Electroweak Physics

Jürgen R. Reuter

DESY Theory Group

HELMHOLTZ GEMEINSCHAFT

DESY

PHYSICS AT THE TERA SCALE Helmholtz Alliance

LC School 2013
DESY Hamburg, 10/2013

Literature

- Georgi: Weak Interactions and Modern Particle Physics, Dover, 2009
- Quigg: Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, Perseus 1997
- Peskin: An Introduction to Quantum Field Theory Addison Wesly, 1994
- Weinberg, The Quantum Theory of Fields, Vol. II Cambridge Univ. Press, 1995-98
- Böhm/Denner/Joos, Gauge Theories of Strong and Electroweak Interactions, Springer, 2000
- ...and many more

Basics of Hidden Symmetries

- Hidden symmetry is obeyed by the Lagrangian (and the E.O.M.)
- It is not respected by the spectrum, especially the ground state
- In principle only possible in a system of infinite volume
- **Nambu-Goldstone Theorem** Goldstone, 1961; Nambu, 1960; Goldstone/Salam/Weinberg, 1962
  For any broken symmetry generator of a global symmetry there is a massless boson (Nambu-Goldstone boson) in the theory.

  Two cases:
  i) $Q^a |0\rangle = 0 \forall a$ unbroken or Wigner-Weyl phase
  ii) $Q^a |0\rangle \neq 0$ for at least one a $\Rightarrow$ Nambu-Goldstone phase

▶ Simple proof:

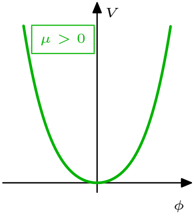
$$\phi_i \to \delta\phi_i i\theta^a T^a_{ik}\phi_k \quad \Rightarrow \quad \delta\mathcal{V} = \frac{\partial\mathcal{V}}{\partial\phi_i}\delta\phi_i = \frac{\partial\mathcal{V}}{\partial\phi_i} i\theta^a T^a_{ij}\phi_j = 0 \quad \Rightarrow$$

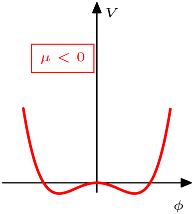
$$\underbrace{\frac{\partial^2\mathcal{V}}{\partial\phi_i\partial\phi_j}\bigg|_{\langle 0|\phi|0\rangle}}_{=(m^2)_{ij}} T^a_{jk}\langle 0|\phi_k|0\rangle + \underbrace{\frac{\partial\mathcal{V}}{\partial\phi_j}\bigg|_{\langle 0|\phi|0\rangle}}_{=0} T^a_{ji} = 0$$

The Nambu-Goldstone Theorem

- N-component real scalar field, possesses $O(N)$ symmetry

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^T)(\partial^\mu \phi) - \frac{\mu^2}{2}\phi^T\phi - \frac{g}{4}(\phi^T\phi)^2 \quad \text{with} \quad \phi = (\phi_1, \ldots, \phi_N)$$





Minimizing the potential:
$\langle \phi \rangle = 0$ (metastable) or
$\langle \phi^T \phi \rangle \sim \langle \phi \rangle^T \langle \phi \rangle = -\mu^2/g > 0$

- Without loss of generality: $\langle \phi_i \rangle = (0, 0, \ldots, 0, \langle \phi_N \rangle)$ VEV in n-th comp.
- Mass squared matrix:

$$(M^2)_{ij} = \frac{\partial^2 V(\phi)}{\partial \phi_i \partial \phi_j}\Bigg|_{\phi = \langle \phi \rangle} = 2g \langle \phi_i \rangle \langle \phi_j \rangle = \left(\begin{array}{c|c} 0_{(N-1)\times(N-1)} & 0_{1\times(N-1)} \\ \hline 0_{(N-1)\times 1} & 2g \langle \phi \rangle^2 \end{array} \right)$$

- $O(N)$ symmetry group broken down to $O(N-1)$ symmetry group
- # broken symmetry generators = # Goldstone bosons = $\frac{1}{2}N(N-1) - \frac{1}{2}(N-1)(N-2) = N-1$

Hidden Local Symmetries

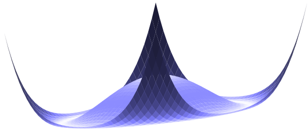
Anderson, 1961; Higgs, 1964; Brout/Englert, 1964; Kibble 1964

- Consider scalar electrodynamics:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$$

$$V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}(|\phi|^2)^2$$

- Remember the gauge trafos: $A_\mu \to A_\mu + \partial_\mu\theta(x), \quad \phi(x) \to \exp[-ie\theta(x)]\phi(x)$



  - Minimize the potential $\Rightarrow \langle\phi\rangle = v/\sqrt{2}e^{i\alpha}$ where $v/\sqrt{2} = \mu/\sqrt{\lambda}$
  - Radial excitation: "Higgs field"
  - Phase is the NGB

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))e^{\frac{i}{v}\pi(x)}$$

- Evaluating the kinetic term

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial h)^2 + \frac{e^2}{2}(v+h)^2\left(A_\mu - \frac{1}{ev}\partial_\mu\pi\right)^2 A_\mu$$

- Mixture between gauge boson and NGB. Define $B_\mu := A_\mu - \frac{1}{ev}\partial_\mu\pi$
- Field strength term does not change under this redefinition

Hidden Local Symmetries

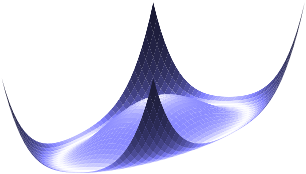
Anderson, 1961; Higgs, 1964; Brout/Englert, 1964; Kibble 1964

- Consider scalar electrodynamics:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

$$V(\phi) = -\mu^2 |\phi|^2 + \frac{\lambda}{2} (|\phi|^2)^2$$

- Remember the gauge trafos: $A_\mu \to A_\mu + \partial_\mu \theta(x), \quad \phi(x) \to \exp[-ie\theta(x)]\phi(x)$



- Minimize the potential $\Rightarrow \langle \phi \rangle = v/\sqrt{2} e^{i\alpha}$ where $v/\sqrt{2} = \mu/\sqrt{\lambda}$
- Radial excitation: "Higgs field"
- Phase is the NGB

$$\phi(x) = \frac{1}{\sqrt{2}} (v + h(x)) e^{\frac{i}{v}\pi(x)}$$

- Evaluating the kinetic term

$$|D_\mu \phi|^2 = \frac{1}{2} (\partial h)^2 + \frac{e^2}{2} (v+h)^2 \left(A_\mu - \frac{1}{ev} \partial_\mu \pi \right)^2 A_\mu$$

- Mixture between gauge boson and NGB. Define $B_\mu := A_\mu - \frac{1}{ev} \partial_\mu \pi$
- Field strength term does not change under this redefinition

Hidden Local Symmetries

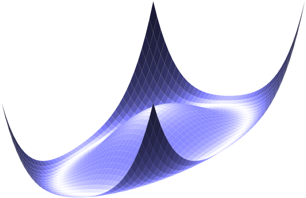
Anderson, 1961; Higgs, 1964; Brout/Englert, 1964; Kibble 1964

- Consider scalar electrodynamics:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi)$$

$$V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}(|\phi|^2)^2$$

- Remember the gauge trafos: $A_\mu \to A_\mu + \partial_\mu\theta(x), \quad \phi(x) \to \exp[-ie\theta(x)]\phi(x)$



- Minimize the potential $\Rightarrow \langle\phi\rangle = v/\sqrt{2}e^{i\alpha}$ where $v/\sqrt{2} = \mu/\sqrt{\lambda}$
- Radial excitation: "Higgs field"
- Phase is the NGB

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))e^{\frac{i}{v}\pi(x)}$$

- Evaluating the kinetic term

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial h)^2 + \frac{e^2}{2}(v+h)^2\left(A_\mu - \frac{1}{ev}\partial_\mu\pi\right)^2 A_\mu$$

- Mixture between gauge boson and NGB. Define $B_\mu := A_\mu - \frac{1}{ev}\partial_\mu\pi$
- Field strength term does not change under this redefinition

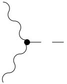
The Higgs Mechanism

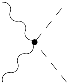
- VEV generates mass term for the gauge boson
- Gauge boson mass: only consistent (renormalizable) way 't Hooft/Veltman, 1971
- $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M_B^2 B_\mu B^m u + \frac{1}{2} (\partial h)^2 - \frac{1}{2} m_h h^2 - g_{h,3} h^3 - g_{h,4} h^4$$

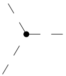
  with $\boxed{m_h^2 = \lambda v^2}$ $\boxed{M_B = ev}$ $\boxed{g_{h,3} = \frac{m_h^2}{2v}}$ $\boxed{m_h^2 = \frac{m_h^2}{8v^2}}$

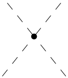
- Higgs field generates particle masses proportional to its VEV and its coupling to that particle

Feynman rules


$2\frac{M_B^2}{v}$


$2\frac{M_B^2}{v^2}$

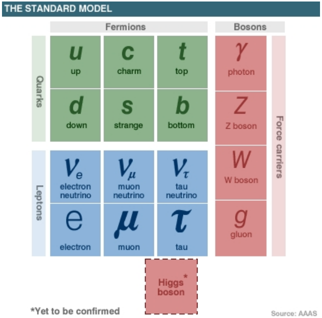

$3\frac{m_h^2}{v}$


$3\frac{m_h^2}{v^2}$

- Hey, what happened to the Nambu-Goldstone theorem??

  Longitudinal polarisation now becomes physical, Goldstone boson takes over its place in cancelling unphysical degrees of freedom.

The Electroweak Standard Model



- Standard Model (SM) is $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory
- Nuclear forces known since 1930s
- QCD ($SU(3)_c$) proven to be the correc theory in 1968-1980 (DIS, $e^+e^- \to$ jets at SLAC/DESY)
- Weak interactions known since 1896 (beta decay)
- Charged current weak processes, e.g. muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ Fermi, 1934
- Weak interactions couple only to left-handed particles Wu, 1957; Goldhaber, 1958
- Discovery of neutral currents in ν-nucleus scattering 1973, discrepancy in strength to charged current $\Rightarrow$ weak mixing angle
- Production of W, Z bosons (CERN, 1983)

The Lagrangian and its particles *in totaliter*

- Building blocks $(SU(3)_c, SU(2)_L)_{U(1)_Y}$ quantum numbers:

Q_L	u_R	d_R	L_L	e_R	H	ν_R
$(\mathbf{2},\mathbf{3})_{\frac{1}{3}}$	$(\mathbf{1},\mathbf{3})_{\frac{4}{3}}$	$(\mathbf{1},\mathbf{3})_{-\frac{2}{3}}$	$(\mathbf{2},\mathbf{1})_1$	$(\mathbf{1},\mathbf{1})_{-2}$	$(\mathbf{2},\mathbf{1})_1$	$(\mathbf{1},\mathbf{1})_0$

- All renormalizable interactions possible with these fields:

$$\begin{aligned}\mathcal{L}_{SM} = & \sum_{\psi=Q,u,d,L,e,H,\nu} \overline{\psi} \not{D} \psi - \frac{1}{2} \operatorname{tr}\left[G_{\mu\nu} G^{\mu\nu}\right] - \frac{1}{2} \operatorname{tr}\left[W_{\mu\nu} W^{\mu\nu}\right] \\ & - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + Y^u \overline{Q}_L \epsilon H^\dagger u_R + Y^d \overline{Q}_L H d_R \\ & + Y^e \overline{L}_L H e_R \quad \left[+ Y^n \overline{L}_L \epsilon H^\dagger \nu_R\right] + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2\end{aligned}$$

Electroweak Symmetry Breaking

- Higgs vev $\langle H \rangle = (0, v/\sqrt{2})$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{em.}$
  $D_\mu \phi = (\partial_\mu + ig\vec{W}_\mu \frac{\vec{\sigma}}{2} + ig' Y B_\mu)\phi$
- Electroweak gauge boson mass term:

$$\Delta\mathcal{L} = \tfrac{1}{2}(0, v)\left(g\vec{W}_\mu \frac{\vec{\sigma}}{2} + \frac{g'}{2} B_\mu\right)\left(g\vec{W}^\mu \frac{\vec{\sigma}}{2} + \frac{g'}{2} B^\mu\right)\begin{pmatrix}0\\ v\end{pmatrix}$$

- Three massive vector bosons $W^\pm$, Z

$$W^\pm_\mu = \tfrac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right) \qquad m_W = \tfrac{1}{2} g v$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g W^3_\mu - g' B_\mu\right) \qquad m_Z = \tfrac{v}{2}\sqrt{g^2 + g'^2}$$

- Orthogonal combination remains massless photon

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right) \qquad m_A = 0$$

- Rewrite the covariant derivative: $\sigma^\pm = \tfrac{1}{2}(\sigma^1 \pm \sigma^2)$

$$D_\mu = \partial_\mu + i\tfrac{g}{\sqrt{2}}(W^+_\mu \sigma^+ + W^-_\mu \sigma^-) + i\frac{1}{\sqrt{g^2+g'^2}} Z_\mu (g^2 T^3 - g'^2 Y) + i\frac{g g'}{\sqrt{g^2+g'^2}} A_\mu (T^3 + Y)$$

Weak mixing angle: $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$, $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

Gell-Mann–Nishijima relation: $Q = T^3 + Y$

Electroweak Symmetry Breaking

- Higgs vev $\langle H \rangle = (0, v/\sqrt{2})$ breaks $SU(2)_L \times U(1)_Y \to U(1)_{em.}$

  $D_\mu \phi = (\partial_\mu + ig\vec{W}_\mu \frac{\vec{\sigma}}{2} + ig' Y B_\mu)\phi$
- Electroweak gauge boson mass term:

$$\Delta\mathcal{L} = \tfrac{1}{2}(0, v) \left(g\vec{W}_\mu \frac{\vec{\sigma}}{2} + \frac{g'}{2} B_\mu \right) \left(g\vec{W}^\mu \frac{\vec{\sigma}}{2} + \frac{g'}{2} B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Three massive vector bosons $W^\pm$, Z

$$W^\pm_\mu = \tfrac{1}{\sqrt{2}} \left(W^1_\mu \mp i W^2_\mu \right) \qquad m_W = \tfrac{1}{2} g v$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g W^3_\mu - g' B_\mu \right) \qquad m_Z = \tfrac{v}{2} \sqrt{g^2 + g'^2}$$

- Orthogonal combination remains massless photon

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W^3_\mu + g B_\mu \right) \qquad m_A = 0$$

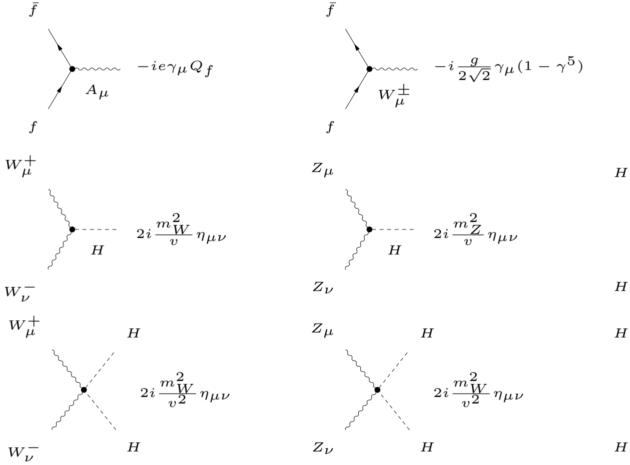
- Rewrite the covariant derivative: $\sigma^\pm = \frac{1}{2}(\sigma^1 \pm \sigma^2)$

$$D_\mu = \partial_\mu + i \tfrac{g}{\sqrt{2}} (W^+_\mu \sigma^+ + W^-_\mu \sigma^-) + i \frac{1}{\cos\theta_W} Z_\mu (T^3 - \sin^2\theta_W Q) + ie A_\mu Q$$

Weak mixing angle: $\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$, $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

Gell-Mann–Nishijima relation: $Q = T^3 + Y$

Electroweak Feynman Rules (all momenta outgoing)



$-ie\gamma_\mu Q_f$

$-i\frac{g}{2\sqrt{2}}\gamma_\mu(1-\gamma^5)$

$-i\frac{g}{2\cos\theta_W}\gamma_\mu$ $[(T^3_f - 2\sin^2$ $-\gamma^5 T^3_f]$

$2i\frac{m_W^2}{v}\eta_{\mu\nu}$

$2i\frac{m_Z^2}{v}\eta_{\mu\nu}$

$-3i\frac{m_H^2}{v}$

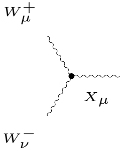
$2i\frac{m_W^2}{v^2}\eta_{\mu\nu}$

$2i\frac{m_W^2}{v^2}\eta_{\mu\nu}$

$-3i\frac{m_H^2}{v^2}$

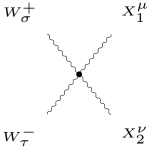
Easily derivable: $\left(m_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}m_Z^2 Z^2\right)\left(1+\frac{H}{v}\right)^2$

Electroweak Feynman Rules (all momenta outgoing)



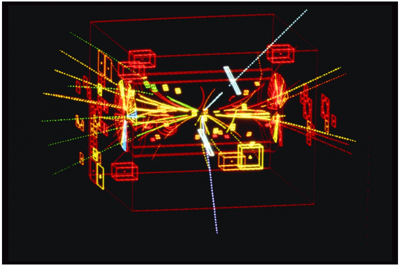
$$-i g_{WWX} \left[(k_- - k_+)^\rho \eta^{\mu\nu} + (q - k_-)^\mu \eta^{\nu\rho} + (k_+ - q)^\nu \eta^{\mu\rho} \right]$$

$$g_{WWZ} = g \cos\theta_W \qquad g_{WW\gamma} = e$$



$$-i g_{WWX_1X_2} \left[2\eta^{\mu\nu}\eta^{\sigma\tau} - \eta^{\mu\sigma}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\tau} \right]$$

$$g_{WW\gamma\gamma} = e^2$$
$$g_{WWZZ} = g^2 \cos^2\theta_W$$
$$g_{WW\gamma Z} = g^2 \cos\theta_W \sin\theta_W$$
$$g_{WWWW} = -g^2$$



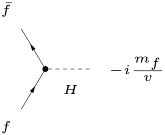
Fermion masses: Yukawa terms

- Fermion mass terms $-m_f(\overline{f}_L f_R + \overline{f}_R f_L)$ forbidden by $U(2)_L \times U(1)_Y$ gauge invariance
- Yukawa coupling is gauge invariant dimension-4 operator:

$$\Delta\mathcal{L}_{Yuk.} = -Y_e(\overline{L}_L \cdot \phi)e_R \to -\frac{vY_e}{\sqrt{2}}\overline{e}_L e_R \left(1 + \frac{H}{v}\right)$$

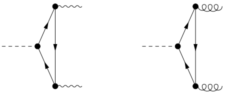
- Again, Higgs boson couples proportional to mass:

$$\boxed{m_f = \frac{1}{\sqrt{2}} Y_f v}$$

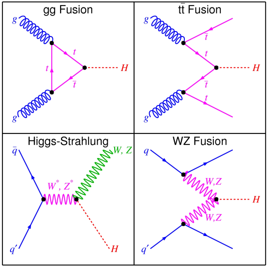


- Hierarchy of Yukawa couplings according to fermion masses: $Y_t \approx 1$, $Y_{c,\tau,b} \approx 10^{-2}$, $Y_{\mu,s} \approx 10^{-3}$, $Y_{e,\nu,d} \approx 10^{-5}$
- $Y_\nu \lesssim 10^{-10}$, but Majorana mass term $\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}m_\nu \overline{\nu^c}_R \nu_R$ possible
-

$H\gamma\gamma, Hgg$ couplings: $\frac{1}{v}\frac{Yg^2}{16\pi^2} \cdot c \cdot HF_{\mu\nu}F^{\mu\nu}$



Higgs: Properties and Search



Production: gluon/vector boson fusion

decays predominantly into the heaviest particles

$b\bar{b}$ hopeless: background!

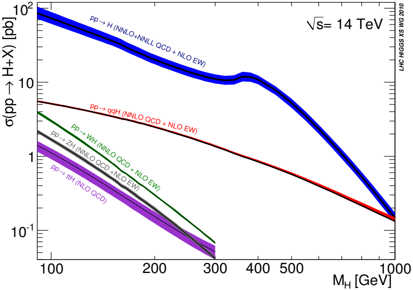
Detection of rare decays

Complicated search: many channels

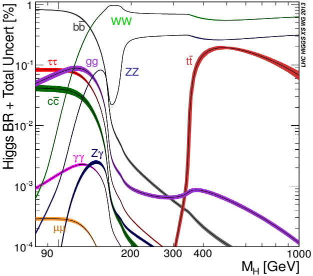
high statistics necessary

$\gamma\gamma$: mass determination

$M_H \gtrsim 125$ GeV: $ZZ^* \to \ell\ell\ell\ell$



Higgs: Properties and Search



Production: gluon/vector boson fusion

decays predominantly into the heaviest particles

$b\bar{b}$ hopeless: background!

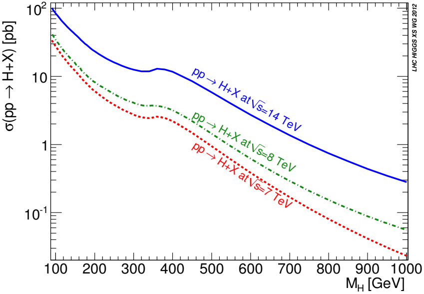
Detection of rare decays

Complicated search: many channels

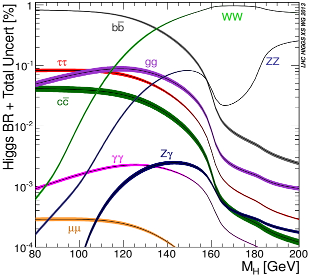
high statistics necessary

$\gamma\gamma$: mass determination

$M_H \gtrsim 125$ GeV: $ZZ^* \to \ell\ell\ell\ell$



Higgs: Properties and Search



Production: gluon/vector boson fusion

decays predominantly into the heaviest particles

$b\bar{b}$ hopeless: background!

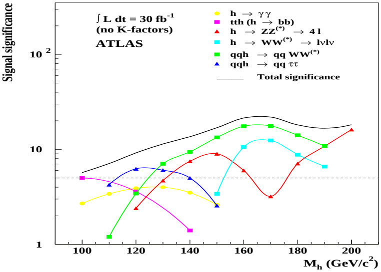
Detection of rare decays

Complicated search: many channels

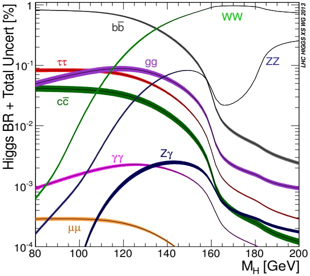
high statistics necessary

$\gamma\gamma$: mass determination

$M_H \gtrsim 125$ GeV: $ZZ^* \to \ell\ell\ell\ell$



Higgs: Properties and Search



Production: gluon/vector boson fusion

decays predominantly into the heaviest particles

$b\bar{b}$ hopeless: background!

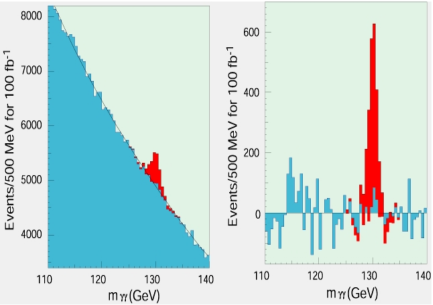
Detection of rare decays

Complicated search: many channels

high statistics necessary

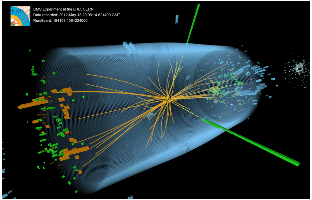
$\gamma\gamma$: mass determination

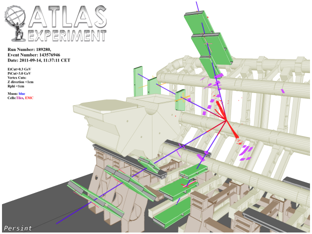
$M_H \gtrsim 125$ GeV: $ZZ^* \to \ell\ell\ell\ell$



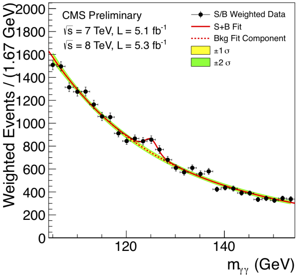
4.7.2012: The Discovery of the Higgs (?)

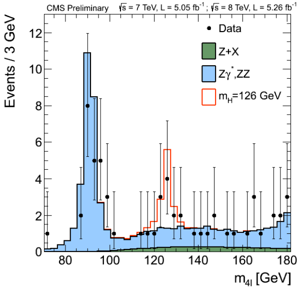
- After roughly 5 fb^{-1} data from 2011 and 2012:
  4.7.2012 CERN-Seminar: *We have found a scalar boson at* 125.3 ± 0.6 *GeV*

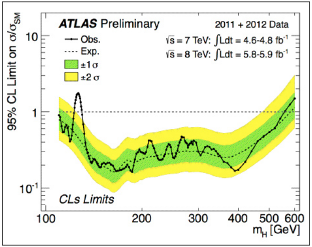


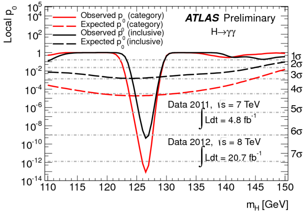


▶ Rolf Heuer (CERN DG, 4.7.12): *As a layman I would say we have it!*



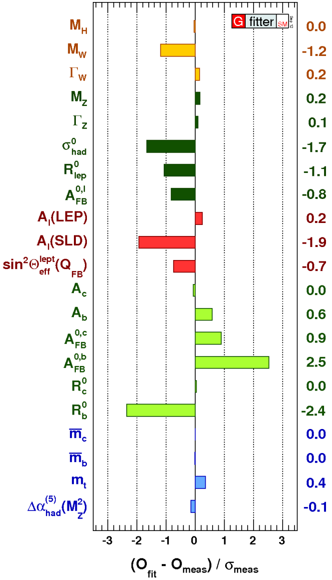




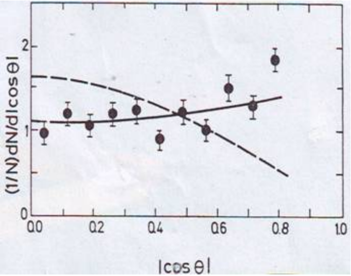


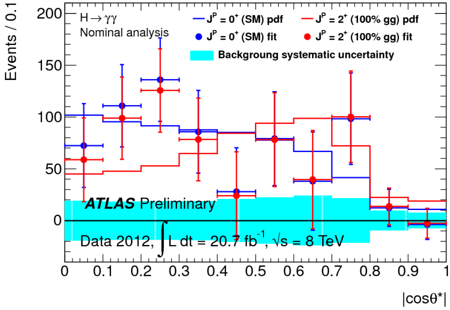
2013: It's the/a/sort of Higgs(-like) WTF

- 2012 data: 25 fb^{-1} $\Rightarrow$ compatible with EW precision
- Higgs compatible with EW precision measurements:
- *Higgs measurement by far not precise enough!!!*
- More: $\longrightarrow$ K. Fujii's talk



Digitization Science Fiction





Flavor, the CKM matrix, and CP violation

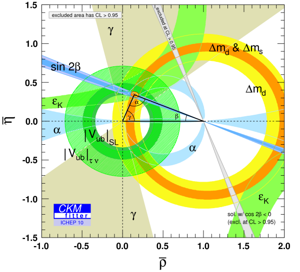
- Three generations of fermions in Nature
- Diagonalization of fermion mass matrices:

$$v^2 Y_u Y_u^\dagger = L_u \text{diag}(m_u^2, m_c^2, m_t^2) L_u^\dagger \qquad v^2 Y_d Y_d^\dagger = L_d \text{diag}(m_d^2, m_s^2, m_b^2) L_d^\dagger$$

- Rotation of quark fields leaves a trace in the charged current:

$$\overline{u}_L \not{W} (L_u^\dagger L_d) d_L = \overline{u}_L \not{W} V^{CKM} d_L$$

- CKM matrix: unitary, experimentally almost diagonal
- Three angles $\theta_{12}, \theta_{13}, \theta_{23}$, one phase
- Phase violates CP (charge conjugation and parity)
- After discovery of neutrino oscillations: MNS matrix
- More: $\longrightarrow$ J. Dingfelder's talk



Properties of W and Z

- M_W = 80.385(15) GeV

$$\Gamma(W \to f_i \overline{f}_j) \approx N_C^f \frac{\alpha M_W}{12 s_W^2} \times \left| V_{ij}^{CKM} \right|^2$$

$\Gamma_W = \sum_{ij} \Gamma(W \to f_i \overline{f}_j) \approx 2.0$ GeV

- BR($W \to \ell \overline{\nu}_\ell$) $\approx 1/9$, BR($W \to u_i \overline{d}_j$) $\approx 3/9$

BR($W \to e\overline{\nu}_e$)	10.80(09) %
BR($W \to \mu\overline{\nu}_\mu$)	10.75(13) %
BR($W \to \tau\overline{\nu}_\tau$)	10.57(15) %
BR($W \to$ hadrons)	67.60(20) %

- $\Gamma_W = 2.085(42)$ GeV

  Γ_W / M_W = 2.59 %

EW, mainly QED rad. corr.

- M_Z = 91.1876(21) GeV

$$\Gamma(Z \to f\overline{f}) \approx N_C^f \frac{\alpha M_Z}{24 c_W^2 s_W^2} \times \left(1 - 4 s_W^2 |Q_f| + 8 s_W^4 |Q_f|^2\right)$$

$\Gamma_Z = \sum_f \Gamma(Z \to f\overline{f}) \approx 2.5$ GeV

- BR($Z \to \ell\overline{\ell}$) $\approx 1/30$, BR($Z \to d\overline{d}$) $\approx 1/6$, BR($Z \to \nu\overline{\nu}$) $\approx 2/30$, BR($Z \to u\overline{u}$) $\approx 1/10$,

BR($Z \to e\overline{e}$)	3.363(3) %
BR($Z \to \mu\overline{\mu}$)	3.366(7) %
BR($Z \to \tau\overline{\tau}$)	3.370(8) %
BR($Z \to$ invis.)	20.00(6) %
BR($Z \to$ hadrons)	69.91(6) %

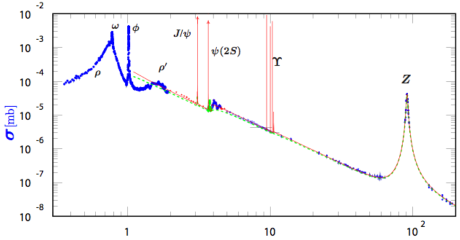
- $\Gamma_Z = 2.4952(23)$ GeV

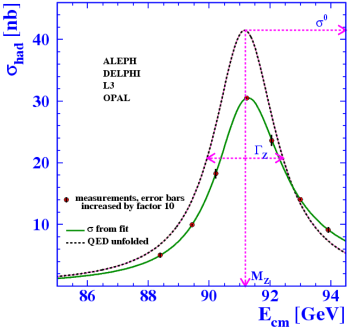
  Γ_Z / M_Z = 2.74 %

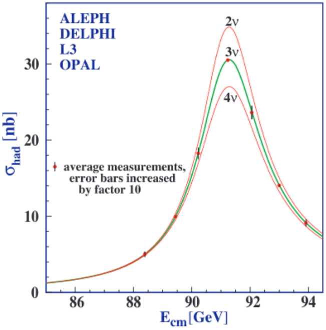
Z pole and lineshape

LEP I, SLC:

- $\sigma_0^{\text{had}} = \frac{12\pi\Gamma(e^+e^-)\Gamma(\text{had})}{M_Z^2\Gamma_Z} = 41.477(9)$ nb
- 30% QED corrections
- $N_\nu = 3$







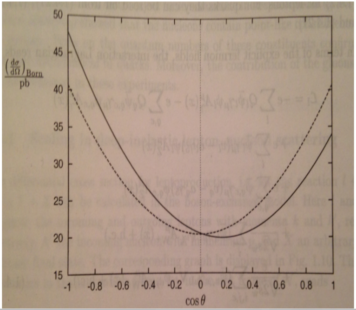
Z asymmetries

$$\frac{d\sigma(e^+e^- \to f\overline{f})}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \times \left[\left(1 + \cos^2\theta + \frac{4m_f^2}{s}\sin^2\theta\right) F_1(s) - \frac{8m_f^2}{s} F_2(s) + 2\cos\theta F_3(s) \sqrt{1 - \frac{4m_f^2}{s}} \right]$$

$$F_1(s) = Q_e^2 Q_f^2 + 2 Q_e Q_f v_e v_f \mathrm{Re} P(s) + (v_e^2 + a_e^2)(v_f^2 + a_f^2)|P(s)|^2$$

$$F_2(s) = (v_e^2 + a_e^2) a_f^2 |P(s)|^2$$

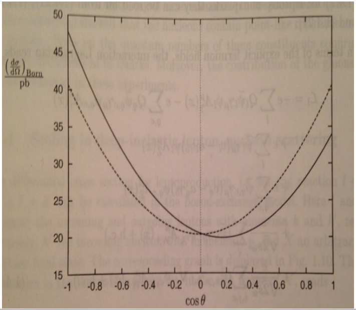
$$F_3(s) = 2 Q_e Q_f a_e a_f \mathrm{Re} P(s) + 4 v_e v_f a_e a_f |P(s)|^2 \qquad P(s) = \frac{s}{s - M_Z^2 + i\Gamma_Z M_Z}$$



Z asymmetries

$$\frac{d\sigma(e^+e^- \to f\overline{f})}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \times$$
$$\left[\left(1+\cos^2\theta + \frac{4m_f^2}{s}\sin^2\theta\right)F_1(s) - \frac{8m_f^2}{s}F_2(s) + 2\cos\theta F_3(s)\sqrt{1-\frac{4m_f^2}{s}}\right]$$

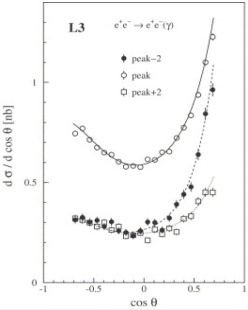
$$F_1(s) = Q_e^2 Q_f^2 + 2Q_e Q_f v_e v_f \text{Re}P(s) + (v_e^2+a_e^2)(v_f^2+a_f^2)|P(s)|^2$$
$$F_2(s) = (v_e^2+a_e^2)a_f^2|P(s)|^2$$
$$F_3(s) = 2Q_e Q_f a_e a_f \text{Re}P(s) + 4v_e v_f a_e a_f |P(s)|^2 \qquad P(s) = \frac{s}{s-M_Z^2+i\Gamma_Z M_Z}$$

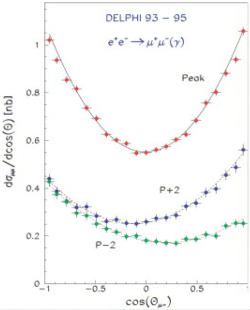


Z asymmetries

$$\frac{d\sigma(e^+e^- \to f\overline{f})}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \times$$
$$\left[\left(1 + \cos^2\theta + \frac{4m_f^2}{s}\sin^2\theta \right) F_1(s) - \frac{8m_f^2}{s} F_2(s) + 2\cos\theta F_3(s) \sqrt{1 - \frac{4m_f^2}{s}} \right]$$

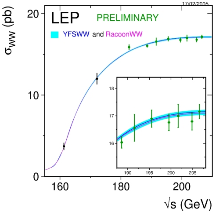
$$F_1(s) = Q_e^2 Q_f^2 + 2Q_eQ_f v_e v_f \mathrm{Re} P(s) + (v_e^2 + a_e^2)(v_f^2 + a_f^2)|P(s)|^2$$
$$F_2(s) = (v_e^2 + a_e^2) a_f^2 |P(s)|^2$$
$$F_3(s) = 2Q_eQ_f a_e a_f \mathrm{Re} P(s) + 4 v_e v_f a_e a_f |P(s)|^2 \qquad P(s) = \frac{s}{s - M_Z^2 + i\Gamma_Z M_Z}$$

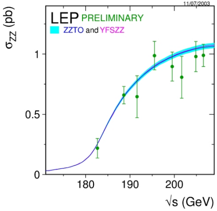




WW/ZZ threshold

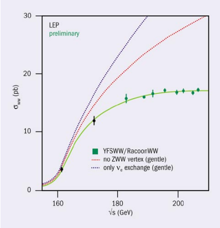
- W threshold scan: M_W with 24 MeV precision





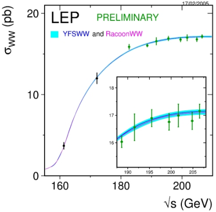
- Evidence for non-Abelian structure of weak interactions

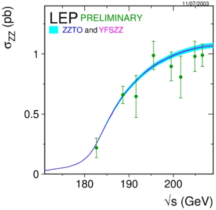
- Careful with propaganda plots:



WW/ZZ threshold

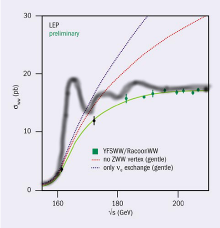
- W threshold scan: M_W with 24 MeV precision



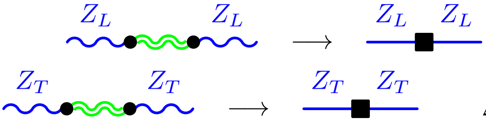


- Evidence for non-Abelian structure of weak interactions

- Careful with propaganda plots:



Radiative Corrections

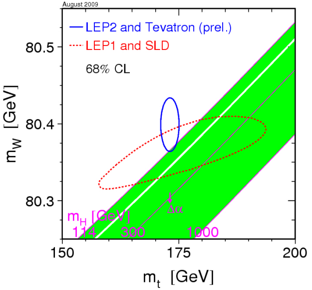


$$\Delta T \sim \Delta\rho \sim \Delta M_Z^2 Z \cdot Z$$

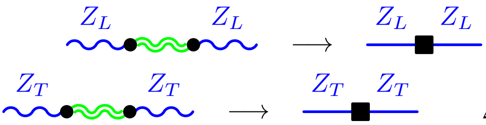
$$\Delta S \sim W^0{}_{\mu\nu} B^{\mu\nu}, \Delta U \sim W^0{}_{\mu\nu} W^{0\,\mu\nu}$$

Electroweak radiative corrections:

$$M_Z^2 = M_W^2 / c_W^2 \qquad M_W^2 = \frac{\pi\alpha/\sqrt{2}G_F}{s_W^2(1-\Delta r)}$$



Radiative Corrections

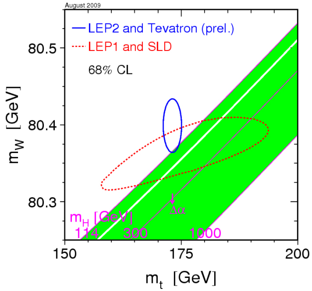


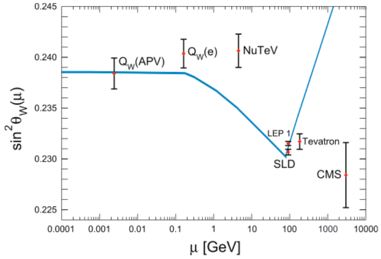
$$\Delta T \sim \Delta\rho \sim \Delta M_Z^2 Z \cdot Z$$

$$\Delta S \sim W^0{}_{\mu\nu} B^{\mu\nu}, \Delta U \sim W^0{}_{\mu\nu} W^{0\,\mu\nu}$$

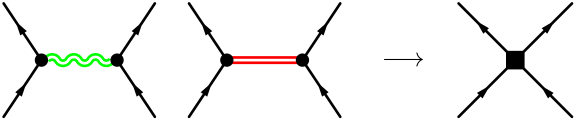
Electroweak radiative corrections:

$$M_Z^2 = M_W^2 / c_W^2 \qquad M_W^2 = \frac{\pi\alpha/\sqrt{2}G_F}{s_W^2(1-\Delta r)}$$

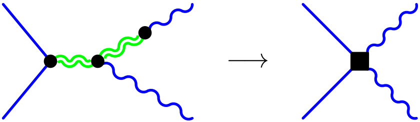




Effective Dim. 6 Operators



$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{\Lambda^2} \mathrm{tr}[J^{(I)} \cdot J^{(I)}]$$



$$\mathcal{O}'_{WW} = -\frac{1}{\Lambda^2} \frac{1}{2} (h^\dagger h - v^2/2) \,\mathrm{tr}\, W_{\mu\nu} W^{\mu\nu}$$

$$\mathcal{O}_B = \frac{1}{\Lambda^2} \frac{i}{2} (D_\mu h)^\dagger (D_\nu h) B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{\Lambda^2} \frac{1}{4} (h^\dagger h - v^2/2) B_{\mu\nu} B^{\mu\nu}$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = \mathrm{i}e \left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu} \right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ A^{\rho\mu} \right]$$
$$+ \mathrm{i}e \frac{c_W}{s_W} \left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu} \right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ Z^{\rho\mu} \right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1,\ \lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_W^2 - s_W^2}$ $\quad g_{1/2}^{VV'} = 1,\ h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$
$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_W^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_W^4}(\alpha_5 + \alpha_7)$$
$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_W^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_W^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$
$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_W^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_W^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$
$$h^{ZZ} = g^2 \left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{QGC} = e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+ \right] + e^2 \frac{c_W}{s_W} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^- \right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+ \right] + e^2 \frac{c_W^2}{s_W^2} \left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+ \right] + \frac{e^2}{2 s_W^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW} \left(W^{-\mu} W_\mu^+ \right)^2 \right] + \frac{e^2}{4 s_W^2 c_W^4} h^{ZZ} (Z^\mu Z_\mu)^2$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_W^2 - s_W^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta \kappa^\gamma = g^2 (\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2 (\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_W^2} \alpha_3 \qquad \Delta \kappa^Z = \delta_Z - g'^2 (\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2 (\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2 \Delta g_1^{\gamma Z} - \frac{g^2}{c_W^4} (\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_W^2} \alpha_3 \qquad \Delta g_1^{WW} = 2 c_W^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) + g^2 \alpha_4$$

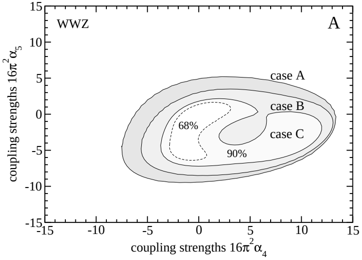
$$\Delta g_1^{ZZ} = 2 \Delta g_1^{\gamma Z} + \frac{g^2}{c_W^4} (\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2 c_W^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) - g^2 (\alpha_4 + 2 \alpha_5)$$

$$h^{ZZ} = g^2 \left[\alpha_4 + \alpha_5 + 2 (\alpha_6 + \alpha_7 + \alpha_{10}) \right]$$

ILC: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JRR

1 TeV, $1\ \mathrm{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

$16\pi^2 \times$	WWZ no pol.	WWZ e^- pol.	WWZ both pol.	ZZZ no pol.	best
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

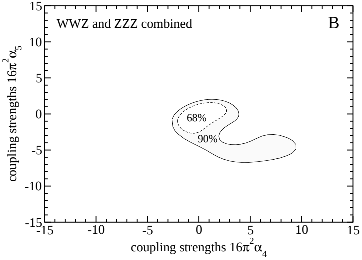
Veto against $E^2_{\mathrm{mis}} + p^2_{\perp,\mathrm{mis}}$

No angular correlations yet

ILC: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\mathrm{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

$16\pi^2\times$	WWZ no pol.	WWZ e^- pol.	WWZ both pol.	ZZZ no pol.	best
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E^2_{\mathrm{mis}} + p^2_{\perp,\mathrm{mis}}$

No angular correlations yet

ILC: Vector Boson Scattering

1 TeV, $1\,\mathrm{ab}^{-1}$, full $6f$ final states, 80 % e^-_R, 60 % e^+_L polarization, binned likelihood

Contributing channels: $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$

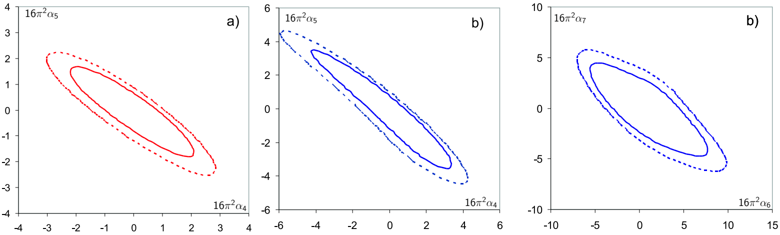
Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e\bar{\nu}_e q\bar{q}q\bar{q}$	$WW \to WW$	23.19
$e^+e^- \to \nu_e\bar{\nu}_e q\bar{q}q\bar{q}$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar{\nu}q\bar{q}q\bar{q}$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar{q}q\bar{q}$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar{q}q\bar{q}$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar{q}q\bar{q}$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar{b}X$	$e^+e^- \to t\bar{t}$	331.768
$e^+e^- \to q\bar{q}q\bar{q}$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar{q}q\bar{q}$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar{q}$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar{q}$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar{q}$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55

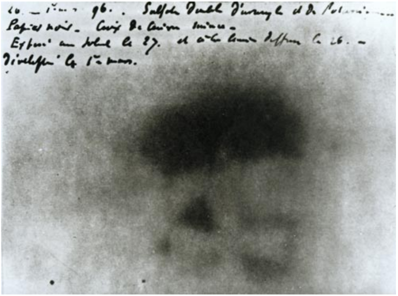


Summary

- LHC: paved the path to Electroweak Symmetry Breaking
- Microscopic picture of EWSB not yet understood
- (Precision) properties of W, Z, H crucial
- Interesting times!

Summary

- LHC: paved the path to Electroweak Symmetry Breaking
- Microscopic picture of EWSB not yet understood
- (Precision) properties of W, Z, H crucial
- Interesting times!



Summary

- LHC: paved the path to Electroweak Symmetry Breaking
- Microscopic picture of EWSB not yet understood
- (Precision) properties of W, Z, H crucial
- Interesting times!