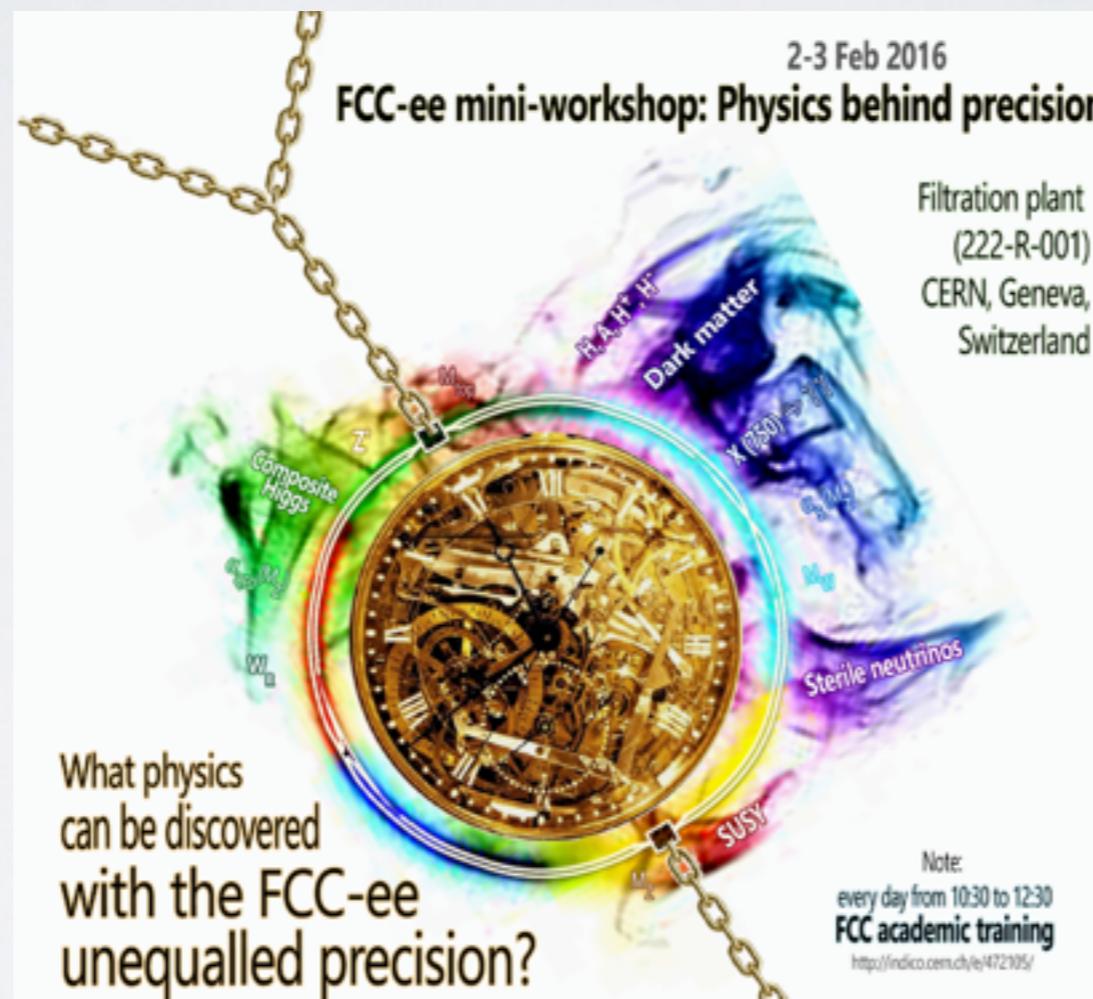




The WHIZARD Generator for precision measurements



Jürgen R. Reuter, DESY



J.R.Reuter

Event generator WHIZARD

FCC-ee “Physics behind precision”, CERN, 2.2.16



WHIZARD: Past and recent timeline

- Original scope: electroweak (multi-fermion) studies at 1.6 TeV TESLA [\approx 1998-2000]
 - Used for many TESLA studies and most ILC CDR and TDR, CLIC CDR and detector L0L studies (versions v1.24, v1.50, v1.95) [\approx 2002-2013]
 - Color flow formalism [\approx 2005]
 - Major refactoring phase I: **LHC physics** \rightarrow **v2.0.0** [\approx 2007-2010]
 - Validation inside ATLAS and CMS [\approx 2011-2014]
 - 2nd refactoring phase II: **NLO automation / maintainability** \rightarrow **v2.2.0** [\approx 2012-2014]
 - Strong interest of CEPC/FCC-ee group(s) for CEPC/FCC-ee simulations [\approx 2013-2015]
 - 04/2015, ALCW'15 Tokyo: LC generator group endorsed v2.2 for new mass productions
 - Ongoing validation for [ee] physics between v1.95 and v2 [until ca. 04/2016]
- Special thanks to: [beam spectra, photon background, event formats, shower/hadronization, tau decays]





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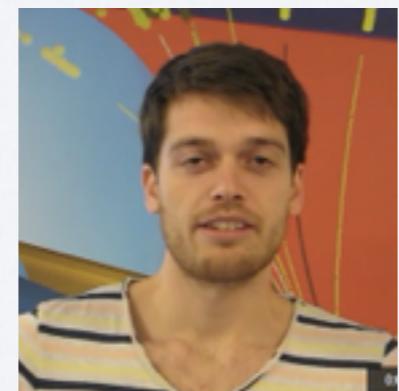
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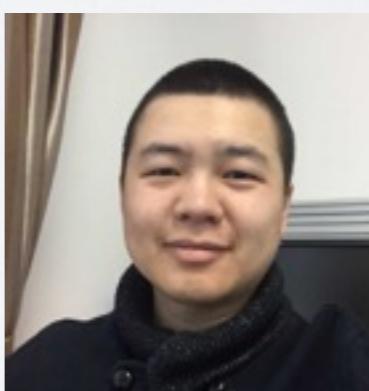
Mikael Berggren



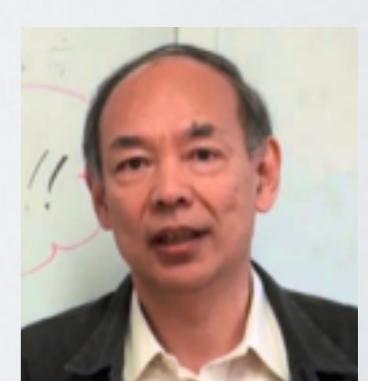
Jean-Jacques Blaising



Moritz Habermehl



Mo Xin



Akiya Miyamoto





WHIZARD: Introduction

WHIZARD v2.2.8 (22.11.2015)

<http://whizard.hepforge.org>

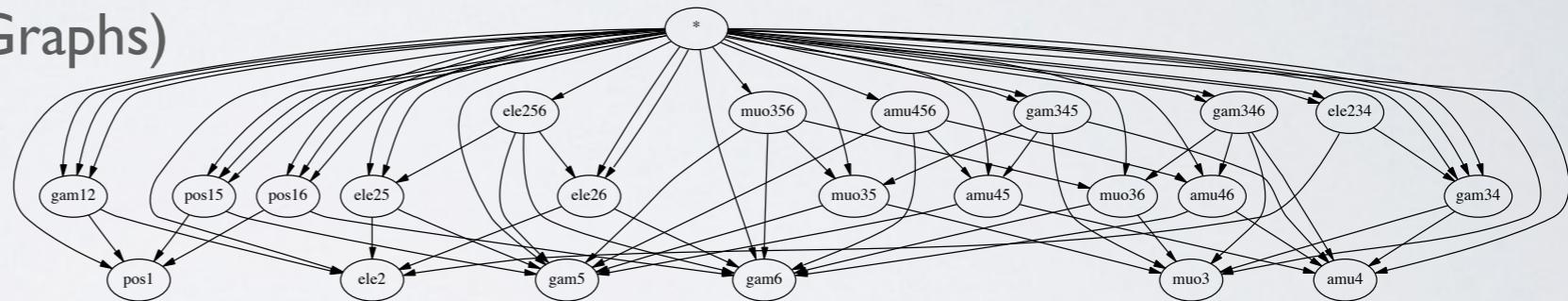
<whizard@desy.de>

WHIZARD Team: Wolfgang Kilian, Thorsten Ohl, JRR, Simon Braß/Bijan Chokoufé/Marco Sekulla/Soyoung Shim/Christian Weiss/Florian Staub/Zhijie Zhao + 2 Master students

Publication: EPJ C71 (2011) 1742

- Universal event generator for lepton and hadron colliders
- Modular package:
 - Phase space parameterization (resonances, collinear emission, Coulomb etc.)
 - O'Mega optimized matrix element generator (recursiveness via Directed Acyclical Graphs)

$$\Omega$$



- VAMP: adaptive multi-channel Monte Carlo integrator
 - CIRCE1/2: generator/simulation tool for lepton collider beam spectra
 - Lepton beam ISR Kuraev/Fadin, 1986; Skrzypek/Jadach, 1991
 - Color flow formalism Stelzer/Willenbrock, 2003; Kilian/Ohl/JRR/Speckner, 2011
 - Parton shower Kilian/JRR/Schmidt/Wiesler, 2011
- Interfaces to external packages for Feynman rules, hadronization, tau decays, event formats, analysis, jet clustering etc.: FastJet, GoSam, GuineaPig(++), HepMC, HOPPET, LCIO, LHAPDF(4/5/6), LoopTools, OpenLoops, PYTHIA6, [PYTHIA8], StdHep [internal]





WHIZARD: Manual

The screenshot shows a web browser window for the URL whizard.hepforge.org. The page title is "WHIZARD: Manual". A banner at the top right states "WHIZARD is hosted by HepForge, IPPP Durham". The main content area has a green background and features the title "WHIZARD 2.2" and subtitle "A generic Monte-Carlo integration and event generation package for multi-particle processes MANUAL 1". Below this, the authors are listed: Wolfgang Kilian,² Thorsten Ohl,³ Jürgen Reuter,⁴ with contributions from Fabian Bach,⁵ Bijan Chokoufé Nejad,⁶ Sebastian Schmidt, Christian Speckner⁷, Florian Staub⁸. The left sidebar contains a navigation menu with links to "HOME", "MANUAL, WIKI, NEWS", "REPOSITORY, BUG TRACKER", "DOWNLOADS", "CONTACT", and "INTERNAL WHIZARD PAGE". The "MANUAL, WIKI, NEWS" section includes links to "Manual", "Wiki Page", "News", "Tutorials", "WHIZARD talks", and "ChangeLog". The "REPOSITORY, BUG TRACKER" section includes links to "Subversion Repository", "SVN Browser", and "Bug Tracker". The "DOWNLOADS" section includes links to "Download Page" and "Patches/Unofficial versions". The "CONTACT" section includes a link to "Contact us". The "INTERNAL WHIZARD PAGE" section includes a link to "You Shall Not Pass!". At the bottom right of the main content area, there is a blue text overlay that reads "WHIZARD Manual @ HepForge".





General structure of SINDARIN input

```
model = SM

alias ll = "e-":"e+":"mu+":mu-
alias nu = n1:N1:n2:N2:n3:N3
alias jet = u:U:d:D:s:S:g

process tth = e1, E1 => t, tbar, h
process tthfull =
    e1, E1 => ll, nu, ll, nu, b, bbar, jet, jet
process inclusive =
    e1, E1 => (Z, h) + (Z, Z) + (Wp, Wm)
process t_dec = t => E1, nubar, b

sqrtS = 500 GeV
beams = e1, E1 => circe1 => ISR

cuts = all M > 10 GeV [jet, jet]

integrate (tthfull)
{ iterations = 15:500000, 5:1000000 }

n_events = 10000

unstable t (t_dec)

sample_format = lhef, stdhep, hepmc
sample = "mydata"
```





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LCWS '14, Belgrade, Simulation summary talk:

WHIZARD Task to implement LCIO format





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WHIZARD Task to implement LCIO format

WHIZARD v2.2.4, 02/2015:

```
sample_format = lcio
simulate (<process>)
```





General structure of SINDARIN input

```

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    e1, E1 => (Z, h) + (Z, Z) + (Wp, Wm)
process t_dec = t => E1, nubar, b

```

```

-----  

- Event : 1  

- run: 42  

- timestamp 14293873900000000000  

- weight 1  

-----  

date: 18.04.2015 20:03:10.000000000  

detector : unknown  

event parameters:  

parameter ProcessID [int]: 20,

```

collection name : MCParticle
parameters:

----- print out of MCParticle collection -----

```

flag: 0x0
simulator status bits: [sbvtcls] s: created in simulation b: backscatter v: vertex is not endpoint of parent t: decayed in tracker c: decayed in calorimeter l: has left detector s: stopped o: overlay

[ id ] [index] PDG | px, py, pz | energy [gen][simstat] | vertex x, y , z | endpoint x, y , z | mass | charge | spin | colorflow | [parents] - [daughters]
[00000004] 0| 2212| 0.00e+00, 0.00e+00, 7.00e+03| 7.00e+03| 3|[s ] || 0.00e+00, 0.00e+00, 0.00e+00| 0.00e+00, 0.00e+00| 0.00e+00| 0.00e+00, 0.00e+00, 0.00e+00| (0, 0) | [] - [2,3]
[00000005] 1| 2212| 0.00e+00, 0.00e+00,-7.00e+03| 7.00e+03| 3|[s ] || 0.00e+00, 0.00e+00, 0.00e+00| 0.00e+00, 0.00e+00| 0.00e+00| 0.00e+00, 0.00e+00, 0.00e+00| (0, 0) | [] - [2,3]
[00000006] 2| 1| 7.50e-01,-1.57e+00, 3.22e+01| 3.22e+01| 3|[s ] || 0.00e+00, 0.00e+00, 0.00e+00| 0.00e+00, 0.00e+00| 0.00e+00| 6.25e-02| 0.00e+00| 0.00e+00, 0.00e+00| (501, 0) | (0,1) - [4,5]
[00000007] 3| -2|-3.05e+00,-1.90e+01,-5.46e+01| 5.79e+01| 3|[s ] || 0.00e+00, 0.00e+00, 0.00e+00| 0.00e+00, 0.00e+00| 0.00e+00| 3.38e-01| 0.00e+00| 0.00e+00, 0.00e+00| (0, 501) | (0,1) - [4,5]
[00000009] 4| -24| 1.52e+00,-2.07e+01,-2.06e+01| 8.59e+01| 3|[s ] || 0.00e+00, 0.00e+00, 0.00e+00| -3.00e-01, 5.00e-02, 4.00e-03| 8.08e+01| 0.00e+00| 0.00e+00, 0.00e+00| (0, 0) | [2,3] - [6,7]
[00000008] 5| 22|-3.81e+00, 1.13e-01,-1.83e+00| 4.23e+00| 1|[s ] || 0.00e+00, 0.00e+00, 0.00e+00| 0.00e+00, 0.00e+00| 0.00e+00| 8.16e-02| 0.00e+00| 6.00e-01, 1.00e+00, 5.00e-01| (0, 0) | [2,3] - []
[00000010] 6| 1|-2.44e+00, 2.88e+01, 6.08e+00| 2.96e+01| 1|[s ] || -3.00e-01, 5.00e-02, 4.00e-03| 0.00e+00, 0.00e+00| -9.95e-02| 0.00e+00| 0.00e+00, 0.00e+00| (0, 0) | [4] - []
[00000011] 7| -2| 3.96e+00,-4.95e+01,-2.67e+01| 5.64e+01| 1|[s ] || -3.00e-01, 5.00e-02, 4.00e-03| 0.00e+00, 0.00e+00| -1.74e-01| 0.00e+00| 0.00e+00, 0.00e+00| (0, 0) | [4] - []

```

LCWS '14, Belgrade, Simulation summary talk:

WHIZARD Task to implement LCIO format

WHIZARD v2.2.4, 02/2015:

```

sample_format = lcio
simulate (<process>)

```





BSM Models in WHIZARD

MODEL TYPE	with CKM matrix	trivial CKM
QED with e, μ, τ, γ	—	QED
QCD with d, u, s, c, b, t, g	—	QCD
Standard Model	SM_CKM	SM
SM with anomalous gauge coupl.	SM_ac_CKM	SM_ac
SM with anomalous top coupl.	SMtop_CKM	SMtop
SM for e^+e^- top threshold	—	SM_tt_threshold
SM with anom. Higgs coupl.	—	SM_rx / NoH
SM ext. for VV scattering	—	SSC / SSC2/ AltH
SM ext. for unitarity limits	—	SM_ul
SM with Z'	—	Zprime
2HDM	2HDM_CKM	2HDM
MSSM	MSSM_CKM	MSSM
MSSM with gravitinos	—	MSSM_Grav
NMSSM	NMSSM_CKM	NMSSM
extended SUSY models	—	PS/E/SSM
Littlest Higgs	—	Littlest
Littlest Higgs with ungauged $U(1)$	—	Littlest_Eta
Littlest Higgs with T parity	—	Littlest_Tpar
Simplest Little Higgs (anomaly-free/univ.)	—	Simplest[_univ]
3-site model	—	Threeshl
UED	—	UED
SM with gravitino and photino	—	GravTest
Augmentable SM template	—	Template

2.2.8: **SM_dim6**





BSM Models in WHIZARD

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QED with e, μ, τ, γ	—	QED
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2HDM	2HDM_CKM	2HDM
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2.2.8: **SM_dim6**

- Automated models: interface to SARAH/BSM Toolbox [Staub, 0909.2863](#); [Ohl/Porod/Staub/Speckner, 1109.5147](#)
- Automated models: interface to FeynRules [Christensen/Duhr; Christensen/Duhr/Fuks/JRR/Speckner, 1010.3251](#)





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- Automated models: interface to FeynRules [Christensen/Duhr; Christensen/Duhr/Fuks/JRR/Speckner, 1010.3251](#)
- Automated models: UFO interface [in connection with new WHIZARD/0' Mega model format]





Decay processes / auto_decays

WHIZARD cannot only do scattering processes, but also decays

Example Energy distribution electron in muon decay:

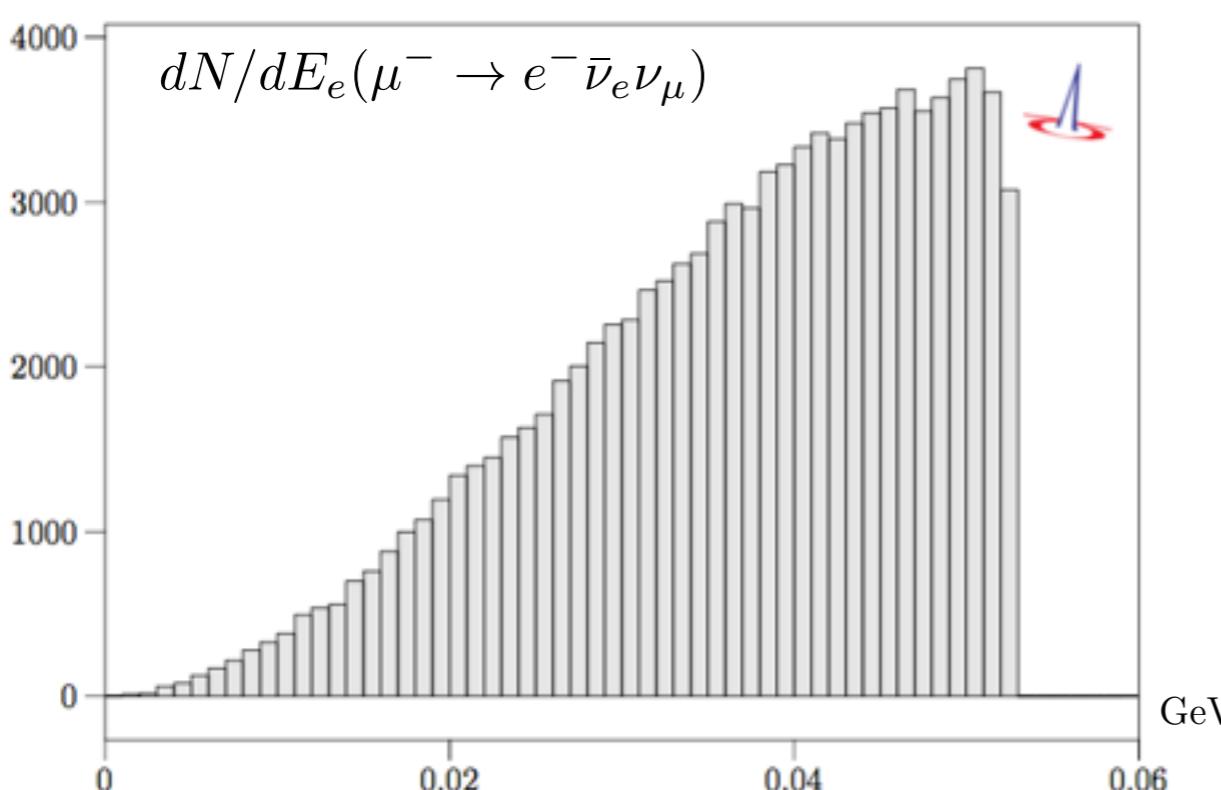
```
model = SM
process mudec = e2 => e1, N1, n2
integrate (mudec)

histogram e_e1 (0, 60 MeV, 1 MeV)
analysis = record e_e1 (eval E [e1])

n_events = 100000

simulate (mudec)

compile_analysis { $out_file = "test.dat" }
```





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Example Energy distribution electron in muon decay:

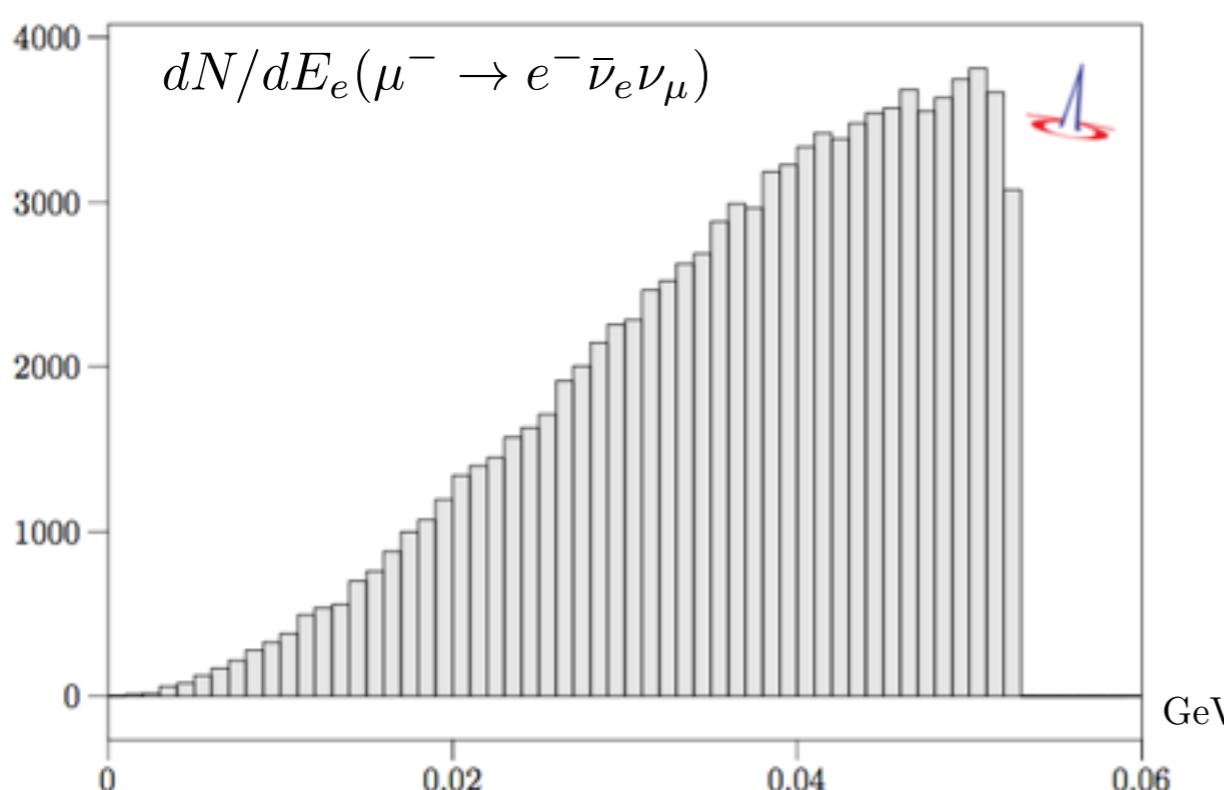
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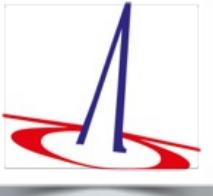
Automatic integration of particle decays

```
auto_decays_multiplicity = 2
?auto_decays_radiative = false

unstable Wp () { ?auto_decays = true }
```

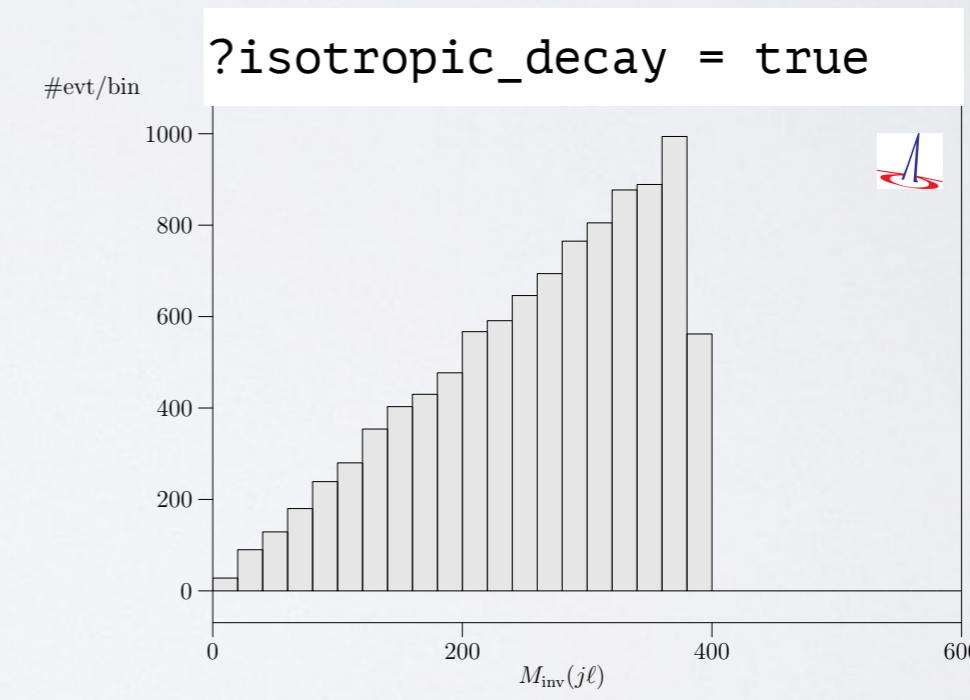
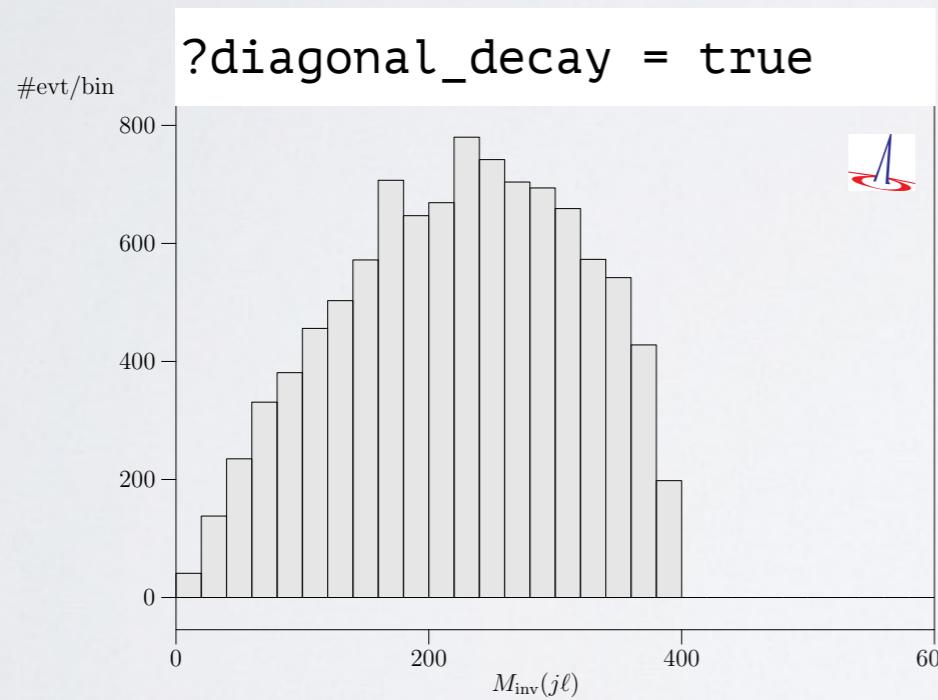
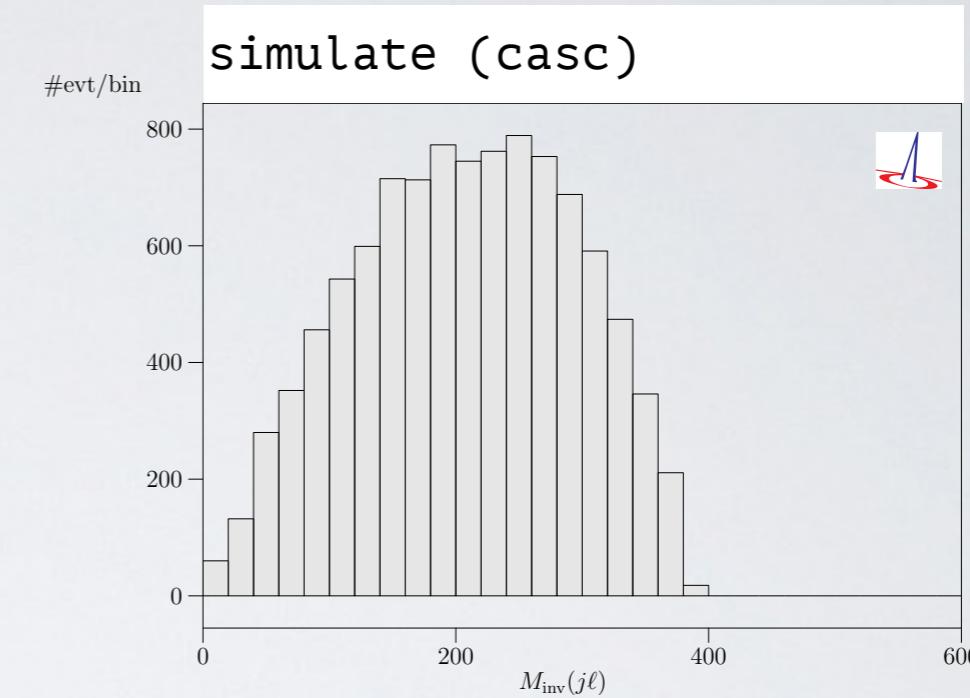
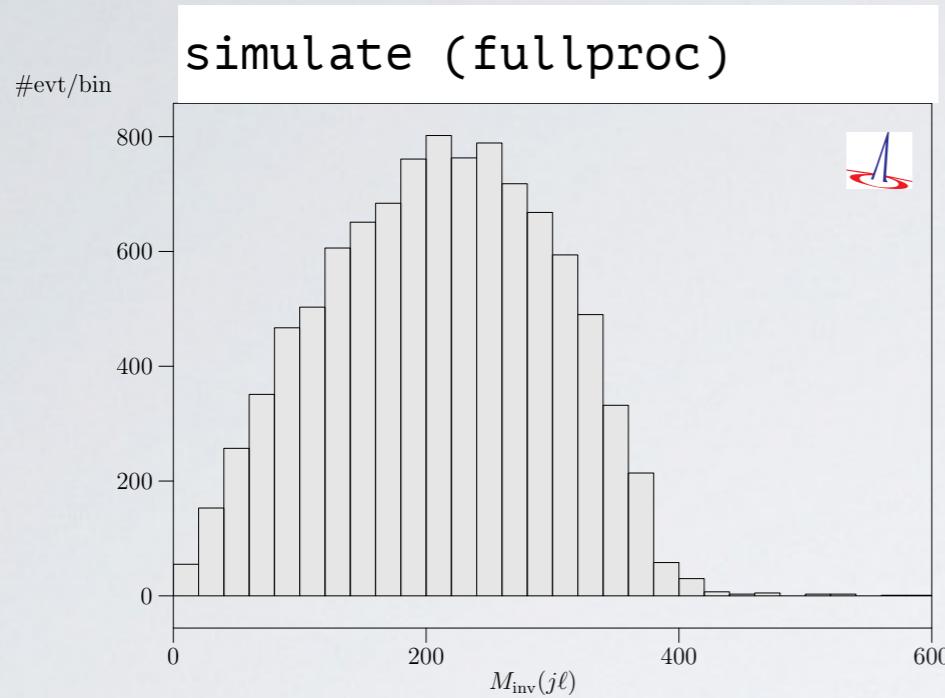
```
=====
| It      Calls  Integral[GeV] Error[GeV] Err[%]   Acc
| -----
|   1      100   2.2756406E-01  0.00E+00  0.00   0.00*
| -----
|   1      100   2.2756406E-01  0.00E+00  0.00   0.00
| -----
| Unstable particle W+: computed branching ratios:
|   decay_p24_1: 3.3337068E-01  dbar, u
|   decay_p24_2: 3.3325864E-01  sbar, c
|   decay_p24_3: 1.1112356E-01  e+, nue
|   decay_p24_4: 1.1112356E-01  mu+, numu
|   decay_p24_5: 1.1112356E-01  tau+, nutau
|   Total width = 2.0478471E+00 GeV (computed)
|                           = 2.0490000E+00 GeV (preset)
| Decay options: helicity treated exactly
```





Spin Correlation and Polarization in Cascades

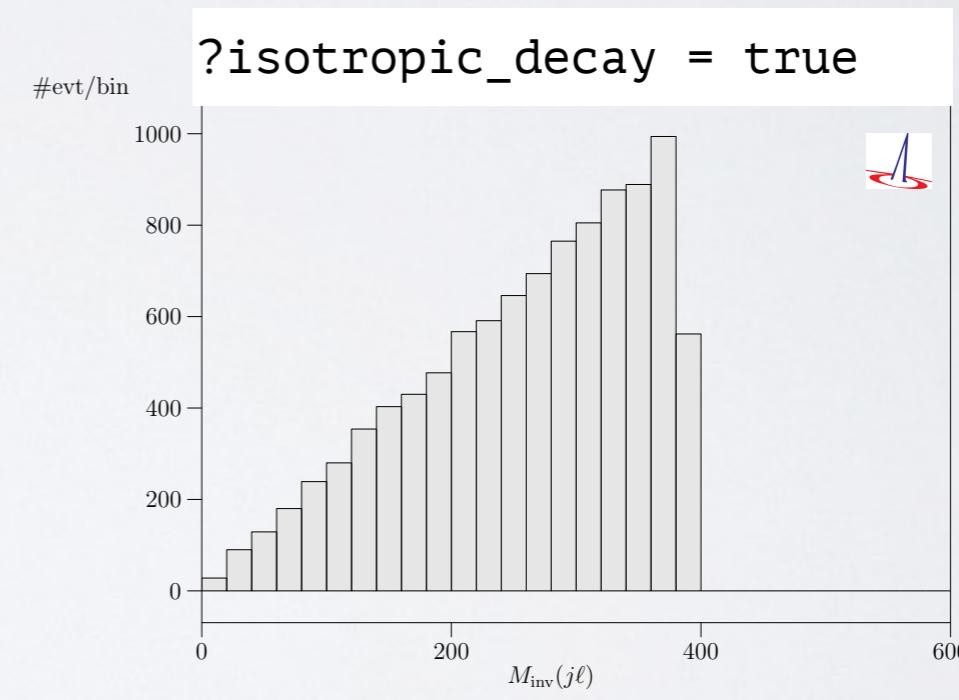
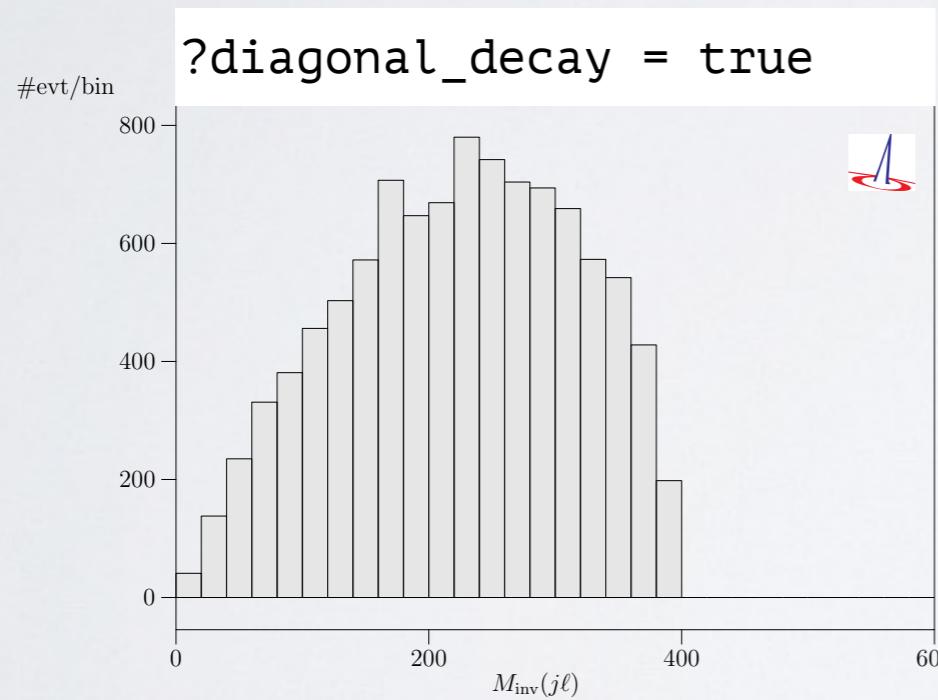
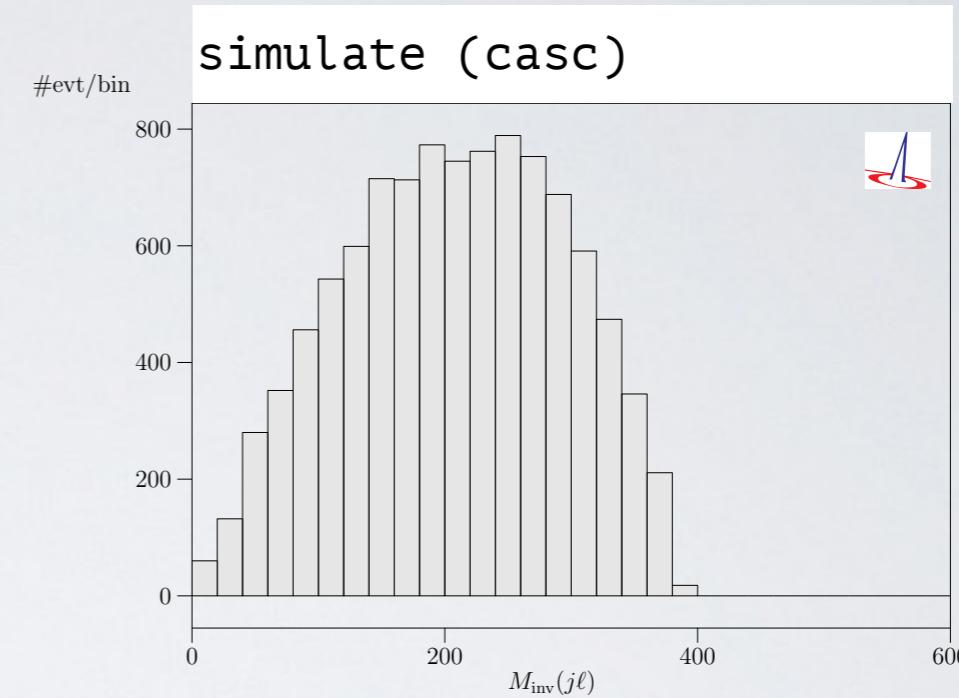
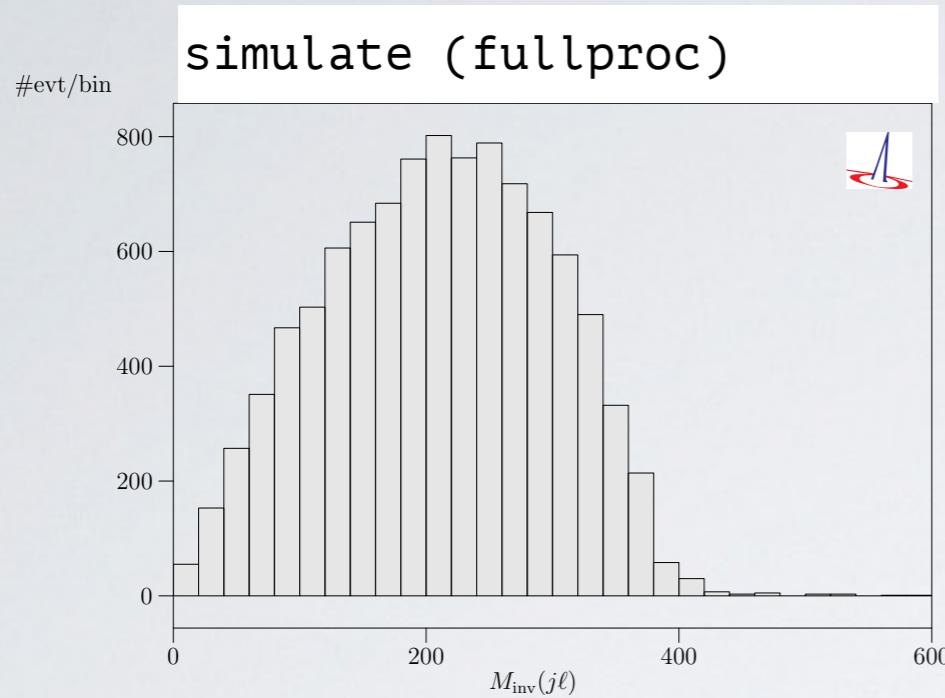
Cascade decay, factorize production and decay





Spin Correlation and Polarization in Cascades

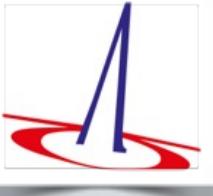
Cascade decay, factorize production and decay



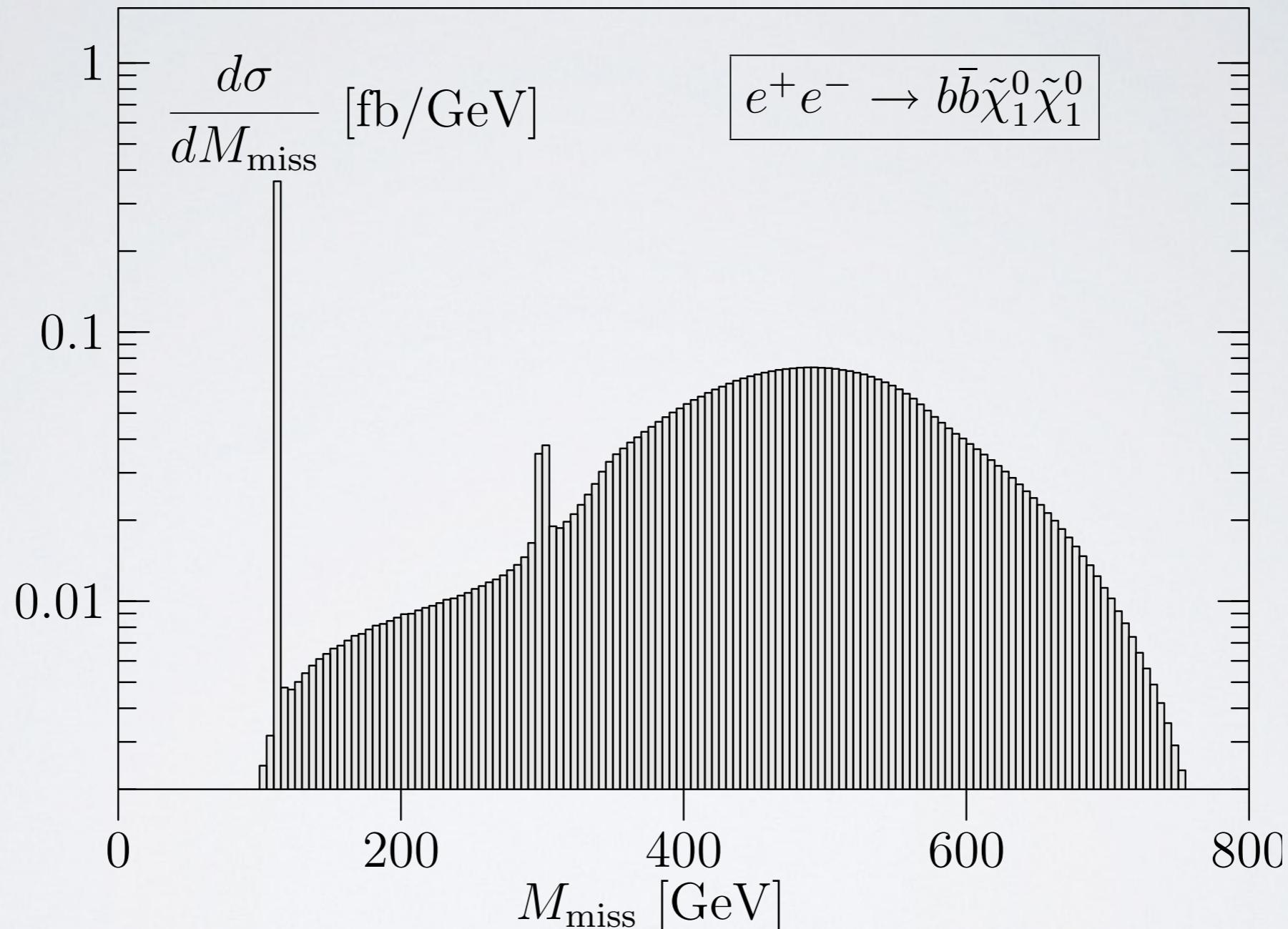
NEW: possibility to select specific helicity in decays!

unstable "W+" { decay_helicity = 0 }



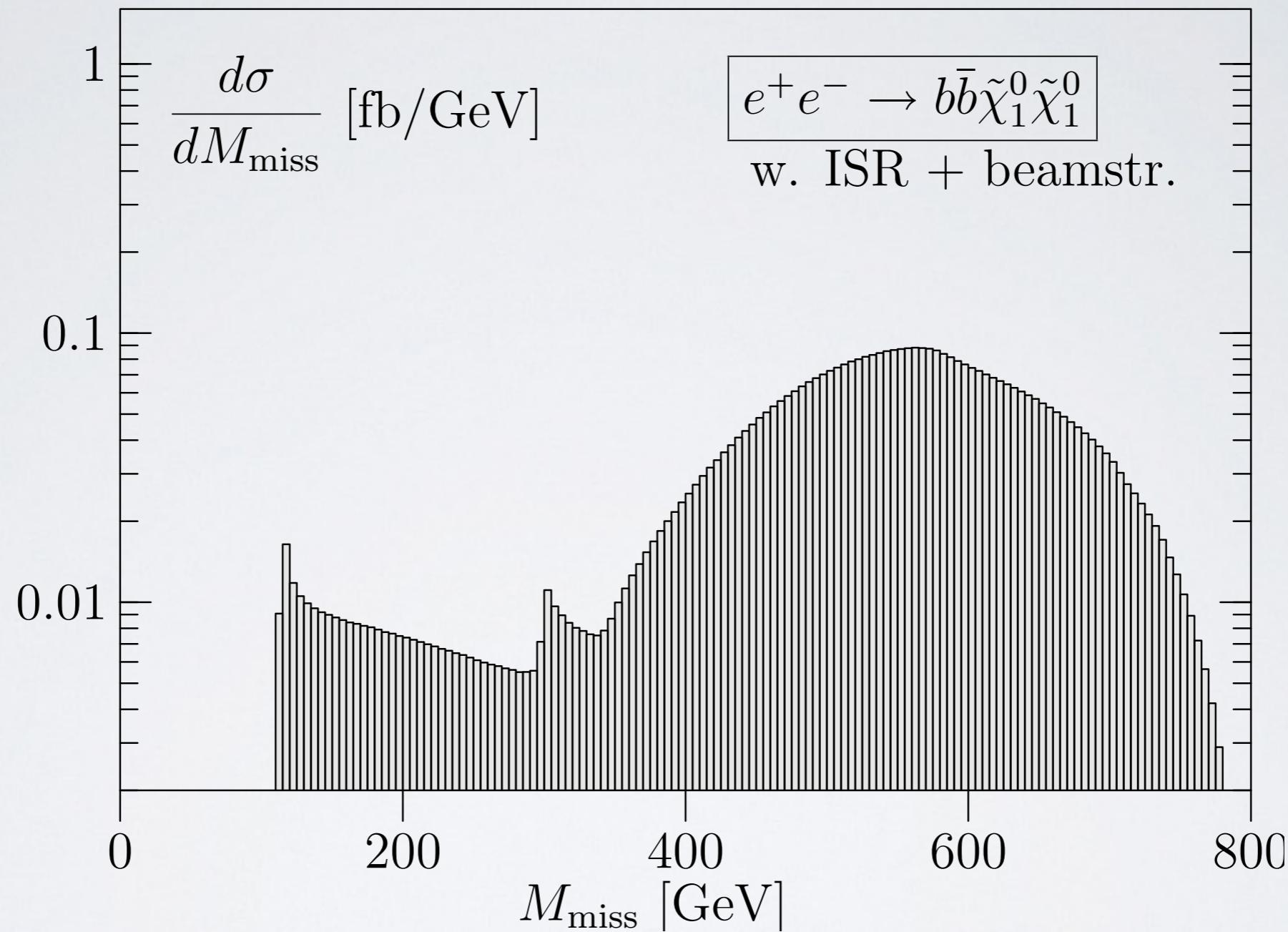


Lepton Collider Beamspectra





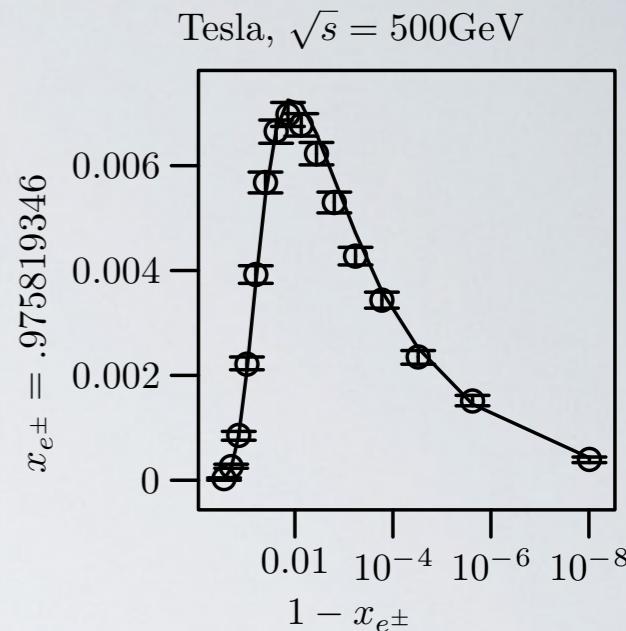
Lepton Collider Beamspectra





Lepton Collider Beam Simulation

- Another demand: adapt GuineaPig beam spectra for WHIZARD v2
- For WHIZARD v1.95 simulations done by Lumilinker [\[T. Barklow\]](#)
- TESLA/SLC spectra were rather simple
- Fits with 6 or 7 parameters possible [CIRCE1]
- **Beams not factorizable:** $D_{B_1 B_2}(x_1, x_2) \neq D_{B_1}(x_1) \cdot D_{B_2}(x_2)$
- **No simple power law:** $D_{B_1 B_2}(x_1, x_2) \neq x_1^{\alpha_1} (1 - x_1)^{\beta_1} x_2^{\alpha_2} (1 - x_2)^{\beta_2}$

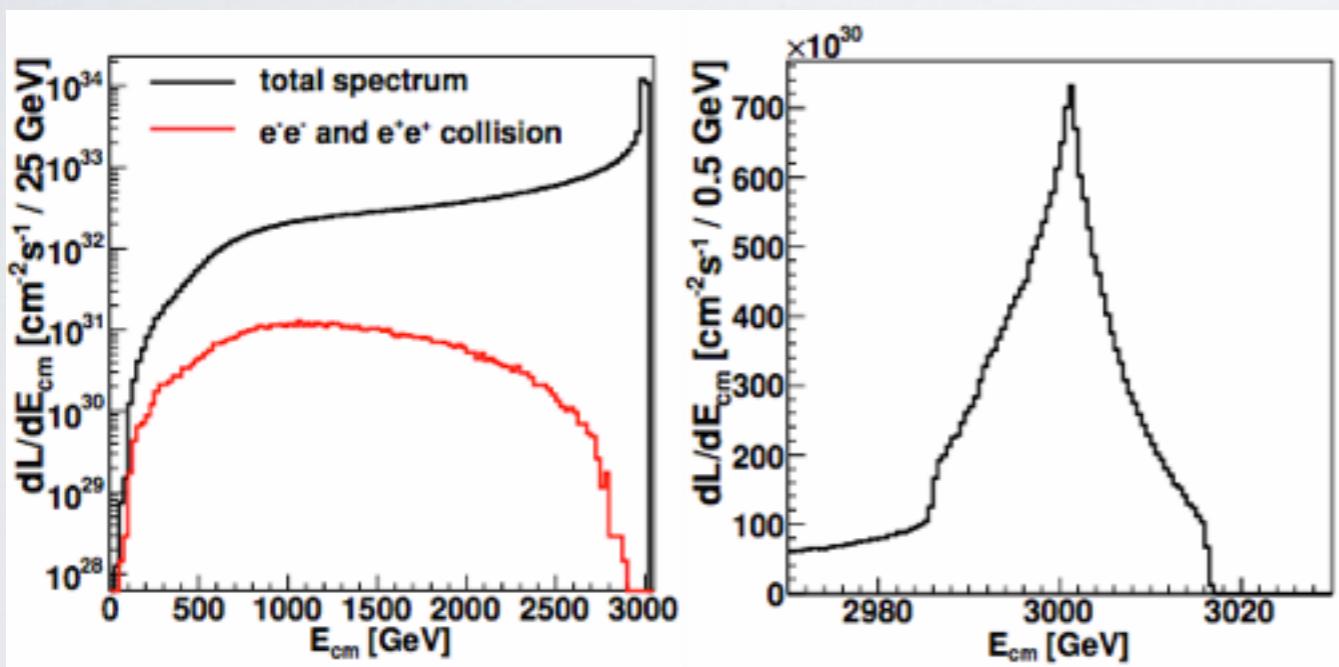
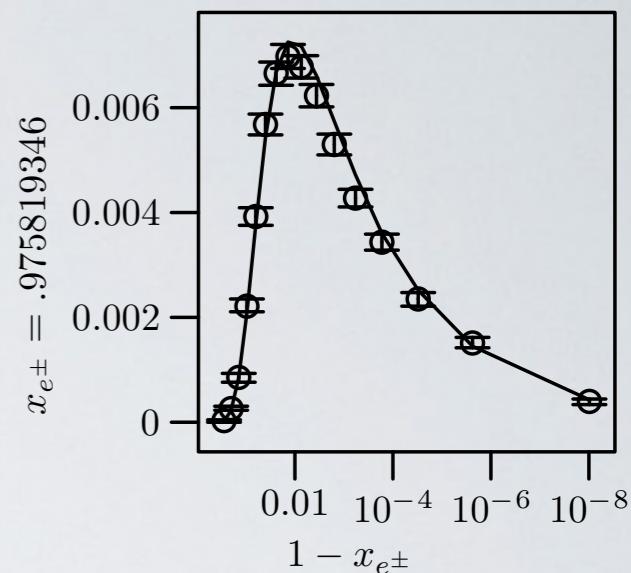




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- Beams not factorizable: $D_{B_1 B_2}(x_1, x_2) \neq D_{B_1}(x_1) \cdot D_{B_2}(x_2)$
- No simple power law: $D_{B_1 B_2}(x_1, x_2) \neq x_1^{\alpha_1} (1 - x_1)^{\beta_1} x_2^{\alpha_2} (1 - x_2)^{\beta_2}$

Tesla, $\sqrt{s} = 500\text{GeV}$



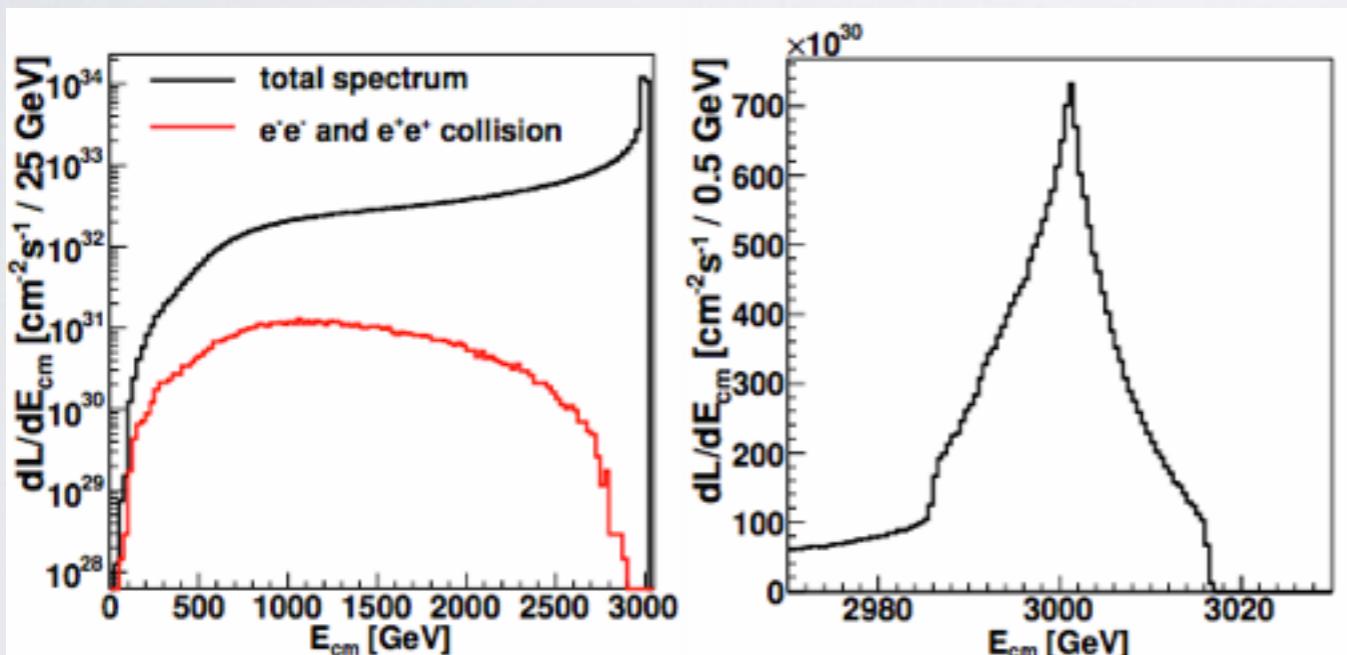
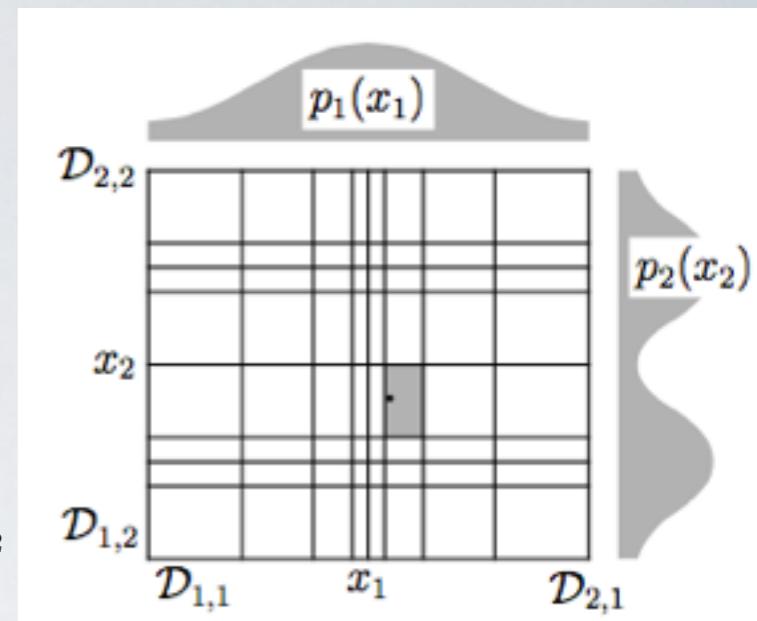
Tails @ CLIC much more complicated (wakefields)





Lepton Collider Beam Simulation

- Another demand: adapt GuineaPig beam spectra for WHIZARD v2
- For WHIZARD v1.95 simulations done by Lumilinker [T. Barklow]
- TESLA/SLC spectra were rather simple
- Fits with 6 or 7 parameters possible [CIRCE1]
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Dalena/Esbjerg/Schulte [LCWS 2011]

Tails @ CLIC much more complicated (wakefields)

CIRCE2 algorithm (WHIZARD 2.2.5, 02/15)

- Adapt 2D factorized variable width histogram to steep part of distribution
- Smooth correlated fluctuations with moderate Gaussian filter [suppresses artifacts from limited GuineaPig statistics]
- Smooth continuum/boundary bins separately [avoid artificial beam energy spread]





Workflow GuineaPig/CIRCE2/WHIZARD

1. Run Guinea-Pig++ with

```
do_lumi=7;num_lumi=100000000;num_lumi_eg=100000000;num_lumi_gg=100000000;
```

to produce lumi.[eg][eg].out with (E_1, E_2) pairs.

[Large event numbers, as Guinea-Pig++ will produce only a small fraction!]

2. Run circe2_tool.opt with steering file

```
{ file="ilc500/beams.circe"                                # to be loaded by WHIZARD
  { design="ILC" roots=500 bins=100 scale=250 # E in [0,1]
    { pid/1=electron pid/2=positron pol=0      # unpolarized e-/e+
      events="ilc500/lumi.ee.out" columns=2    # <= Guinea-Pig
      lumi = 1564.763360                      # <= Guinea-Pig
      iterations = 10                          # adapting bins
      smooth = 5 [0,1) [0,1)                   # Gaussian filter 5 bins
      smooth = 5 [1] [0,1) smooth = 5 [0,1) [1] } }
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to produce correlated beam description

3. Run WHIZARD with SINDARIN input:

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$circe2_design = "ILC"
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3 simulation options

1. Unpolarized simulation with unpol. spectra
2. Pol. simulation: unpol. spectra + pol. beams
3. Polarized spectrum with helicity luminosities





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ILC 200/230/250/350/500 GeV

CEPC 240/250 GeV

spectra **directly included** in CIRCE2





NLO Development in WHIZARD

- Need for precision predictions that match (sub-) percent experimental accuracy
- mainly NLO corrections, but also QED and electroweak (ee)

[Binoth Les Houches Interface \(BLHA\): Workflow](#)

1. Process definition in SINDARIN (contract to One-Loop Program [OLP])
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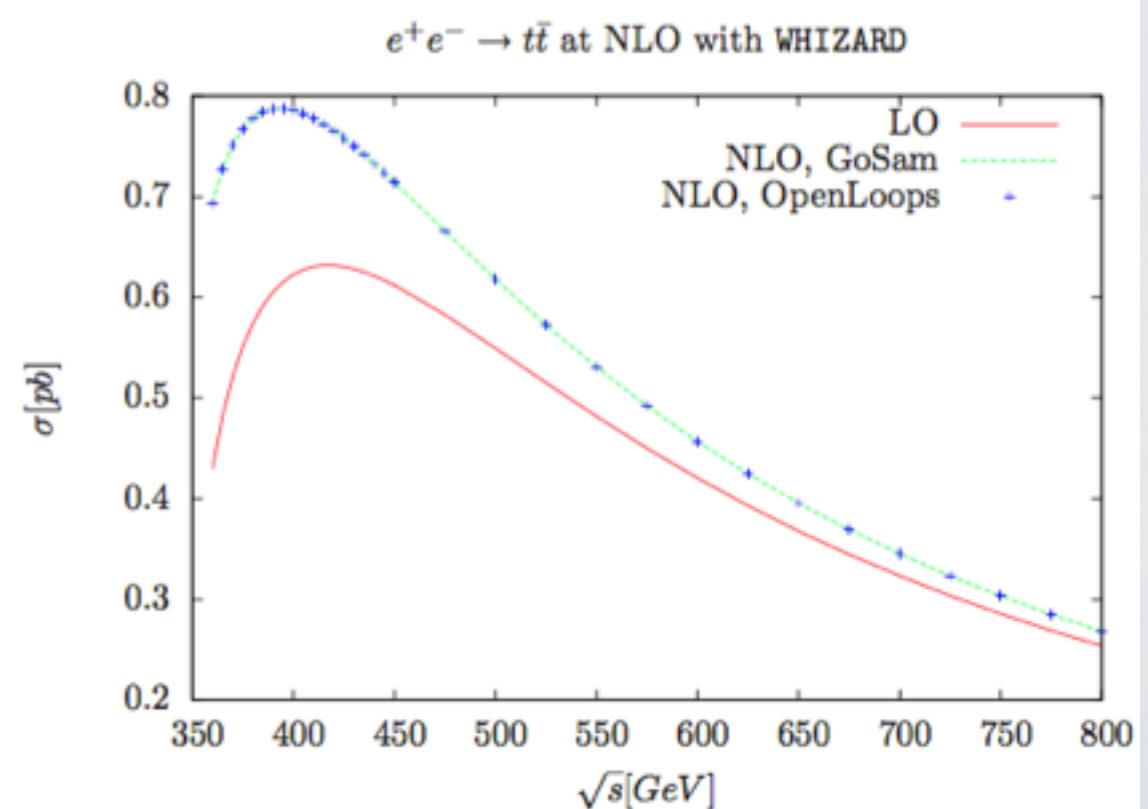
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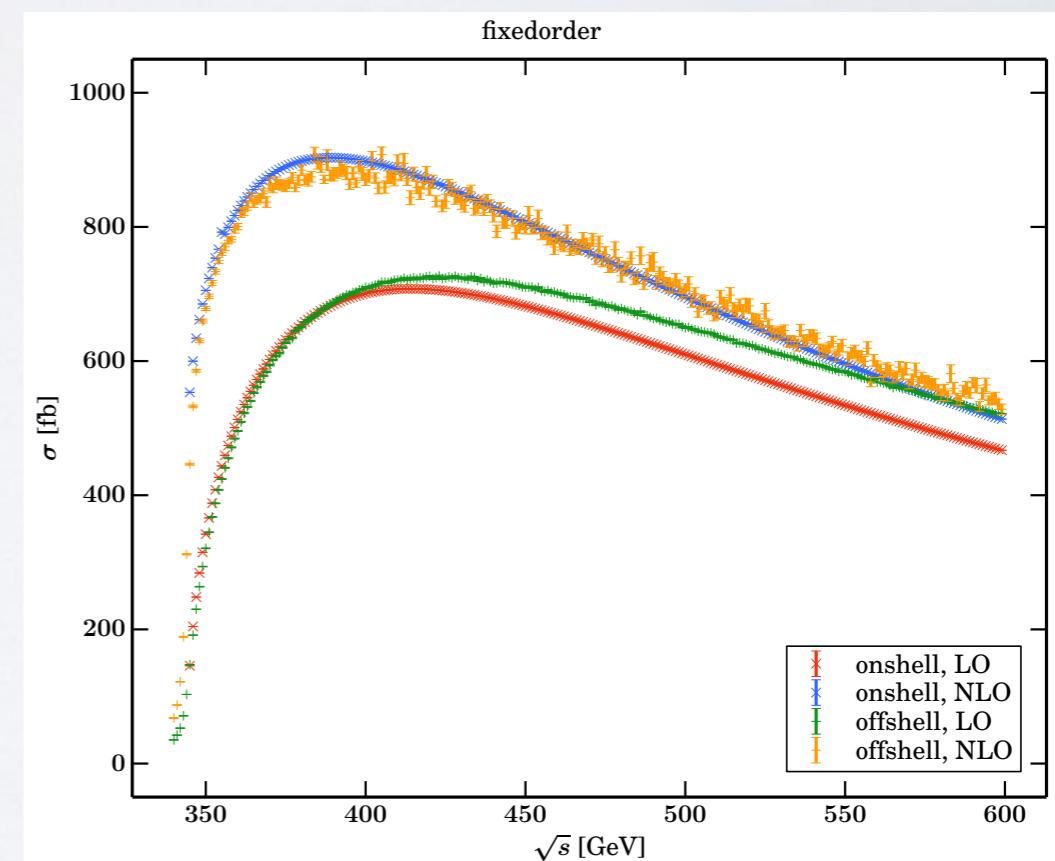
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FKS Subtraction (Frixione/Kunszt/Signer)

Subtraction formalism to make real and virtual contributions separately finite

$$d\sigma^{\text{NLO}} = \underbrace{\int_{n+1} (d\sigma^R - d\sigma^S)}_{\text{finite}} + \underbrace{\int_{n+1} d\sigma^S + \int_n d\sigma^V}_{\text{finite}}$$

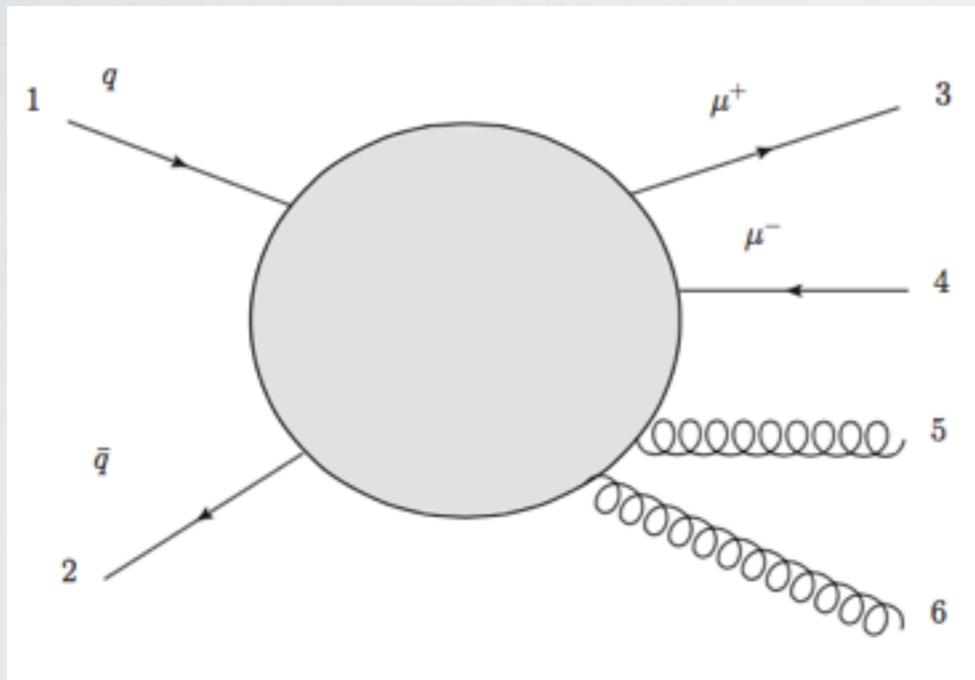




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Automated subtraction terms in WHIZARD,
algorithm:

- * Find all singular pairs
 $\mathcal{I} = \{(1, 5), (1, 6), (2, 5), (2, 6), (5, 6)\}$
- * Partition phase space according to singular regions
 $\mathbb{1} = \sum_{\alpha \in \mathcal{I}} S_\alpha(\Phi)$
- * Generate subtraction terms for singular regions

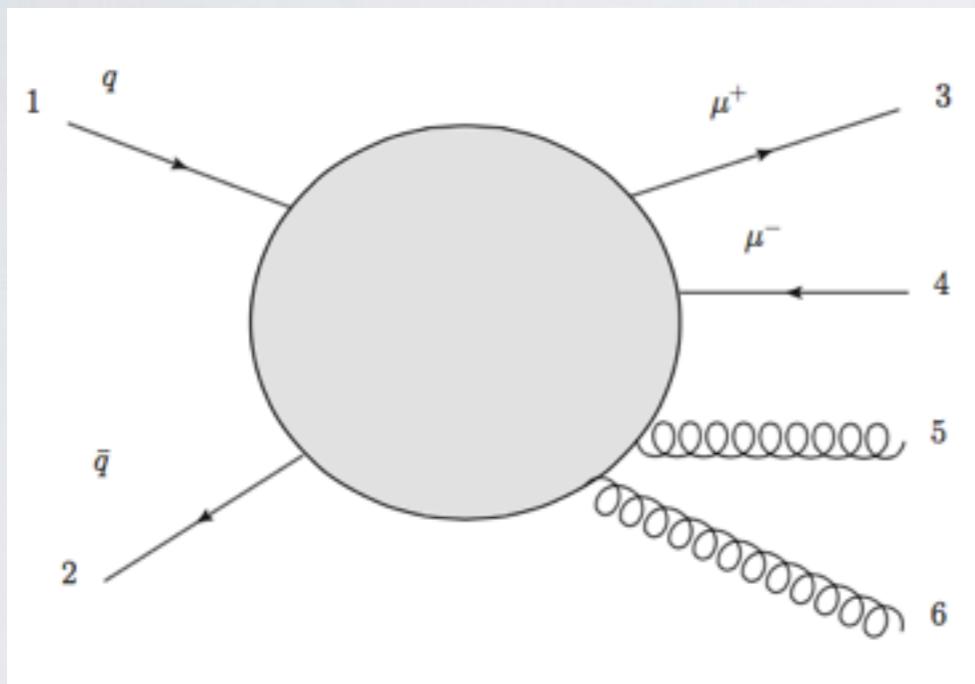




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Soft subtraction involves color-correlated matrix elements:

$$\mathcal{B}_{kl} \sim - \sum_{\substack{\text{color} \\ \text{spin}}} \mathcal{A}^{(n)} \vec{\mathcal{Q}}(\mathcal{I}_k) \cdot \vec{\mathcal{Q}}(\mathcal{I}_l) \mathcal{A}^{(n)*},$$

Collinear subtraction involves spin-correlated matrix elements:

$$\mathcal{B}_{+-} \sim \text{Re} \left\{ \frac{\langle k_{\text{em}} k_{\text{rad}} \rangle}{[k_{\text{em}} k_{\text{rad}}]} \sum_{\substack{\text{color} \\ \text{spin}}} \mathcal{A}_+^{(n)} \mathcal{A}_-^{(n)*} \right\}$$





Examples and Validation

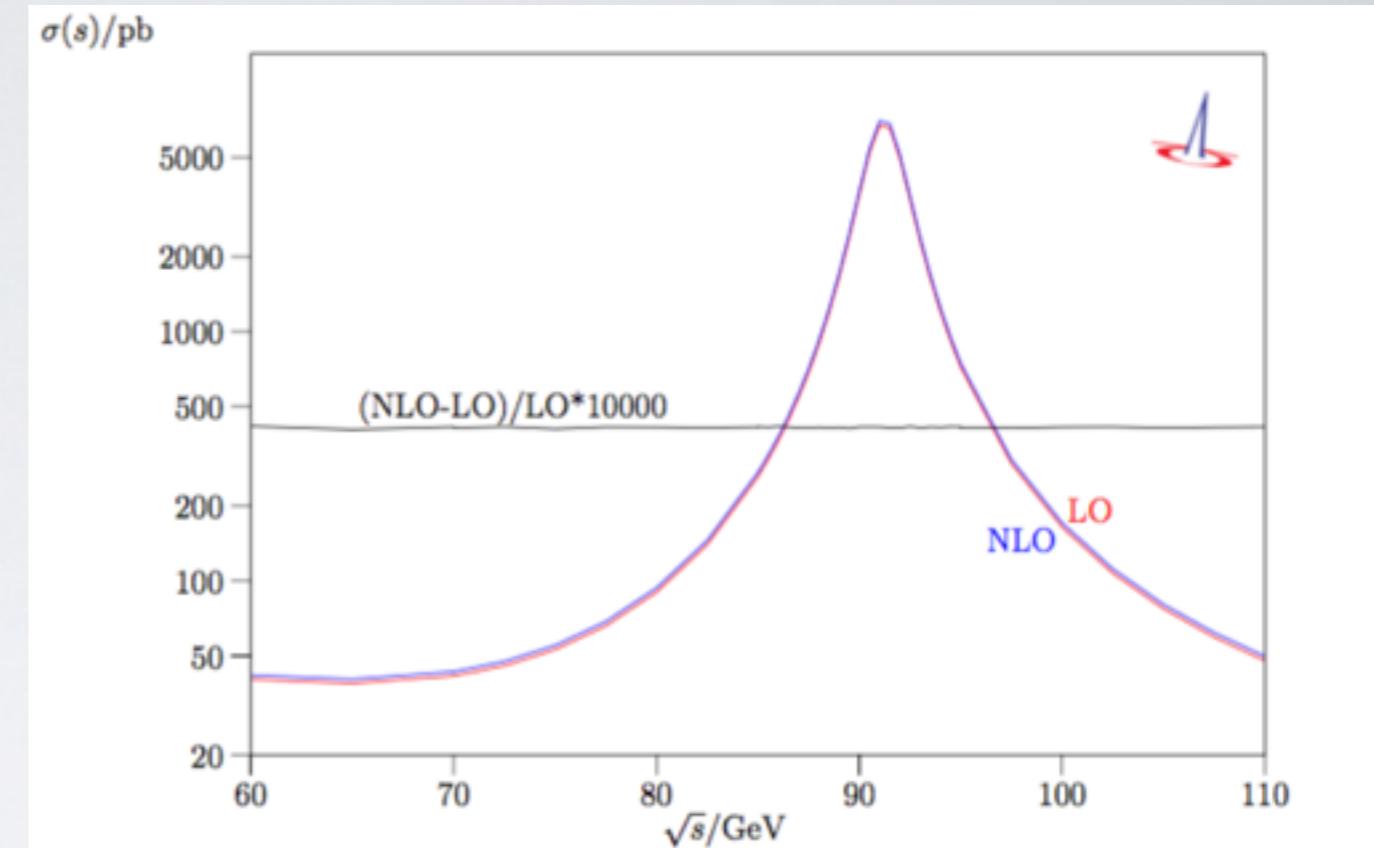
Simplest benchmark process:

$$e^+ e^- \rightarrow q\bar{q} \quad \text{with} \quad (\sigma^{\text{NLO}} - \sigma^{\text{LO}}) / \sigma^{\text{LO}} = \alpha_s / \pi$$

Plot for total cross section for fixed strong coupling constant

List of validated QCD NLO processes

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- $e^+ e^- \rightarrow q\bar{q}g$
- $e^+ e^- \rightarrow \ell^+ \ell^- q\bar{q}$
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- $e^+ e^- \rightarrow tW^- b$
- $e^+ e^- \rightarrow W^+ W^- b\bar{b}$
- $e^+ e^- \rightarrow t\bar{t}H$



- Cross-checks with MG5_aMC@NLO
- Phase space integration for virtuals performs great





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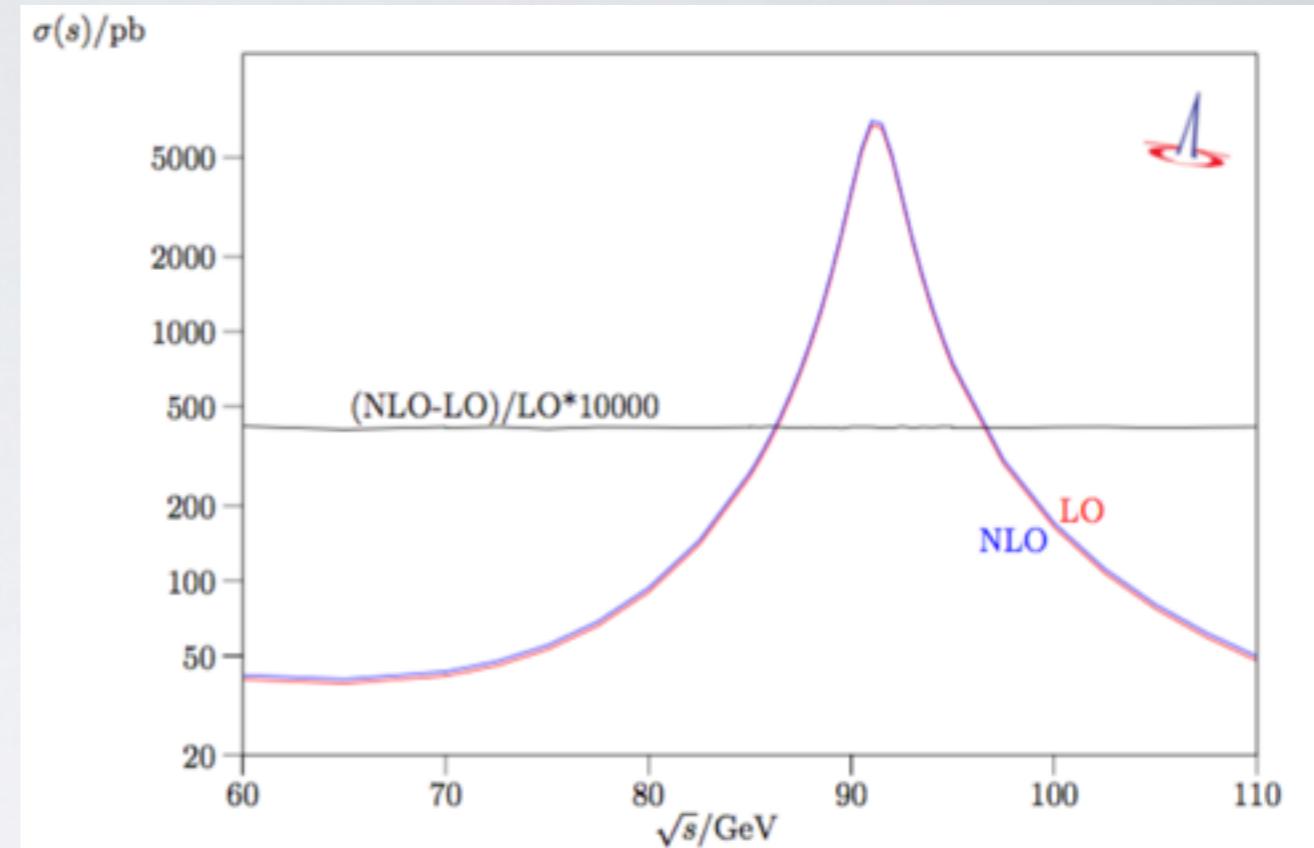
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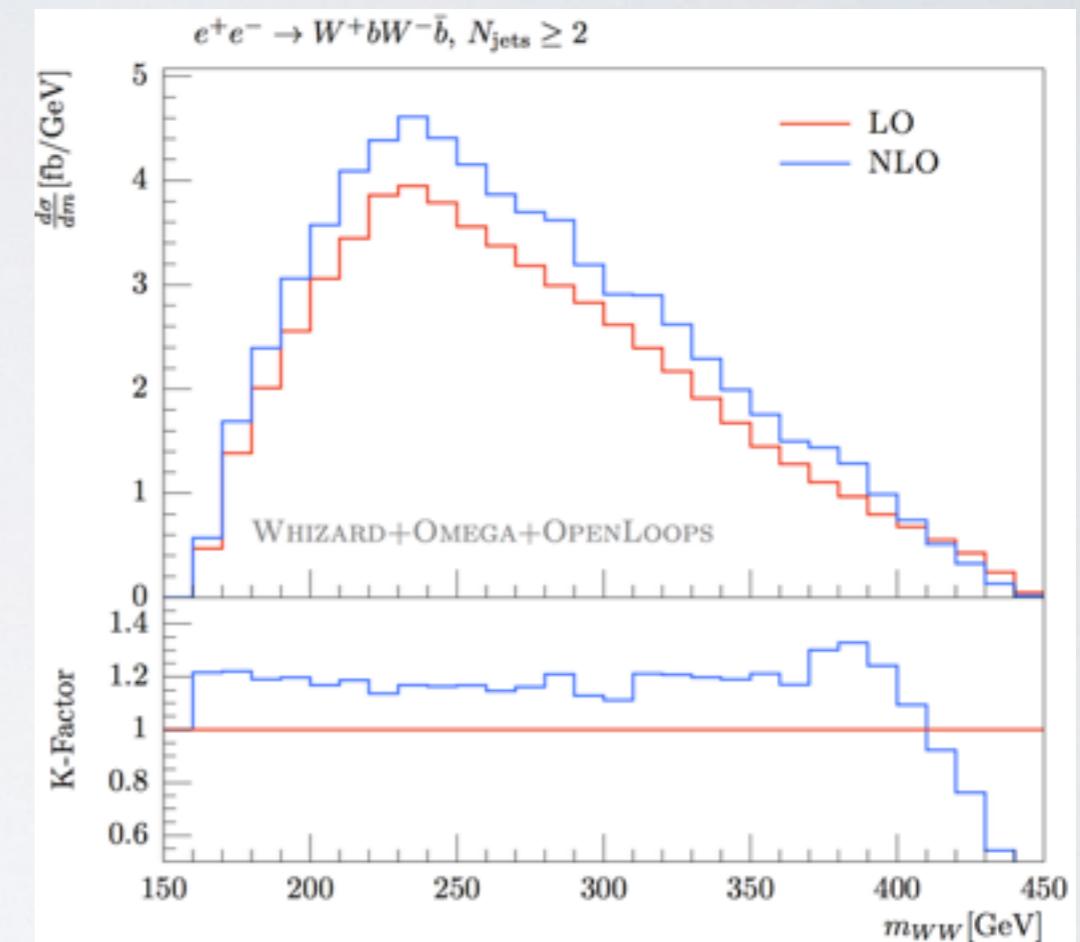
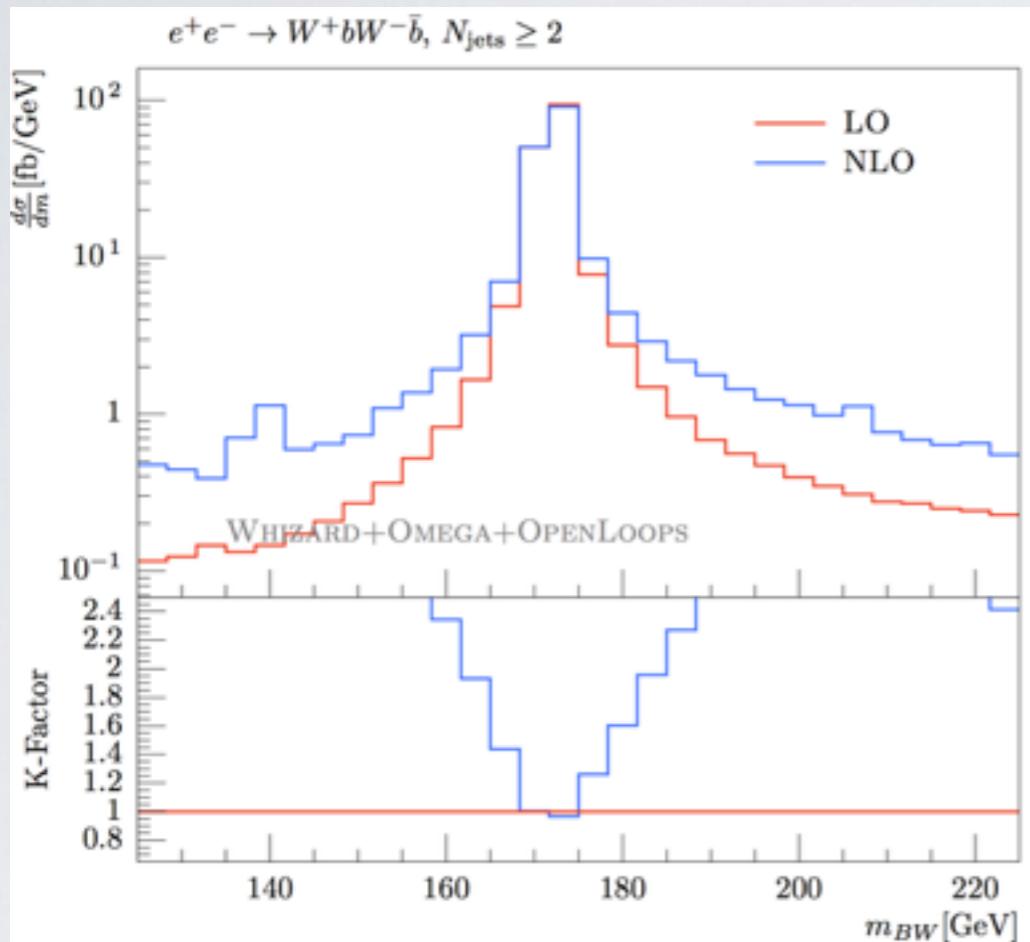
- ◆ QCD NLO infrastructure in pp almost complete
- ◆ First attempts on electroweak corrections, interfacing the RECOLA code [Denner et al.]





NLO Fixed-Order Events

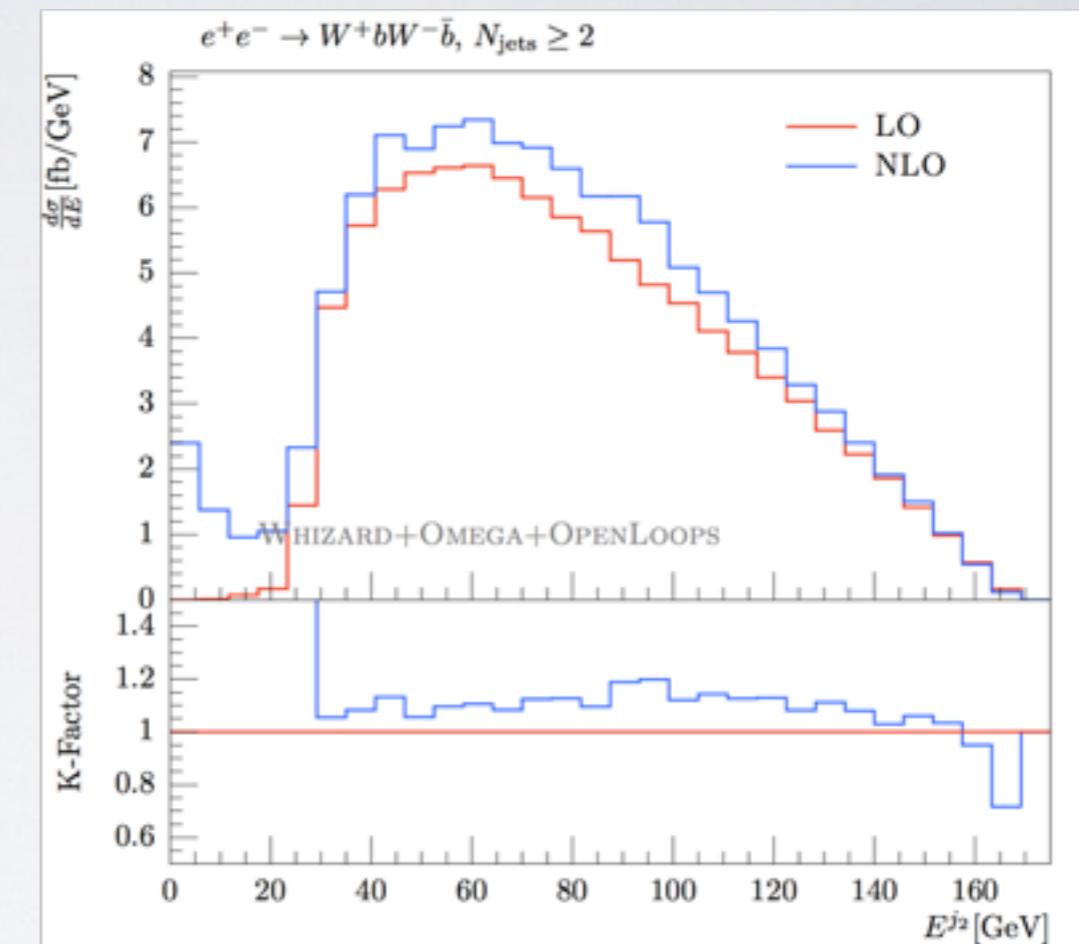
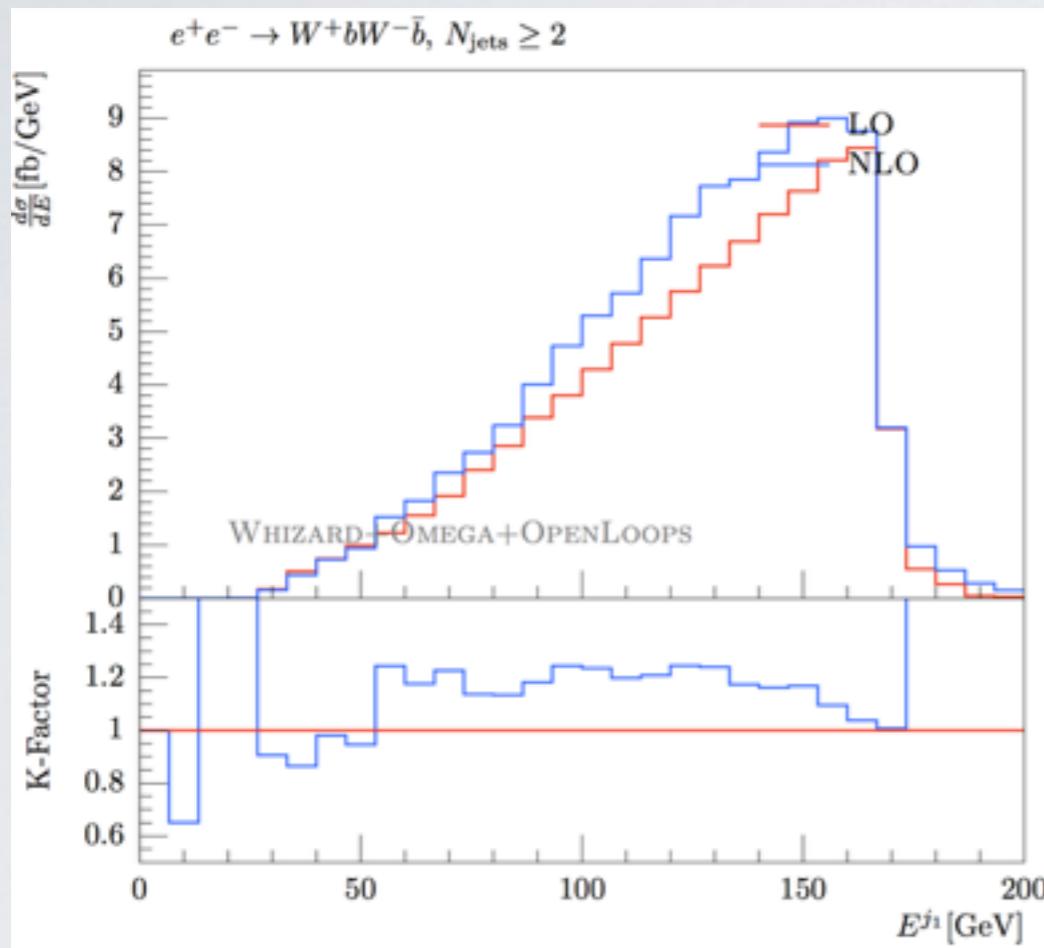
- Add weights of real emission events to weight of Born kinematics using the FKS mapping
- Output weighted events in WHIZARD (e.g. using HepMC), then analysis with Rivet
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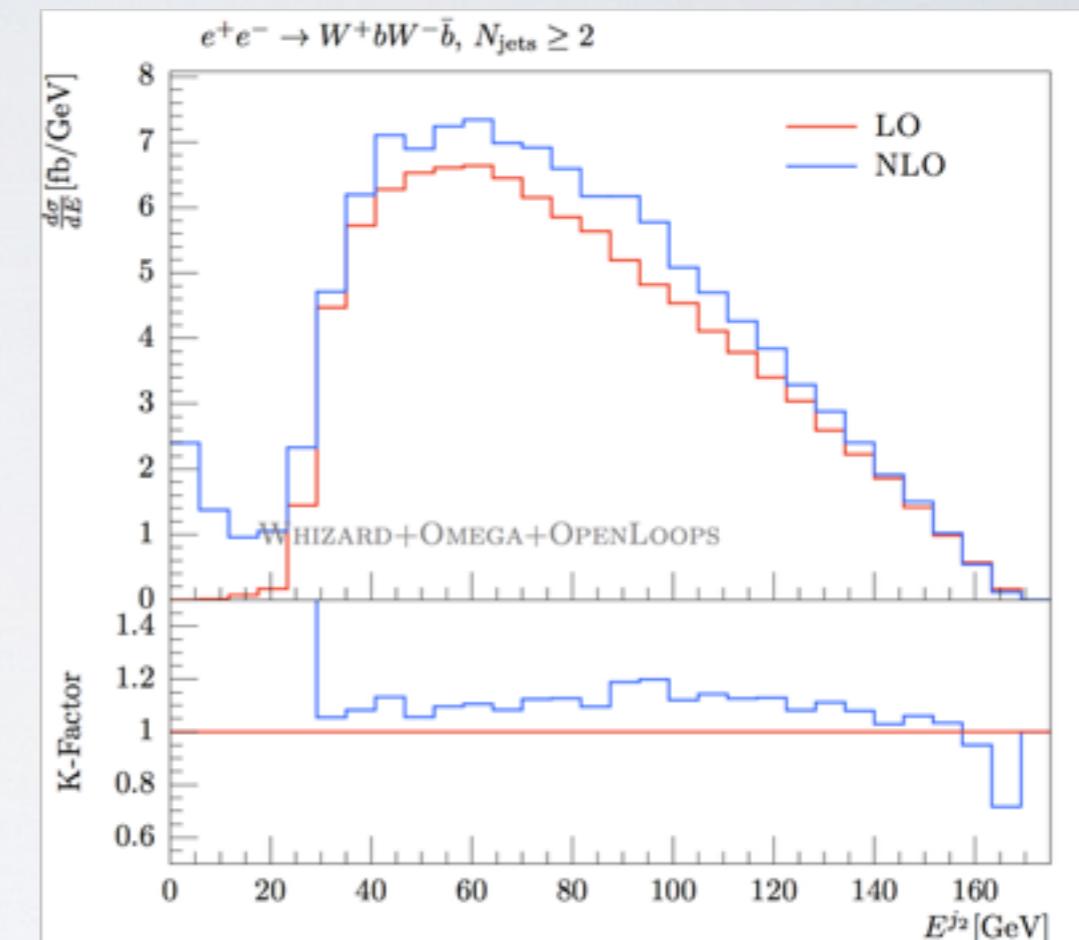
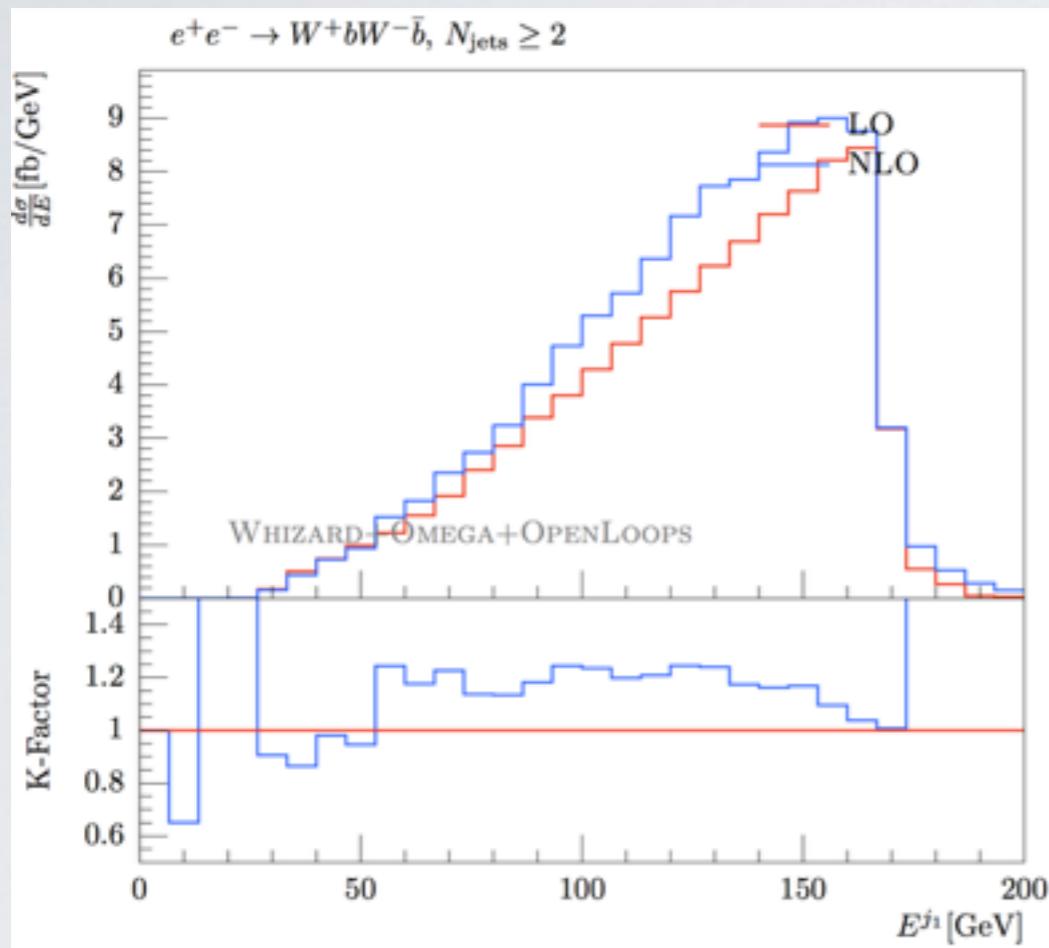
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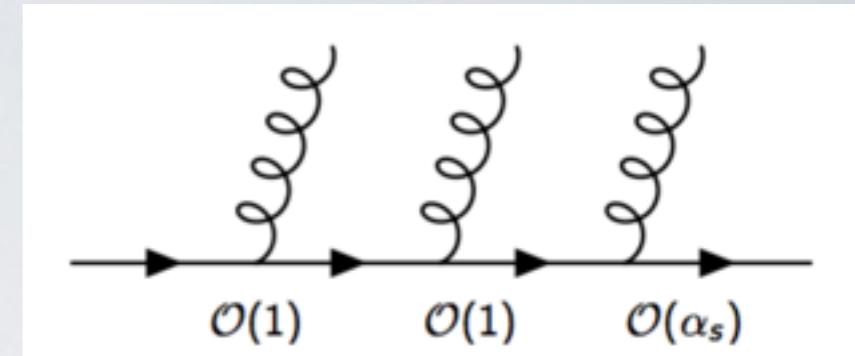
- Completed: **polarized NLO results** (remember: ILC/CLIC will always run with polarization)
- Combine NLO with complete ISR photon radiation and beamstrahlung
- **NLO decays also available** (Initial state Jacobian, important for consistent widths)
- **Investigate the full $2 \rightarrow 7$ process: $e^+e^- \rightarrow b\bar{b}\mu\nu\gamma\gamma\gamma$** [Chokouf  /Kilian/Lindert/JRR/Pozzorini/Weiss]





Automated POWHEG Matching in WHIZARD

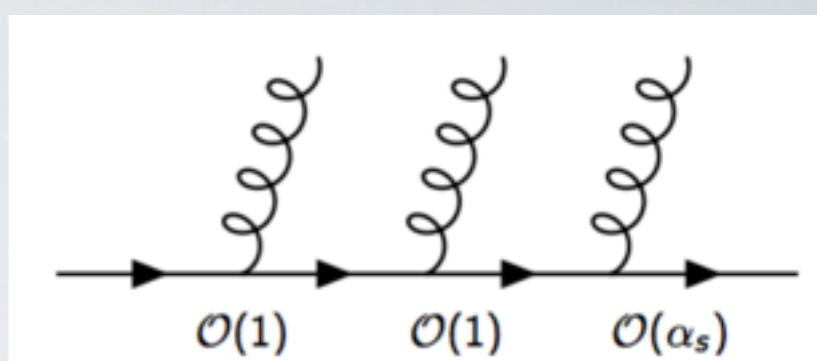
- Soft gluon emissions before hard emission generate large logs
- Perturbative α_s : $|\mathcal{M}_{\text{soft}}|^2 \sim \frac{1}{k_T^2} \rightarrow \log \frac{k_T^{\max}}{k_T^{\min}}$
- Consistent matching of NLO matrix element with shower
- **POWHEG method:** hardest emission first [Nason et al.]





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- Complete NLO events

$$\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_{\text{rad}} R(\Phi_{n+1})$$

- POWHEG generate events according to the formula:

$$d\sigma = \bar{B}(\Phi_n) \left[\Delta_R^{\text{NLO}}(k_T^{\min}) + \Delta_R^{\text{NLO}}(k_T) \frac{R(\Phi_{n+1})}{B(\Phi_n)} d\Phi_{\text{rad}} \right]$$

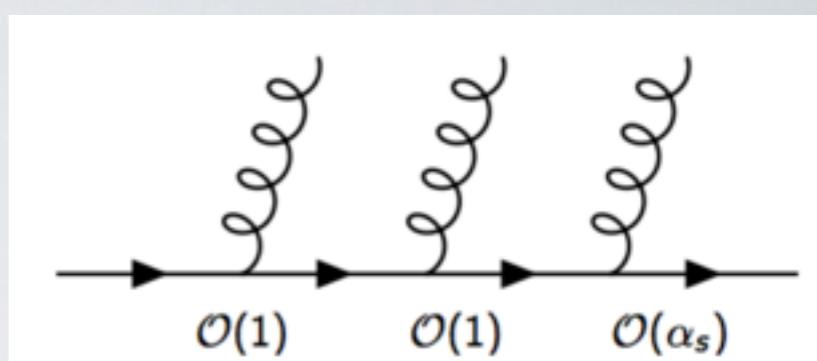
- Uses the modified Sudakov form factor:

$$\Delta_R^{\text{NLO}}(k_T) = \exp \left[- \int d\Phi_{\text{rad}} \frac{R(\Phi_{n+1})}{B(\Phi_n)} \theta(k_T(\Phi_{n+1}) - k_T) \right]$$



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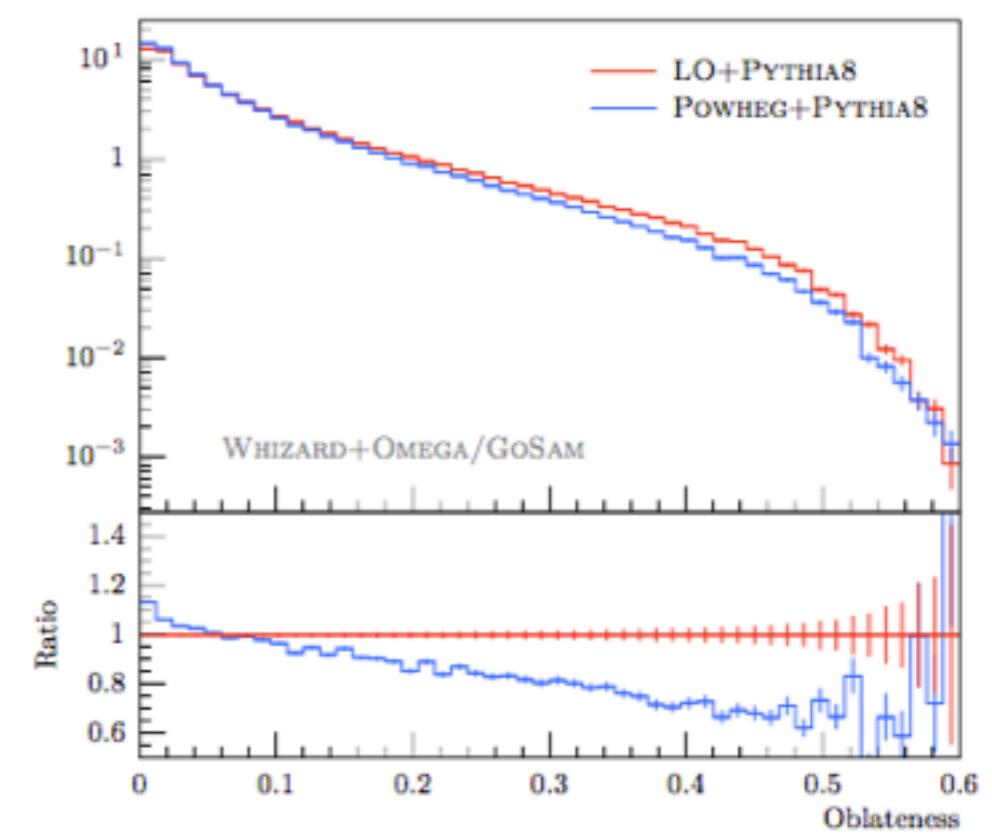
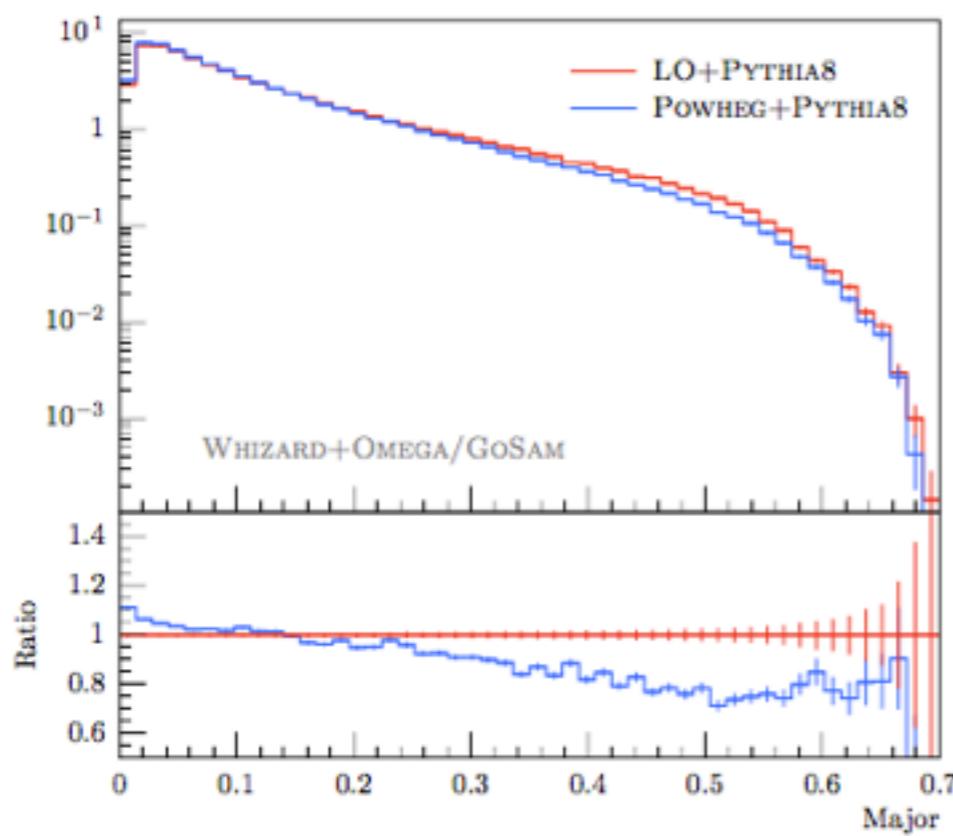
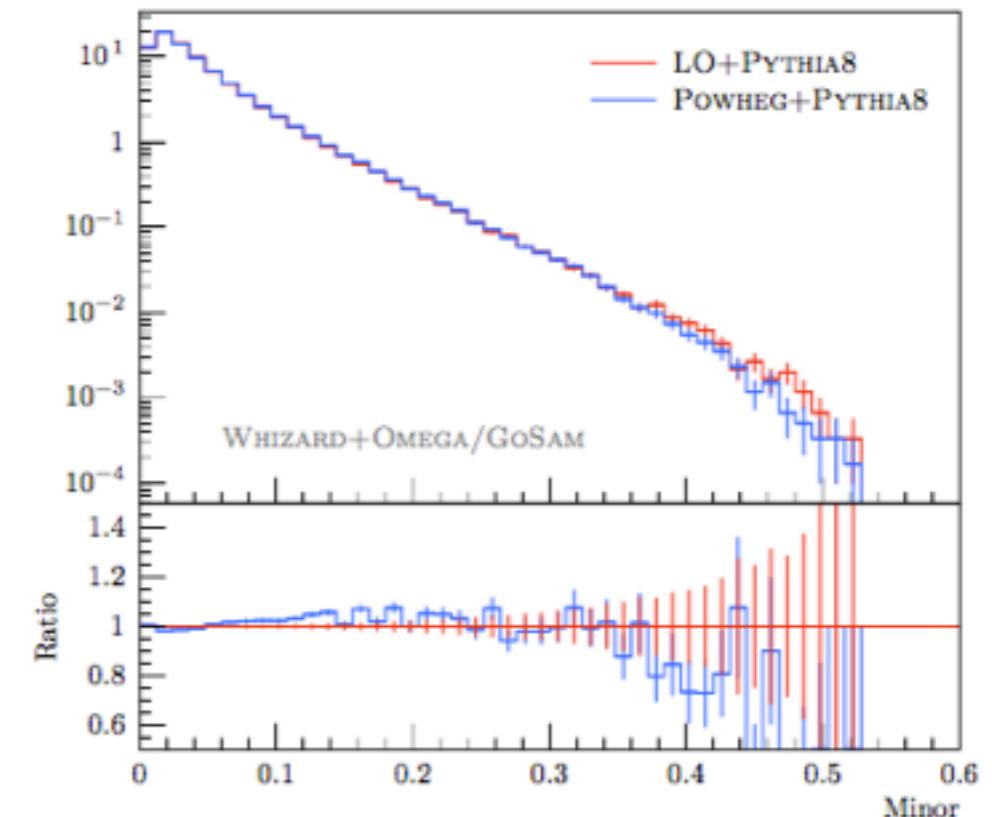
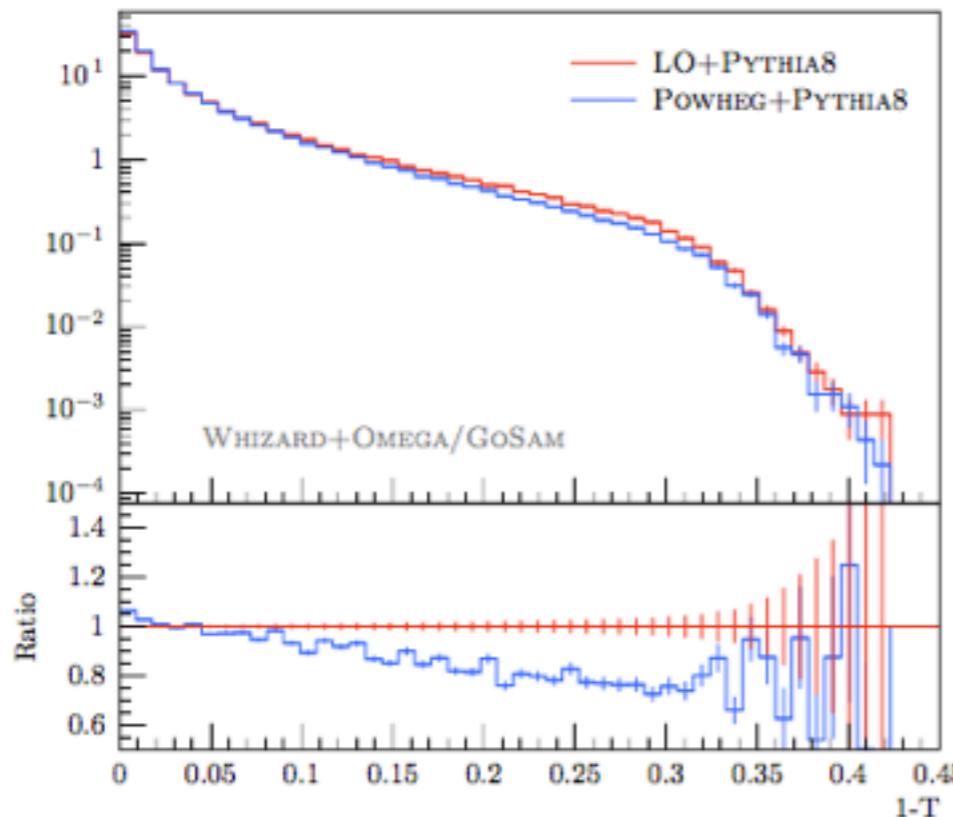
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- Hardest emission: k_T^{\max} ; shower with **imposing a veto**
- $\bar{B} < 0$ if virtual and real terms larger than Born: shouldn't happen in perturbative regions
- Reweighting such that $\bar{B} > 0$ for all events
- **POWHEG: Positive Weight Hardest Emission Generator** own implementation in WHIZARD



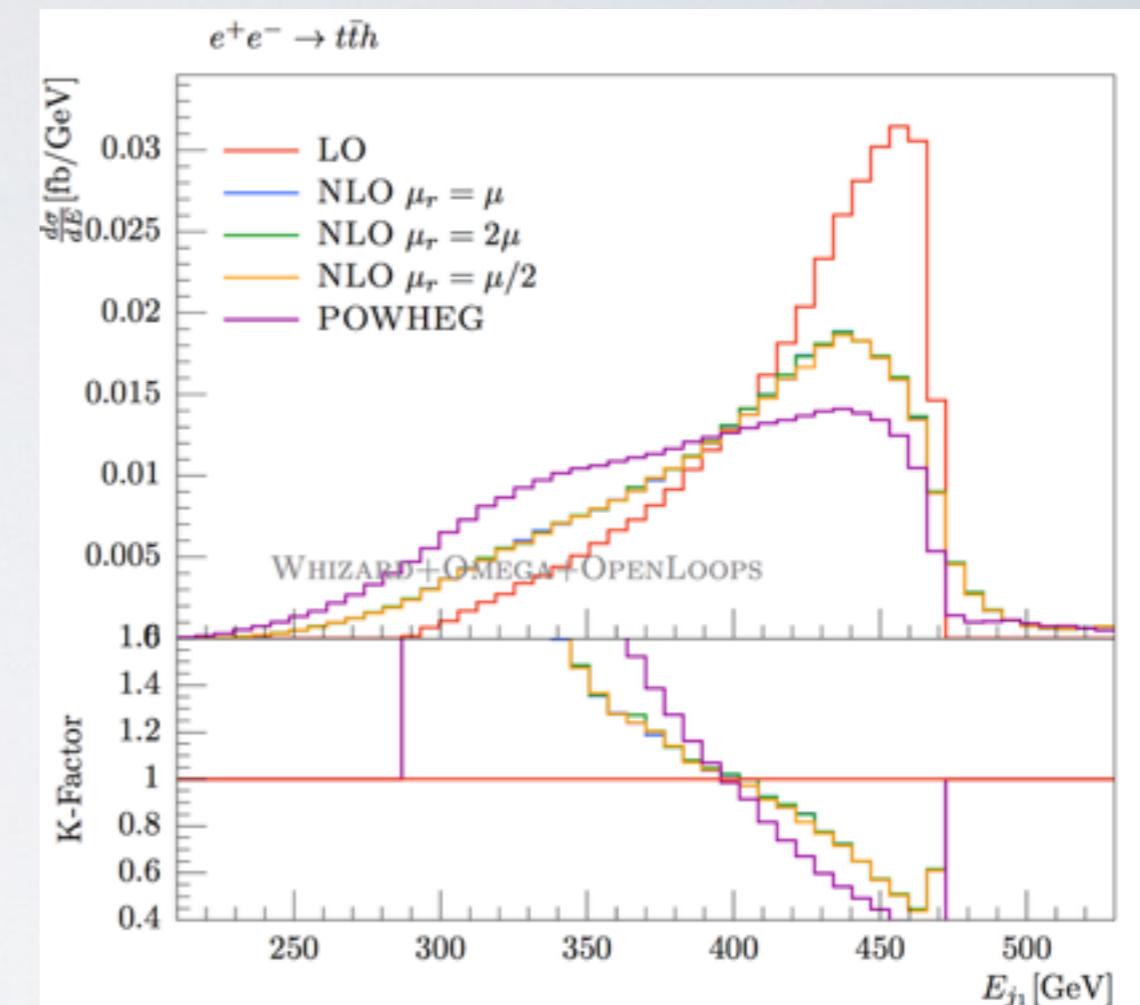
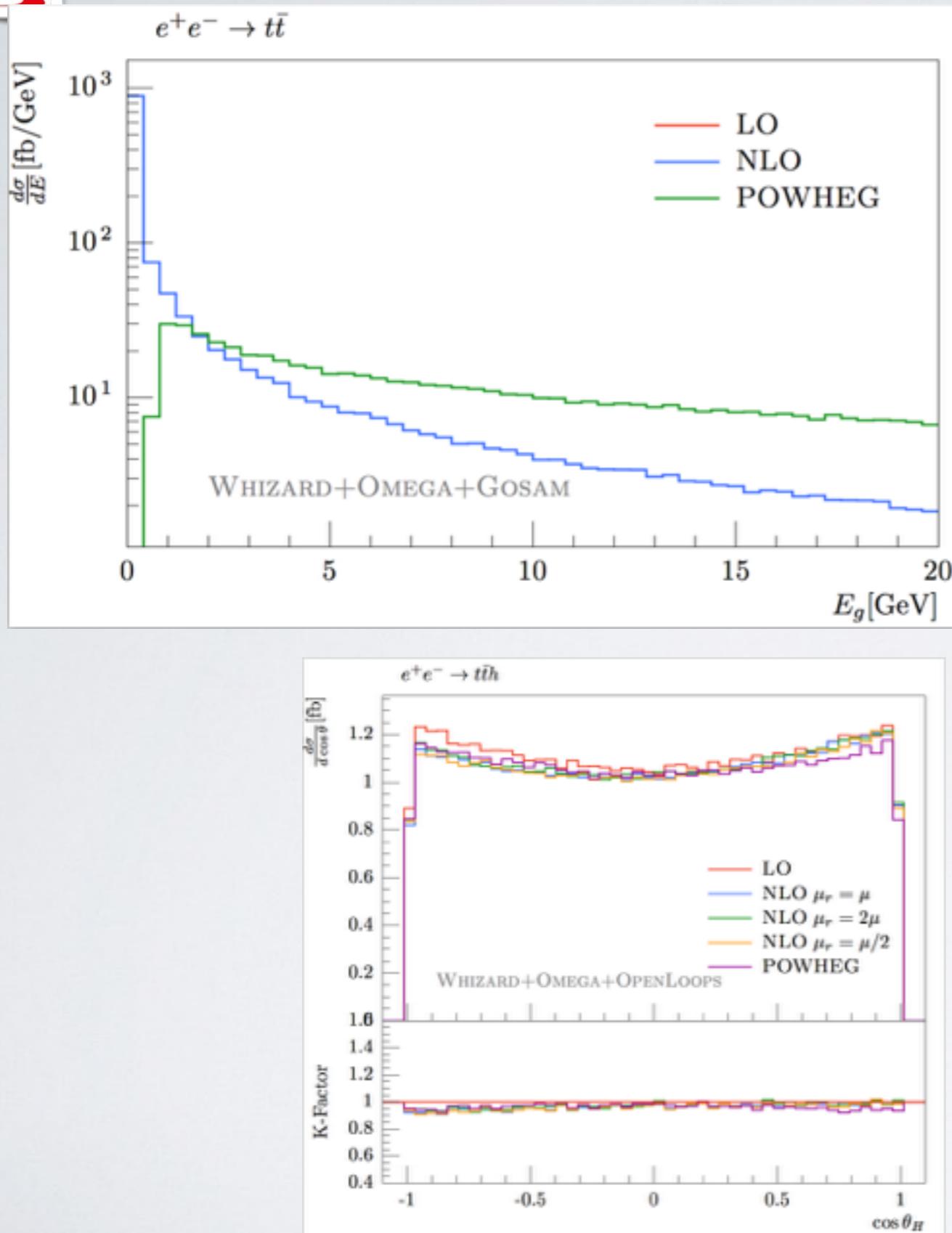


POWHEG Matching, example: e⁺e⁻ to dijets





Examples: Top pairs and tth production





Resonance mappings for NLO processes

- Amplitudes (except for pure QCD/QED) contain **resonances (Z, W, H, t)**
- In general: **resonance masses *not* respected by modified kinematics of subtraction terms**
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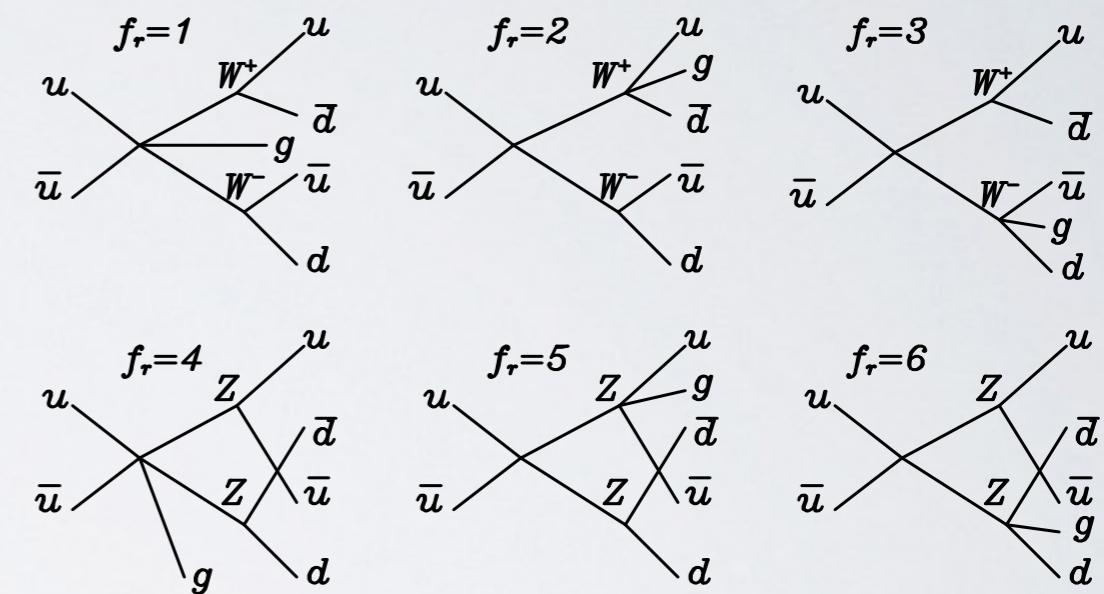
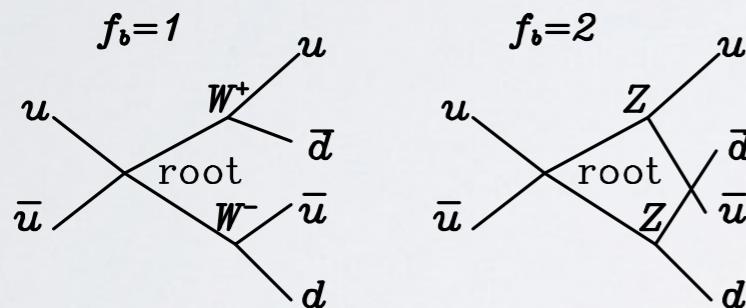
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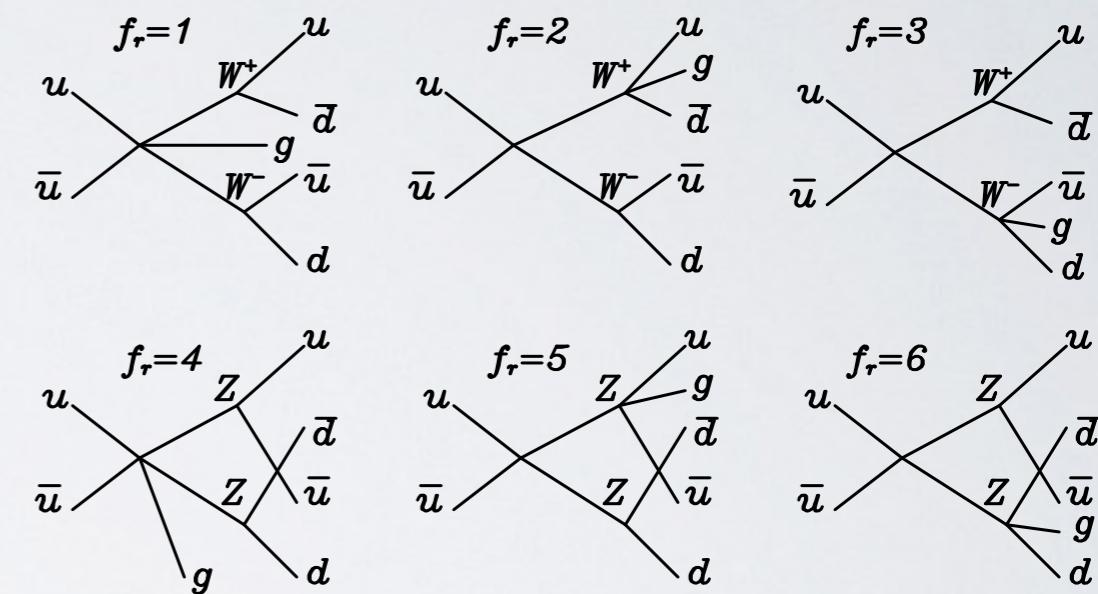
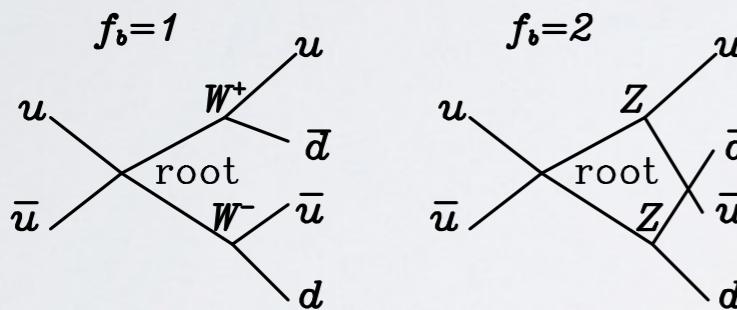
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- WHIZARD complete automatic implementation: example $e^+ e^- \rightarrow \mu\mu bb$ (ZZ, ZH histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
	5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49
								5

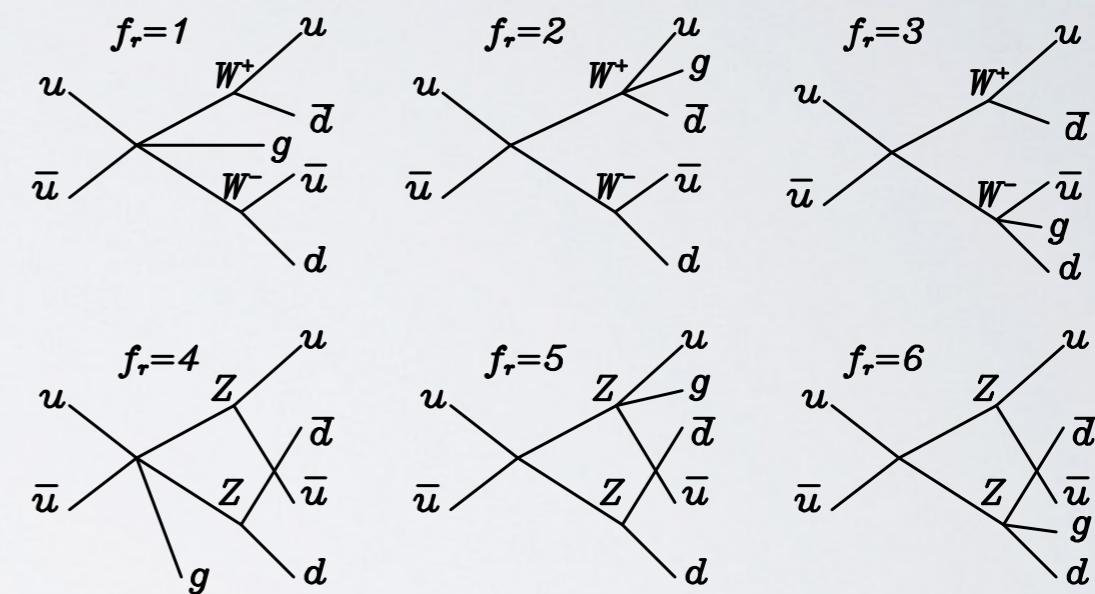
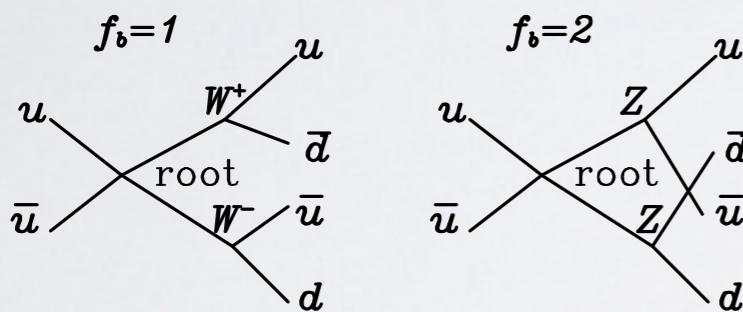
standard FKS





Resonance mappings for NLO processes

- Amplitudes (except for pure QCD/QED) contain **resonances (Z, W, H, t)**
- In general: resonance masses *not* respected by modified kinematics of subtraction terms
- Collinear (and soft) radiation can lead to mismatch between Born and subtraction terms
- Algorithm to include resonance histories** [Ježo/Nason, I509.09071]
- Avoids double logarithms in the resonances' width
- Most important for narrow resonances ($H \rightarrow bb$)
- Separate treatment of Born and real terms, soft mismatch**



WHIZARD complete automatic implementation: example $e^+ e^- \rightarrow \mu\mu bb$ (ZZ, ZH histories)

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standard FKS

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1	11988	2.9057032E+00	8.35E-02	2.87	3.15*	7.90		
2	11962	2.8591952E+00	5.20E-02	1.82	1.99*	10.91		
3	11936	2.9277880E+00	4.09E-02	1.40	1.52*	14.48		
4	11902	2.8512337E+00	3.98E-02	1.40	1.52*	13.70		
5	11874	2.8855399E+00	3.87E-02	1.34	1.46*	17.15		
5	59662	2.8842006E+00	2.04E-02	0.71	1.72	17.15	0.53	5

FKS with resonance mappings

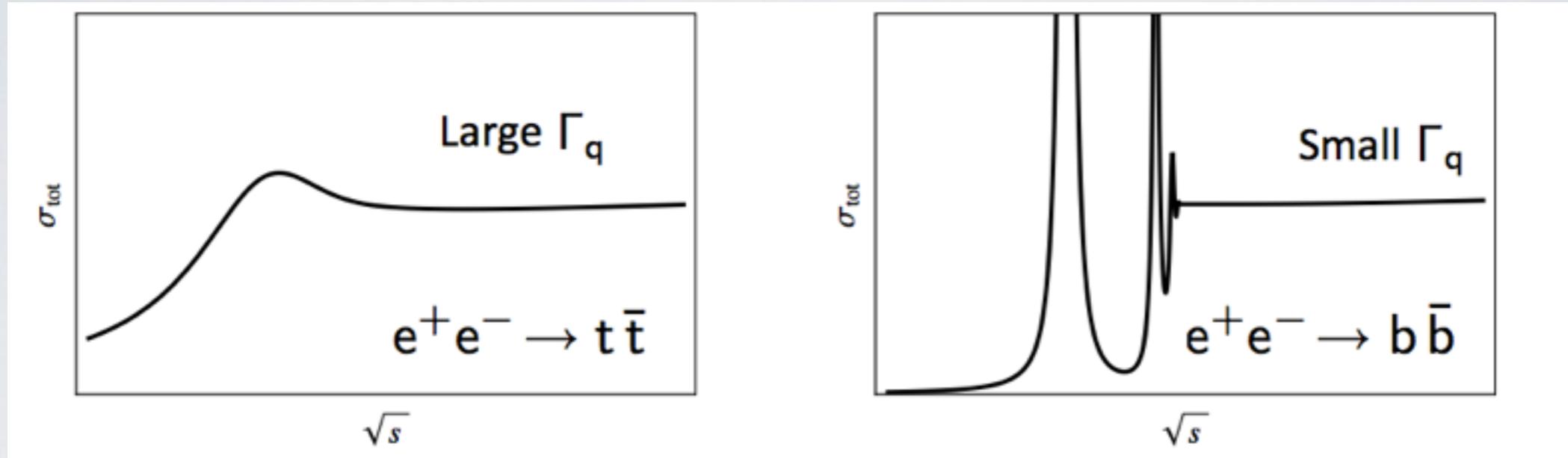




Top Threshold at lepton colliders

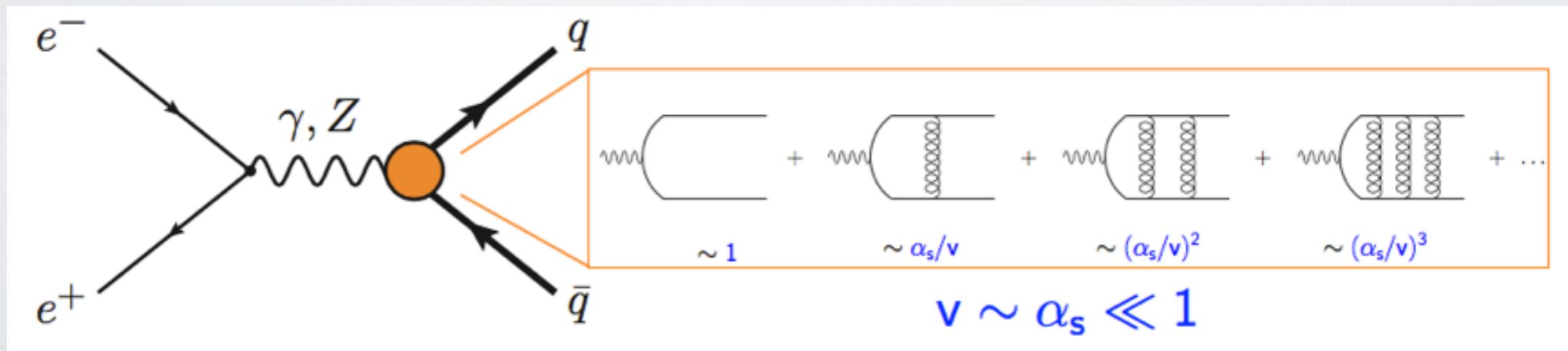
e^+e^- top threshold scan best-known method to measure top quark mass, $\Delta M \sim 30\text{-}50 \text{ MeV}$

Heavy quark production at lepton colliders, qualitatively:



Threshold region: top velocity $v \sim \alpha_s \ll 1$

$$v = \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m}}$$





Top Threshold Resummation in (v)NRQCD

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.: **vNRQCD** talk by M. Steinhauser
- Resummation of singular terms close to threshold ($v = 0$) Hoang et al. '99-'01; Beneke et al., '13-'14

Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v} \right)^k \sum_i (\alpha_s \ln v)^i \times \\ \times \{ 1 (\text{LL}); \alpha_s, v (\text{NLL}); \alpha_s^2, \alpha_s v, v^2 (\text{NNLL}) \}$$

(p/v)NRQCD EFT w/ RG improvement





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$R^{\gamma, Z}(s) = \underbrace{F^v(s)R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s)R^a(s)}_{\text{p-wave} \sim v^2: \text{NNLL}}$

but contributes
at NLL differentially!

$(p/v)\text{NRQCD EFT w/ RG improvement}$





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Coulomb potential gluon ladder resummation

~ 1 $\sim \alpha_s/v$ $\sim (\alpha_s/v)^2$ $\sim (\alpha_s/v)^3$

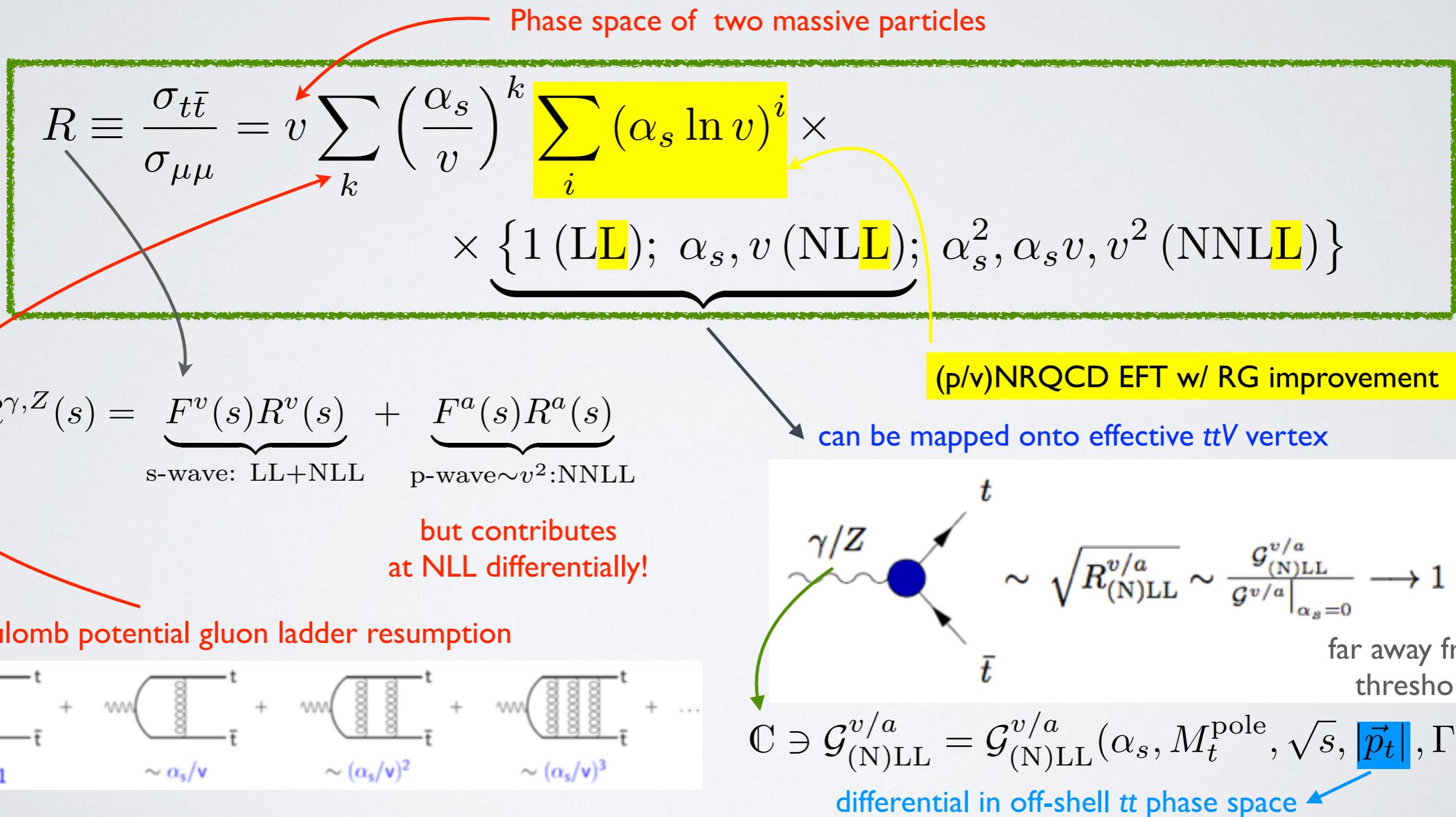
(p/v)NRQCD EFT w/ RG improvement





Top Threshold Resummation in (v)NRQCD

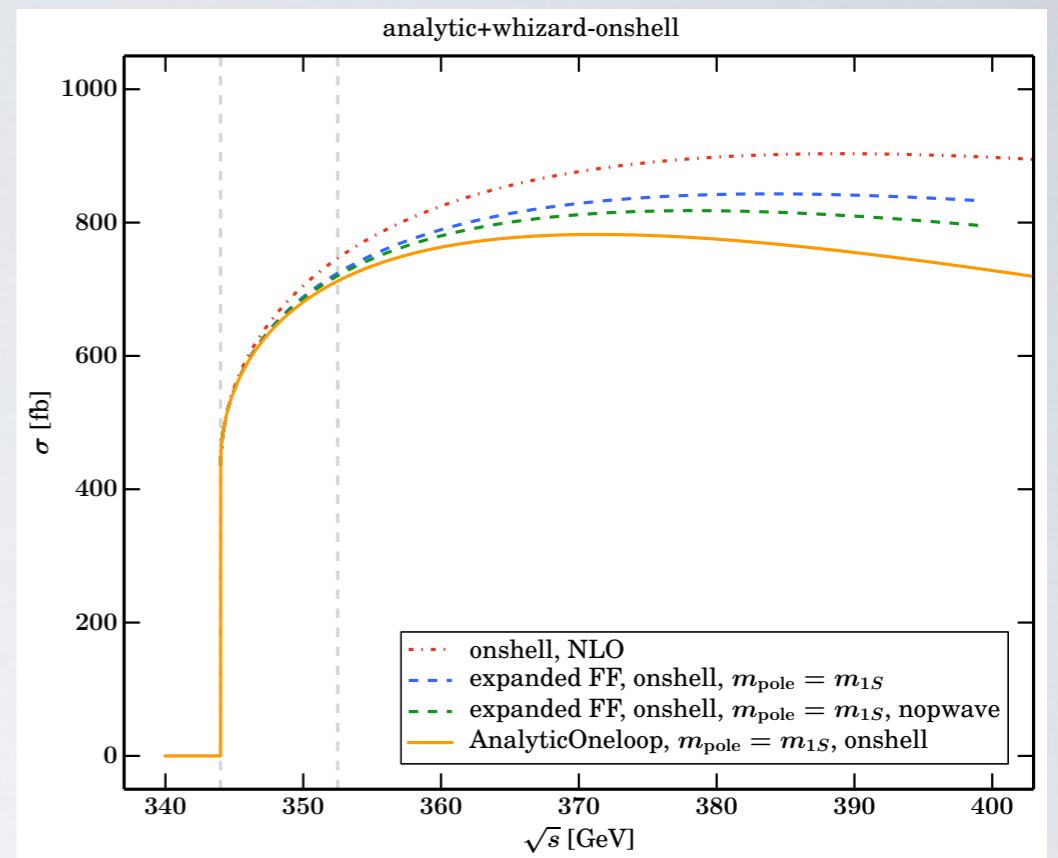
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Matching to continuum at (LO and) NLO

- Transition region between relativistic and resummation effects
- CLIC benchmark energies:
0.38 TeV, 1.4 TeV, 3.0 TeV
- Remove double-counting NLO / (N)LL



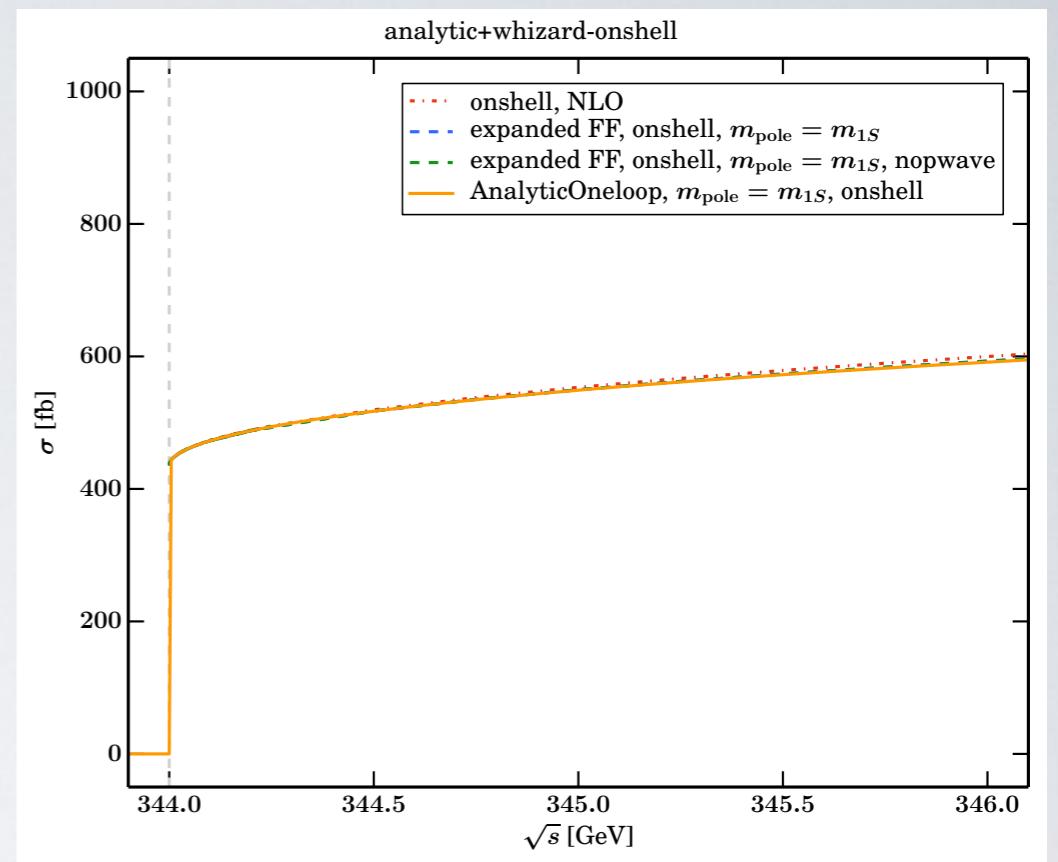


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$$\nu = \sqrt{\frac{\sqrt{s} - 2m_t + i\Gamma_t}{m}} \quad p = |\vec{p}| \quad p_0 = E_t - m_t$$



$$F^{\text{expanded}} [\alpha_H, \alpha_S] = \alpha_H \left(-\frac{2C_F}{\pi} \right) + \alpha_S \left(\frac{i C_F m \log \frac{mv+p}{mv-p}}{2p} \right)$$



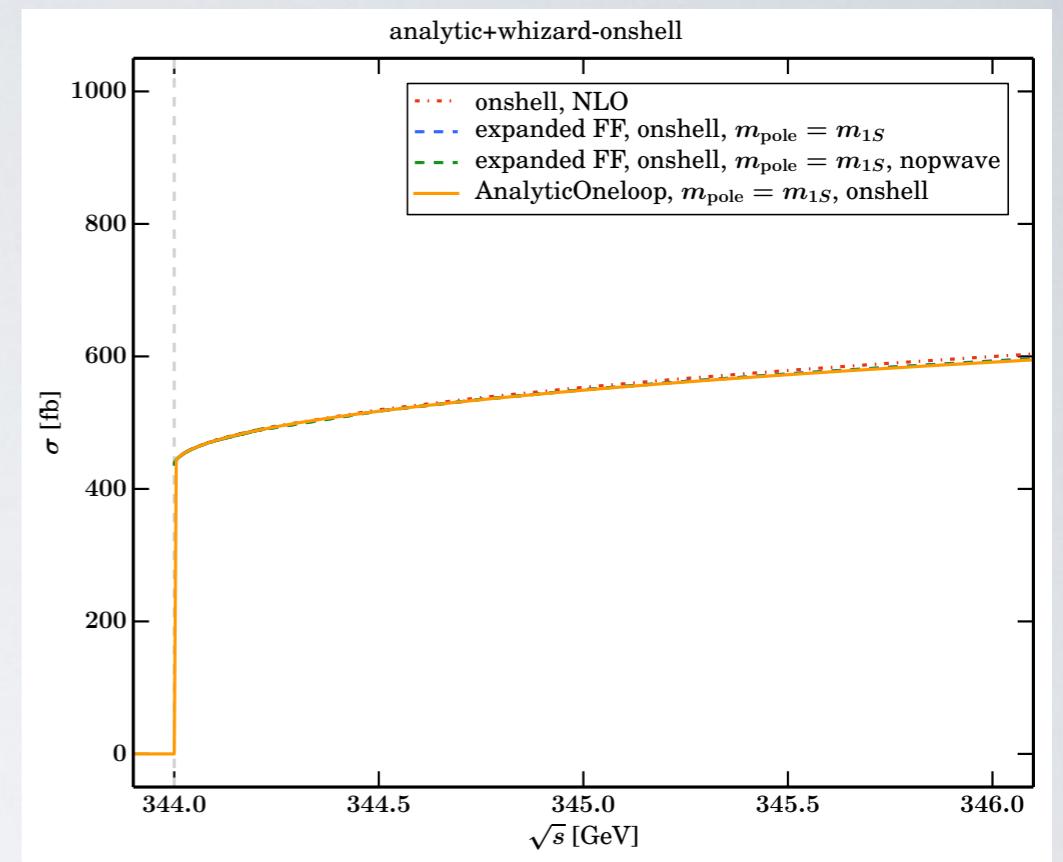


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$$\begin{aligned} \sigma_{\text{matched}} = & \sigma_{\text{QCD}} [\alpha_H] - \sigma_{\text{NRQCD}}^{\text{expanded}} [\alpha_H, \alpha_H] \\ & + \sigma_{\text{NRQCD}}^{\text{expanded}} [\alpha_H, f_s \alpha_S + (1 - f_s) \alpha_H] \\ & + \sigma_{\text{NRQCD}}^{\text{full}} [f_s \alpha_H, f_s \alpha_S, f_s \alpha_{\text{US}}] - \sigma_{\text{NRQCD}}^{\text{expanded}} [f_s \alpha_H, f_s \alpha_S] \end{aligned}$$



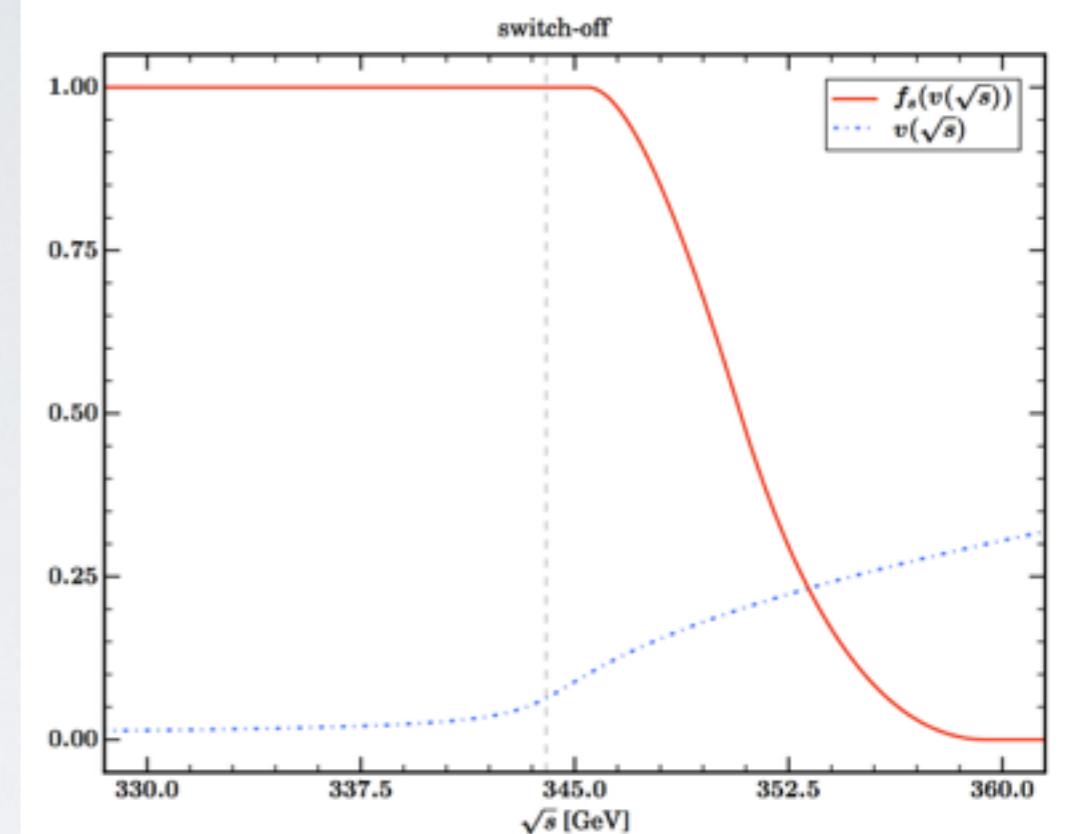


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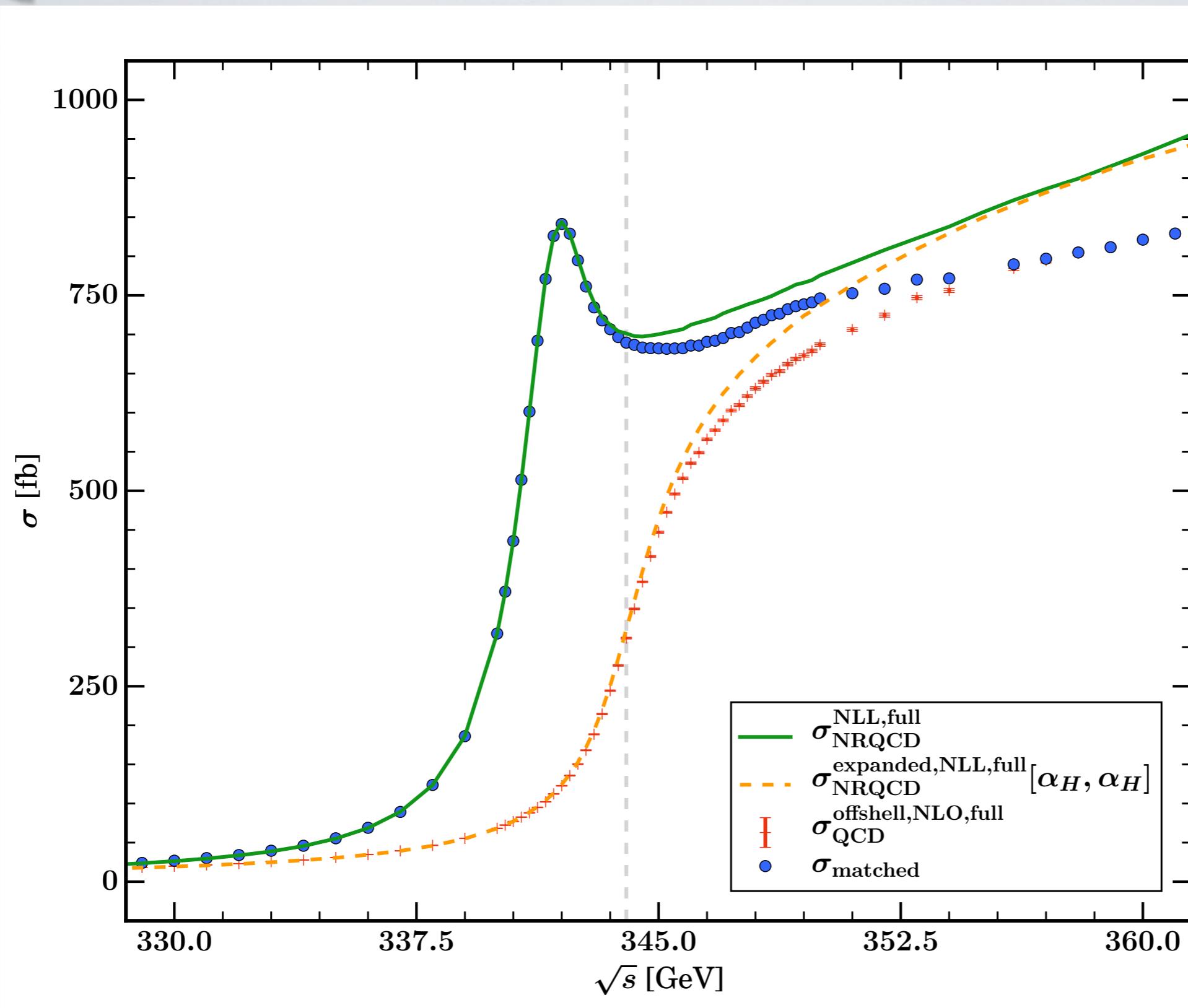
Switch-off function

$$f_s(v) = \begin{cases} 1 & v < v_1 \\ 1 - 2 \frac{(v-v_1)^2}{(v_2-v_1)^2} & v_1 < v < \frac{v_1+v_2}{2} \\ 2 \frac{(v-v_2)^2}{(v_2-v_1)^2} & \frac{v_1+v_2}{2} < v < v_2 \\ 0 & v > v_2 \end{cases}$$





Threshold-continuum matching



Bach/Chokouf  /Hoang/
Kilian/JRR/Stahlhofen/
Teubner/Weiss
work in progress





Conclusions & Outlook

- WHIZARD 2.2 event generator for collider physics (ee, pp, ep)
- Allows to simulate all possible BSM models
- Allows for all SM backgrounds
- ee physics: beamspectra, LCIO, ee top threshold
- NLO automation: reals and subtraction terms (FKS) [+ virtuals externally] → WHIZARD 3.0
 - allows to produce NLO fixed-order histograms
 - Automated POWHEG matching (other schemes in progress)
 - Top threshold in e+e-: NLL NRQCD threshold / NLO continuum matching
- Ongoing projects: Lorentz structure, showers, merging
- Tell us what is missing, insufficient, annoying, desirable





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- Tell us what is missing, insufficient, annoying, desirable
 - even if it is in an international conference summary talk ⇒ Challenge accepted !





New



WHIZARD
Quantum
HIGH PERFORMANCE GREASE

**Higher Performance
Superior Protection**

▶ **Learn More**





Projects, Plans, Performance and all that

- O'Mega Virtual Machine (OVM): ME via bytecode interpreter than compiled code ✓
- Parton shower: LO merging (MLM ✓), NLO matching
- QED shower (FSR)
- QED shower (ISR); exclusive part of ISR spectrum
- pT spectrum of ISR radiation
- automated massless/massive QCD NLO corrections: FS ✓ / Initial state in preparation
→ WHIZARD 3.0
- QED/electroweak NLO automation: longer time scale
- complete NLL NRQCD top threshold/NLO continuum matching; extension to ttH [✓]
- POWHEG matching implemented ✓ ; maybe also MC@NLO or Nagy-Soper matching
- Monte Carlo over helicities and colors, work on performance (MPI)
- Modified algorithm for multi-leg (tree) matrix elements: includes high-color flow amplitudes, QCD/EW coupling orders, completely general Lorentz structures, UFO format
- Automatic generation of decays (and calculation of decay widths) ✓
- New syntax for nested decay chains

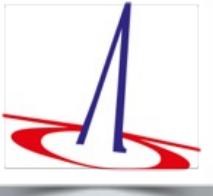
```
process = e1, E1 => (t => (Wp => E2, nu2), b), tbar
```





BACKUP SLIDES



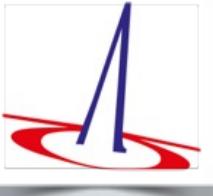


WHIZARD: Installation and Run

- Download: <http://www.hepforge.org/archive/whizard/whizard-2.2.8.tar.gz>
- Unpack it, intended to be installed in /usr/local (or locally)
- Create build directory and do ./configure
- make, [make check], make install
- Working directory: create SINDARIN steering file <input>.sin
- Working directory: run whizard <input>.sin
- Supported event formats: LHA, StdHep, LHEF (i-iii), HepMC, LCIO, div.ASCII
- Interfaces to external packages: FastJet, GoSam, GuineaPig(++) , HepMC, HOPPET, LCIO, LHAPDF(4/5/6), LoopTools, OpenLoops, PYTHIA6, [PYTHIA8], StdHep [internal]

```
PASS: circez_2.run
PASS: ewa_1.run
PASS: ewa_2.run
PASS: ewa_3.run
PASS: ewa_4.run
PASS: ilc.run
PASS: gaussian_1.run
PASS: gaussian_2.run
PASS: beam_events_1.run
PASS: beam_events_2.run
PASS: beam_events_3.run
PASS: beam_events_4.run
PASS: energy_scan_1.run
PASS: restrictions.run
PASS: process_log.run
PASS: shower_err_1.run
PASS: parton_shower_1.run
PASS: parton_shower_2.run
PASS: mlm_matching_fsr.run
XFAIL: user_cuts.run
XFAIL: user_strfun.run
PASS: hepmc_1.run
PASS: hepmc_2.run
PASS: hepmc_3.run
PASS: hepmc_4.run
PASS: hepmc_5.run
PASS: hepmc_6.run
PASS: hepmc_7.run
PASS: hepmc_8.run
PASS: hepmc_9.run
PASS: hepmc_10.run
PASS: analyze_4.run
SKIP: lhapdf5.run
PASS: lhapdf6.run
PASS: stdhep_1.run
PASS: stdhep_2.run
PASS: stdhep_3.run
PASS: stdhep_4.run
PASS: stdhep_5.run
PASS: pythia6_1.run
PASS: pythia6_2.run
PASS: pythia6_3.run
PASS: pythia6_4.run
PASS: mlm_matching_isr.run
PASS: mlm_pythia6_isr.run
PASS: analyze_3.run
PASS: static_1.run
=====
Testsuite summary for WHIZARD 2.2.7
=====
# TOTAL: 286
# PASS: 281
# SKIP: 2
# XFAIL: 3
# FAIL: 0
# XPASS: 0
# ERROR: 0
```



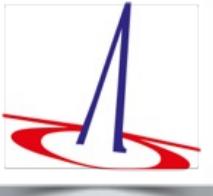


Beams, Fields, Colors, Lorentz structures

Lorentz structures:

- ▶ Large number of hardcoded terms: pure scalar, pure vector, scalar/vector, fermion/scalar, fermion/vector, fermion/tensor, vector/tensor, gravitino couplings, fermion coupl. SUSY Ward id.
- ▶ Growing number of dim. 5/6/7/8 operators: HEFT, aTGCs, aQGCs, anomalous top couplings etc.
- ▶ Completely general Lorentz structures: foreseen for major next release (incl. UFO support), v2.3.0





Beams, Fields, Colors, Lorentz structures

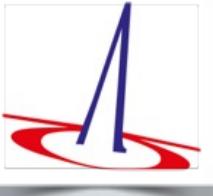
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Beams:

- ▶ Lepton beam ISR Kuraev/Fadin, 2003; Skrzypek/Jadach, 1991
- ▶ Lepton collider beams: CIRCE1/2, also photon beams (more later)
- ▶ PDFs: interface to LHAPDF v4/5/6; internal PDFs: CTEQ6, CT10, MMHT etc.
- ▶ QCD parton shower: 2 own implementations [or ext., more later]

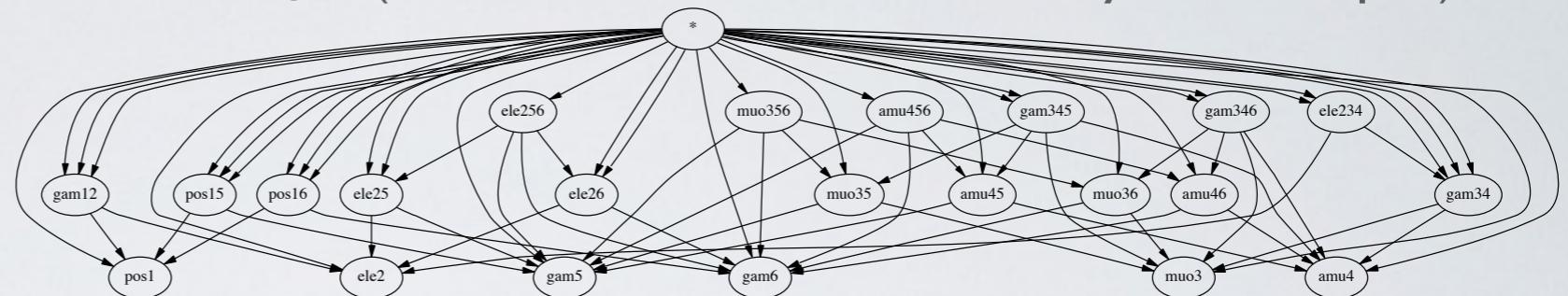




The matrix element generator: O'Mega

- * Built-in matrix element generator O'Mega (recursiveness via Directed Acyclical Graphs)

$$\Omega$$



- * New concept for internal quantum number representation: faster flavor sums, counting of coupling constants (via partial expansion), more speed-up, general Lorentz structures **(in prep.)**

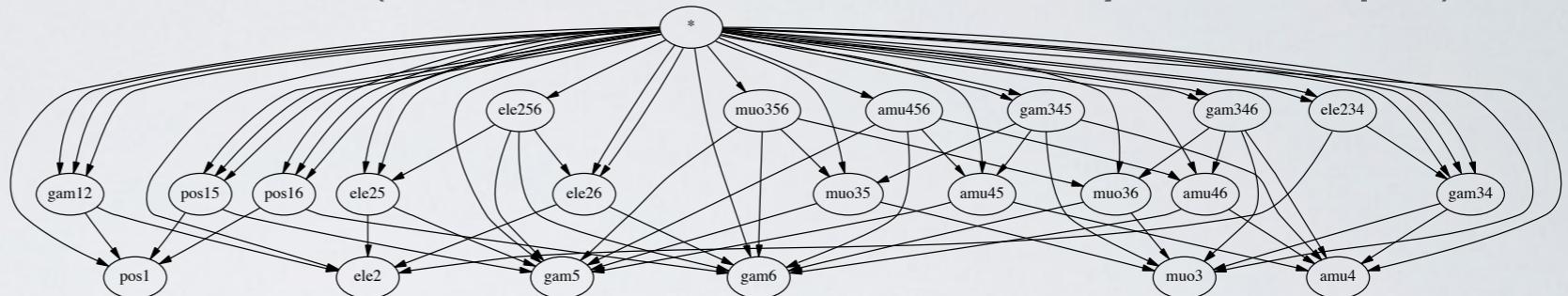




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```
process <proc> = in1, in2 => <out> { $method = "ovm" }
```

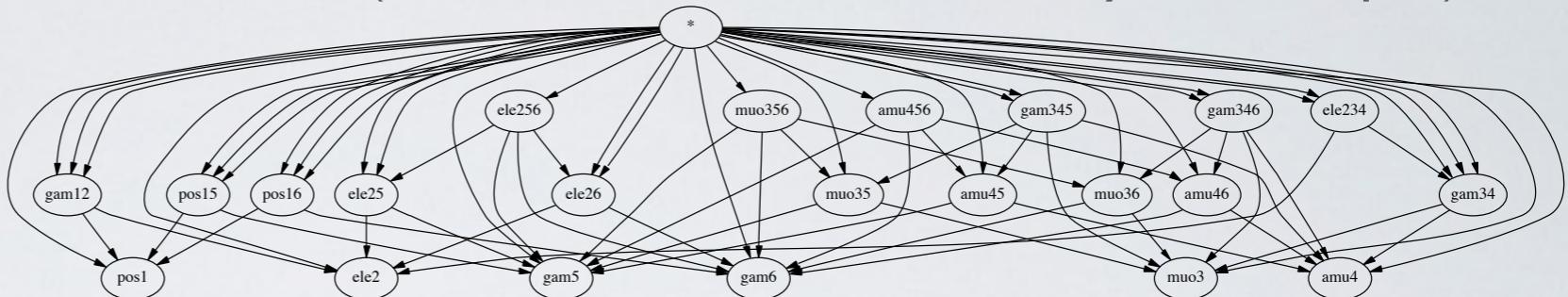
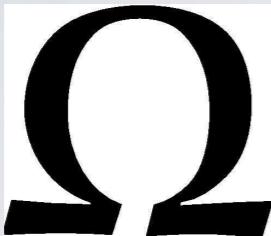
```
Flavor states table
2 -2 11 -11
Color flows table: [ (i, j) (k, l) -> (m, n) ... ]
1 0 0 -1 0 0 0 0
Color ghost flags table:
0 0 0 0
Color factors table: [ i, j: num den power], i, j are indexed color flows
    1      1      1      1      1
Flavor color combination is allowed:
1
OVM instructions for momenta addition, fusions and brackets start here:
0 0 0 0 0 0 0 0
1 0 0 5 1 2 0 0
11 2 0 2 1 0 0 1
13 2 0 2 2 0 0 1
14 11 0 1 3 0 0 1
12 11 0 1 4 0 0 1
0 0 0 0 0 0 0 0
60 22 2 1 5 0 0 1
-1 2 1 1 2 2 0 0
58 23 2 2 5 0 0 1
-4 3 1 2 2 2 0 0
0 0 0 0 0 0 0 0
2 -1 0 1 1 0 0 0
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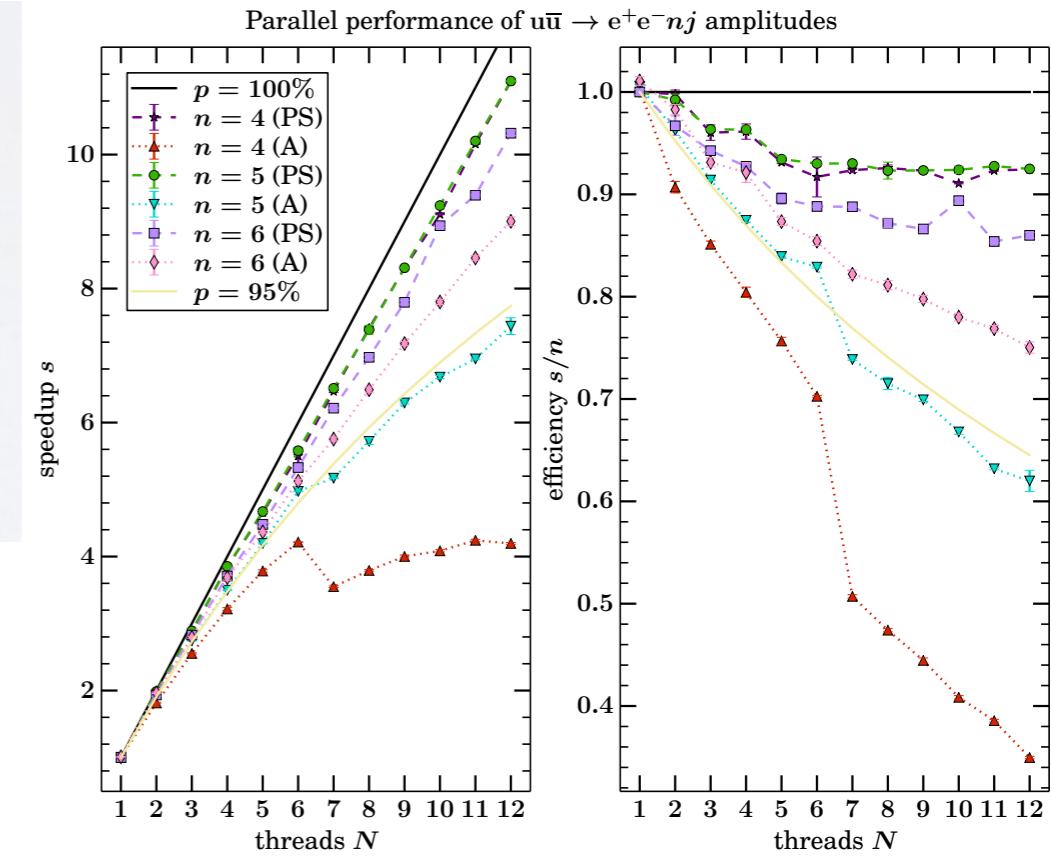
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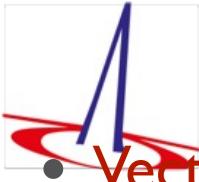
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14 11 0 1 3 0 0 1
12 11 0 1 4 0 0 1
0 0 0 0 0 0 0 0
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-4 3 1 2 2 2 0 0
0 0 0 0 0 0 0 0
2 -1 0 1 1 0 0 0
-4 2 1 2 1 1 0 0
-1 1 1 1 1 1 0 0

```

process	BC size	Fortran size	t_{compile}
$gg \rightarrow gggggg$	428 MiB	4.0 GiB	-
$gg \rightarrow ggggg$	9.4 MiB	85 MiB	483(18) s
$gg \rightarrow q\bar{q}q'\bar{q}'q''\bar{q}''g$	3.2 MiB	27 MiB	166(15) s
$e^+e^- \rightarrow 5(e^+e^-)$	0.7 MiB	1.9 MiB	32.46(13) s





High-Energy Electroweak Sector

- **Vector Boson Scattering:** access to New Physics in W, Z selfcoupl. Beyer/JRR/Mönig, arXiv:hep-ph/0604048
- 1 TeV, 1/ab, full 6-fermion states, P(80% e-, 60% e+), binned likelihood
- Contributing channels: $WW \rightarrow WW$, $WW \rightarrow ZZ$, $WZ \rightarrow WZ$, $ZZ \rightarrow ZZ$

Process	Subprocess	σ [fb]
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q\bar{q}q\bar{q}$	$WW \rightarrow WW$	23.19
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q\bar{q}q\bar{q}$	$WW \rightarrow ZZ$	7.624
$e^+e^- \rightarrow \nu \bar{\nu} q\bar{q}q\bar{q}$	$V \rightarrow VVV$	9.344
$e^+e^- \rightarrow \nu e q\bar{q}q\bar{q}$	$WZ \rightarrow WZ$	132.3
$e^+e^- \rightarrow e^+e^- q\bar{q}q\bar{q}$	$ZZ \rightarrow ZZ$	2.09
$e^+e^- \rightarrow e^+e^- q\bar{q}q\bar{q}$	$ZZ \rightarrow W^+W^-$	414.
$e^+e^- \rightarrow bbX$	$e^+e^- \rightarrow t\bar{t}$	331.768
$e^+e^- \rightarrow q\bar{q}q\bar{q}$	$e^+e^- \rightarrow W^+W^-$	3560.108
$e^+e^- \rightarrow q\bar{q}q\bar{q}$	$e^+e^- \rightarrow ZZ$	173.221
$e^+e^- \rightarrow e\nu q\bar{q}$	$e^+e^- \rightarrow e\nu W$	279.588
$e^+e^- \rightarrow e^+e^- q\bar{q}$	$e^+e^- \rightarrow e^+e^- Z$	134.935
$e^+e^- \rightarrow X$	$e^+e^- \rightarrow q\bar{q}$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
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$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55



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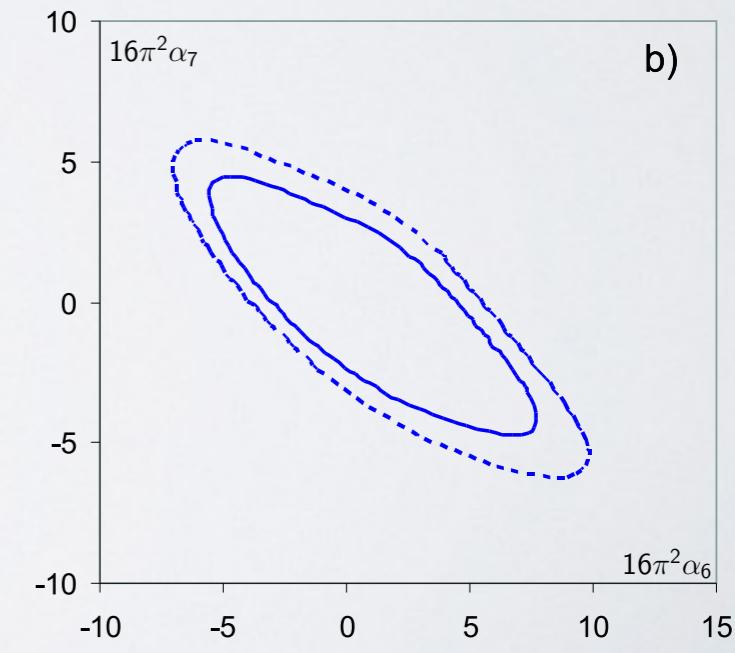
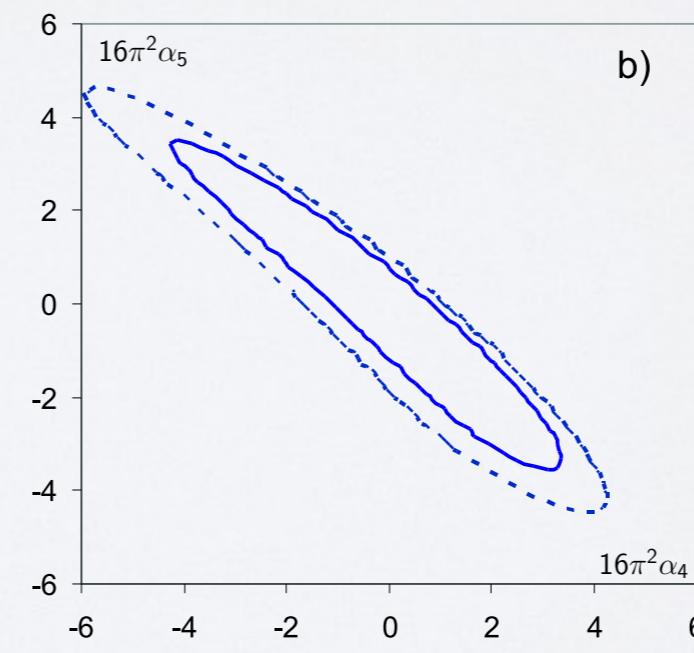
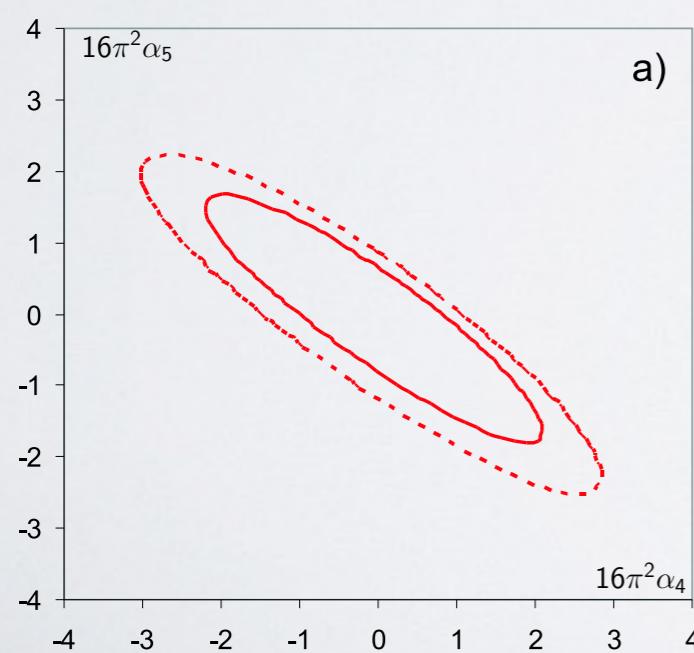
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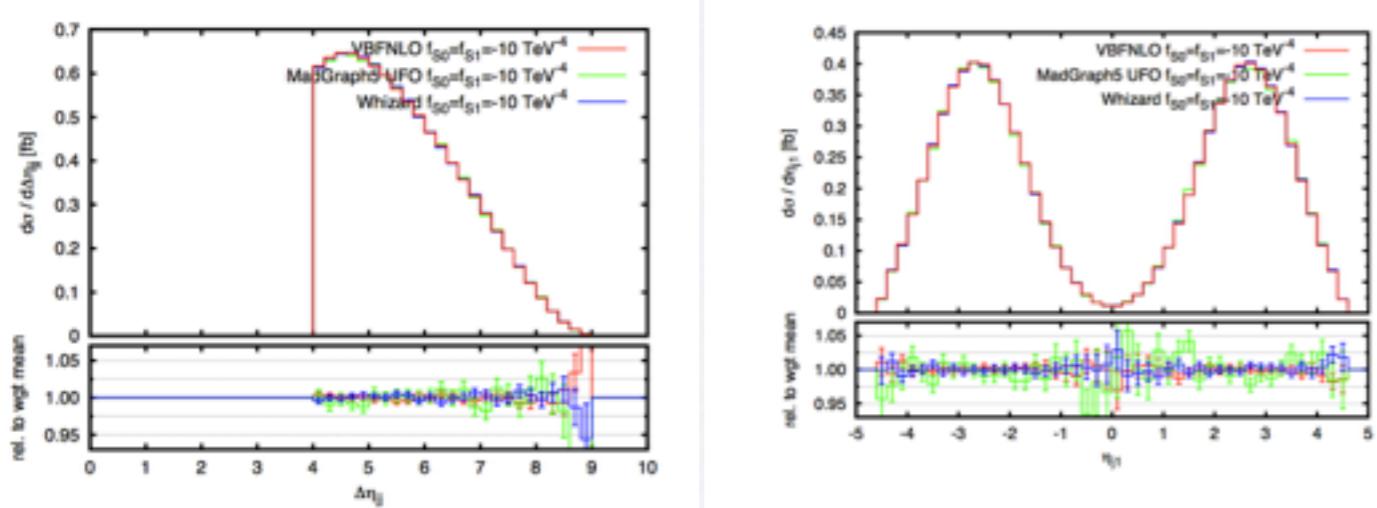




New Physics in Vector Boson Scattering

- Vector Boson Scattering (VBS) major measurement of LHC runs II/III [Gianotti, CERN 01/2014](#)
- Light Higgs suppression makes VBS prime candidate for BSM searches
- Model-independent EFT descriptions (almost) useless: either weakly-coupled resonances in reach or strongly-coupled sectors [Alboteanu/Kilian/JRR, 2008; Kilian/Ohl/JRR/Sekulla, 2014](#)
- Parameterize new physics by dim 6/dim 8 operators, calculate unitarity limits
- K-matrix unitarization implemented in WHIZARD (both for operators and resonances)

For the pure operators: full agreement
between WHIZARD, Madgraph5, VBFNLO



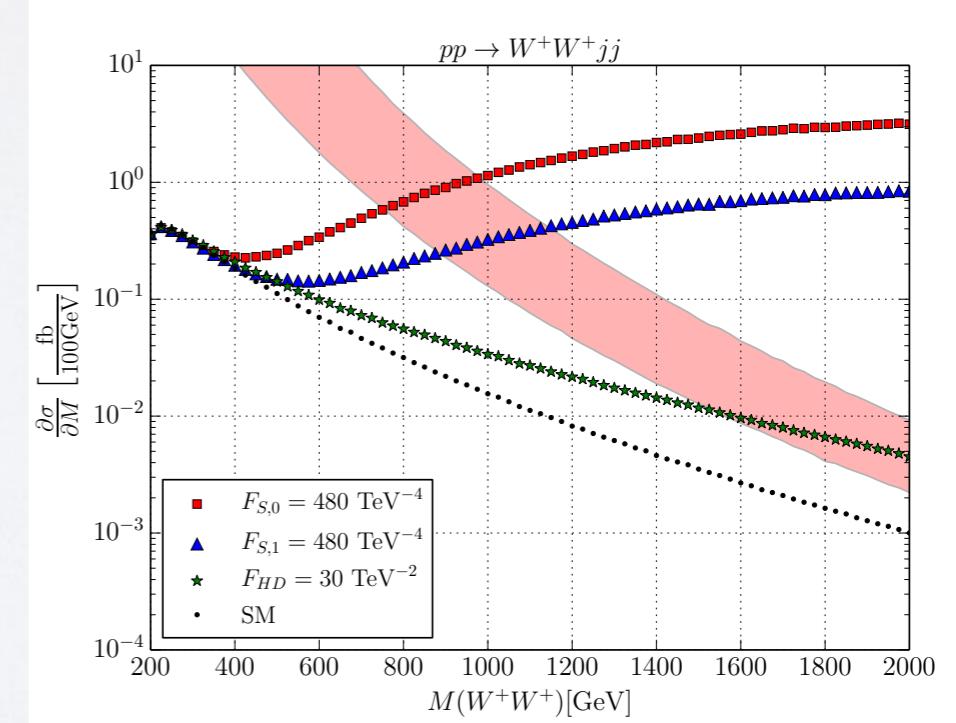
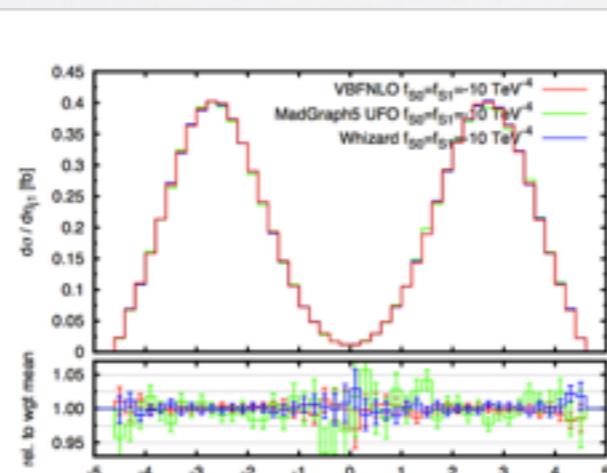
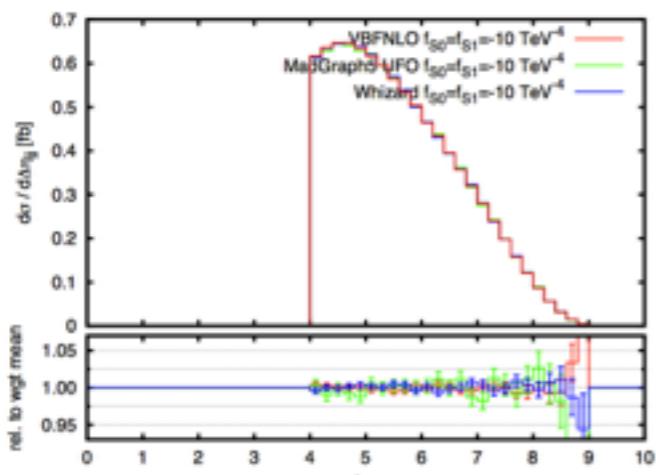


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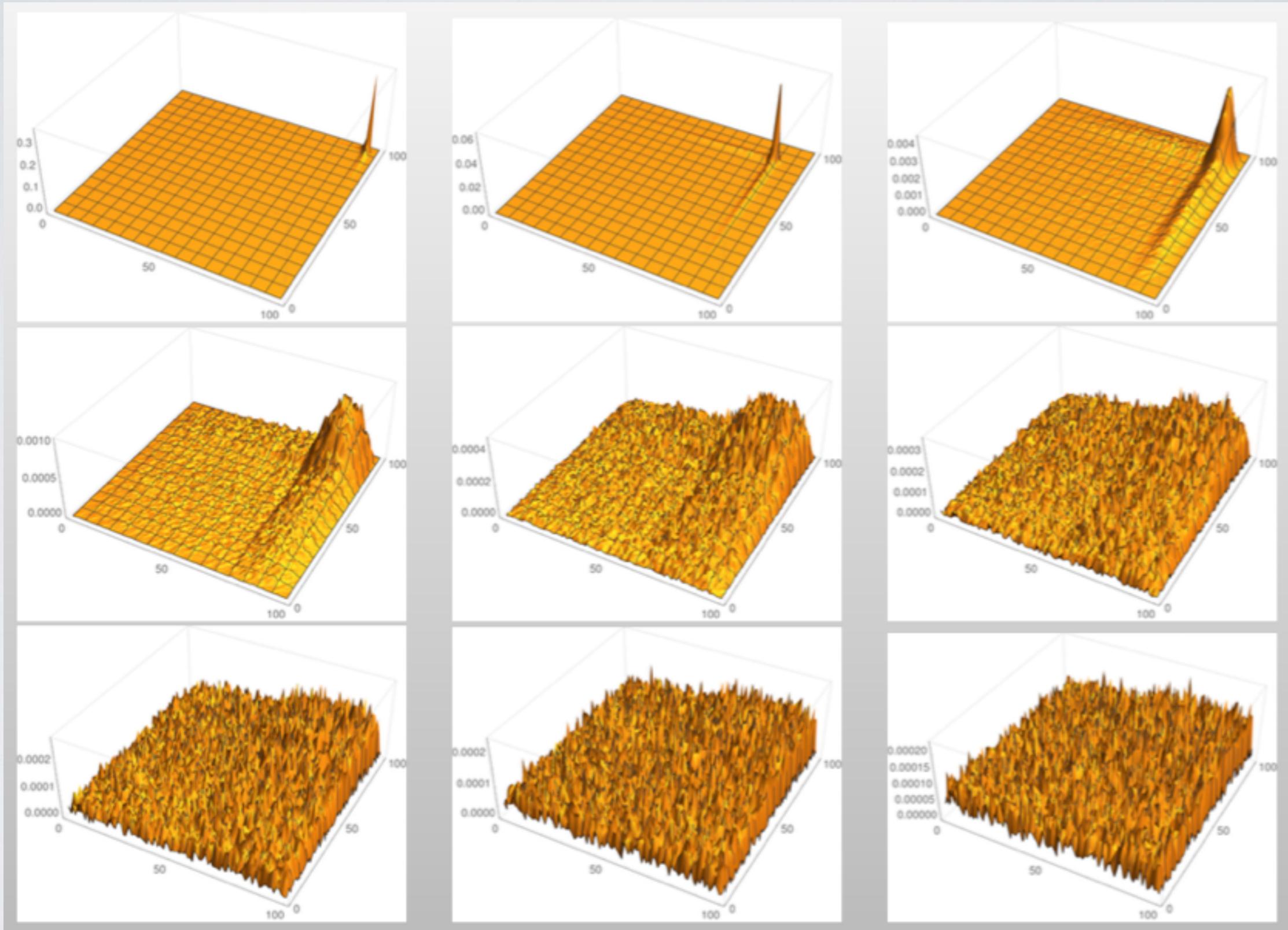
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$$\begin{aligned}\mathcal{L}_{HD} &= F_{HD} \operatorname{tr} \left[\mathbf{H}^\dagger \mathbf{H} - \frac{v^2}{4} \right] \cdot \operatorname{tr} \left[(\mathbf{D}_\mu \mathbf{H})^\dagger (\mathbf{D}^\mu \mathbf{H}) \right] \\ \mathcal{L}_{S,0} &= F_{S,0} \operatorname{tr} \left[(\mathbf{D}_\mu \mathbf{H})^\dagger \mathbf{D}_\nu \mathbf{H} \right] \cdot \operatorname{tr} \left[(\mathbf{D}^\mu \mathbf{H})^\dagger \mathbf{D}^\nu \mathbf{H} \right] \\ \mathcal{L}_{S,1} &= F_{S,1} \operatorname{tr} \left[(\mathbf{D}_\mu \mathbf{H})^\dagger \mathbf{D}^\mu \mathbf{H} \right] \cdot \operatorname{tr} \left[(\mathbf{D}_\nu \mathbf{H})^\dagger \mathbf{D}^\nu \mathbf{H} \right]\end{aligned}$$





Iterations of Beam Spectrum



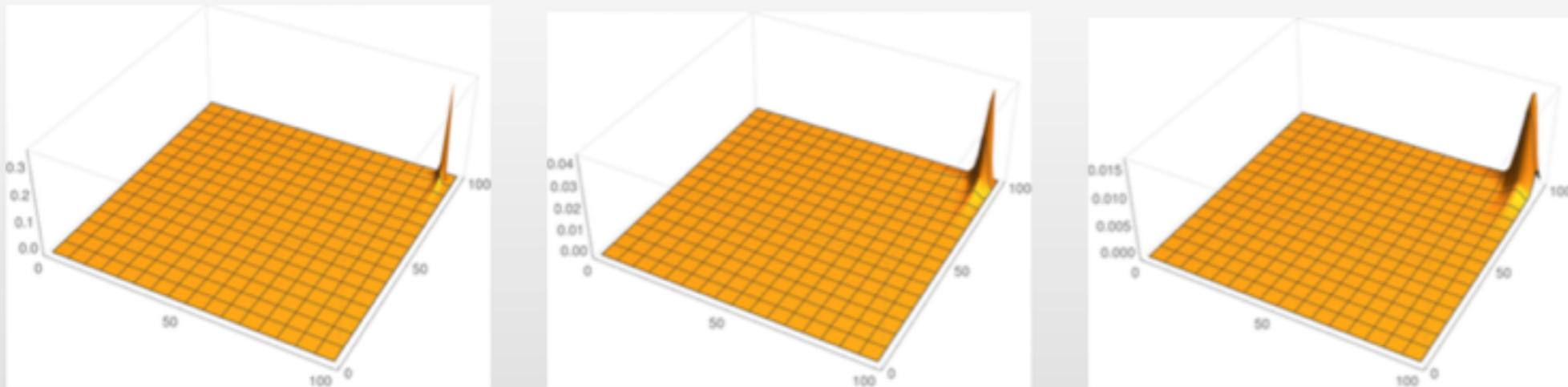
(171,306 GuineaPig events in 10,000 bins)



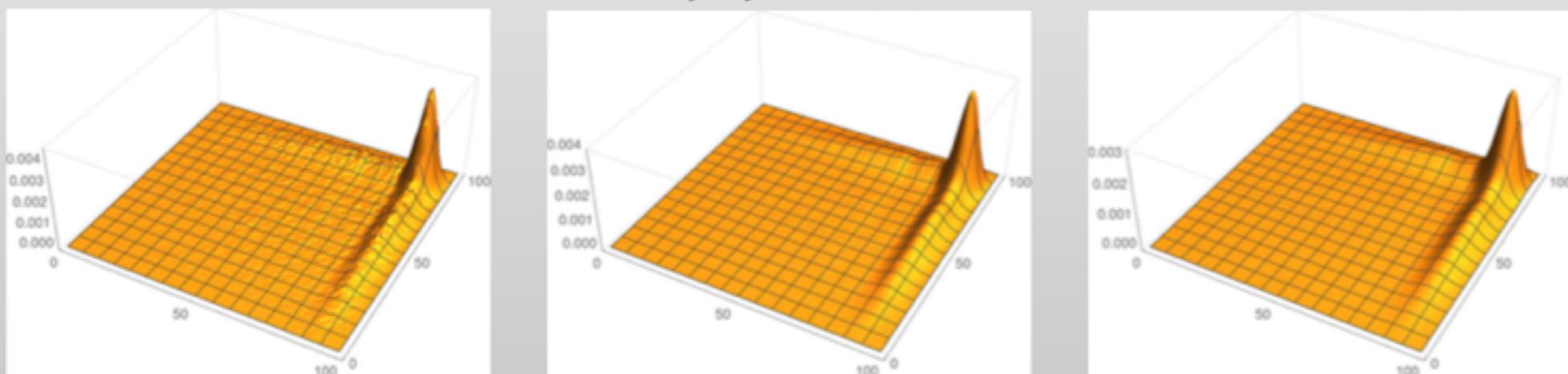


Iterations of Beam Spectrum

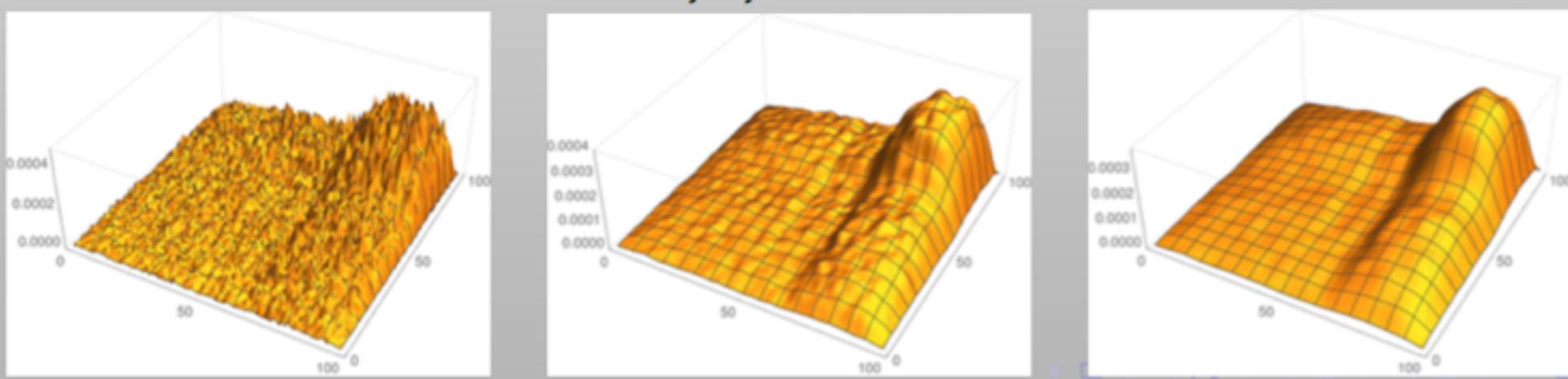
- ▶ **iterations = 0 and smooth = 0, 3, 5:**



- ▶ **iterations = 2 and smooth = 0, 3, 5:**

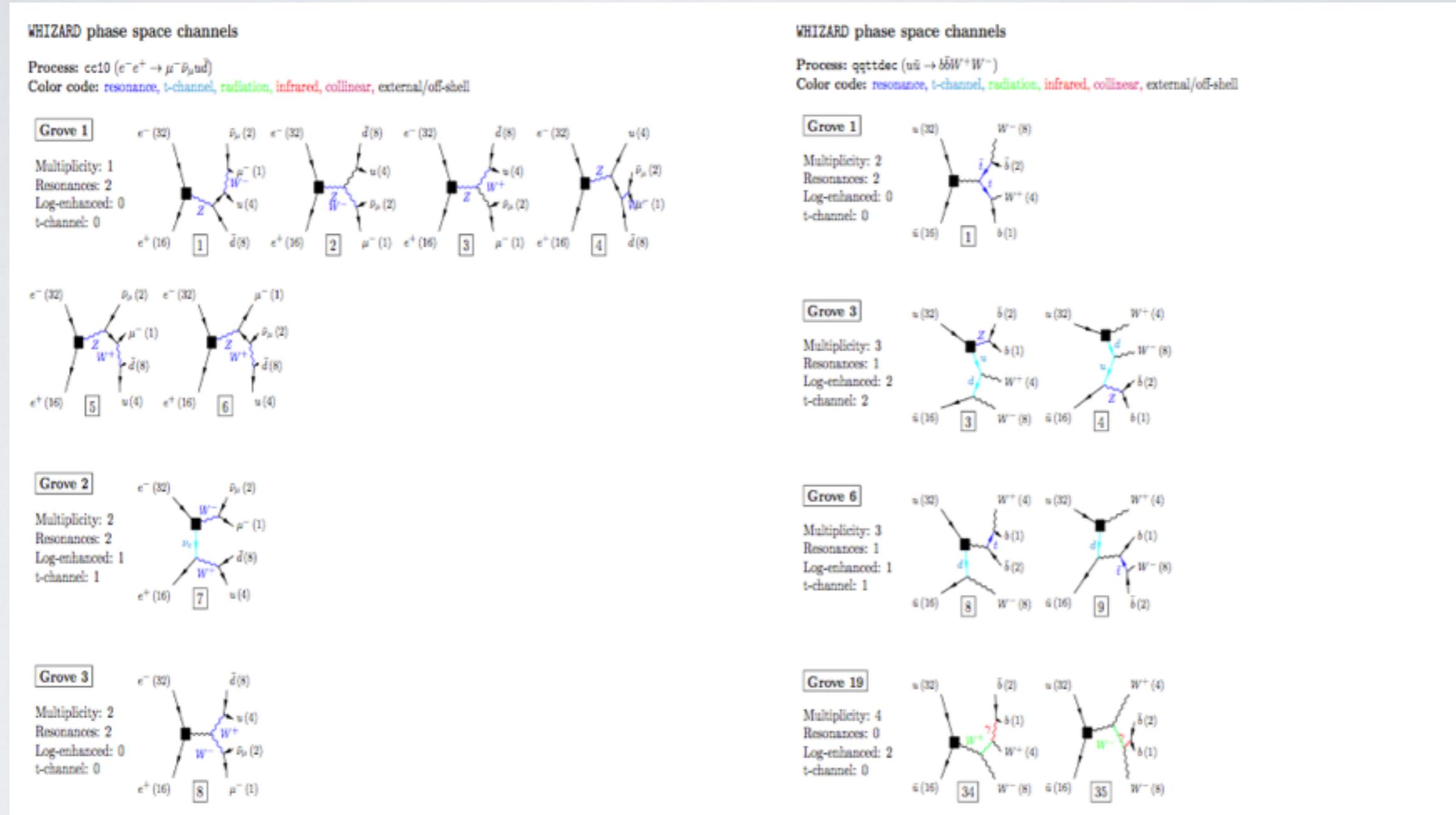


- ▶ **iterations = 4 and smooth = 0, 3, 5:**



Phase Space Setup

WHIZARD algorithm: heuristics to classify phase-space topology, adaptive multi-channel mapping \implies resonant, t-channel, radiation, infrared, collinear, off-shell



Complicated processes: **factorization into production and decay** with the unstable option

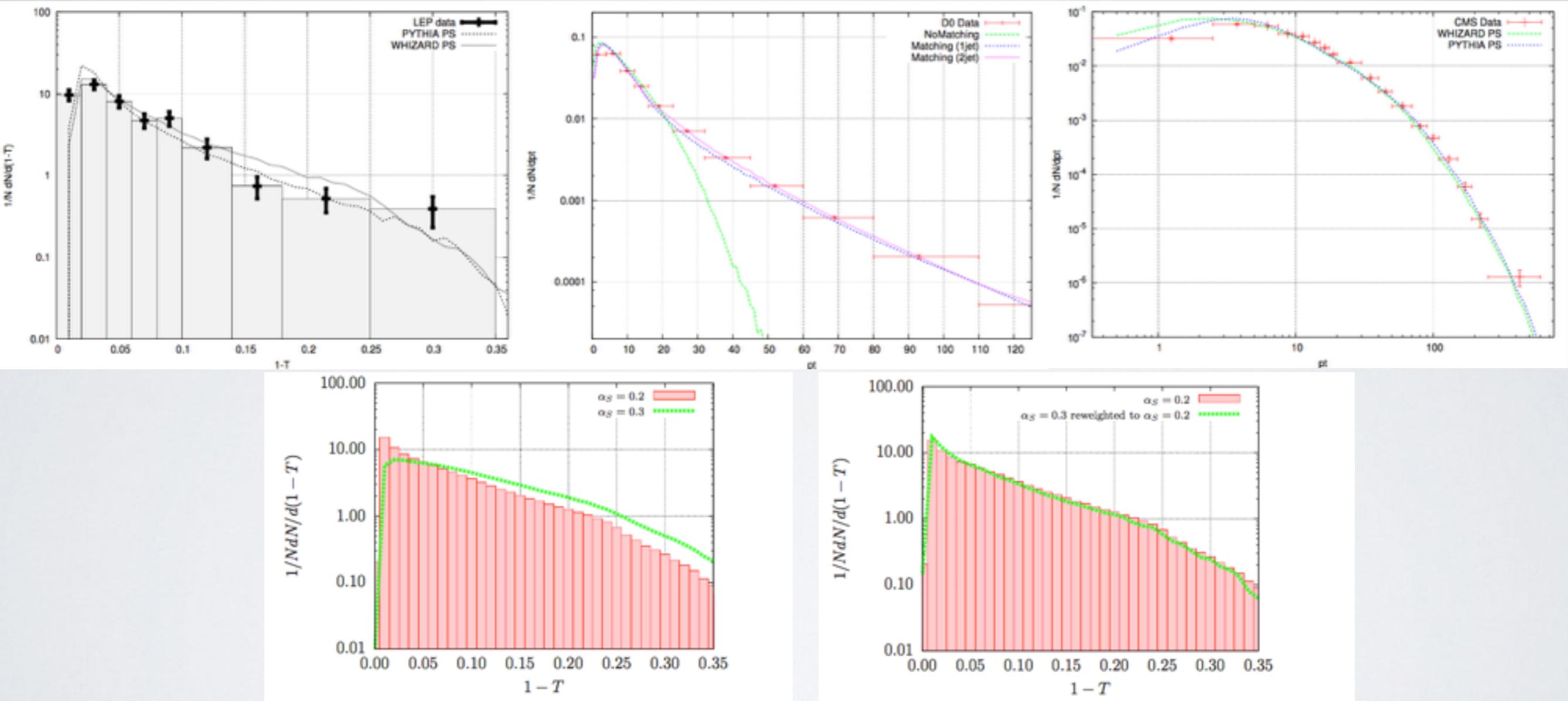




WHIZARD Parton Shower

- Two independent implementations: kT-ordered QCD and Analytic QCD shower
- Analytic shower: no shower veto \Rightarrow exact shower history known, allows reweighting

Kilian/JRR/Schmidt/Wiesler, JHEP 1204 013 (2012)



- Technical overhaul of the shower / merging part
- Plans: implement GKS matching, QED shower (also interleaved, infrastructure ready)





Tuning of the WHIZARD Parton Shower

- ▶ First tunes of both kT-ordered QCD and Analytic QCD shower
- ▶ Di- and Multijet data from LEP as given in RIVET analysis
- ▶ Usage of the PROFESSOR tool for determining the best fit

Chokoufe/Englert/JRR, 2015

Buckley et al., 2009

