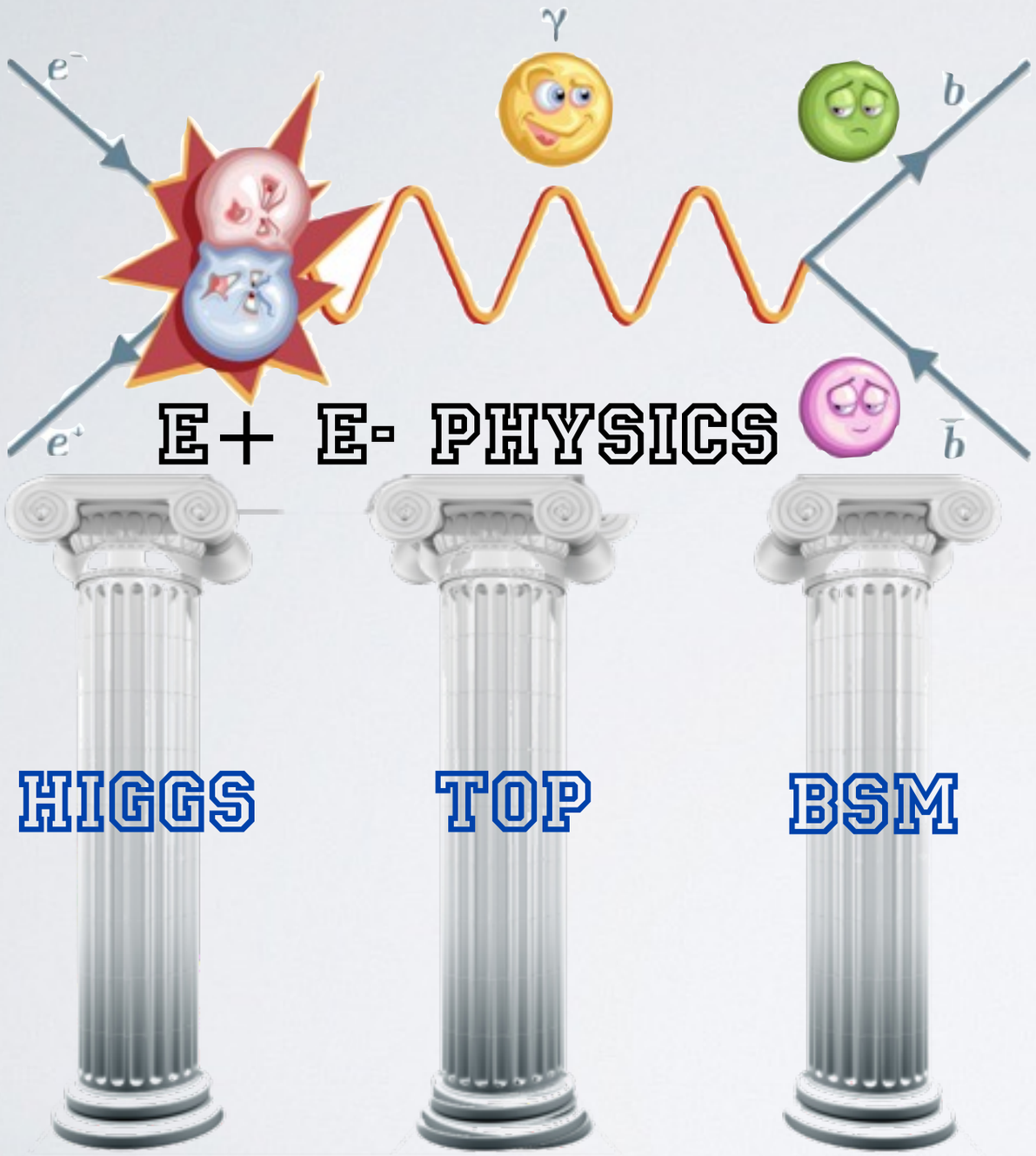
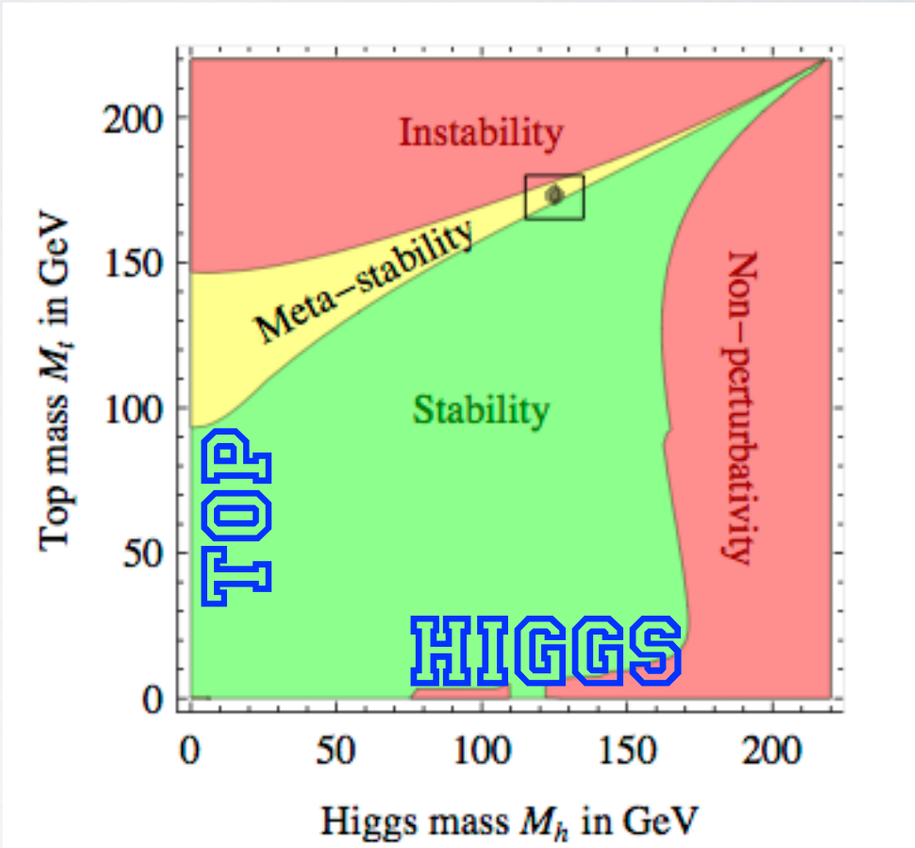
tt(h) production in e+e- collisions (a theorist's view)

Jürgen R. Reuter, DESY

The importance of top / Higgs



Electroweak vacuum & excitations:



(note: plot under assumptions of NO additional BSM)

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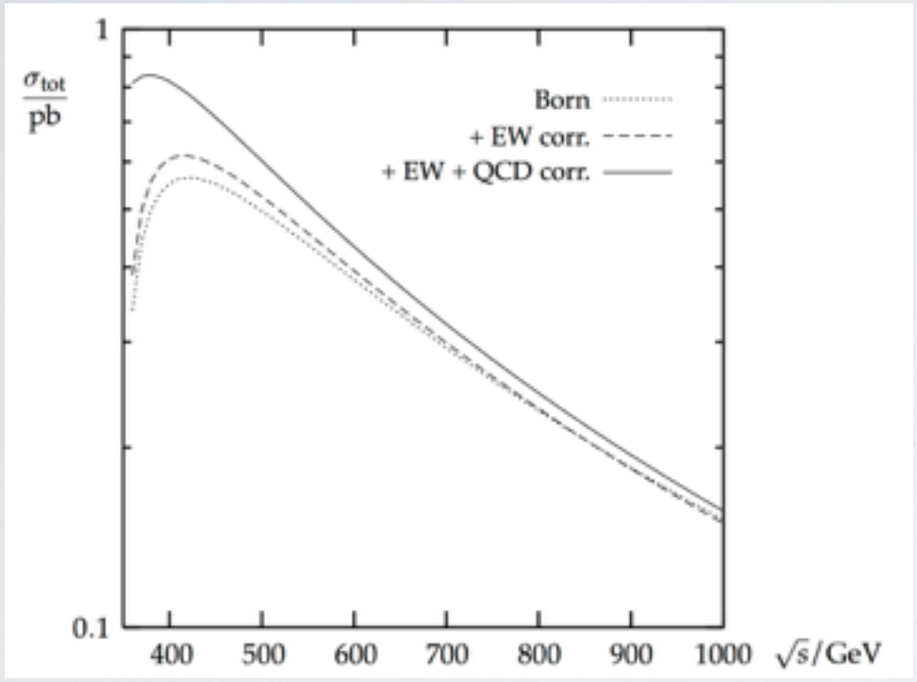

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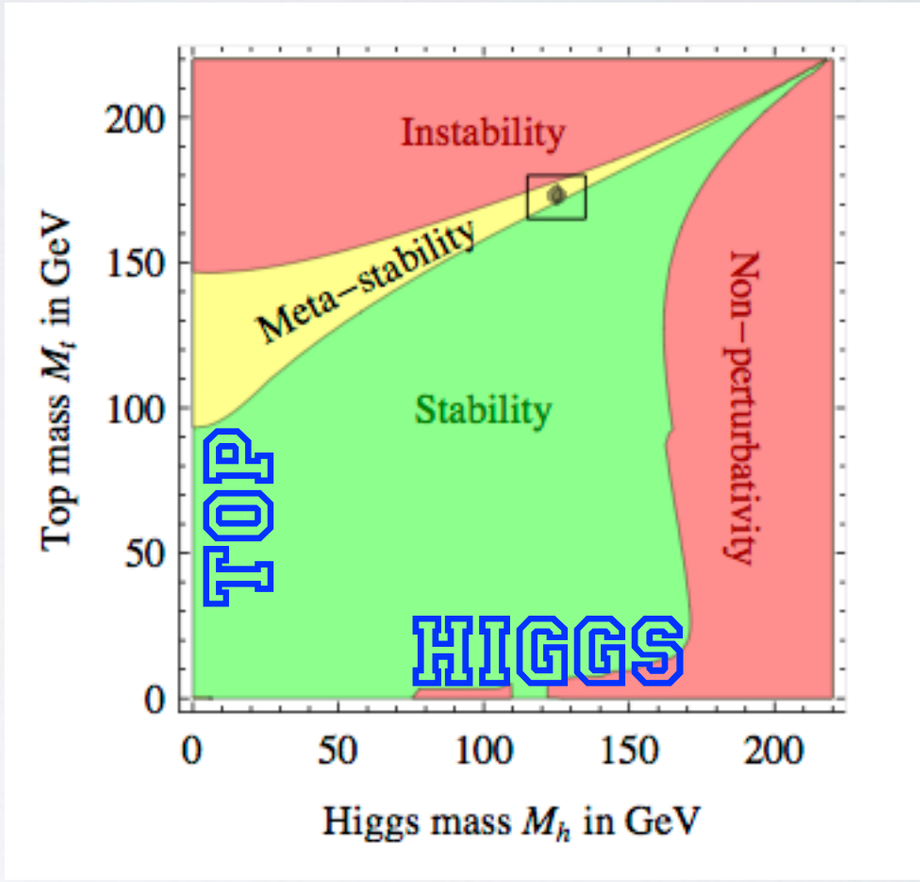

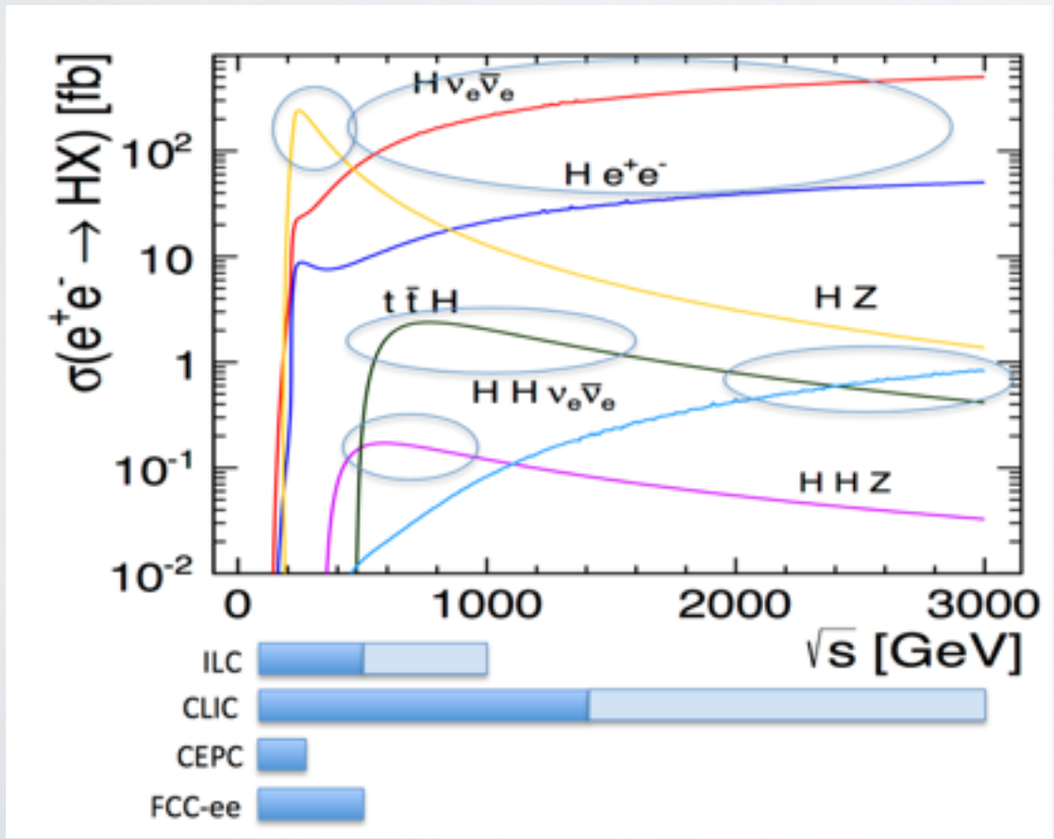

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Top pair production

Top pair production (top mass) and tth production (top Yukawa coupling)

⇒ **Flagship measurements @ e+e- colliders (350 GeV and 500+ GeV)**

On-Shell process: $e^+e^- \to t\bar{t}$

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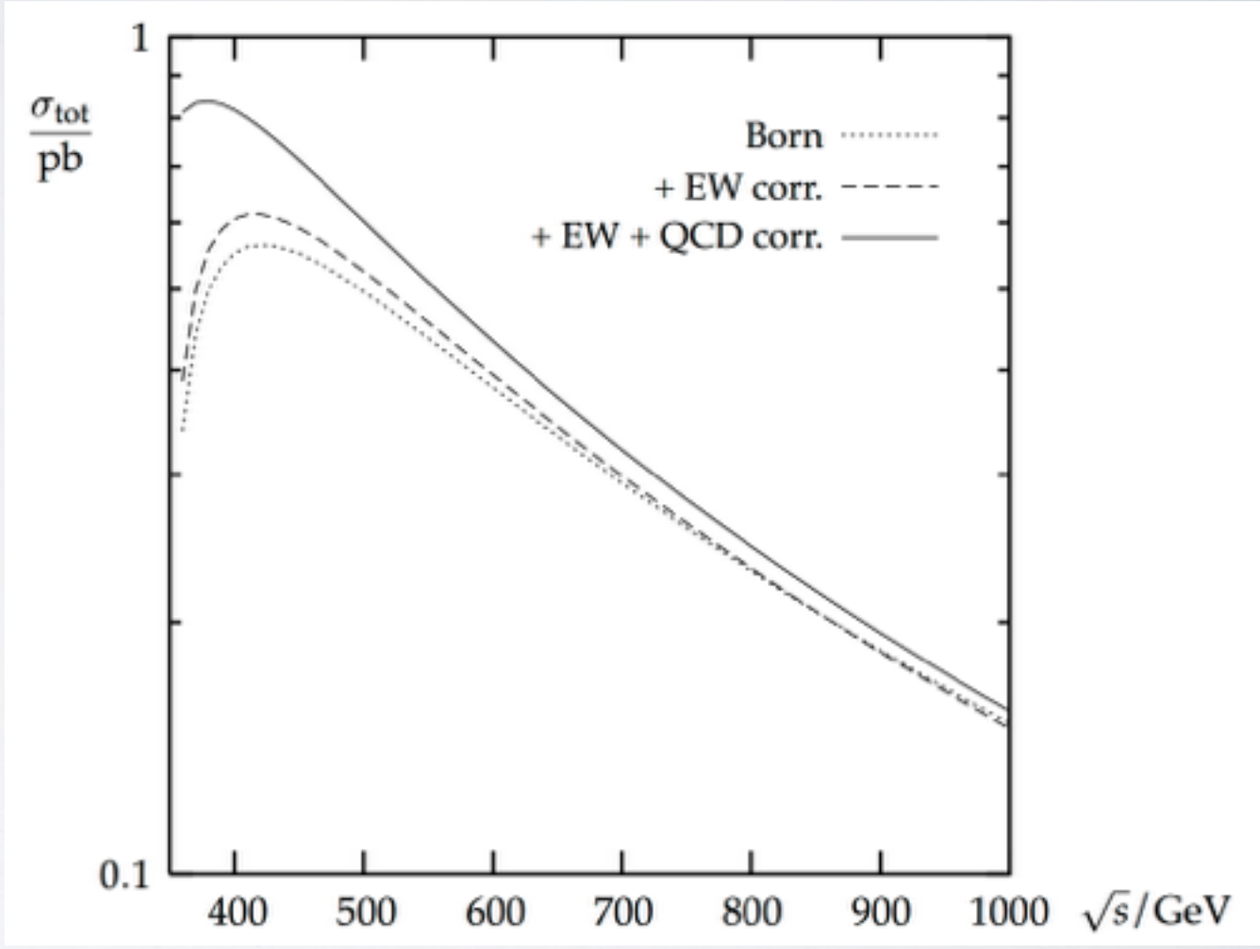

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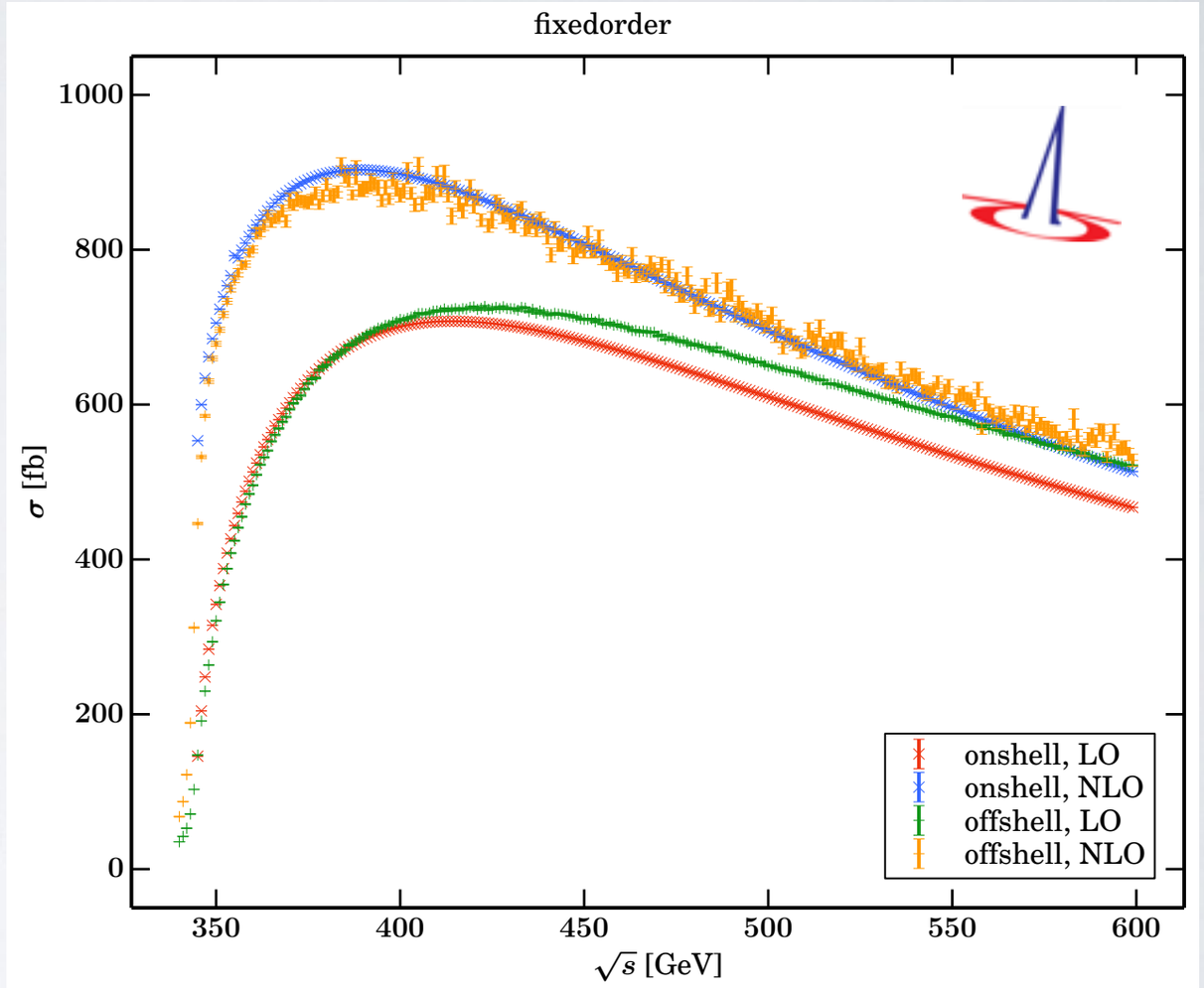

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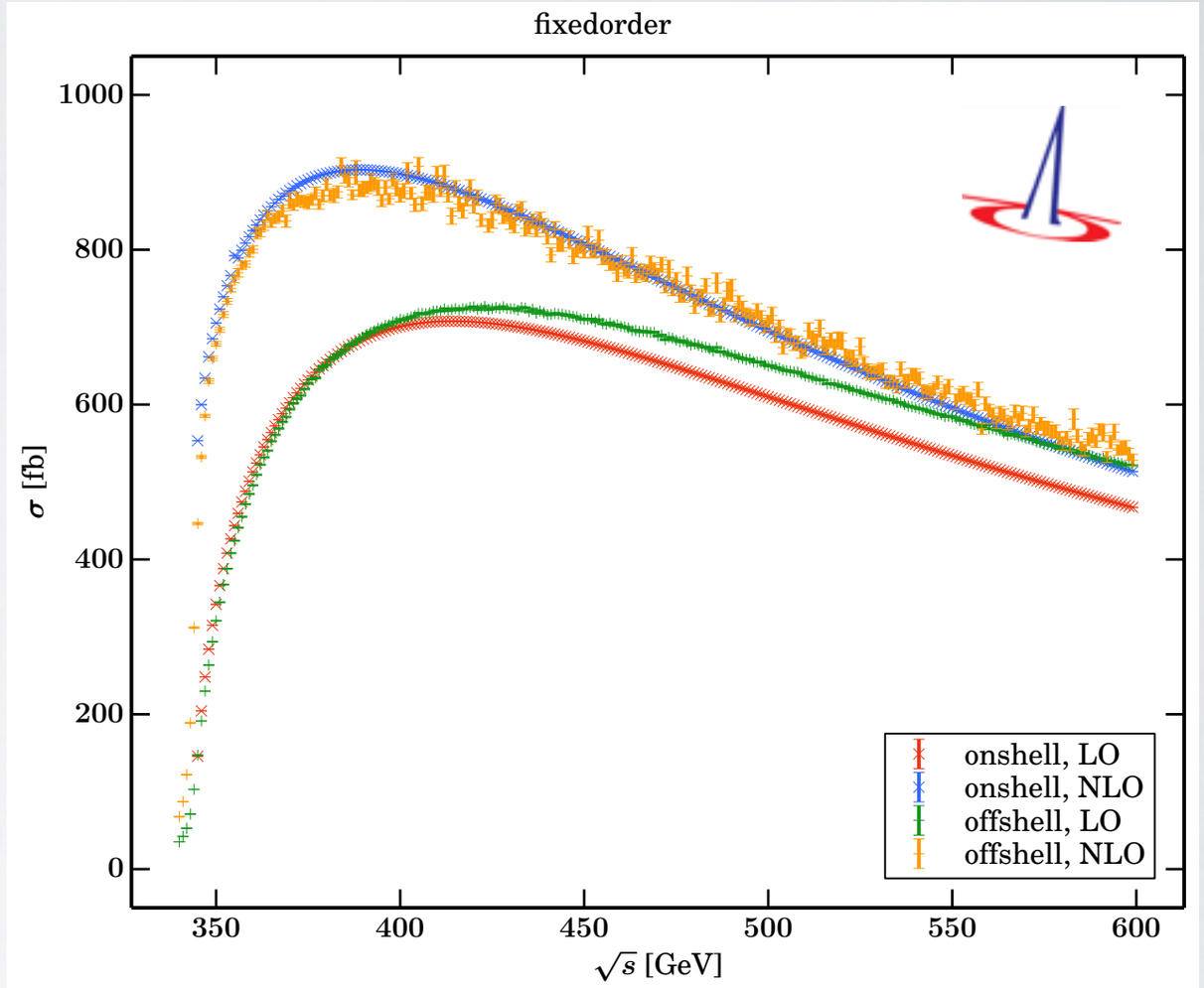

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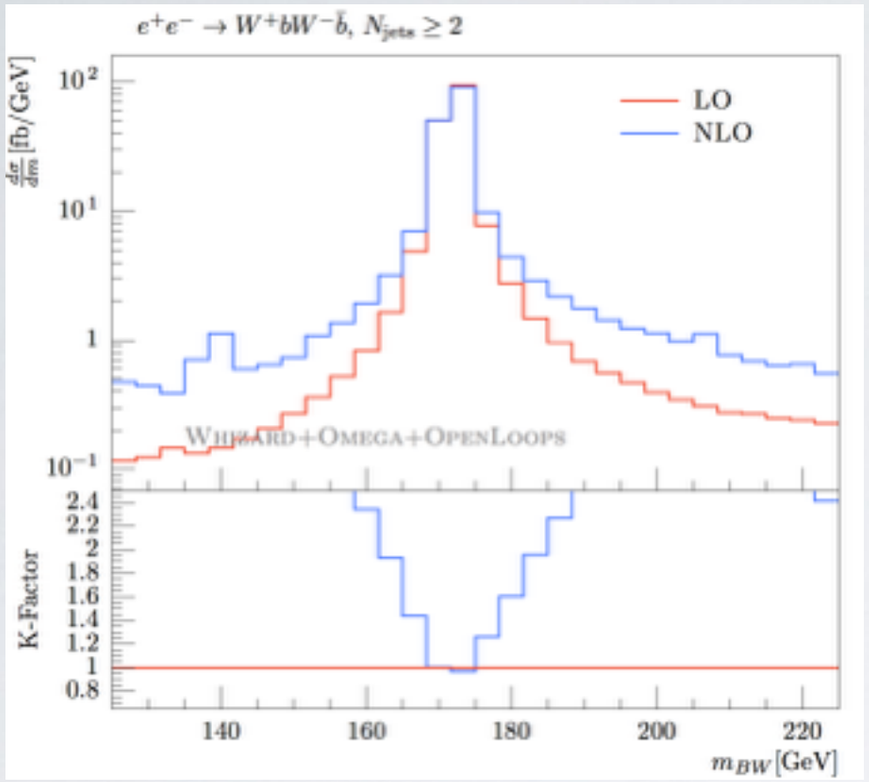
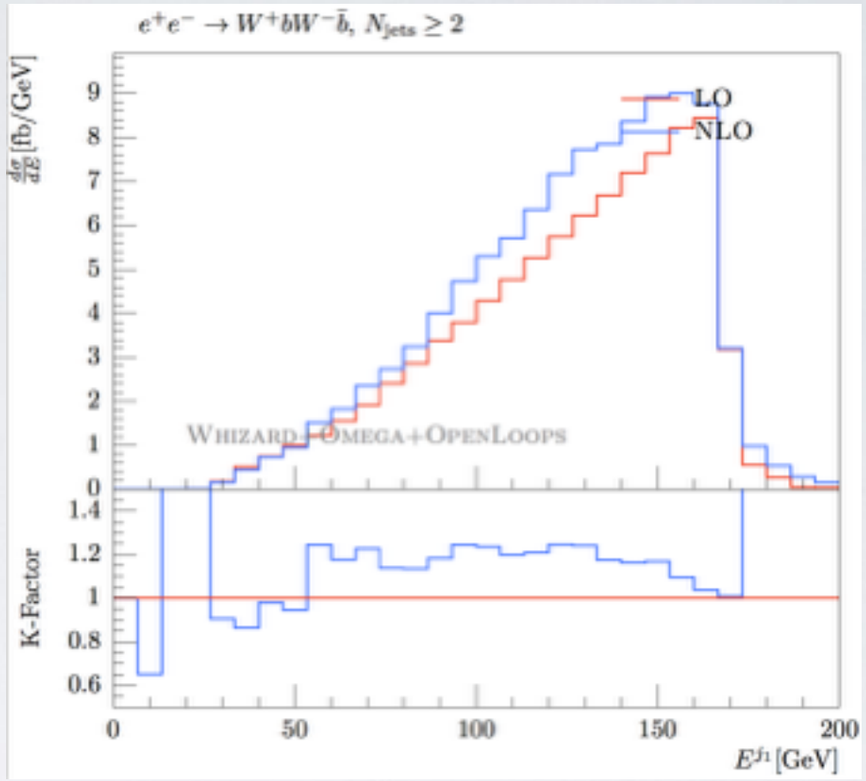
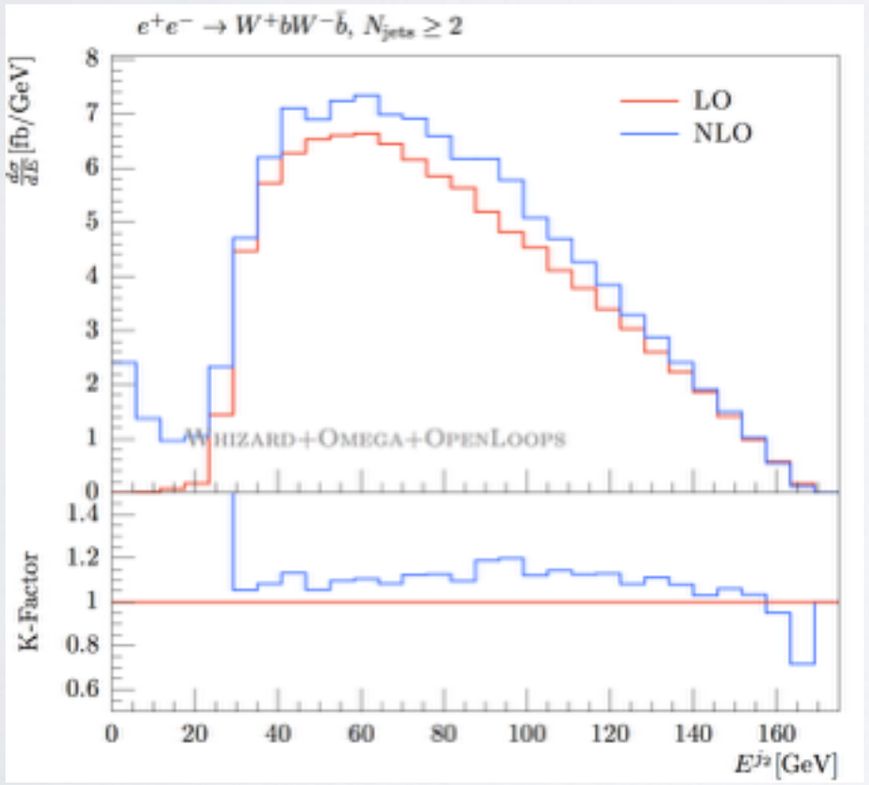
Leptonic decays: $e^+e^- \to \bar{b}b\ell^+\nu_\ell\ell^-\bar{\nu}_\ell$

- NLO QCD 2 → 6 : `WHIZARD+OpenLoops`

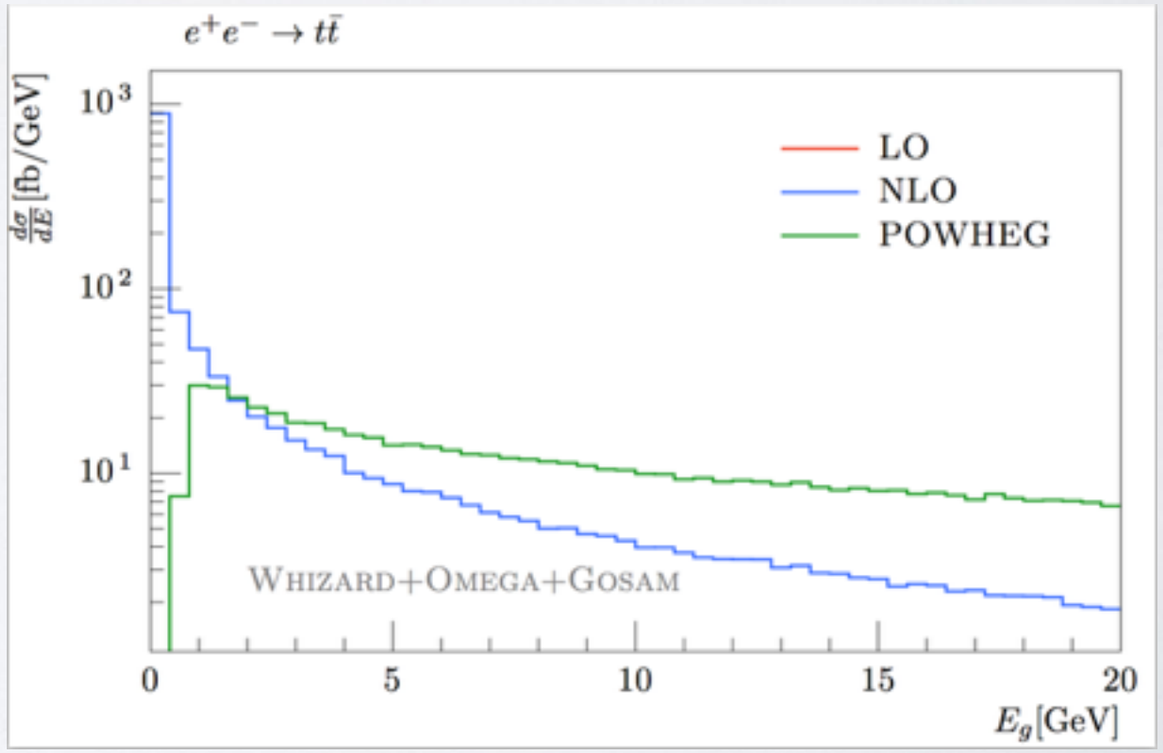
Top pair production







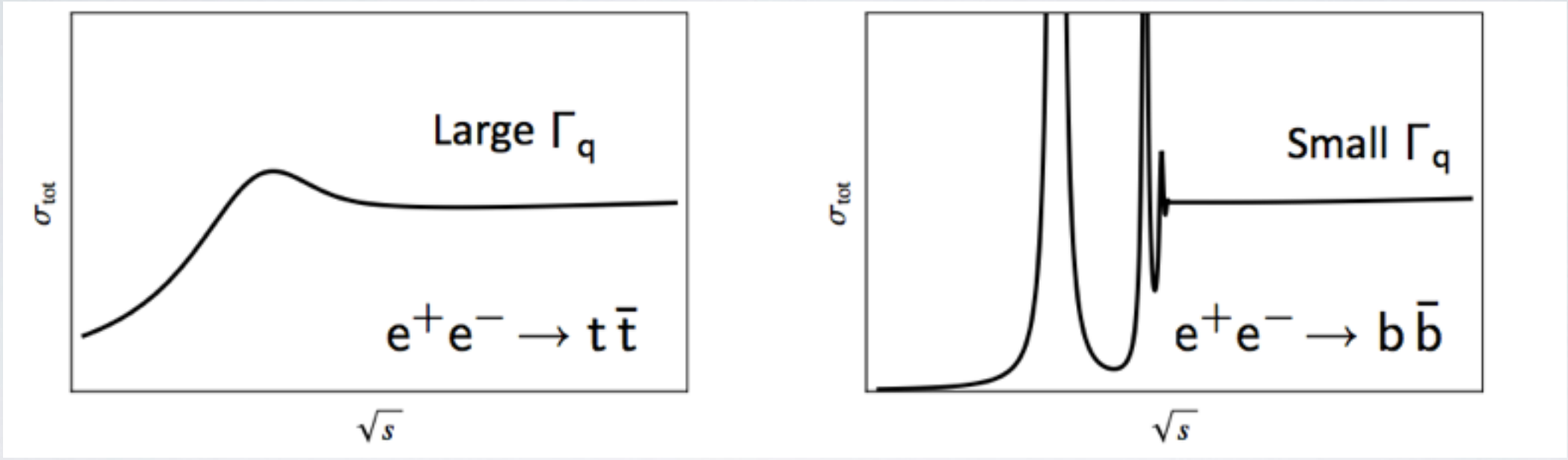
Chokoufe/JRR/Weiss, EPS 2015



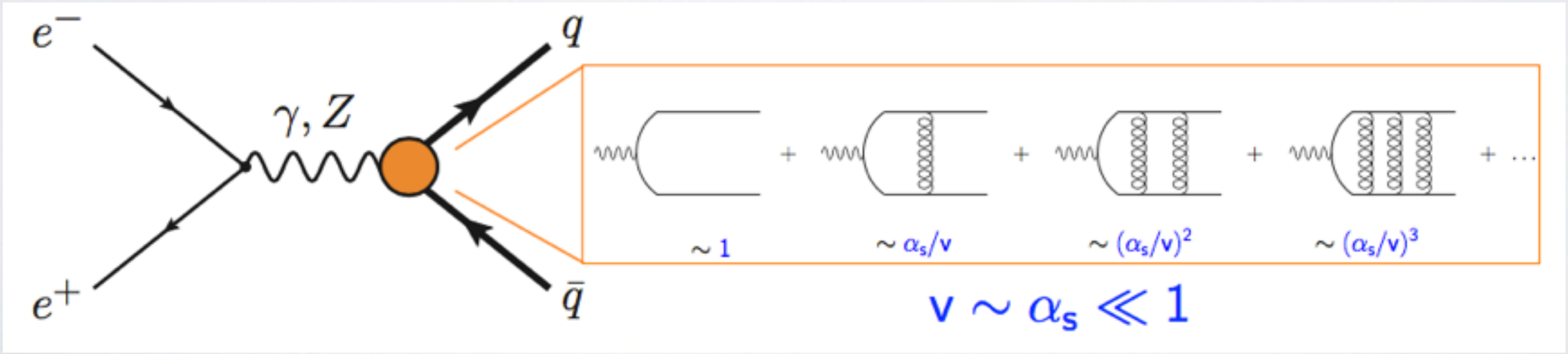
(v)NRQCD for Top (threshold) physics

ILC top threshold scan best-known method to measure top quark mass, $\Delta M \sim 30$-50 MeV

Heavy quark production at lepton colliders, qualitatively:



Threshold region: top velocity $v \sim \alpha_s \ll 1$



Top Threshold Resummation in (p/v)NRQCD

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold (v = 0) Hoang et al. '99-'01; Beneke et al., '13-'14

Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i \left(\alpha_s \ln v\right)^i \times$$

$$\times \left\{1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL})\right\}$$

(p/v)NRQCD EFT w/ RG improvement

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$$R^{\gamma,Z}(s) = \underbrace{F^v(s)R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s)R^a(s)}_{\text{p-wave}\sim v^2\text{:NNLL}}$$

but contributes
at NLL differentially!

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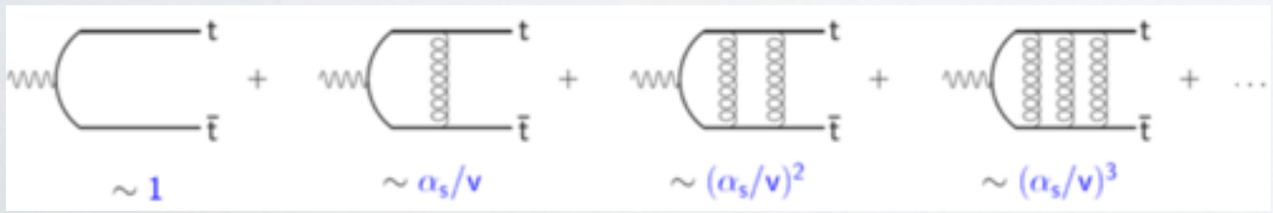
Coulomb potential gluon ladder resumption



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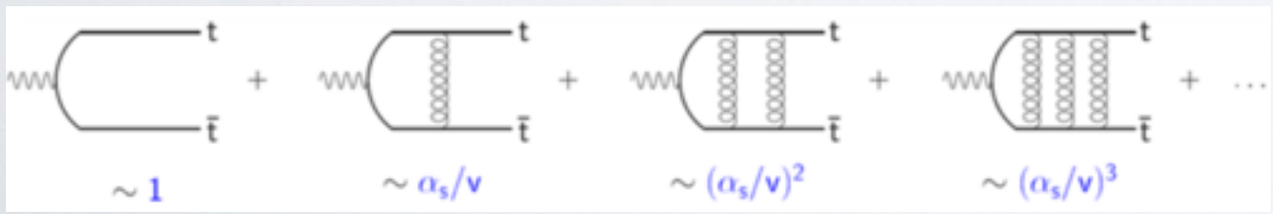
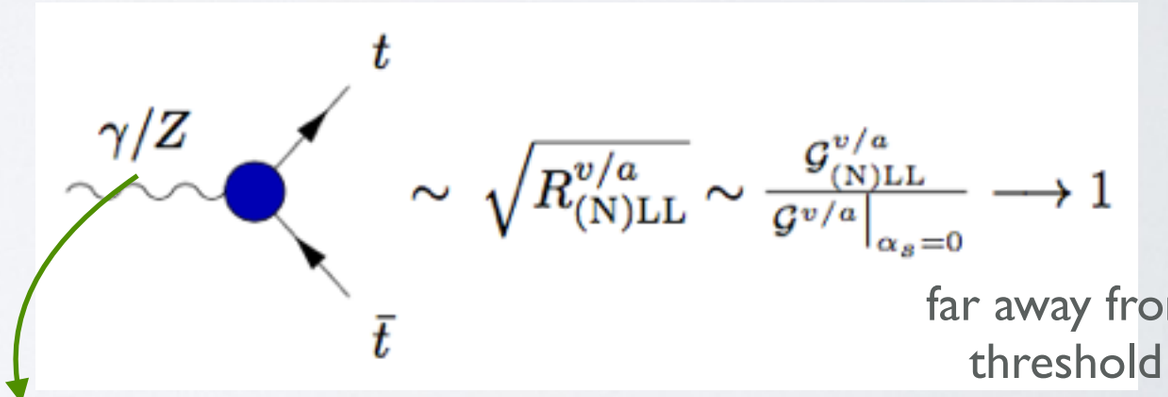


can be mapped onto effective *ttV* vertex



$$\sim \sqrt{R^{v/a}_{(\mathrm{N})\mathrm{LL}}} \sim \frac{\mathcal{G}^{v/a}_{(\mathrm{N})\mathrm{LL}}}{\mathcal{G}^{v/a}\big|_{\alpha_s=0}} \longrightarrow 1$$

far away from threshold

$$\mathbb{C} \ni \mathcal{G}^{v/a}_{(\mathrm{N})\mathrm{LL}} = \mathcal{G}^{v/a}_{(\mathrm{N})\mathrm{LL}}(\alpha_s, M_t^{\mathrm{pole}}, \sqrt{s}, |\vec{p}_t|, \Gamma_t)$$

differential in off-shell *tt* phase space

Top Threshold Physics

- $G^{v,a}(0, p_t, E + i\Gamma_t, \nu)$ from TOPPIK [Jezabek/Teubner]
- Solution of non-relativistic Schrödinger equation with Coulombic potential

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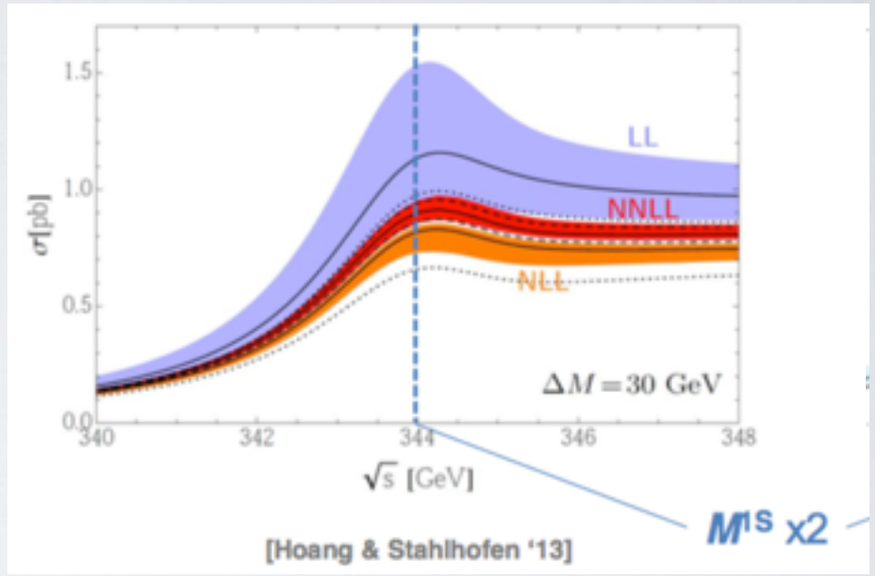
available @ 4 loops: Talk by P. Marquard (Thursday)



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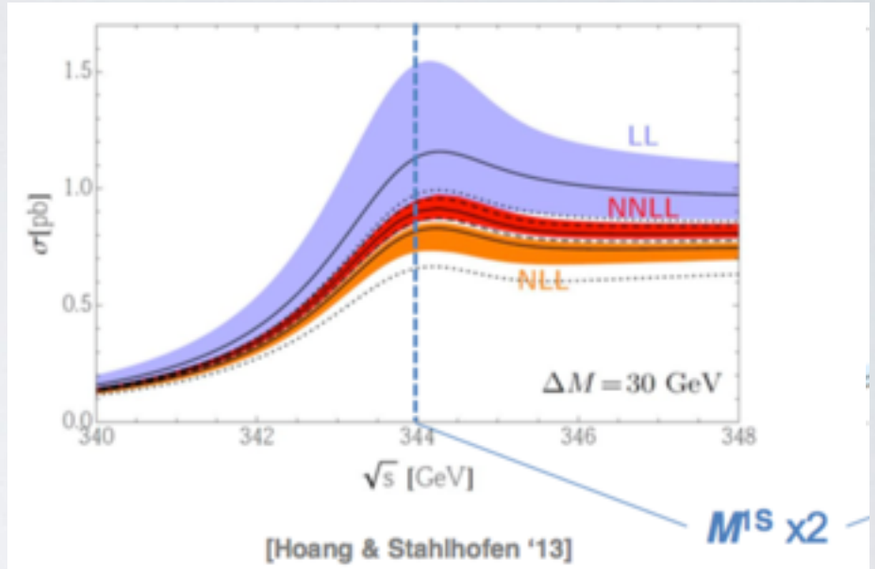

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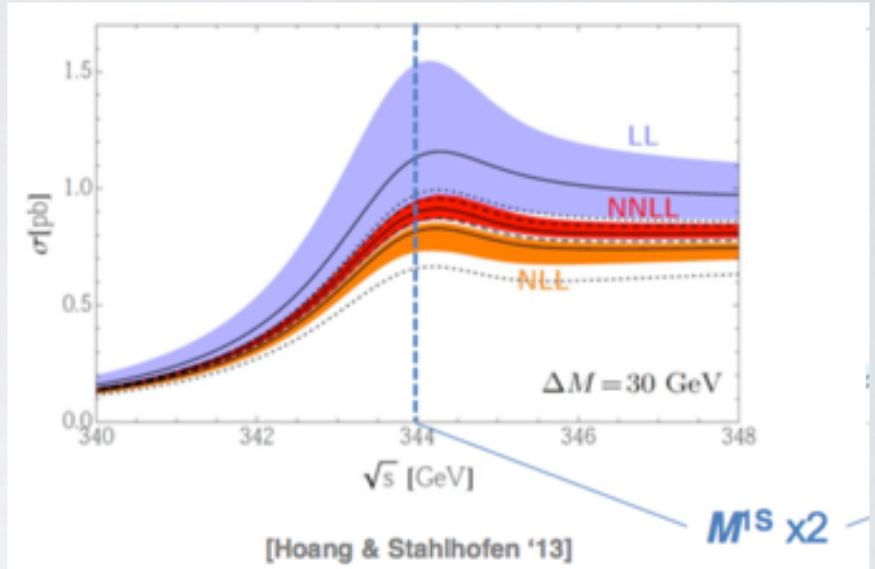

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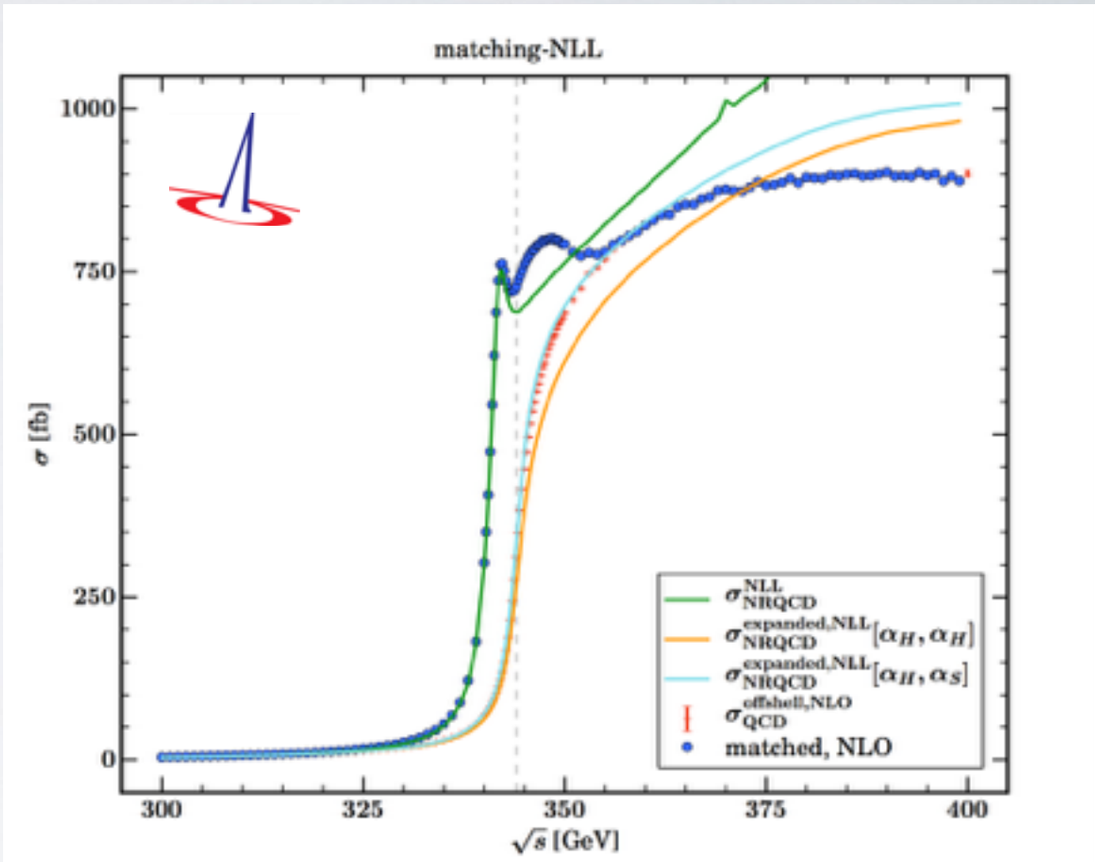

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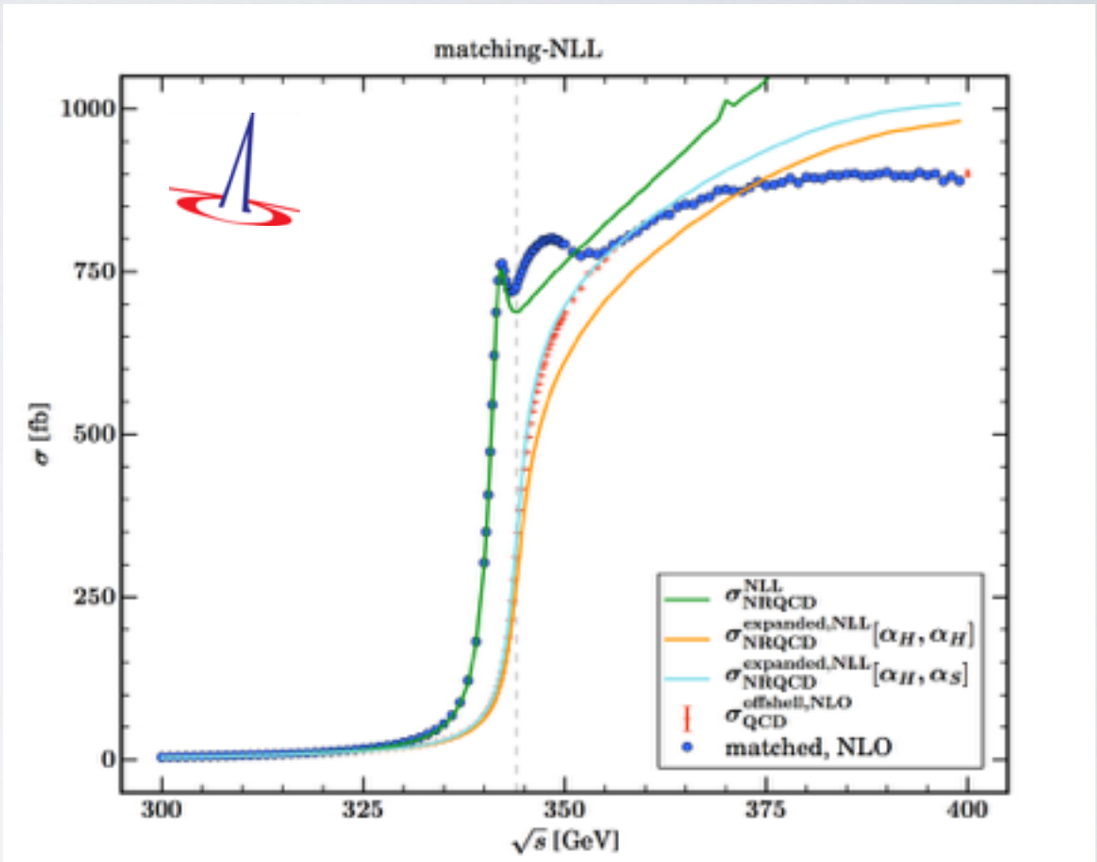

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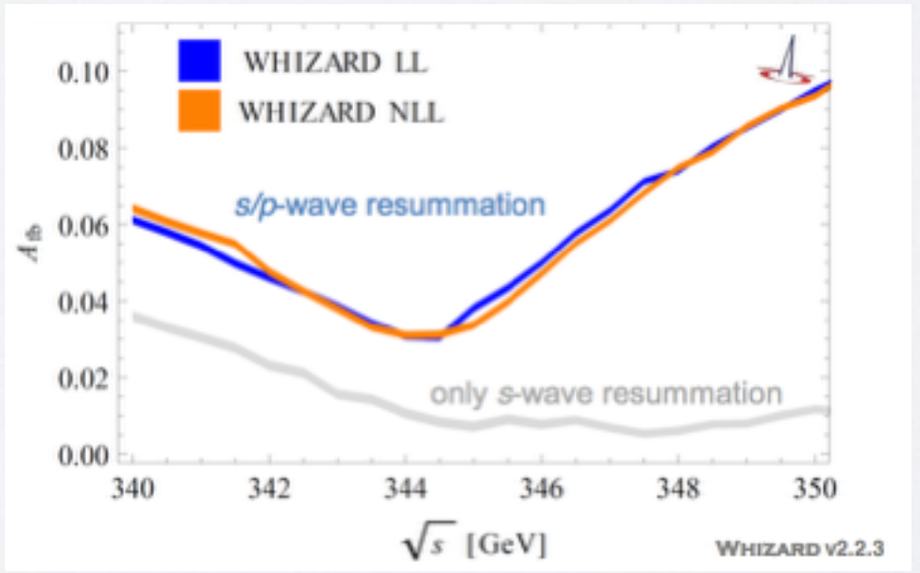
Forward-backward asymmetry
(norm. ⇒ good shape stability)

$$A_{fb} := \frac{\sigma(p_z^t > 0) - \sigma(p_z^t) < 0)}{\sigma(p_z^t > 0) + \sigma(p_z^t < 0)}$$



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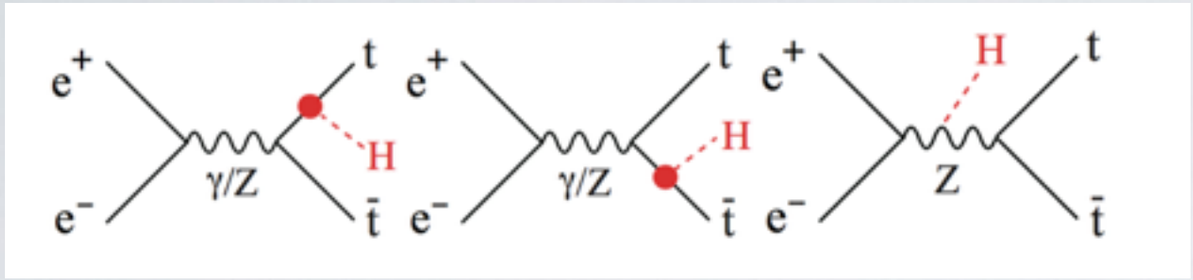

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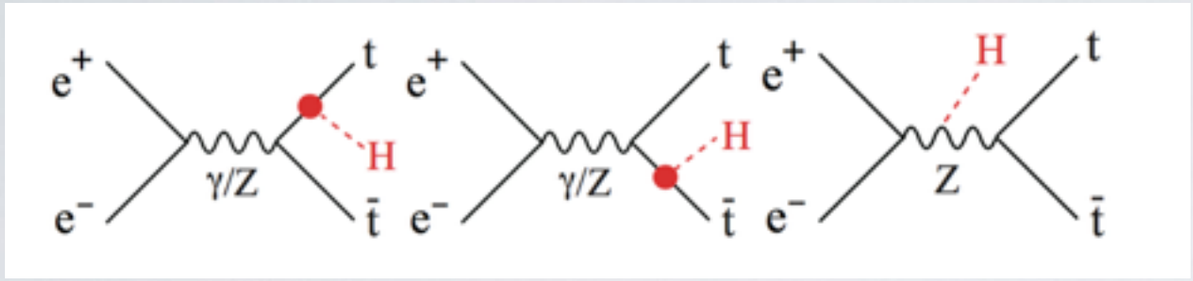

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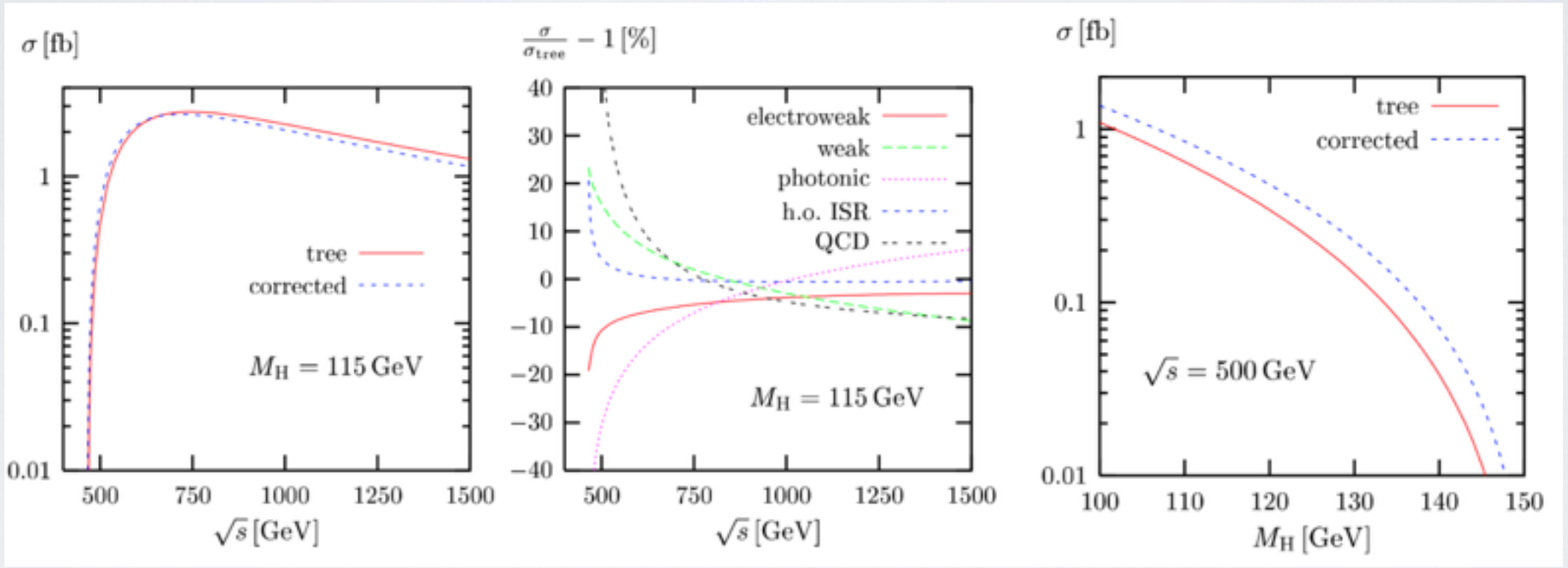

- Photonic and gluonic corrections largely cancel
- Large effects at threshold (ISR, but more so QCD → more soon)
- Depends crucially on Higgs mass: sweet spot for *M(h)* = 125 GeV is for *E* ~ 800 GeV

Matched NLO QCD results

- Precise predictions of multi-parton final states require properly matched samples
- NLO QCD including POWHEG matching already available [WHIZARD+OpenLoops]
- All descriptions at NLO at the moment for the on-shell process
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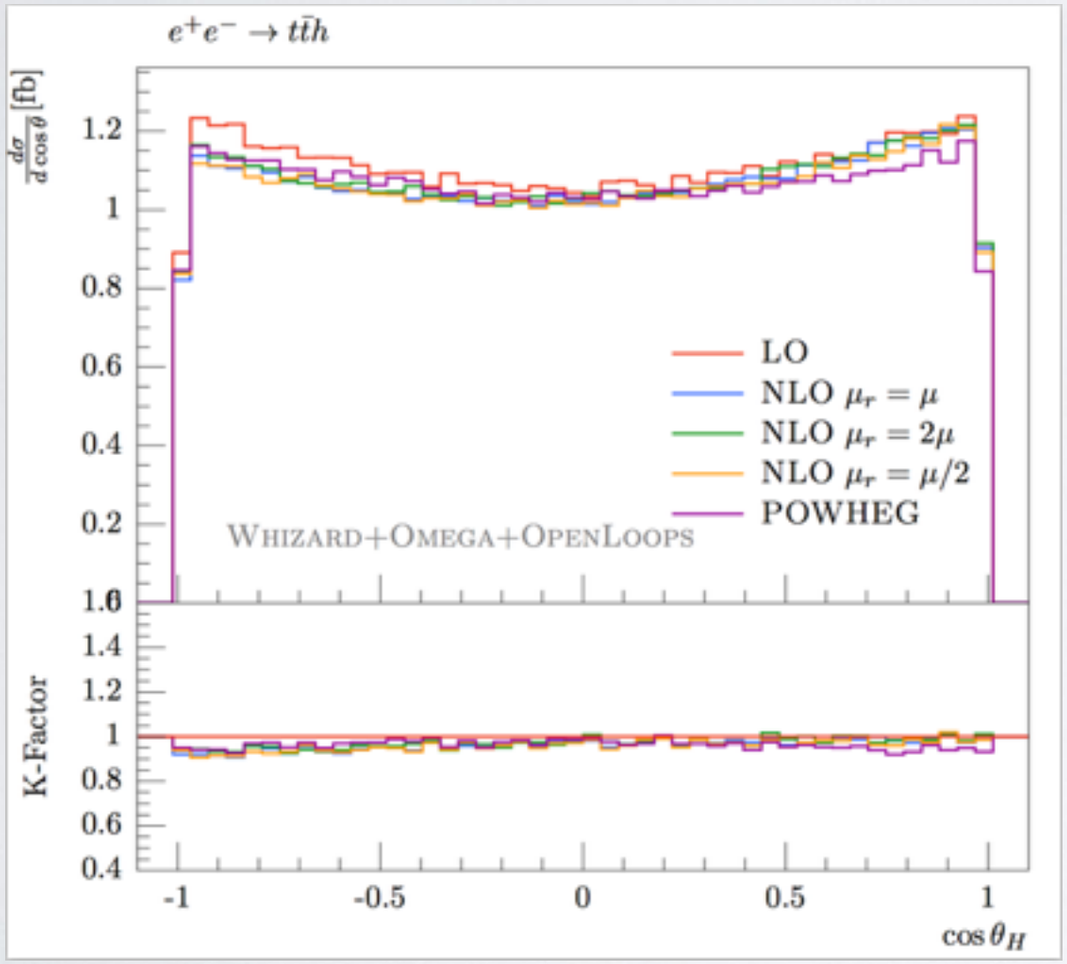
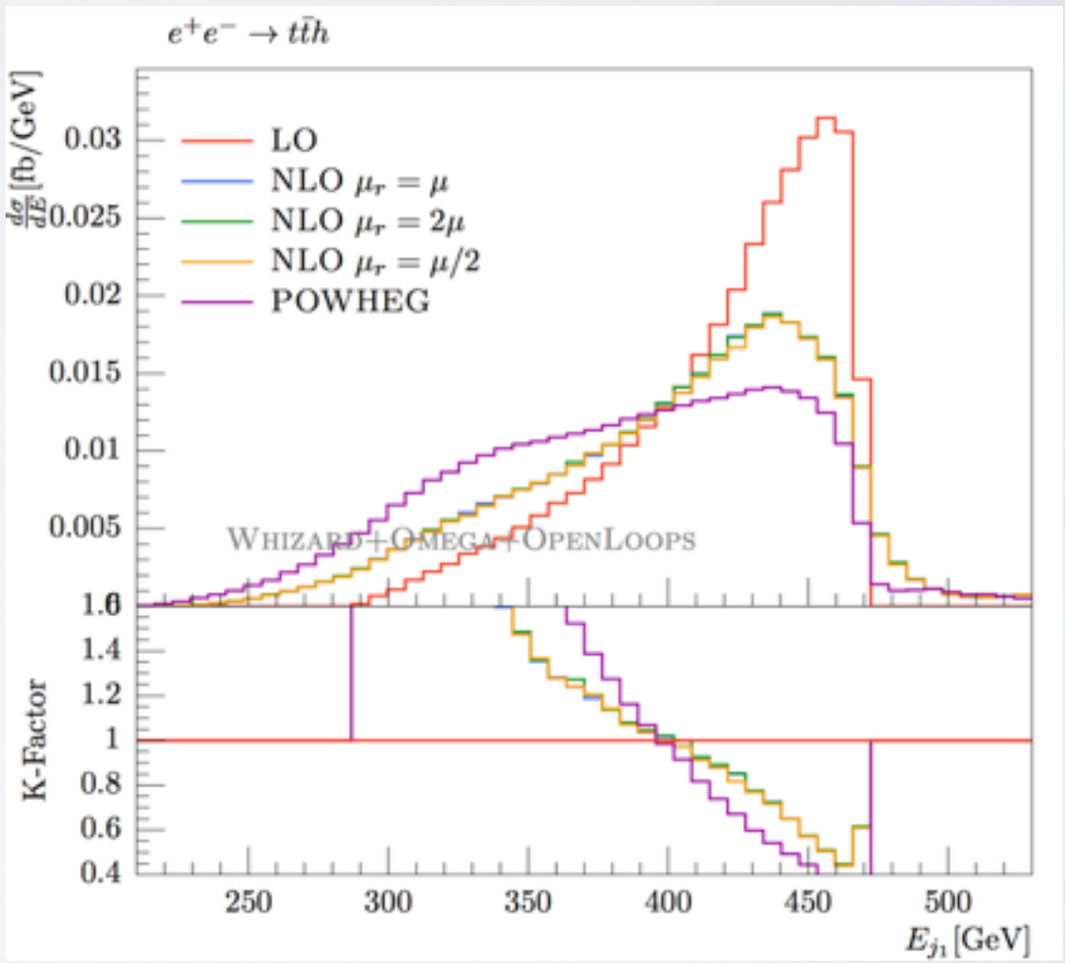
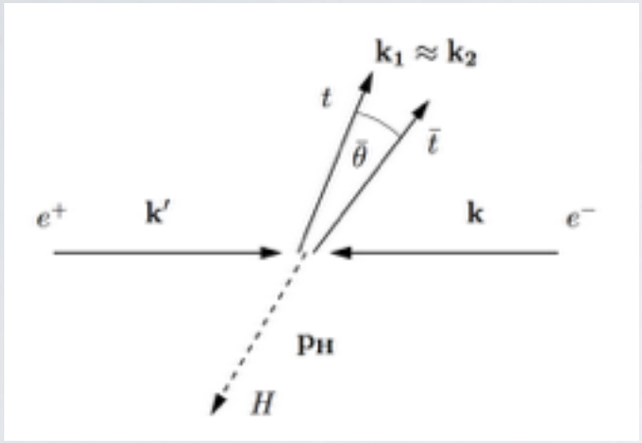




[Chokoufe/JRR/Weiss]

Threshold enhancement @ 500 GeV for *tth*

- Close to threshold: Coulomb enhancement of non-relativistic top pair
- Energy distribution of Higgs enhanced in top pair recoil [Farrell/Hoang, 2005-2006]



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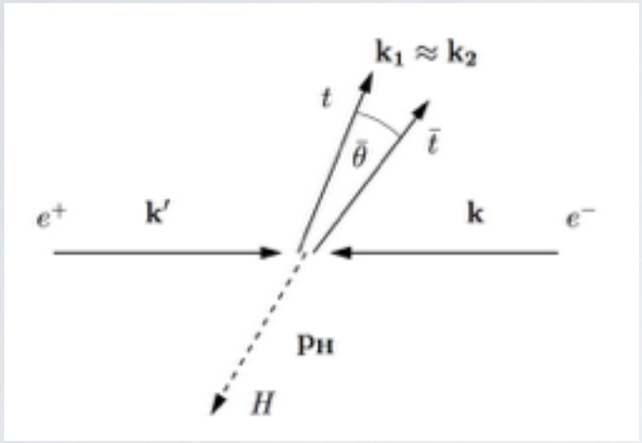

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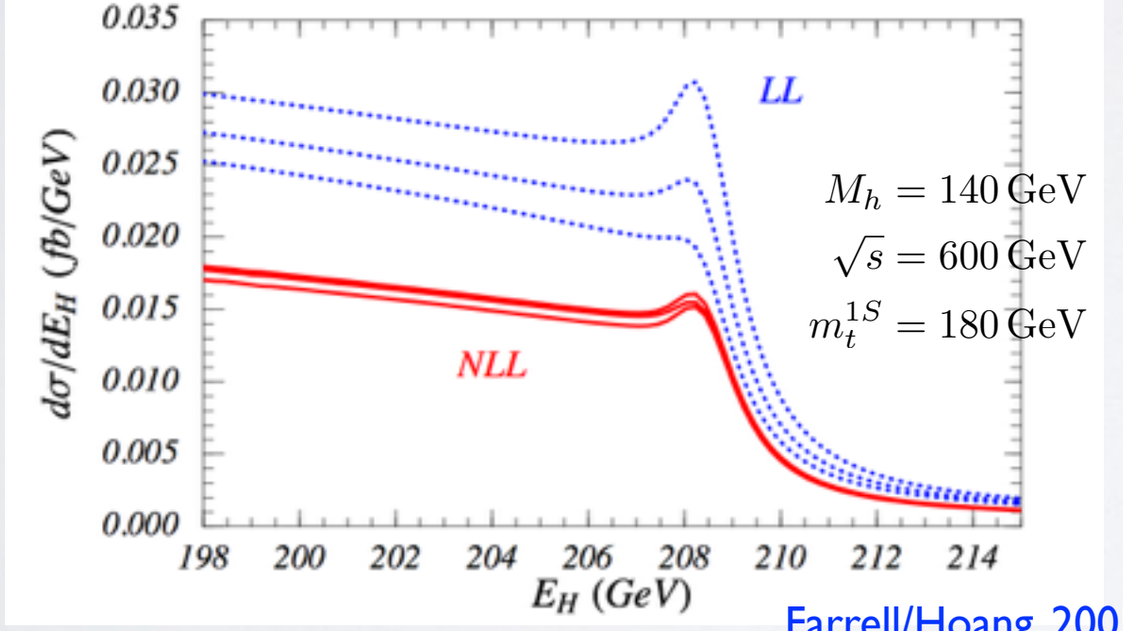

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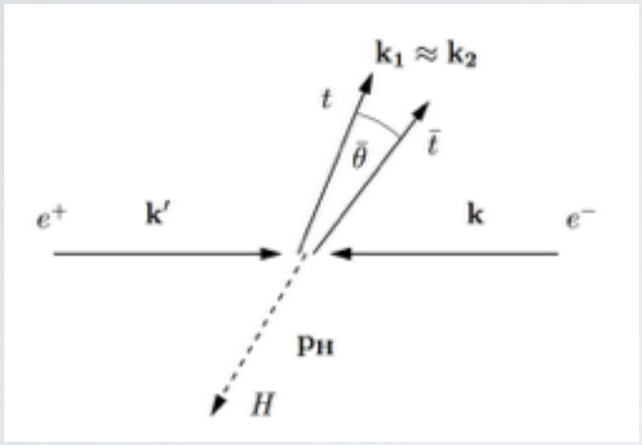

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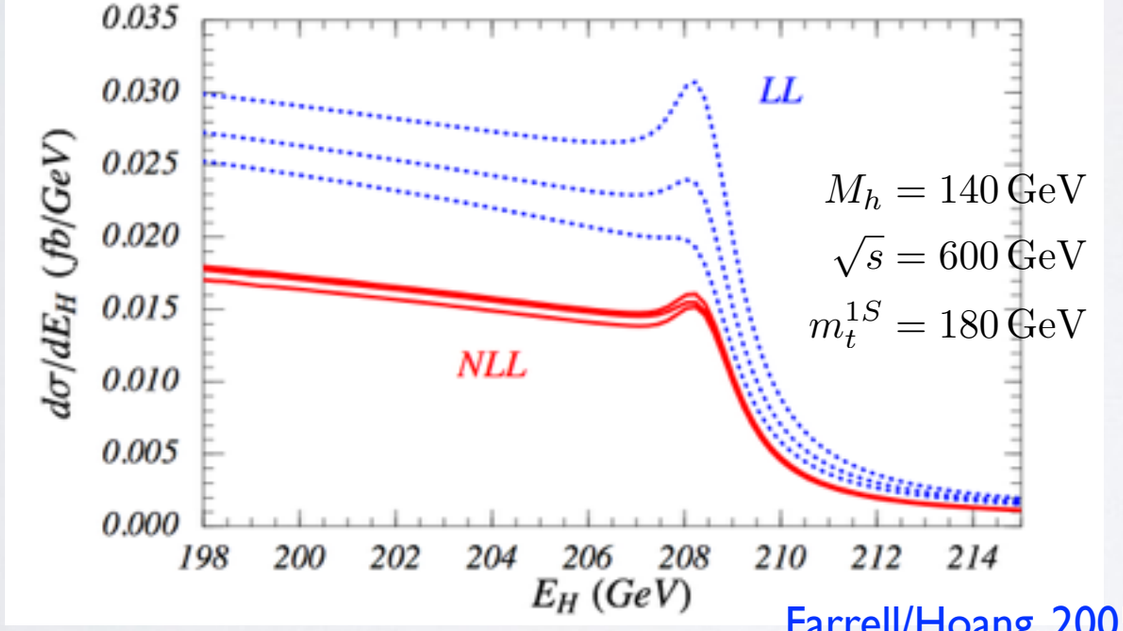

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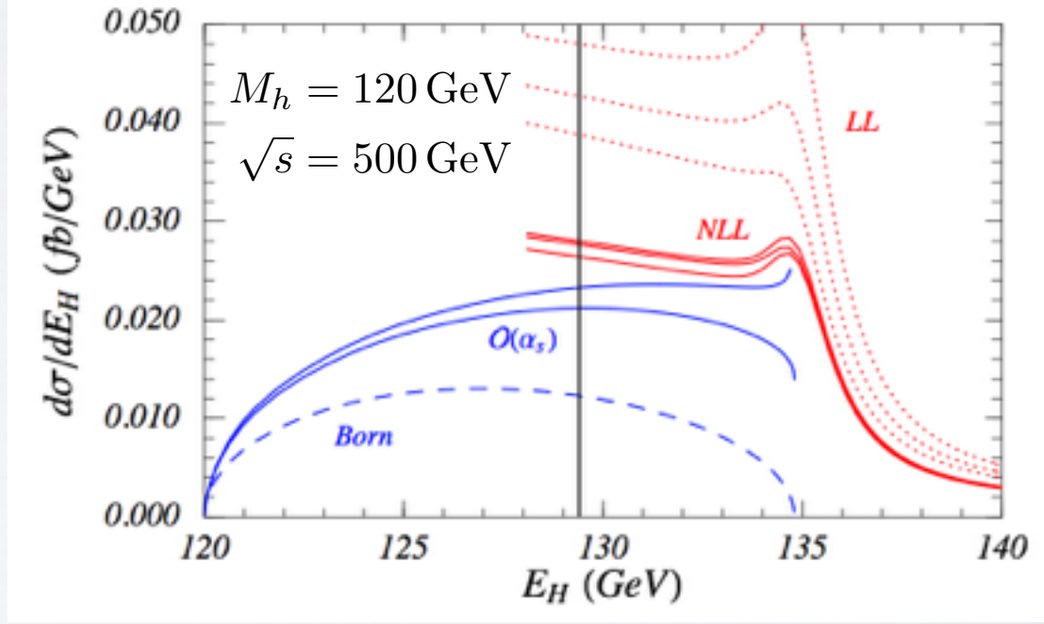

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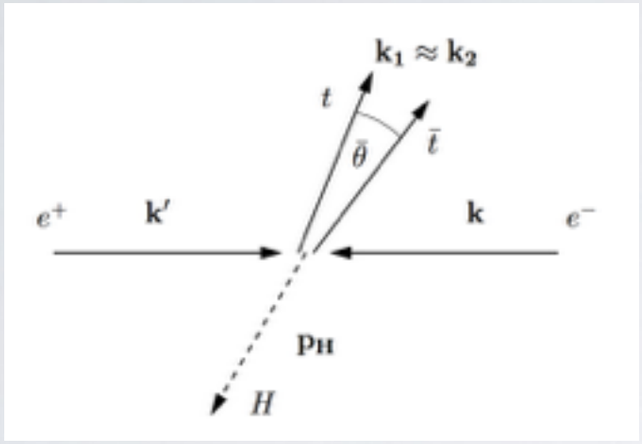

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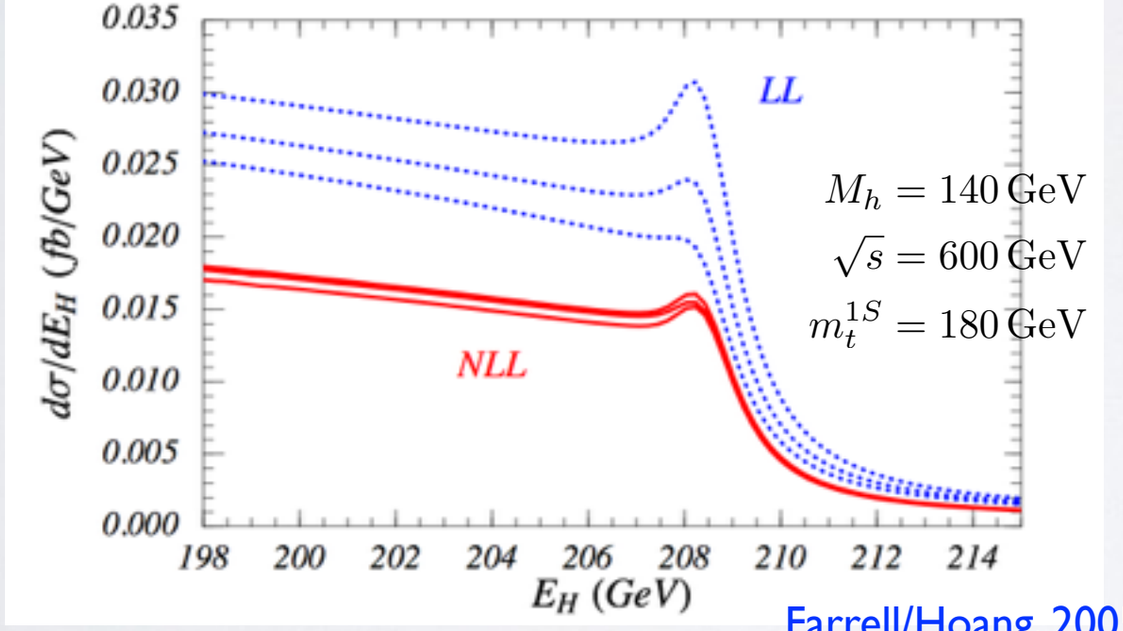

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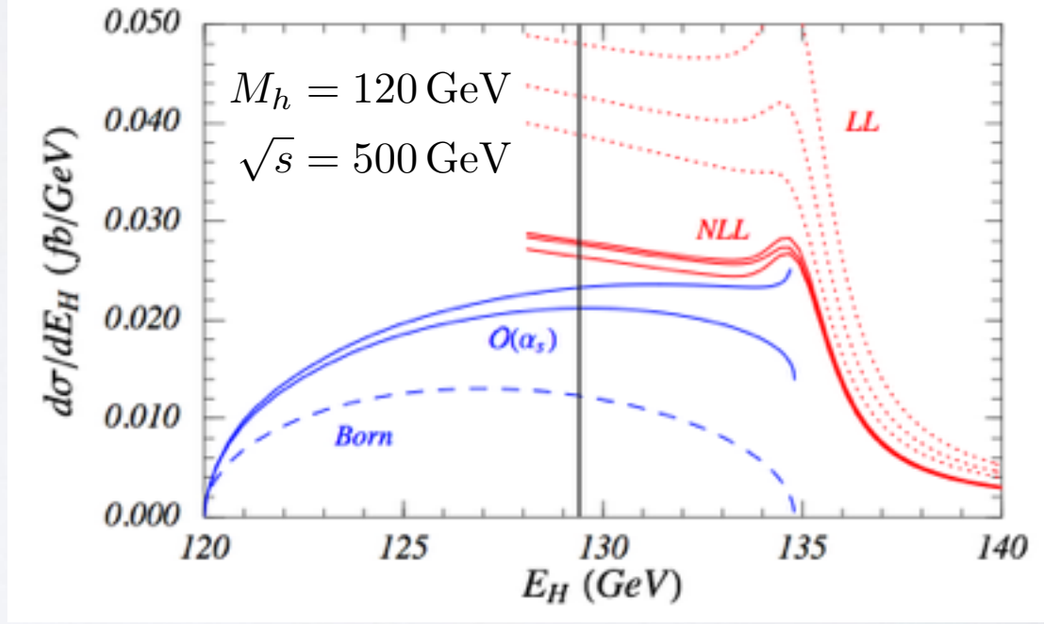

- Goal: properly matched NLL threshold/NLO cont. [Chokoufe/Hoang/JRR/Stahlhofen/Weiss, *in prep.*]

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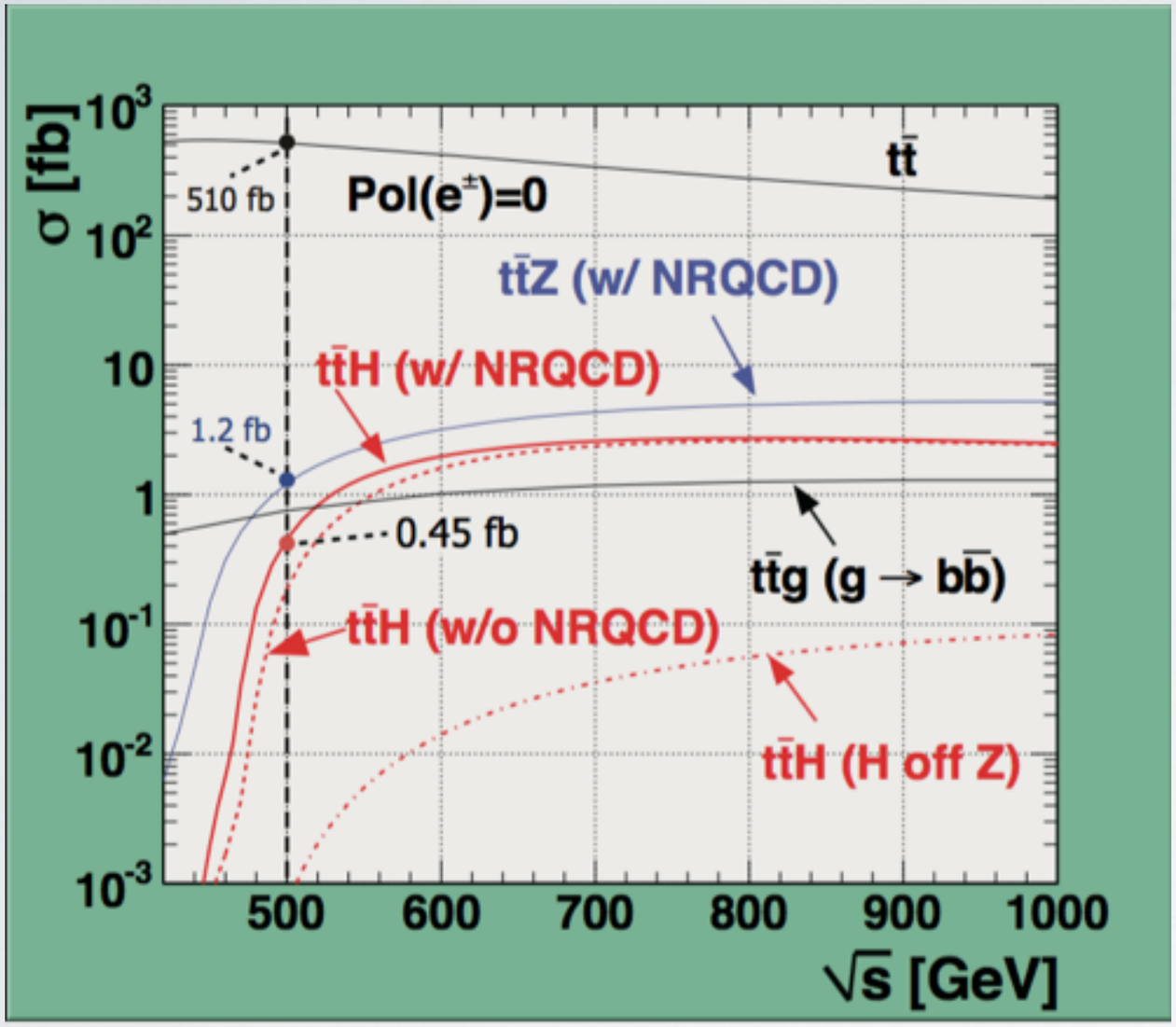

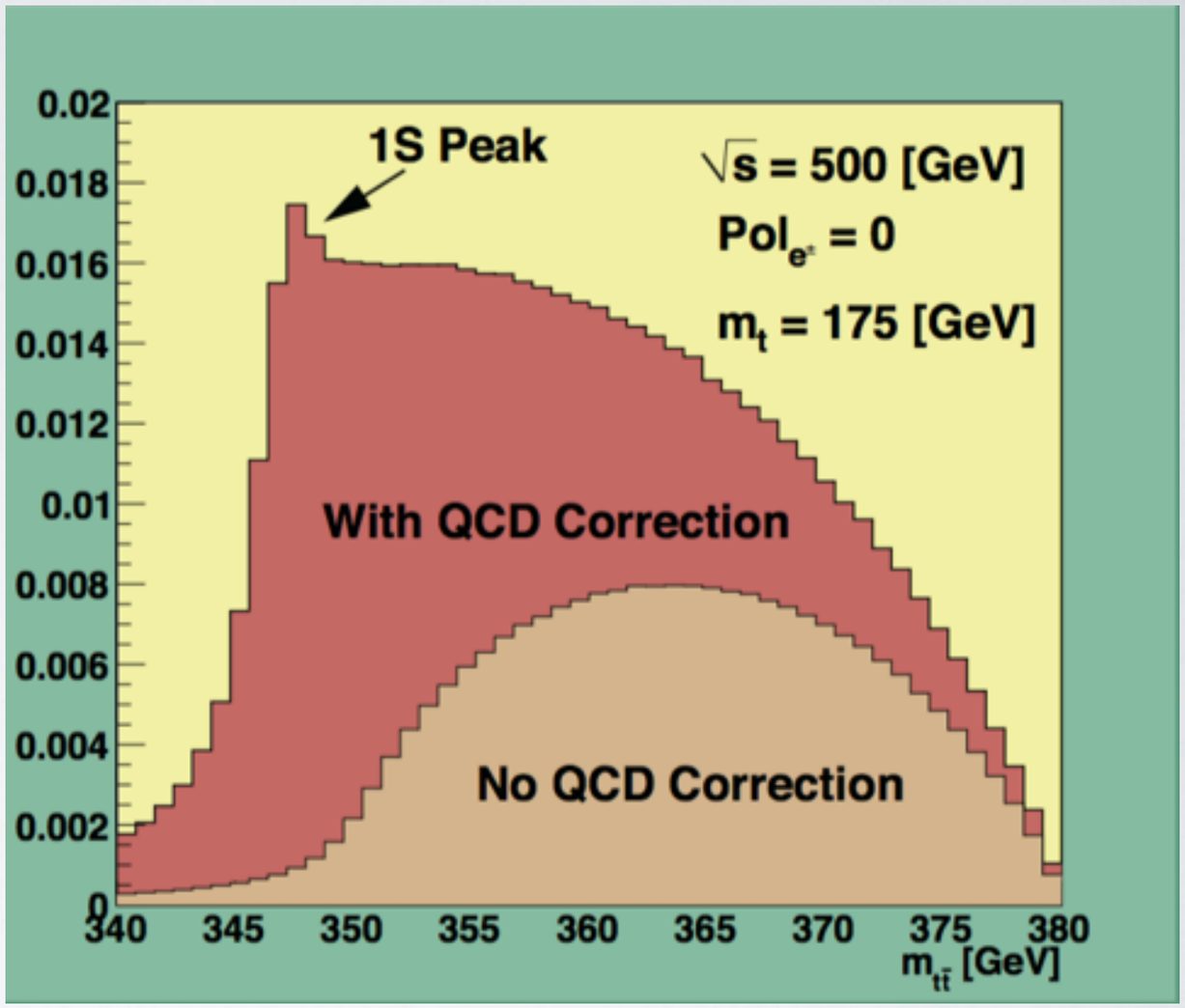

Yonamine/Ikematsu/Tanabe/Fujii/Kiyo/Sumino/Yokoya, 2011

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- Top threshold: top mass to 30-50 MeV [→ Top session]
- NLO QCD / Matching available in Monte Carlos
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