VW}, \mathcal{O}_{VB} \Rightarrow$ Modified rotation of neutral gauge fields to diagonalize kinetic energies $\Rightarrow$ Change in gauge couplings g and g'

$$g = \frac{e}{s_w}\left[1 + M_W^2(f_{VW} + 2f_{VB})\right],$$
$$g' = \frac{e}{c_w}\left[1 + [M_Z^2 - M_W^2](f_{VW} + 2f_{VB})\right],$$

S parameter $(\mathcal{O}_{VW}, \mathcal{O}_{VB})$

$$\boxed{\Delta S = 8\pi v^2(f_{VW} + 2f_{VB})}$$

$SU(2)_c$-violating sector $(\mathcal{O}'_{h,1})$

$$\boxed{\alpha\Delta T = \Delta M_W^2/M_W^2 = -2v^2 f_{VV}^{(1)} - \frac{2v^2\lambda_{2\phi}^2}{M_\phi^4}}$$

Shift in physical vector masses:

$$M_W^2 = \left(\frac{ev}{2s_w}\right)^2 (1+x) \qquad M_Z^2 = \left(\frac{ev}{2s_w c_w}\right)^2 (1+y)$$

$$x = \alpha\left(\frac{\Delta S}{4s_w^2} + \Delta T\right)$$
$$y = \alpha\left(\frac{\Delta S}{4s_w^2 c_w^2}\right)$$

Four-Fermion Interactions

Very low energies $\Rightarrow$ Fermi theory

$$\sqrt{2}\,G_F = \frac{1}{v^2}(1+z) \quad \text{with} \quad z = -\alpha\Delta T - \frac{v^2}{4} f_{JJ}^{(3)}.$$

Choose G_F-M_Z-α **scheme** (muon decay, LEP I, Bhabha) and define the parameters $\hat{v}_0$ and $\hat{s}_0$ by the relations

$$\hat{v}_0 = (\sqrt{2}\,G_F)^{-1/2} \quad \text{and} \quad M_Z = \frac{e\hat{v}_0}{2\hat{s}_0\hat{c}_0}.$$

Two definitions of mixing angles:

$$s_w^2 = \hat{s}_0^2\left(1 + \frac{\hat{c}_0^2}{\hat{c}_0^2 - \hat{s}_0^2}(y+z)\right),$$
$$c_w^2 = \hat{c}_0^2\left(1 - \frac{\hat{s}_0^2}{\hat{c}_0^2 - \hat{s}_0^2}(y+z)\right).$$

Constraints from contact IA: ($f_{JJ}^{(3)}$ and $f_{JJ}^{(1)}$)

$$c^2 \lesssim F/4.5\,\text{TeV} \qquad c'^2 \lesssim F/10\,\text{TeV}\ .$$

◊ **Constraints evaded** $\Longleftrightarrow$ $\mathbf{c, c' \ll 1}$.

$B', Z', W'^{,\pm}$ superheavy ($\mathcal{O}(\Lambda)$)

decouple from fermions

ΔS, ΔT in the Littlest Higgs model, violation of Custodial SU(2):

Csáki et al., 2002; Hewett et al., 2002; Han et al., 2003; Kilian/JR, 2003

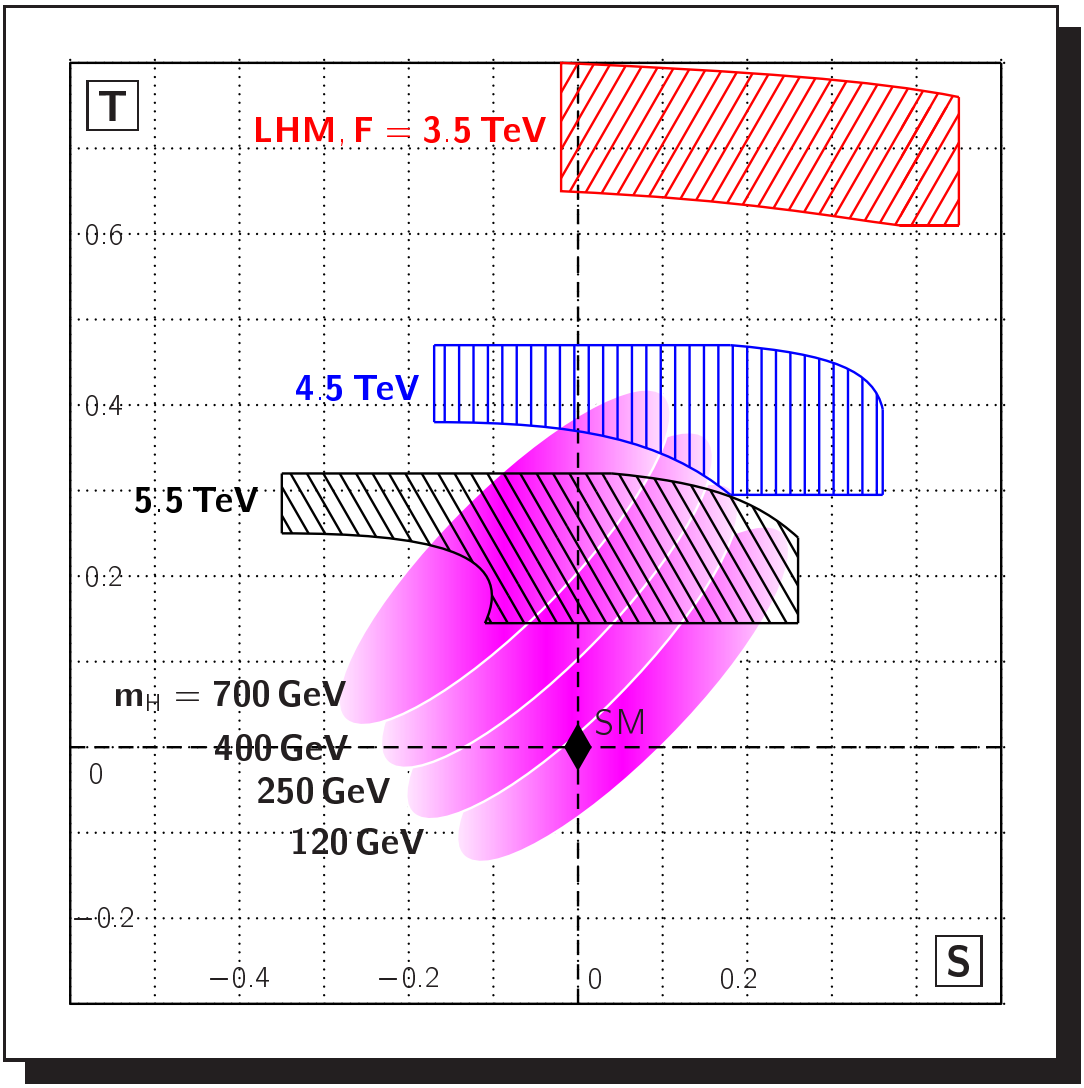
◇ Mixing of (Z, B', Z') and $(W^\pm, W'^\pm)$

$$\Delta S = -8\pi\left[\frac{c^2(c^2-s^2)}{g^2} + 5\frac{c'^2(c'^2-s'^2)}{g'^2}\right]\frac{v^2}{F^2} \longrightarrow 0$$

$$\alpha\Delta T \longrightarrow \frac{5}{4}\frac{v^2}{F^2} - \frac{2v^2\lambda_{2\phi}^2}{M_\phi^4} \gtrsim \frac{v^2}{F^2}$$

General models

- Triplet sector: (almost) identical to Littlest Higgs (ΔS only)
- More freedom in $U(1)$ sector: (ΔT)



Higgs mass *variable* (Coleman-Weinberg, UV completion)

$$\Delta S = \frac{1}{12\pi} \ln \frac{m_H^2}{m_0^2} \qquad \Delta T = -\frac{3}{16\pi c_w^2} \ln \frac{m_H^2}{m_0^2}$$

Peskin/Takeuchi, 1992; Hagiwara et al., 1992

Making the Higgs heavier reduces amount of fine-tuning

◇ *Tree-level contributions to TGC from heavy particles exchange impossible in dim.-6 operators*

Arzt/Einhorn/Wudka, 1994

◇ Gauge couplings measured in processes w. external fermions. Use EOM for normalization of fermion gauge interactions. ⇒ Anom. TGC

- $\boxed{g_1^Z = \kappa_Z = 1 - 2M_Z^2 f_{VW}}$ $\qquad$ $\boxed{g_1^\gamma = \kappa_\gamma = 1}$
- Generated by renormalization of gauge couplings ⇒

$$g_1^Z = \kappa_Z \qquad g_1^\gamma = \kappa_\gamma$$

- $U(1)_{EM}$ unbroken $\Leftrightarrow g_1^\gamma = 1$
- $SU(2)_c$ + gauge relation $\Delta\kappa_\gamma = -\frac{c_w^2}{s_w^2}(\Delta\kappa_Z - \Delta g_1^Z)$ automatically satisfied — no contribution from $SU(2)_c$-violating coefficient f_{VB}

Parameters of the effective Lagrangian e, s_w,(M_W, M_Z, v).

Differences in scheme of higher order

G_F-M_Z-α scheme: Define the 3-gauge-boson vertices in terms of $\hat{s}_0$ and $\hat{c}_0$:

$$\boxed{g_1^Z = \kappa_Z = 1 - 2M_Z^2 f_{VW} - \frac{y+z}{2(\hat{c}_0^2 - \hat{s}_0^2)}}$$

Induced by vector and scalar exchange

1. Couplings to longitudinal gauge bosons:

$$\mathcal{L}_{HVV} = \chi_W \frac{2M_W^2}{v} H W^{+,\mu} W^-{}_\mu + \chi_Z \frac{M_Z^2}{v} H Z^\mu Z_\mu$$

SM: tree-level, Anomalous contribution

$$\chi_W = 1 + 4M_W^2 f_{VW} + \frac{v^2}{2}\left(3f_{VV}^{(3)} + 2f_{VV}^{(1)}\right) + v^2 \frac{2\lambda_{2\phi}^2}{M_\phi^4},$$

$$\chi_Z = \ldots$$

G_F-M_Z-α scheme $\Rightarrow$ indirect contrib. $\Delta\chi_{W/Z} = -\frac{1}{2}z$

2. Coupl. to transversal gauge bosons: @ loop-level only

3. Contact terms

$$\mathcal{L}_{HVJ} = M_W f_{VJ}^{(3)} H W^{\pm,\mu} J^\mp{}_\mu + \\ + M_Z f_{VJ}^{(3)} H Z^\mu J^{(3),0}{}_\mu - 2M_Z f_{VJ}^{(1)} H Z^\mu J^{(1)}{}_\mu.$$

4. Anomalous couplings H to massive fermions. Operator $\mathcal{O}'_{J_s}$: all such couplings modified commonly
$\chi_f = 1 - v^2 f_{J_s}$

Higgs pair production

1. Quartic $HHWW$ and $HHZZ$ couplings modified: depend on $\mathcal{O}_{VW}$, $f_{VV}^{(1/3)}$, $\lambda_{2\phi}/M_\phi^2$

2. Cubic Higgs self-coupling affected by $\mathcal{O}_{h,3}$, $\mathcal{O}_{h,1}$, $\mathcal{O}_{hh}$

Presence of heavy quark T affects interactions of t with gauge bosons and Higgs bosons in a non-universal way

1. Modified weak current IA ($\mathcal{O}_{Vq}$: T exchange, redefinition of vector fields due to heavy-vector exchange)

$$\mathcal{L}_{tV} = -\frac{e}{4c_w s_w}\left[\bar{t}\not{Z}\left(v_t - a_t\gamma_5\right)t - \bar{b}\not{Z}\left(v_b - a_b\gamma_5\right)b\right] - \frac{e}{2\sqrt{2}\,s_w}c_{tb}\left[\bar{t}\not{W}^+(1-\gamma_5)b + \bar{b}\not{W}^-(1-\gamma_5)t\right]$$

   - $(f_{VW} + 2f_{VB})$ corrections universal to all fermions
   - Corrections $\sim v^2 f_{Vq}$ specific to top-quark vertices

2. Modified top-quark Yukawa coupling (T, $\mathcal{O}_{hq}$, $\mathcal{O}_{VV}^{(3)}$, $\mathcal{O}_{VV}^{(1)}$)

$$\chi_t = 1 + \frac{v^2}{2}\left(f_{VV}^{(3)} + 2f_{VV}^{(1)}\right) + \frac{v^3}{\sqrt{2}\,m_t}f_{hq}.$$

3. $\exists$ quartic $ttZH$ and $tbWH$ vertices

   Note: Couplings left-handed (right-handed top quark mixes T)

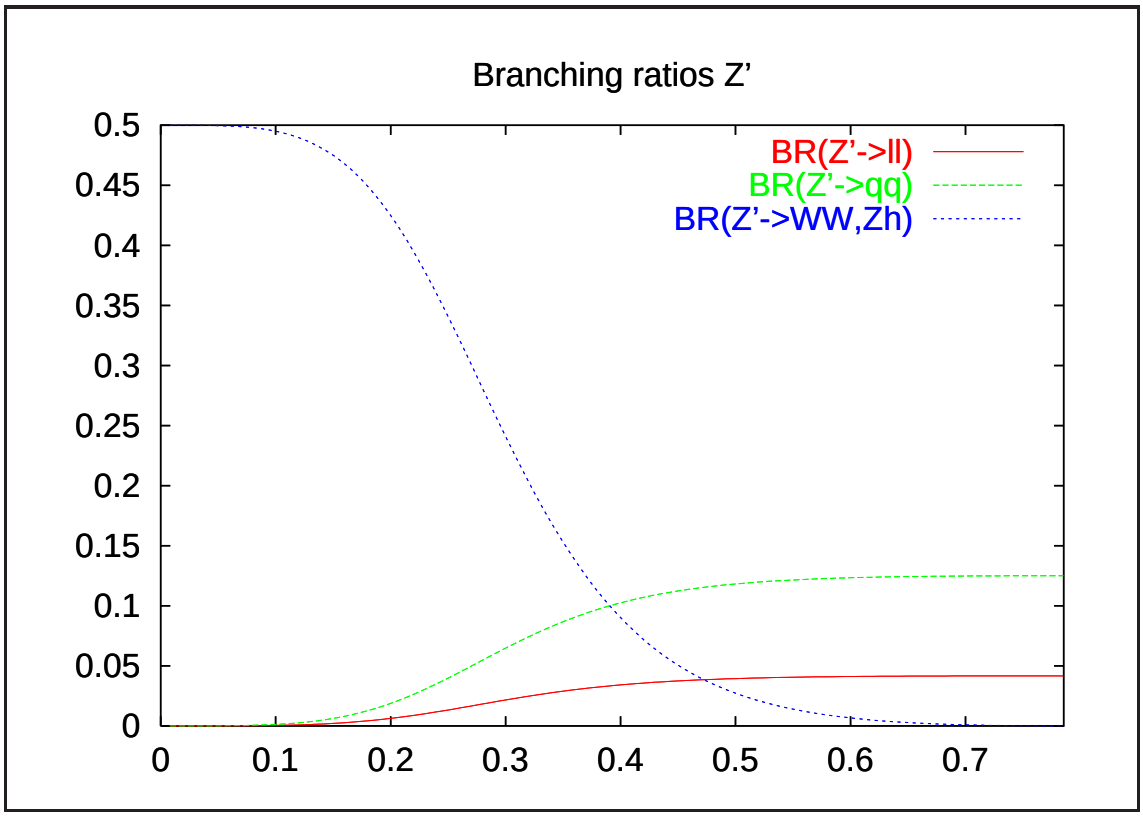
- **Heavy Gauge Bosons:**

Detection: Resonance in $e^+e^- \to f\bar{f}$, Drell-Yan $\Rightarrow$

Tevatron: $M_{B'} \gtrsim 1.5\,\text{TeV}$

Total width Z', fixed $M_{Z'}$:

$$\Gamma_{Z'} = \frac{g^2}{96\pi}\left(\cot^2 2\phi + 24\tan^2\phi\right) M_{Z'}$$



Determination of **F**, **c**, **s**

$\mathcal{O}(10^2)$ lepton events @ LHC w. 300 fb^{-1}

- **Heavy Scalars:**
  - ◇ Φ_P : $e^+e^-, q\bar{q} \to \Phi_P h, \quad \Phi_P \to h Z_L$
  - ◇ $\Phi^\pm$: $e^+e^-, q\bar{q} \to \Phi^+ W_L^-, \quad \Phi^\pm \to W_L Z_L$
  - ◇ $\Phi^{\pm\pm}$: $e^-e^- \to \nu\nu\Phi^{--}, \quad \Phi^{\pm\pm} \to W_L^- W_L^-$
  - ◇ Φ^0 : $e^+e^-, q\bar{q} \to Z_L \Phi,$
    $\Phi \to Z_L Z_L, hh,$ *not* W^+W^-

- **Heavy Quarks:**

  T production @ LHC: $bq \to T q'$

  Decay $T \to W_L^+ b, th, tZ$ Perelstein/Peskin/Pierce, 2003

  Total cross section

$$\Gamma_T = \frac{m_T \lambda_T^2}{16\pi}$$

  Branching ratios (limit $g \to 0$):

$$\Gamma(T \to th) \approx \Gamma(T \to tZ) \approx \frac{1}{2}\Gamma(T \to bW^+) \approx \frac{m_T \lambda_T^2}{64\pi}$$

Determination of $\mathbf{m}_T$, λ_T

How can we unravel the structure of LHM @ colliders?

- ◇ Symmetry structure ⇒ Quadr. Div. Cancell.
- ◇ Nonlinear Goldstone boson structure

Signals:

- *Anom.* Triple Gauge Couplings: WWZ, $WW\gamma$
- *Anom.* Higgs Coupl.: $H(H)WW$, $H(H)ZZ$
- *Anom.* Top Couplings: ttZ, tbW

Vectors:

- Direct Search (LHC) M_V, F, c, c'
- LC: Contact Terms $e^+e^- \to e^+e^-, \mu^+\mu^-$
  $\Rightarrow M_{B'} \sim 5 - 10\,\text{TeV}$, $e^+e^- \to \nu\bar{\nu}\gamma \Rightarrow M_{B'} \sim 5\,\text{TeV}$
- Higgs-strahlung, WW fusion: $HZff$, $HWff$ angular distr./energy dependence $\Rightarrow f_{VJ}^{(1/3)}$
- Check from TGC (LC: per mil precision), GigaZ $\Rightarrow f_{JJ}^{(3)}$

Combining ⇒ Determination of all coefficients in the gauge sector

Scalars:

Affected by scalars and vectors

- ΔT, $f_{VV}^{(1)}$, B' known $\Rightarrow$ $(\lambda_{2\phi}/M_\phi^2)^2$
- Higgsstr., WW fusion $\Rightarrow$ Higgs coupl., $f_{VV}^{(3)}$
- Higgs BRs $\Rightarrow$ $f_{VV}^{(1)}, f_{VV}^{(3)}$; (take care of t)
  $f_{VV}^{(3)}$ **Goldstone contr. $\Rightarrow$ Evidence for nonlinear nature**
- HH production $\Rightarrow$ $f_{h,3}$ (difficult!)

LHC $\bowtie$ LC $\Rightarrow$ 1-2 % accuracy @ Higgs measurements
Reconstruction of scalar sector up to $F \sim 2\,\mathrm{TeV}$

Top:

- Direct production @ LHC
- $t\bar{t}$ production $\Rightarrow$ f_{Vq}, v_t, a_t; accuracy 1-2 %
- tbW from t decays, single t production $g_{ttH}/g_{bbH} \Rightarrow$ *anom. Yukawa coupl.* $\Rightarrow$ f_{hq}, nonlin. structure w. $\sim 2.5\%$ accuracy

Include all observables in a combined fit if Little Higgs signals are found (sufficient data from LHC and LC)

◊ Little Higgs elegant alternative to SUSY
Gauge group structure stabilizes EW scale

- Generics: new heavy gauge bosons, scalars, quarks

Clear experimental signatures:
direct search [Gauge & Top sector, LHC (LC)]
$\longleftrightarrow$
precision observables [Gauge, Scalar, Top sector LC (LHC)]

Strategy for Reconstruction
by *Complementarity* of LC & LHC

- *Future work:* Detailed studies of various effects

UV embedding, GUT, Flavor ?

◊ Little Higgs *in accord with EW precision observables* **w/o Fine Tuning** (Higgs mass!)