Anomalous Gauge Couplings at the LHC and ILC

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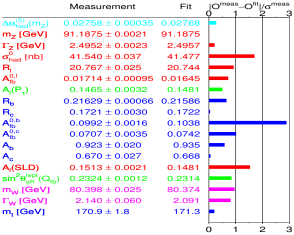
Kilian/JR **PRD 70** (2004), 015004; Beyer/Kilian/Krstonošić/Mönig/Schmitt/Schröder, **EPJC 48** (2006), 353;

M. Mertens, 2005; Alboteanu/Kilian/JR, Kilian/Kobel/Mader/JR/Schumacher, work in progress

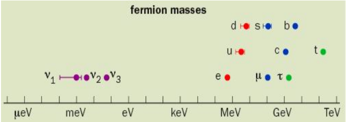
Seminar, Bonn, January 17, 2008

Doubts on the Standardmodel

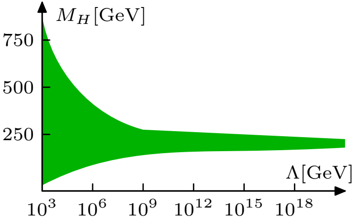


– describes microcosm (too good?)

– 28 free parameters



– Higgs ?, form of Higgs potential ?



Hierarchy Problem

chiral symmetry: $\delta m_f \propto v \ln(\Lambda^2/v^2)$

no symmetry for quantum corrections to Higgs mass

$$\delta M_H^2 \propto \Lambda^2 \sim M^2_{\text{Planck}} = (10^{19})^2\,\text{GeV}^2$$

20000 GeV^2 = (100000000000000000000000000000000020000 –

100000000000000000000000000000000000000) GeV^2

Open Questions

– Unification of all interactions (?)

– Baryon asymmetrie $\Delta N_B - \Delta N_{\bar{B}} \sim 10^{-9}$
missing CP violation

– Flavour: three generations

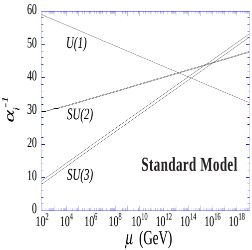
– Tiny neutrino masses: $m_\nu \sim \frac{v^2}{M}$

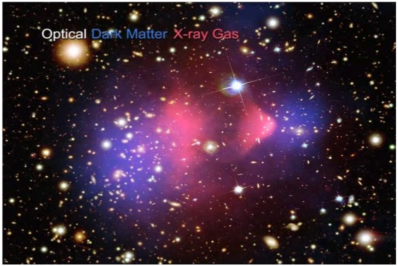
– Dark Matter:
  - stable
  - only weakly interacting
  - $m_{DM} \sim 100$ GeV

– Quantum theory of gravity

– Cosmic inflation

– Cosmological constant





Ideas for New Physics since 1970

(1) **Symmetry for eliminating the quantum corrections**

- Supersymmetry: Spin-Statistics ⇒ Corrections from bosons and fermions cancel each other
- Little-Higgs models: Global symmetries ⇒ Corrections from particles of like statistics cancel each other

(2) **New ingredients/sub-structure**

- Technicolor/Topcolor: Higgs bound state of new strongly interacting partons

(3) **Nontrivial space-time structure eliminates hierarchy**

- Additional space dimensions: gravity appears only weak
- Noncommutative spacetime: space-time is coarse-grained

(4) **Ignoring the hierarchy**

- Anthropic principle: Parameters are the way they are, since we can measure them


bosonic TC
composite Higgs
TC-TC
hypercolor
supercolor
techni-GIM
extended TC
pseudo NGB
NOT YET THOUGHT OF
NOT YET THOUGHT OF
NOT YET THOUGHT OF
NOT YET THOUGHT OF
effective susy
SUSY
NMSSM
MSSM +νR
unified SM
axigluon
sterile ν
6th gen
5th gen
4th gen
S,T,U
vectorlike family
lepto quark
heavy Majorana
fractionally charged
milli-charged
monopole
Z'LR
Z'ψ
WR
Z'SM
Z'χ
Z'η
X
Y
shadow matter
mirror symmetry
triplet Higgs
doubly charged Higgs
general 2HDSM
Type I
Type II
spontaneous CP
superweak
milli-weak
Weinberg's 3HD
Majoron
axion
familon
NGB
saxion
axino
quintessence
k-essence
composite W,Z
contact int.
string
N=2
N=4
N=8
heterotic
matrix
M
IIB
IIA
F
I
Extra dim
worm-hole

Model-Independent Way – Effective Field Theories

How to <u>clearly</u> separate effects of heavy degrees of freedom?

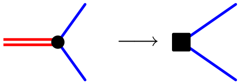
Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j,J] = \int \mathcal{D}[\Phi]\,\mathcal{D}[\varphi]\;\exp\left[i\int dx\left(\tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}\Phi(\Box + M^2)\Phi - \lambda\varphi^2\Phi - \ldots + J\Phi + j\varphi\right)\right]$$

Low-energy effective theory $\Rightarrow$ integrating out heavy degrees of freedom (DOF) in path integrals, set up Power Counting

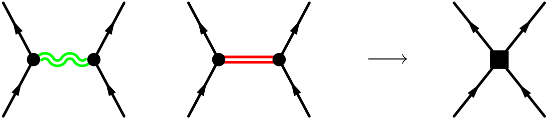
Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2}\left(1 + \frac{\partial^2}{M^2}\right)^{-1}\varphi^2 \Rightarrow$$

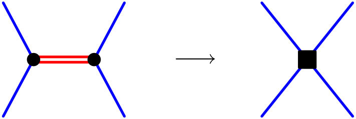


$$\frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\varphi^2\Phi = -\frac{1}{2}\Phi'(M^2+\partial^2)\Phi' + \frac{\lambda^2}{2M^2}\varphi^2\left(1+\frac{\partial^2}{M^2}\right)^{-1}\varphi^2.$$

Effective Dim. 6 Operators

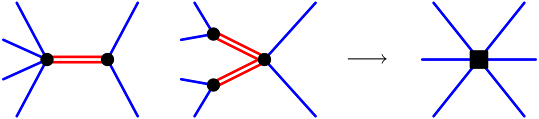


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{F^2}\mathrm{tr}\,[[]\, J^{(I)}\cdot J^{(I)}]$$

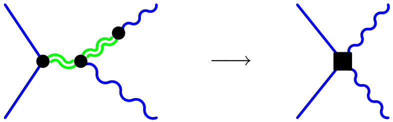


$$\mathcal{O}'_{h,1} = \tfrac{1}{F^2}\left((Dh)^\dagger h\right)\cdot\left(h^\dagger (D^h)\right) - \tfrac{v^2}{2}|Dh|^2$$

$$\mathcal{O}'_{hh} = \tfrac{1}{F^2}(h^\dagger h - v^2/2)\,(Dh)^\dagger\cdot(Dh)$$



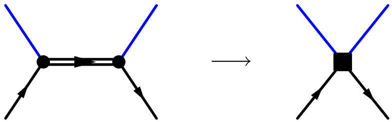
$$\mathcal{O}'_{h,3} = \frac{1}{F^2}\frac{1}{3}(h^\dagger h - v^2/2)^3$$



$$\mathcal{O}'_{WW} = -\frac{1}{F^2}\frac{1}{2}(h^\dagger h - v^2/2)\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

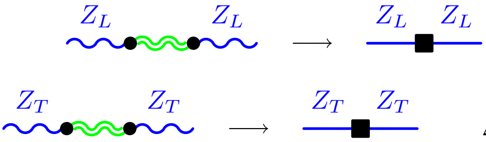
$$\mathcal{O}_B = \frac{1}{F^2}\frac{i}{2}(D_\mu h)^\dagger (D_\nu h) B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{F^2}\frac{1}{4}(h^\dagger h - v^2/2)B_{\mu\nu}B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{F^2}\overline{q}h(\not{D}h)q$$

Oblique Corrections: S, T, U



$$\Delta T \sim \Delta\rho \sim \Delta M_Z^2 Z \cdot Z$$

$$\Delta S \sim W^0{}_{\mu\nu} B^{\mu\nu}, \Delta U \sim W^0{}_{\mu\nu} W^{0\,\mu\nu}$$

- All low-energy effects order v^2/F^2 (Wilson coefficients)
- Low-energy observables with low-energy input G_F, α, M_Z affected by **non-oblique** contributions:

$$G_F = \frac{1}{v} \longrightarrow \frac{1}{v}\left(1 - \alpha\Delta T + \delta\right), \qquad \boxed{\delta \equiv = -\frac{v^2}{4} f_{JJ}^{(3)}}$$

$$\begin{aligned} S_{\text{eff}} &= \Delta S \\ T_{\text{eff}} &= \Delta T - \tfrac{1}{\alpha}\delta \\ U_{\text{eff}} &= [\Delta U = 0] + \tfrac{4s_w^2}{\alpha}\delta \end{aligned}$$

- non-oblique flavour-dependent corrections ⇒ enforce flavour-dependent EW fit

Model-Independent Description of the EW sector

- Higgs boson still not observed
- Aim: describe any physics beyond the SM as generically as possible
- Implement what we know about the SM
- Implements $SU(2)_L \times U(1)_Y$ gauge invariance
- Building blocks (including longitudinal modes):

$$\psi \text{ (SM fermions)}, \quad W_\mu^a \; (a = 1, 2, 3), \quad B_\mu, \quad \Sigma = \exp\left[\frac{-i}{v} w^a \tau^a\right]$$

- Minimal Lagrangian including gauge interactions

$$\mathcal{L}_{\text{min}} = \sum_\psi \overline{\psi}(i\not{D})\psi - \frac{1}{2g^2}\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2g'^2}\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \frac{v^2}{4}\text{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right]$$

The Fundamental Building Blocks

- $\mathbf{V} = \Sigma(\mathbf{D}\Sigma)^\dagger$ (longitudinal vectors), $\mathbf{T} = \Sigma\tau^3\Sigma^\dagger$ (neutral component)
- Unitary gauge (no Goldstones): $\mathbf{w} \equiv 0$, i.e., $\Sigma \equiv 1$.

$$\mathbf{V} \longrightarrow -\frac{\mathrm{i}g}{2}\left[\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_\mathrm{w}} Z\tau^3\right]$$
$$\mathbf{T} \longrightarrow \tau^3$$

- Gaugeless limit (only Goldstones) ($g, g' \to 0$):

$$\mathbf{V} \longrightarrow \frac{\mathrm{i}}{v}\left\{\sqrt{2}\partial w^+\tau^+ + \sqrt{2}\partial w^-\tau^- + \partial z\tau^3\right\} + O(v^{-2})$$
$$\mathbf{T} \longrightarrow \tau^3 + 2\sqrt{2}\frac{\mathrm{i}}{v}\left(w^+\tau^+ - w^-\tau^-\right) + O(v^{-2})$$

So $\mathbf{T}$ projects out the neutral part:

$$\mathrm{tr}\,[\mathbf{TV}] = \frac{2\mathrm{i}}{v}\left[\partial z + \frac{\mathrm{i}}{v}\left(w^+\partial w^- - w^-\partial w^+\right)\right] + O(v^{-3})$$

Electroweak Chiral Lagrangian

Complete Lagrangian contains infinitely many parameters

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{min}} - \sum_\psi \overline{\psi}_L \Sigma M \psi_R + \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \operatorname{tr}[\mathbf{T}\mathbf{V}_\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \operatorname{tr}[\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}] \qquad \mathcal{L}_6 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_2 = i\operatorname{tr}[\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \mathcal{L}_7 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}_\nu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_3 = i\operatorname{tr}[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \mathcal{L}_8 = \tfrac{1}{4}\operatorname{tr}[\mathbf{T}\mathbf{W}_{\mu\nu}] \operatorname{tr}[\mathbf{T}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_4 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \operatorname{tr}[\mathbf{V}^\mu \mathbf{V}^\nu] \qquad \mathcal{L}_9 = \tfrac{i}{2}\operatorname{tr}[\mathbf{T}\mathbf{W}_{\mu\nu}] \operatorname{tr}[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_5 = \operatorname{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \operatorname{tr}[\mathbf{V}_\nu \mathbf{V}^\nu] \qquad \mathcal{L}_{10} = \tfrac{1}{2}\left(\operatorname{tr}[\mathbf{T}\mathbf{V}_\mu] \operatorname{tr}[\mathbf{T}\mathbf{V}^\mu]\right)^2$$

Indirect info on new physics in $\beta_1, \alpha_i, \ldots$ (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1 \qquad \alpha_1 = 0.0026 \pm 0.0020$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}} \qquad \beta_1 = -0.00062 \pm 0.00043$$

$$\Delta U = -16\pi\alpha_8 \qquad \alpha_8 = -0.0044 \pm 0.0026$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = \mathrm{ie}\left[g_1^\gamma A_\mu\left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\,\nu} W_{\nu\rho}^+ A^{\rho\mu}\right]$$
$$+ \mathrm{ie}\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^Z Z_\mu\left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\,\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1,\ \lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $\quad g_{1/2}^{VV'} = 1,\ h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$
$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$
$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$
$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$
$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Anomalous triple and quartic gauge couplings

$$\begin{aligned}\mathcal{L}_{QGC} &= e^2\left[g_1^{\gamma\gamma}A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma}A^\mu A_\mu W^{-\nu}W_\nu^+\right]\\ &+ e^2\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^{\gamma Z}A^\mu Z^\nu\left(W_\mu^-W_\nu^+ + W_\mu^+W_\nu^-\right) - 2g_2^{\gamma Z}A^\mu Z_\mu W^{-\nu}W_\nu^+\right]\\ &+ e^2\frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\left[g_1^{ZZ}Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ}Z^\mu Z_\mu W^{-\nu}W_\nu^+\right]\\ &+ \frac{e^2}{2s_\mathrm{w}^2}\left[g_1^{WW}W^{-\mu}W^{+\nu}W_\mu^-W_\nu^+ - g_2^{WW}\left(W^{-\mu}W_\mu^+\right)^2\right] + \frac{e^2}{4s_\mathrm{w}^2c_\mathrm{w}^4}h^{ZZ}(Z^\mu Z_\mu)^2\end{aligned}$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$

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$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Parameters and Scales, Resonances

α_i measurable at ILC

- $\alpha_i \ll 1$ (LEP)
- $\alpha_i \gtrsim 1/16\pi^2 \approx 0.006$ (renormalize divergencies, $16\pi^2 \alpha_i \gtrsim 1$)

Translation of parameters into new physics scale Λ: $\alpha_i = v^2/\Lambda^2$

- Operator normalization is arbitrary
- Power counting can be intricate

To be specific: consider resonances that couple to EWSB sector
Resonance mass gives detectable shift in the α_i

- Narrow resonances $\Rightarrow$ particles
- Wide resonances $\Rightarrow$ continuum

$\beta_1 \ll 1 \;\Rightarrow SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' \neq 0$ and fermion masses)

	$J=0$	$J=1$	$J=2$
$I=0$	σ^0 (Higgs ?)	ω^0 $(\gamma'/Z'\ ?)$	f^0 (Graviton ?)
$I=1$	$\pi^\pm, \pi^0$ (2HDM ?)	$\rho^\pm, \rho^0$ $(W'/Z'\ ?)$	$a^\pm, a^0$
$I=2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

accounts for weakly and strongly interacting models

Integrating out resonances

Consider leading order effects of resonances on EW sector:

$$\mathcal{L}_\Phi = z\left[\Phi\left(M_\Phi^2 + DD\right)\Phi + 2\Phi J\right] \qquad \Rightarrow \qquad \mathcal{L}_\Phi^{\text{eff}} = -\frac{z}{M^2} JJ + \frac{z}{M^4} J(DD)J + \mathcal{O}(M^{-6})$$

- Simplest example: scalar singlet σ:

$$\mathcal{L}_\sigma = -\frac{1}{2}\left[\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \text{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] - h_\sigma \text{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\text{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right]$$

- Effective Lagrangian

$$\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2}\left[g_\sigma \text{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + h_\sigma \text{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\text{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right]^2$$

- leads to anomalous quartic couplings

$$\alpha_5 = g_\sigma^2\left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_7 = 2 g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_{10} = 2 h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2}\right)$$

- Special case: SM Higgs with $g_\sigma = 1$ and $h_\sigma = 0$

Coupl. strengths, Anomal. Couplings, Power Counting

Scalar resonance width ($M_\sigma \gg M_W, M_Z$):

$$\Gamma_\sigma = \frac{g_\sigma^2 + \frac{1}{2}(g_\sigma^2 + 2h_\sigma^2)^2}{16\pi}\left(\frac{M_\sigma^3}{v^2}\right) + \Gamma(\text{non} - WW, ZZ)$$

Largest allowed coupling for a broad continuum: $\Gamma \sim M \gg \Gamma(\text{non} - WW, ZZ) \sim 0$
translates to bounds for effective Lagrangian (e.g. scalar with no isospin violation):

$$\alpha_5 \leq \frac{4\pi}{3}\left(\frac{v^4}{M_\sigma^4}\right) \approx \frac{0.015}{(M_\sigma \text{ in TeV})^4} \qquad \Rightarrow \qquad 16\pi^2\alpha_5 \leq \frac{2.42}{(M_\sigma \text{ in TeV})^4}$$

Scalar:	$\Gamma \sim g^2 M^3$, $\alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^4$
Vector:	$\Gamma \sim g^2 M$, $\alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^2$
Tensor:	$\Gamma \sim g^2 M^3$, $\alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^4$

Vector triplet (simplified)

$$\mathcal{L}_\rho = -\frac{1}{8}\text{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\text{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{\mathrm{i}g_\rho v^2}{2}\text{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right]$$

$1/M^2$ term renormalizes kinetic energy (i.e. v), hence unobservable:

$$\mathcal{L}_\rho^{\text{eff}} = \frac{g_\rho^2 v^4}{4M_\rho^2}\text{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right] + \mathcal{O}(1/M_\rho^4)$$

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Tensor:	$\Gamma \sim g^2 M^3$, $\alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^4$

Vector triplet (simplified)

$$\mathcal{L}_\rho = -\frac{1}{8} \text{tr}\left[\boldsymbol{\rho}_{\mu\nu} \boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4} \text{tr}\left[\boldsymbol{\rho}_\mu \boldsymbol{\rho}^\mu\right] + \frac{\mathrm{i} g_\rho v^2}{2} \text{tr}\left[\boldsymbol{\rho}_\mu \mathbf{V}^\mu\right]$$

$1/M^2$ term renormalizes kinetic energy (i.e. v), hence unobservable:

$$\mathcal{L}_\rho^{\text{eff}} = \frac{g_\rho^2 v^4}{4 M_\rho^2} \text{tr}\left[(\mathbf{D}_\mu \Sigma)(\mathbf{D}^\mu \Sigma)\right] + \mathcal{O}(1/M_\rho^4)$$

Vector Resonances

$$\mathcal{L}_\rho = -\frac{1}{8}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{\Delta M_\rho^2}{8}\left(\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\right)^2 + \mathrm{i}\frac{\mu_\rho}{2}g\,\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{W}^{\mu\nu}\boldsymbol{\rho}_\nu\right]$$
$$+\,\mathrm{i}\frac{\mu'_\rho}{2}g'\,\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{B}^{\mu\nu}\boldsymbol{\rho}_\nu\right] + \mathrm{i}\frac{g_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right] + \mathrm{i}\frac{h_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]$$
$$+\,\frac{g' v^2 k_\rho}{2M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{B}^{\nu\mu},\mathbf{V}_\nu]\right] + \frac{g v^2 k'_\rho}{4M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{T},\mathbf{V}_\nu]\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^{\nu\mu}\right]$$
$$+\,\frac{g v^2 k''_\rho}{4M_\rho^2}\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\mathrm{tr}\left[[\mathbf{T},\mathbf{V}_\nu]\mathbf{W}^{\nu\mu}\right] + \mathrm{i}\frac{\ell_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right]$$
$$+\,\mathrm{i}\frac{\ell'_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{B}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right] + \mathrm{i}\frac{\ell''_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right]$$

all $\alpha_i \sim 1/M_\rho^4$, except for $\beta_1 \sim \Delta\rho \sim T \sim h_\rho^2/M_\rho^2$

4-fermion contact interaction $j_\mu j^\mu \sim 1/M_\rho^2$ (eff. T and U parameter)

vector coupling $j_\mu V^\mu \sim 1/M_\rho^2$ (eff. S parameter)
Mismatch: measured fermionic vs. bosonic coupling g Nyffeler/Schenk, 2000; Kilian/JR, 2003

Effects on Triple Gauge Couplings

- $\mathcal{O}(1/M^2)$: Renormalization of ZWW coupling
- $\mathcal{O}(1/M^4)$: shifts in Δg_1^Z, $\Delta\kappa^\gamma$, $\Delta\kappa^Z$, λ^γ, λ^Z

Effects on Quartic Gauge Couplings

- $\mathcal{O}(1/M^4)$, orthogonal (in α_4–α_5 space) to scalar case

Simulations: The Event Generator WHIZARD

http://whizard.event-generator.org

Matrix Element Generator O'Mega: Ohl, 2000/01; M.Moretti/Ohl/JR, 2001

Optimized helicity amplitudes: Avoiding all redundancies

Multi-Purpose Event Generator WHIZARD: Ohl, 1996; Kilian, 2000; Kilian/Ohl/JR, 2007

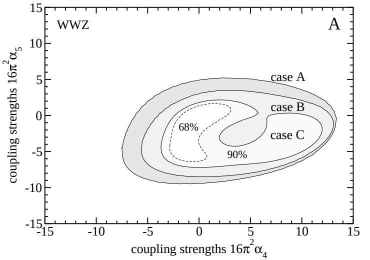
- Adaptive Multi-Channel Monte-Carlo Integration
- very well tested JR et al., 2006; Hagiwara/.../JR..., 2006
- Structure functions, Parton shower/hadronization Kilian, 2001; JR, 2007
- Event formats for detector simulations [STDHEP, HEPEVT, ATHENA, ...]

  Highest degree of complexity:
  - $e^+e^- \to t\bar{t}H \to b\bar{b}b\bar{b}jj\ell\nu$ (110.000 diagrams)
  - $e^+e^- \to ZHH \to ZWWWW \to bb + 8j$ (12.000.000 diagrams)
  - $pp \to \ell\ell + nj, n = 0, 1, 2, 3, 4, \ldots$ (2.100.000 diagrams with 4 jets + flavors)
  - $pp \to \tilde{chi}_1^0 \tilde{chi}_1^0 bbbb$ (32.000 diagrams, 22 color flows, $\sim 10,000$ PS channels)
  - $pp \to VVjj \to jj\ell\ell\nu\nu$ incl. anomalous TGC/QGC
  - Test case $gg \to 9g$ (224.000.000 diagrams)

Results: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\text{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

	WWZ			ZZZ	best
$16\pi^2\times$	no pol.	e^- pol.	both pol.	no pol.	
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E^2_{\text{mis}} + p^2_{\perp,\text{mis}}$

No angular correlations yet

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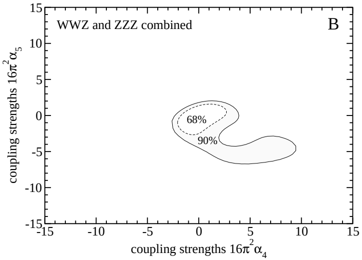

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No angular correlations yet

Vector Boson Scattering

1 TeV, $1\,\mathrm{ab}^{-1}$, full $6f$ final states, 80 % e_R^-, 60 % e_L^+ polarization, binned likelihood

Contributing channels: $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$

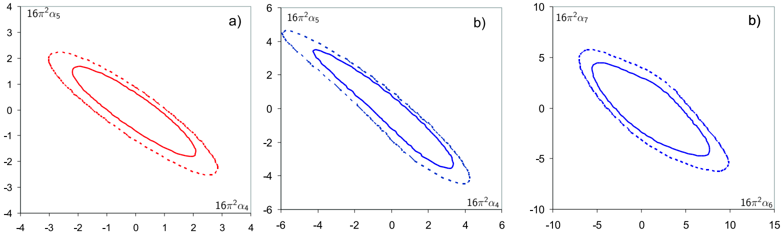
Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e\bar{\nu}_e q\bar{q}q\bar{q}$	$WW \to WW$	23.19
$e^+e^- \to \nu_e\bar{\nu}_e q\bar{q}q\bar{q}$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar{\nu}q\bar{q}q\bar{q}$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar{q}q\bar{q}$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar{q}q\bar{q}$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar{q}q\bar{q}$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar{b}X$	$e^+e^- \to t\bar{t}$	331.768
$e^+e^- \to q\bar{q}q\bar{q}$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar{q}q\bar{q}$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar{q}$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar{q}$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar{q}$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55

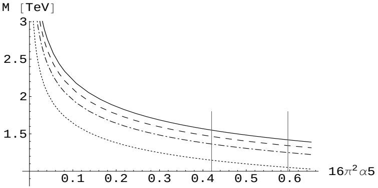


Interpretation as limits on resonances

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma / M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

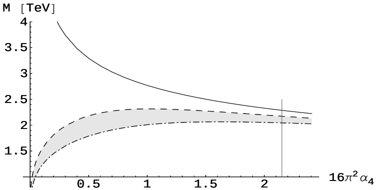


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_{\mathrm{w}}^2(\alpha_4^\lambda)^2/(2c_{\mathrm{w}}^2)} \right)^{\frac{1}{4}}$$



upper/lower limit from λ_Z, grey area: magnetic moments

Final result:

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.55	–	1.95
1	–	2.49	–
2	3.29	–	4.30

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Anomalous Gauge Couplings at LHC

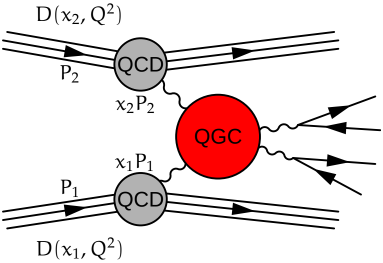
ILC: Beyer/Kilian/Krstonošić/Mönig/JR/Schröder/Schmidt, 2006

LHC: Mertens, 2006; Kilian/Kobel/Mader/JR/Schumacher

Anomalous quartic gauge couplings, by chiral EW Lagrangian:

$$\mathcal{L}_4 = \alpha_4 \frac{g^2}{2} \left\{ \left[(W^+W^+)(W^-W^-) + (W^+W^-)^2 \right] + \frac{2}{c_W^2}(W^+Z)(W^-Z) + \frac{1}{2c_W^4}(ZZ)^2 \right\}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^2}{2} \left\{ (W^+W^-)^2 + \frac{2}{c_W^2}(W^+W^-)(ZZ) + \frac{1}{2c_W^4}(ZZ)^2 \right\}$$



(all leptons, incl. τ):

$$pp \to jj(ZZ/WW) \to jj\ell^-\ell^+\nu_\ell\bar{\nu}_\ell$$

$$\sigma \approx 40\,\text{fb}$$

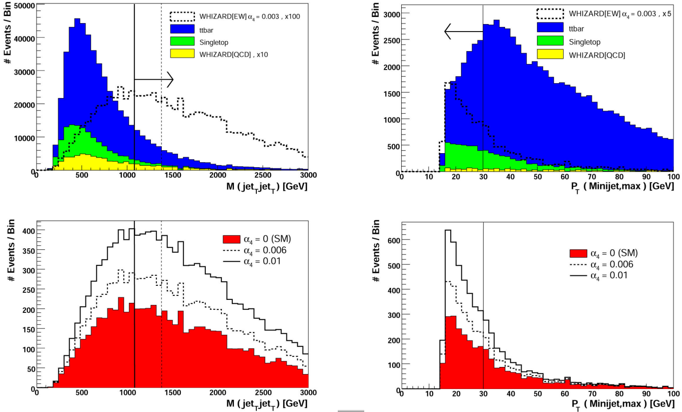
Background:

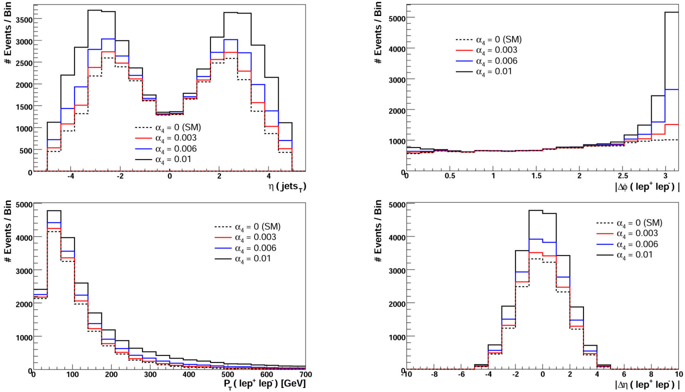
- $t\bar{t} \to WbWb$, $\sigma \approx 52$ pb
- Single t, misrec. jet: $\sigma \approx 4.8$ pb
- QCD: $\sigma \approx 0.21$ pb

Tagging and Cuts:

- $\ell\ell jj$-Tag, $\eta_{tag}^{min} < \eta_\ell < \eta_{tag}^{max}$, b-Veto
- $|\Delta\eta_{jj}| > 4.4$, $M_{jj} > 1080$ GeV
- Minijet-Veto: $p_{T,j} < 30$ GeV
- $E_j > 600, 400$ GeV, $p_{T,j}^1 > 60, 24$ GeV

Improves $S/\sqrt{B}$ from 3.3 to 29.7





Results: (1σ Sensitivity to αs)

Coupl.	ILC ($1\,\mathrm{ab}^{-1}$)	LHC ($100\,\mathrm{fb}^{-1}$)
α_4	0.0088	0.00160
α_5	0.0071	0.00098

Limits for Λ [TeV]:

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Isospin decomposition

- Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + \alpha_4 \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right] + \alpha_5\left(\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right)^2$$

- Leads to the following amplitudes: $s = (p_1 + p2)^2 \quad t = (p1 - p3)^2 \quad u = (p1 - p4)^2$

$$\boxed{\mathcal{A}(s,t,u) =:}\qquad \mathcal{A}(w^+w^- \to zz) = \frac{s}{v^2} + 8\alpha_5\frac{s^2}{v^4} + 4\alpha_4\frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(w^+z \to w^+z) = \frac{t}{v^2} + 8\alpha_5\frac{t^2}{v^4} + 4\alpha_4\frac{s^2+u^2}{v^4}$$

$$\mathcal{A}(w^+w^- \to w^+w^-) = -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5)\frac{s^2+t^2}{v^4} + 8\alpha_4\frac{u^2}{v^4}$$

$$\mathcal{A}(w^+w^+ \to w^+w^+) = -\frac{s}{v^2} + 8\alpha_4\frac{s^2}{v^4} + 4\left(\alpha_4 + 2\alpha_5\right)\frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(zz \to zz) = 8\left(\alpha_4 + \alpha_5\right)\frac{s^2+t^2+u^2}{v^4}$$

- (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\mathcal{A}(I=0) = 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$

$$\mathcal{A}(I=1) = \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t)$$

$$\mathcal{A}(I=2) = \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$

Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section: $\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$

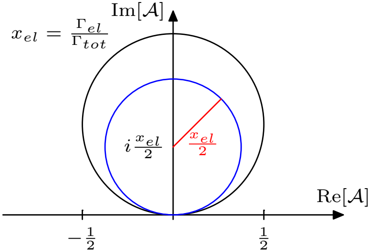
Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im}\left[\mathcal{M}_{ii}(t=0)\right]/s \qquad t = -s(1-\cos\theta)/2$$

Partial wave amplitudes: $\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1) \mathcal{A}_\ell(s) P_\ell(\cos\theta)$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_\ell \frac{32\pi(2\ell+1)}{s} |\mathcal{A}_\ell|^2 \overset{!}{=} \sum_\ell \frac{32\pi(2\ell+1)}{s} \text{Im}\left[\mathcal{A}_\ell\right] \quad \Rightarrow \quad \boxed{|\mathcal{A}_\ell|^2 = \text{Im}\left[\mathcal{A}_\ell\right]}$$



Argand circle $\boxed{\left|\mathcal{A}(s) - \frac{i}{2}\right| = \frac{1}{2}}$

Resonance: $\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s - M^2 + iM\Gamma_{\text{tot}}}$

Counterclockwise circle, radius $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{\text{tot}}$

Unitarity in the EW sector: SM

- Project out isospin eigenamplitudes Lee,Quigg,Thacker, 1973

$$\mathcal{A}_\ell(s) = \frac{1}{32\pi}\int_{-s}^{0}\frac{dt}{s}\mathcal{A}(s,t,u)P_\ell(1+2t/s) \qquad \cos\theta = 1+2t/s$$

Remember the Legendre polynomials:

$P_0(s) = 1 \qquad P_1(s) = \cos\theta \qquad P_2(s) = (3\cos^2\theta - 1)/2$

- SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I,\text{spin}=J}$):

$$\mathcal{A}_{I=0} = 2\frac{s}{v^2}P_0(s) \qquad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2}P_1(s) \qquad \mathcal{A}_{I=2} = -\frac{s}{v^2}P_0(s)$$

$$\boxed{\mathcal{A}_{0,0} = \frac{s}{16\pi v^2}} \qquad \boxed{\mathcal{A}_{1,1} = \frac{s}{96\pi v^2}} \qquad \boxed{\mathcal{A}_{2,0} = -\frac{s}{32\pi v^2}}$$

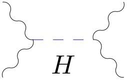
exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$$I = 0: \qquad E \sim \sqrt{8\pi}v = 1.2\,\text{TeV}$$

$$I = 1: \qquad E \sim \sqrt{48\pi}v = 3.5\,\text{TeV}$$

$$I = 2: \qquad E \sim \sqrt{16\pi}v = 1.7\,\text{TeV}$$

Higgs exchange:



$$\mathcal{A}(s,t,u) = -\frac{M_H^2}{v^2}\frac{s}{s-M_H^2}$$

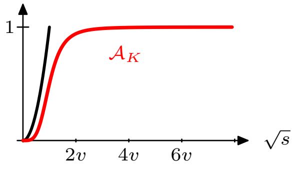
Unitarity: $M_H \lesssim \sqrt{8\pi}v \sim 1.2\,\text{TeV}$

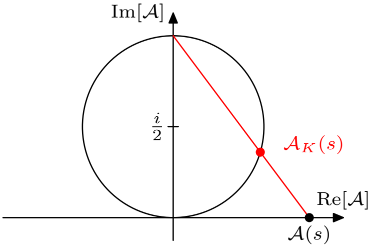
K-Matrix Unitarization and friends

K-Matrix unitarization

$$\mathcal{A}_K(s) = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} = \mathcal{A}(s)\frac{1 + i\mathcal{A}(s)}{1 + \mathcal{A}(s)^2}$$

Unitarization by infinitely heavy and wide resonance





- Low-energy theorem (LET): $\frac{s}{v^2}$
- K-Matrix amplitude: $|\mathcal{A}(s)|^2 = \frac{s^2}{s^2+v^4} \overset{s\to\infty}{\longrightarrow} 1$
- Poles $\pm iv$: M_0, Γ large

Padé unitarization

separates higher chiral orders

$$\mathcal{A}_P(s) = \frac{\mathcal{A}^{(0)}(s)^2}{\mathcal{A}^{(0)}(s) - \mathcal{A}^{(1)}(s) - i\mathcal{A}^{(0)}(s)^2}$$

each partial wave dominated by single resonance

"Naive" Unitarization

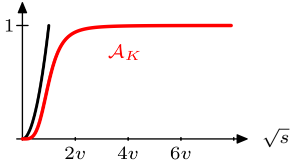
Extreme case:

$$\mathcal{A}_N(s) = e^{i\mathcal{A}(s)} \sin\mathcal{A}(s)$$

Infinitely many resonances becoming denser for $s \to \infty$

LHC Results

- K-Matrix unitarization looks pretty much like the LET case:



- First glance: most brutally possible unitarization weeps out signal
- No significant differences in angular distributions any more

J. Schumacher, 2007

- Next step: Switch on definite (unitarized) resonances
  Look again for angular distributions

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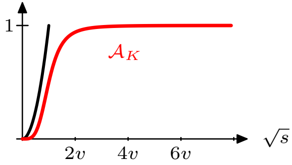

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  J. Schumacher, 2007

- Next step: Switch on definite (unitarized) resonances
  Look again for angular distributions
- Unfortunately: no plots yet

K-Matrix Unitarized Resonance: e.g. Scalar Singlet

Assumptions:

- LHC is able to detect a resonance in the EW sector
- Further resonances might exist, but out of reach or not detectable
- Describe 1st resonance by correct amplitude
- Use K-matrix unitarization to define a consistent model

Example: Scalar Singlet

- $\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \frac{g_\sigma v}{2}\sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]$
- Feynman rules: $\sigma w^+ w^- : -\frac{2ig_\sigma}{v}(k_+ \cdot k_-)$ $\quad \sigma zz : -\frac{2ig_\sigma}{v}(k_1 \cdot k_2)$
- Amplitude (s-channel exchange): $\boxed{\mathcal{A}^\sigma(s,t,u) = \frac{g_\sigma^2}{v^2}\frac{s^2}{s-M^2}}$
- Isospin eigenamplitudes:

$$\begin{aligned}
\mathcal{A}_0^\sigma(s,t,u) &= \frac{g_\sigma^2}{v^2}\left(3\frac{s^2}{s-M^2} + \frac{t^2}{t-M^2} + \frac{u^2}{u-M^2}\right) \\
\mathcal{A}_1^\sigma(s,t,u) &= \frac{g_\sigma^2}{v^2}\left(\frac{t^2}{t-M^2} - \frac{u^2}{u-M^2}\right) \\
\mathcal{A}_2^\sigma(s,t,u) &= \frac{g_\sigma^2}{v^2}\left(\frac{t^2}{t-M^2} + \frac{u^2}{u-M^2}\right)
\end{aligned}$$

Unitarizing the scalar singlet

$$\mathcal{A}^\sigma_{00}(s) = 3\,\frac{g_\sigma^2}{v^2}\,\frac{s^2}{s-M^2} + 2\,\frac{g_\sigma^2}{v^2}\,\mathcal{S}_0(s)$$

$$\mathcal{A}^\sigma_{02}(s) = 2\,\frac{g_\sigma^2}{v^2}\,\mathcal{S}_2(s) \qquad = A^\sigma_{22}(s)$$

$$\mathcal{A}^\sigma_{11}(s) = 2\,\frac{g_\sigma^2}{v^2}\,\mathcal{S}_1(s)$$

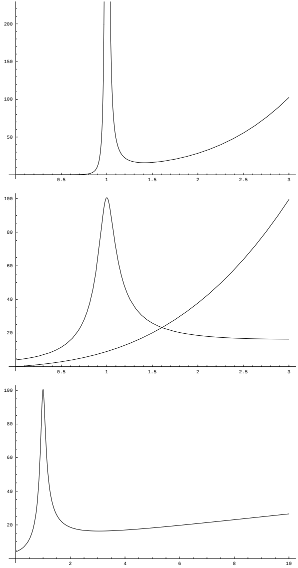
$$\mathcal{A}^\sigma_{13}(s) = 2\,\frac{g_\sigma^2}{v^2}\,\mathcal{S}_3(s)$$

$$\mathcal{A}^\sigma_{20}(s) = 2\,\frac{g_\sigma^2}{v^2}\,\mathcal{S}_0(s)$$

S-wave coefficients no longer polynomial, e.g.:

$$\mathcal{S}_0(s) = M^2 - \frac{s}{2} + \frac{M^4}{s}\log\frac{s}{s+M^2}$$

s-channel pole must be explicitly subtracted:



$$\mathcal{A}^K_{IJ}(s) = \mathcal{A}_{IJ}(s) + \Delta\mathcal{A}_{IJ}(s) \qquad \Delta\mathcal{A}_{IJ}(s) = -3\,\frac{g_\sigma^2}{v^2}\,\frac{s^2}{s-M^2} + \frac{i}{32\pi}\,\frac{\mathcal{A}^2_{\text{no pole}\,IJ}(s)}{1-\frac{i}{32\pi}\mathcal{A}_{\text{no pole}\,IJ}(s)}$$

Summary

New Physics generically encoded in EW Chiral Lagrangian

ILC can measure deviations in quartic gauge couplings

- either via triple boson production
- or via vector boson scattering

interpreted as resonances coupled to EW bosons

Sensitivity rises with number of intermediate states: $1.5 - 6$ TeV

Full analysis including all channels/backgrounds with WHIZARD

LHC sensitivity not yet fully clarified

Unitarization of amplitudes needed (toy UV completion)

Also (very) broad resonances detectable