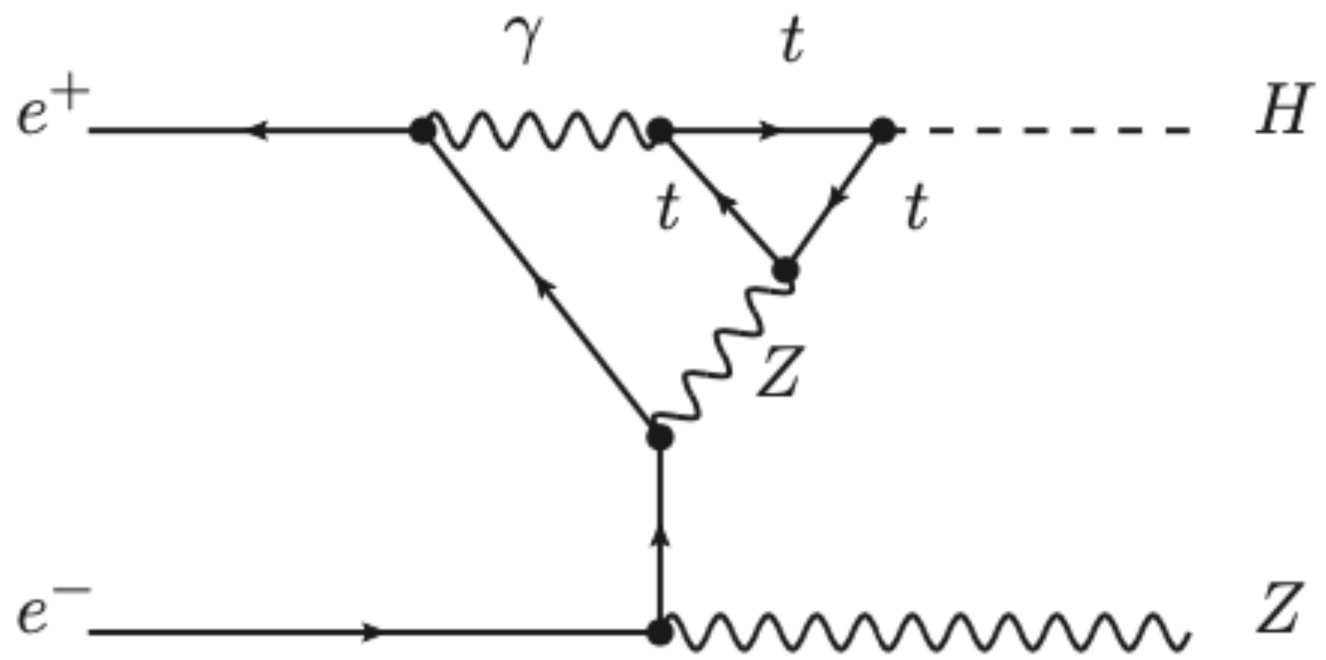
Theory Precision Calculations for future e^+e^- colliders: status and prospects

First ECFA WORKSHOP.

on e+e- Higgs/EW/Top Factories, October 5-7, 2022, in Hamburg



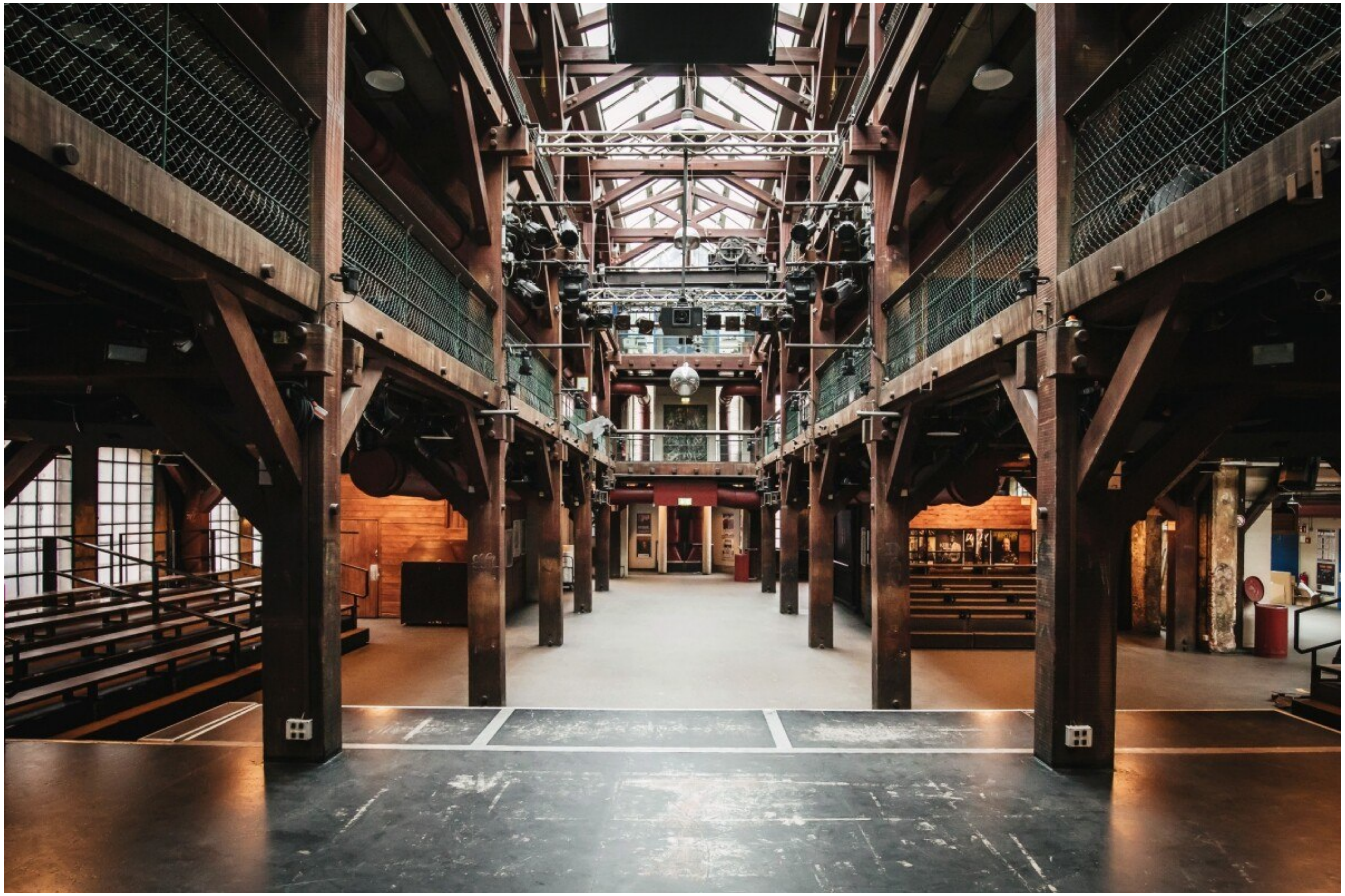
DESY. HELMHOLTZ

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

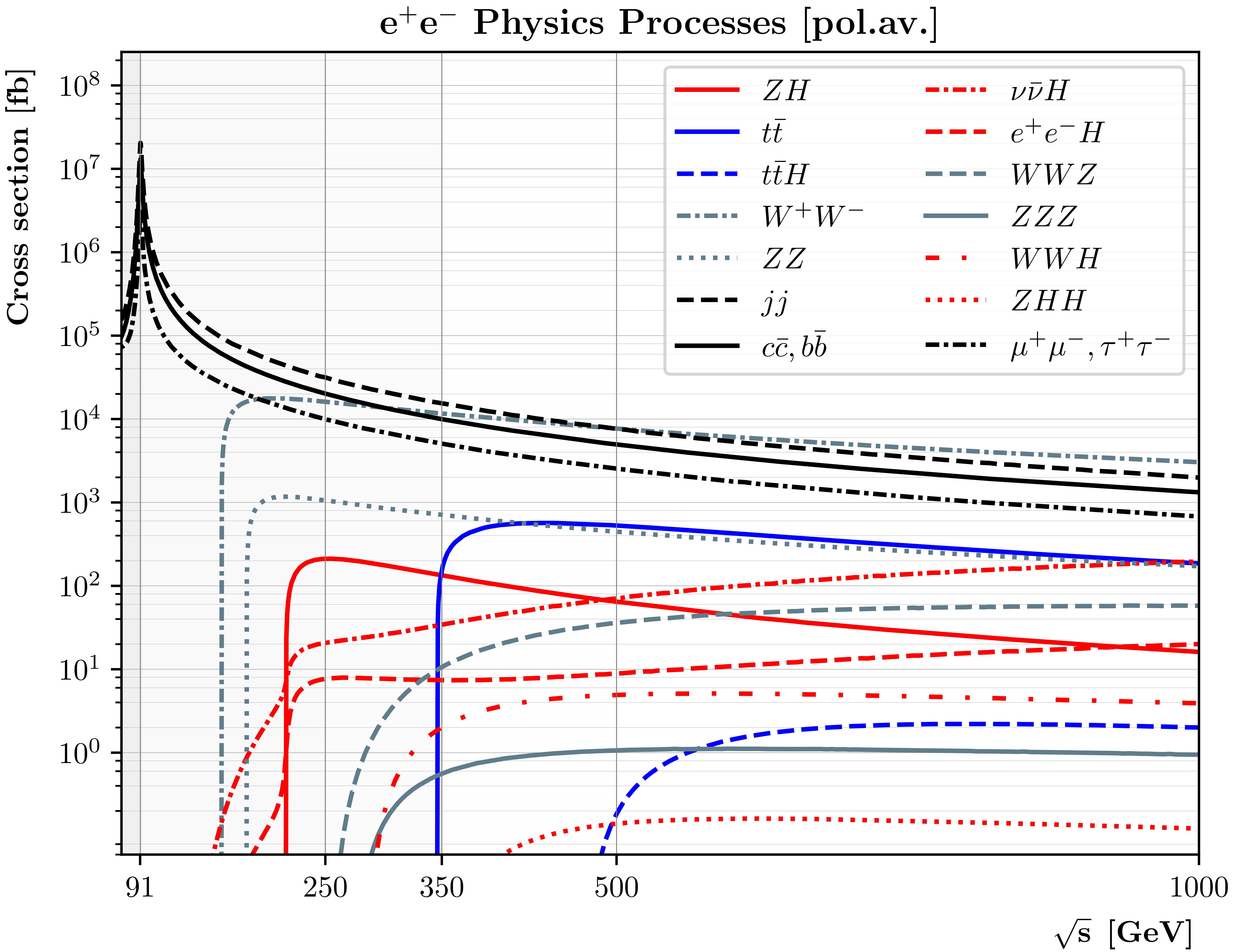
CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Jürgen R. Reuter

Hamburg Factory [Fabrik] — Higgs Factory

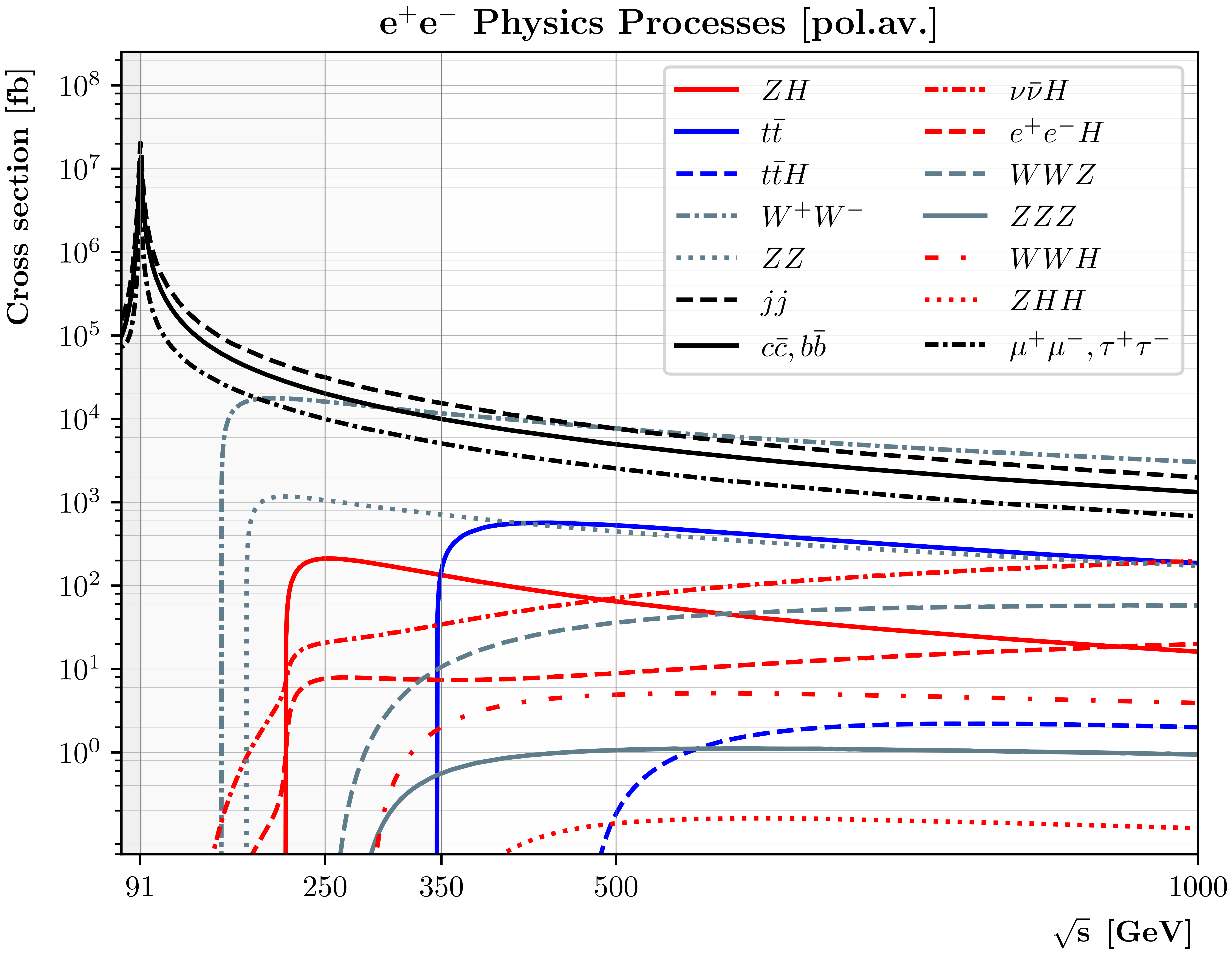


Disclaimer



$\mathcal{O}$*(20 min)* talk: just a faint glimpse

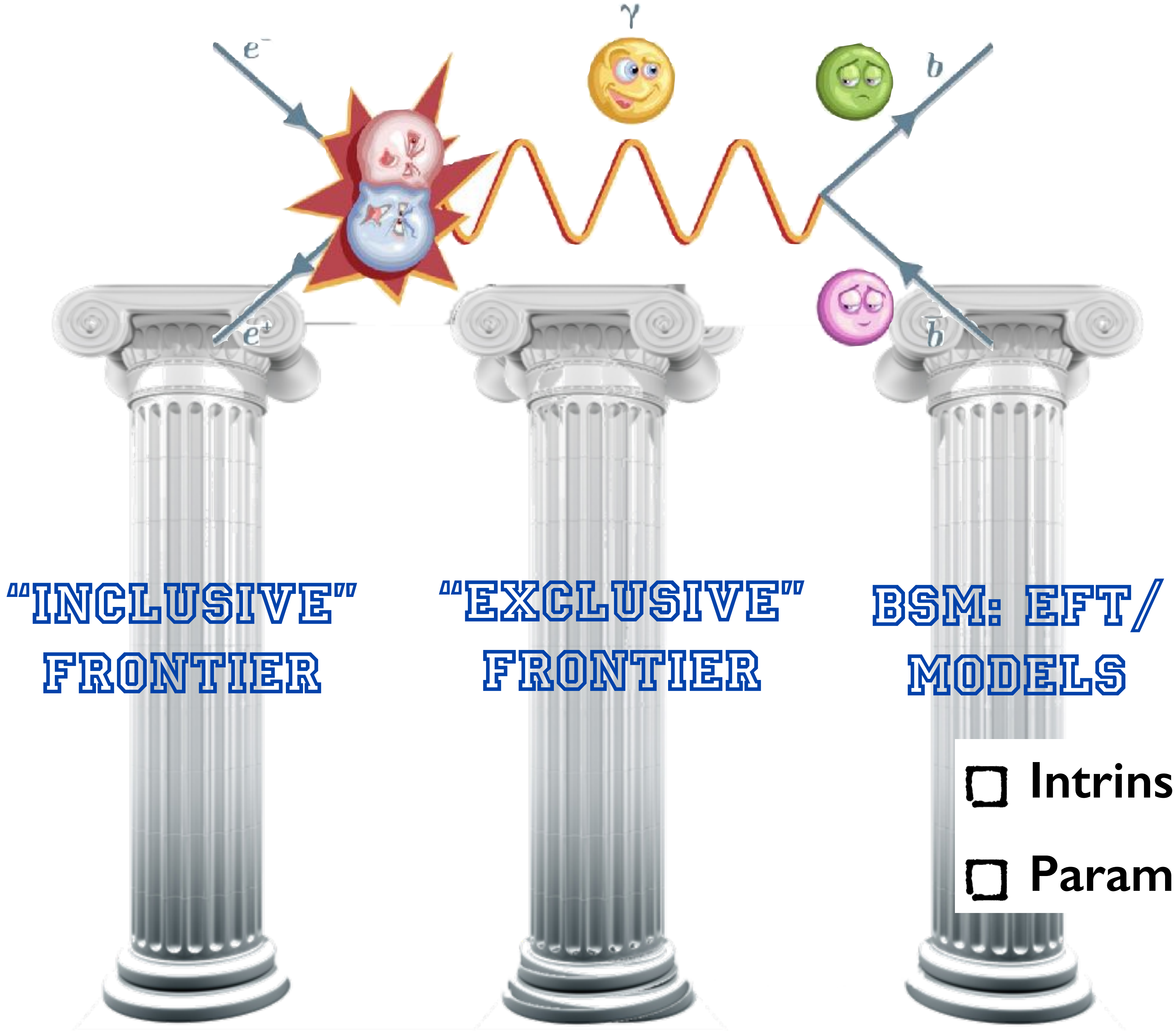
Disclaimer



$\mathcal{O}$*(20 min)* talk: just a faint glimpse

Theory precision landscape

- LHC HXSWG / LHC EWWG / LHC EFTWG: 1101.0593, 1201.3084, 1307.1347, 1610.07922
- LCGenG: focus mostly on complete SM samples for reconstruction
- FCC-ee theory effort: CERN workshops ’18-’22: 1906.05379
- US Snowmass CSS 2021 Reports: 2203.11110, 2209.08078, 2209.14872 etc.



- ECFA HTEF WS:
  - Simulation/MCs 11/21 https://indico.cern.ch/event/1078675/
  - Precision Calc. 05/22 https://indico.cern.ch/event/1140580/
- Other talks at this workshop: A. Freitas (overview precision), S. Heinemeyer (direct vs. indirect), A. Siodmok+A. Price (generators+QED), F. Krauss (hadronization), M. Steinhauser (multi-loop), D. Reichelt+S. Plätzer (QCD: shower+jets etc.), T. Ohl (luminosity spectra)

☐ **Intrinsic uncertainties:** missing higher-order calculations of observables

☐ **Parametric uncertainties:** imperfect knowledge or theoretic data extraction of SM input

Theoretical Uncertainties

- Strip loop amps. of group theory / mass ratios / multiplicities / couplings. → $\mathcal{O}(1)$
- Extrapolate to higher orders from geometric series (beware of renormalons)
- Scale dependence for missing higher order corrections (QCD, $\overline{\mathrm{MS}}$, less useful for EW)
- Compare differences in renormalisation schemes (e.g. On-Shell vs. $\overline{\mathrm{MS}}$)

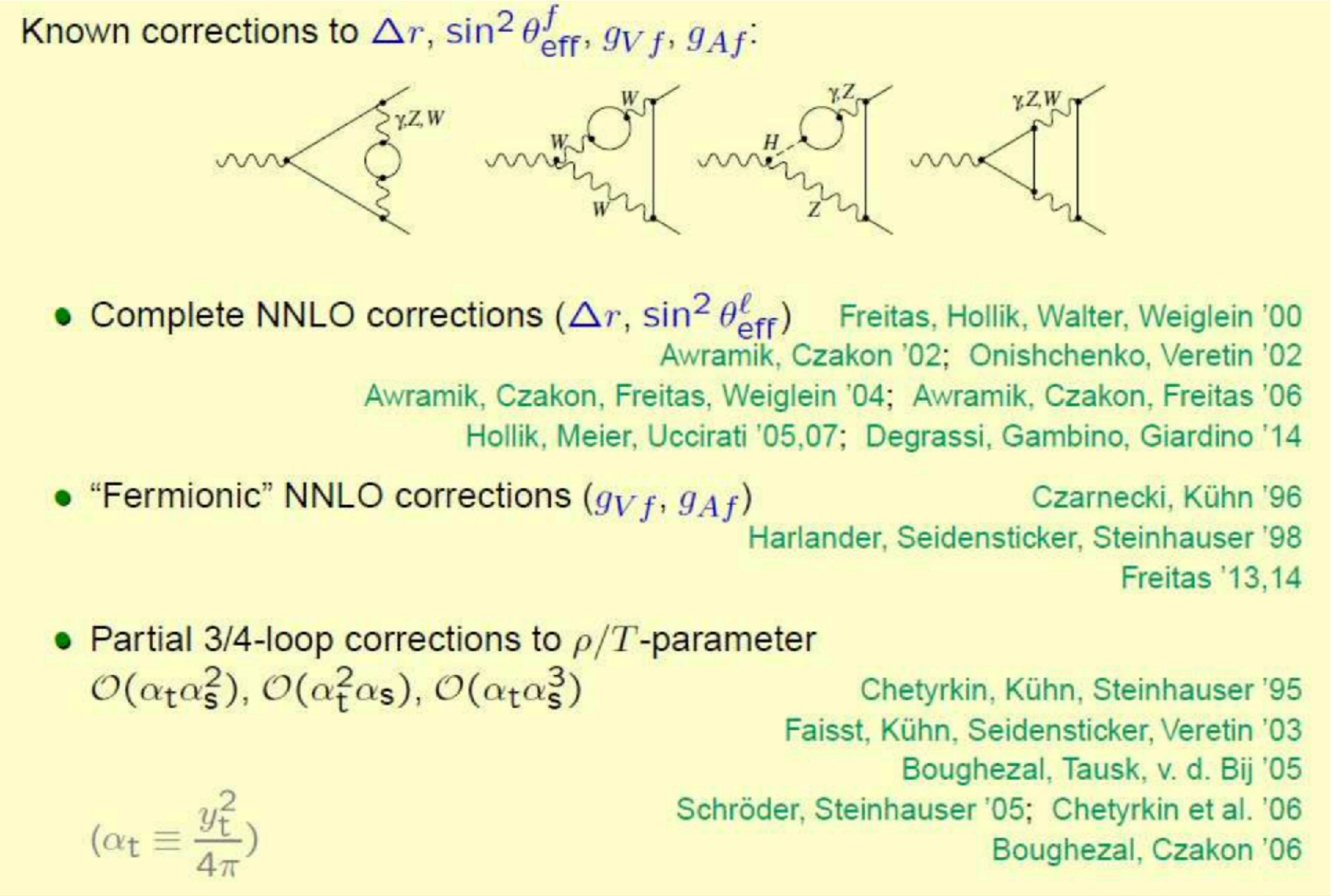
Parametric uncertainties

- M_H: Higgsstrahlung at threshold, 10 MeV uncertainty, leptonic recoil, minor th. uncertainties
- M_Z : Z lineshape, ~0.1 MeV exp., QED ISR+ISR/FSR, EW box diagrams, Jadach/Skrzypek/Pietrzik, 1999
- $\alpha_s (M_Z)$: global fit of overconstrained EW pseudo-observables at Z pole, pert. uncertainties
- $m_t^{MS} (m_t)$: N^{3}LO QCD/NNLO EW, resummed NNLL, 4-loop mass translation., off-shell corr.
  Beneke et al., 1506.06864/1711.10429, Hoang et al. 1309.6323, Marquard et al. 1502.01030, Chokoufé et al. 1609.03390, Bach et al. 1712.02220
- $m_{c/s}^{MS} (m_{c/s})$: lattice QCD, sum rules, NNLO jet ratios. 1404.0319, 1401.7035, 0907.2110, 1411.3132, 1504.07638
- $\Delta\alpha$: extracted from $e^+e^- \to$ *hadrons* , τ *decays* (BESIII, VEPP-2000, Belle II), radiative return
  Proposal for direct measurement below/above Z pole: subtract EW from QED corrections
  available @ 1-loop; needed fermionic 2-loop corr., $\mathcal{O}(\alpha^2, \alpha^2\alpha_s)$ corr. $\Longrightarrow$ 10^{-4}
  2-/3-loop box diagrams: full $\mathcal{O}(\alpha\alpha_s^2)$, double-fermionic $\mathcal{O}(\alpha^3)$ corr. $\Longrightarrow$ 10^{-5}

The “Inclusive” Frontier

Parametric uncertainties

$$\delta m_t = 50\ \mathrm{MeV}, \quad \delta m_b = 13\ \mathrm{MeV}, \quad \delta M_Z = 0.1\ \mathrm{MeV}, \quad \delta\alpha_s = 0.0002\ (0.0001),$$
$$\delta(\Delta\alpha) = 5 \times 10^{-5}\ (3 \times 10^{-5}).$$



Known corrections to Δr, $\sin^2\theta^f_{\text{eff}}$, g_{Vf}, g_{Af}:

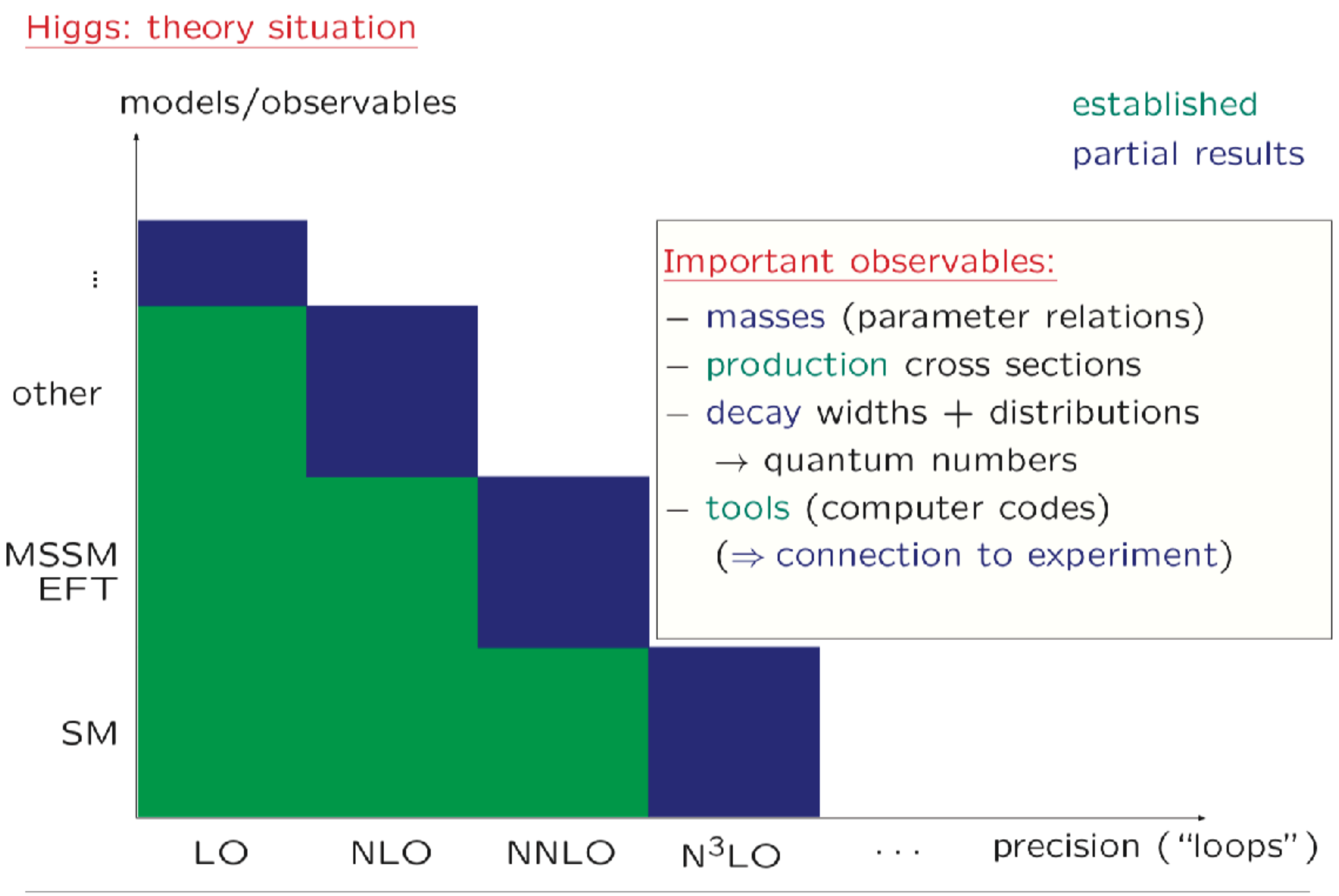
- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\text{eff}}$) Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02; Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- “Fermionic” NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96
  Harlander, Seidensticker, Steinhauser '98
  Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05; Chetyrkin et al. '06
  $(\alpha_t \equiv \frac{y_t^2}{4\pi})$ Boughezal, Czakon '06

A. Freitas, 1604.00406

↪ Talk by A. Freitas

↪ Talk by M. Steinhauser

Higgs Precision Calculations



Higgs: theory situation

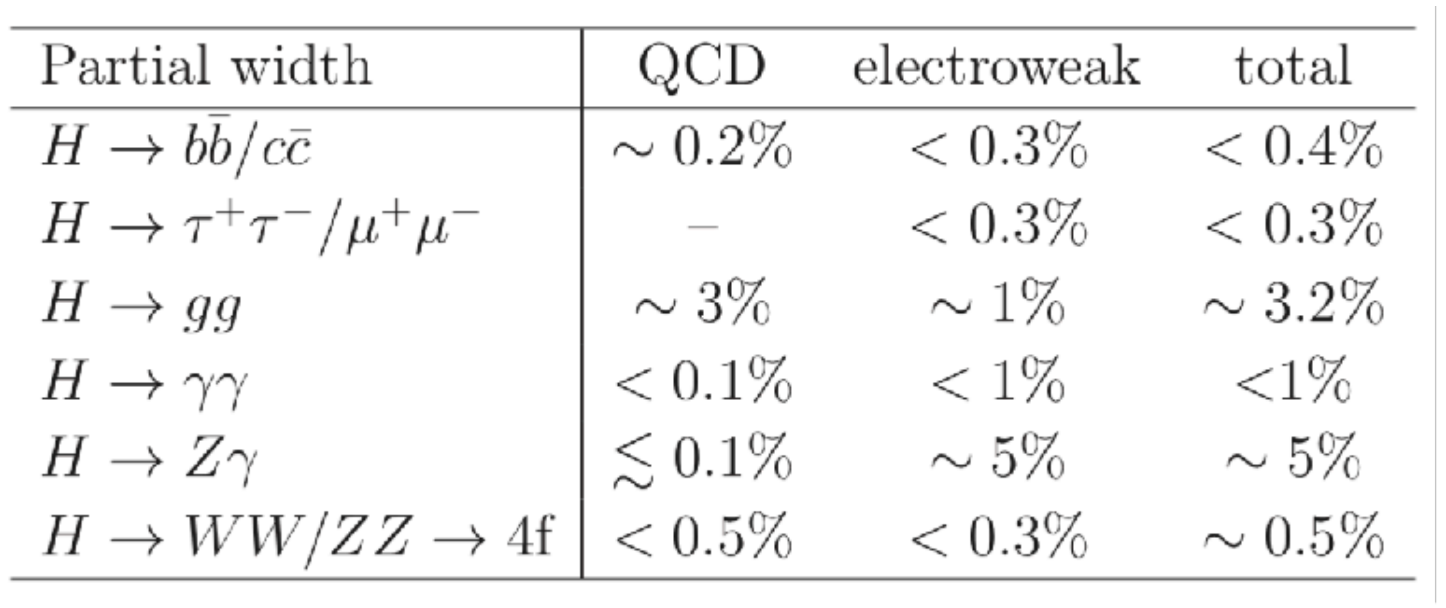
Important observables:

- masses (parameter relations)
- production cross sections
- decay widths + distributions
  → quantum numbers
- tools (computer codes)
  (⇒ connection to experiment)

from S. Heinemeyer

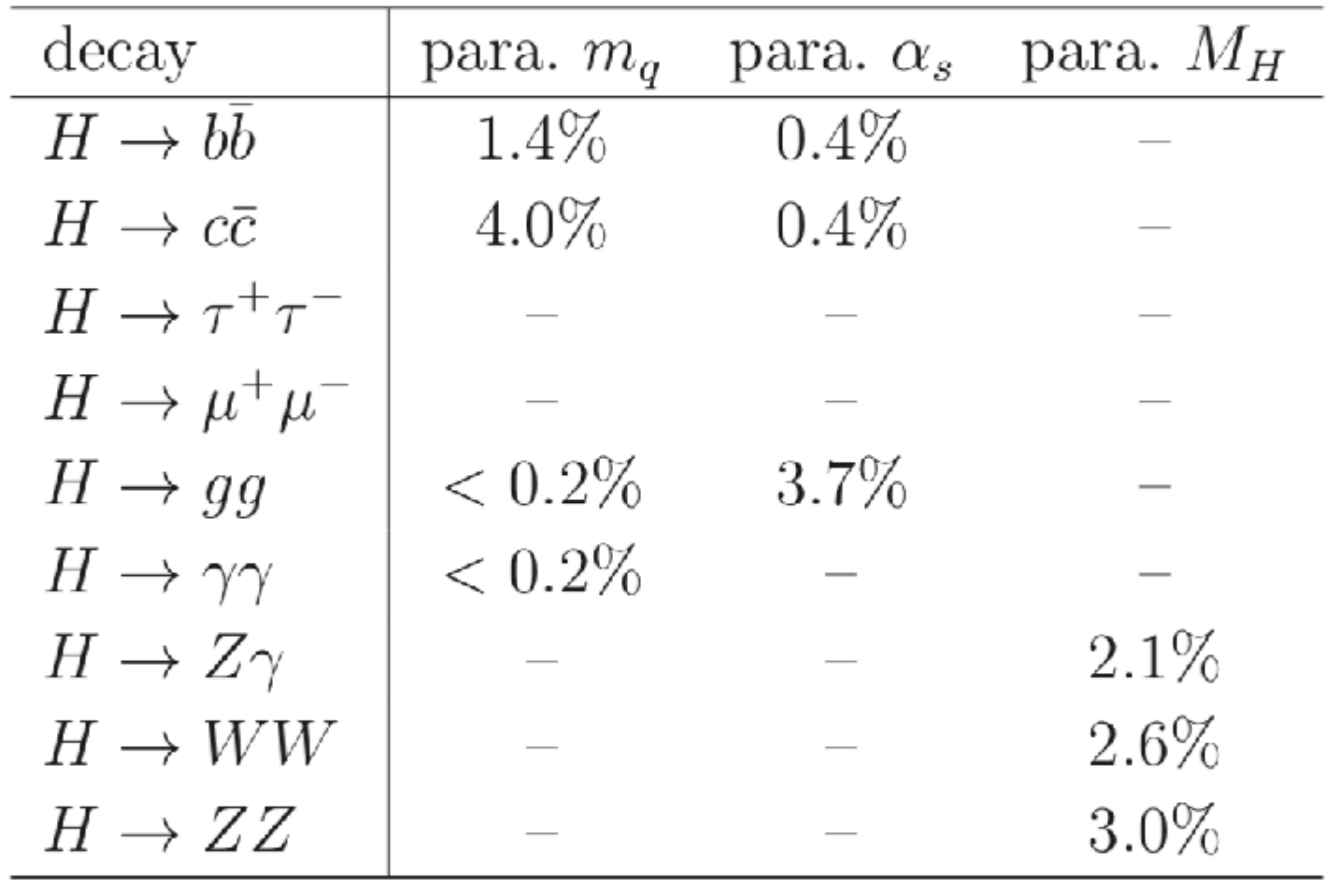
Parametric Higgs decay uncertainties, Lepage/McKenzie/Peskin, 1404.0319

- Full NLO EW exists for *ee* → *ZH*, Denner/Dittmaier/Roth/Weber, hep-ph/0311089
  ee → *ννH* Belanger/Boudjema/Fujimoto/Ishikawa/Kaneki/Kato/Shimizu, hep-ph/0212261



Partial width	QCD	electroweak	total
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$
$H \to \tau^+\tau^-/\mu^+\mu^-$	–	$< 0.3\%$	$< 0.3\%$
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$
$H \to \gamma\gamma$	$< 0.1\%$	$< 1\%$	$<1\%$
$H \to Z\gamma$	$\lesssim 0.1\%$	$\sim 5\%$	$\sim 5\%$
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$

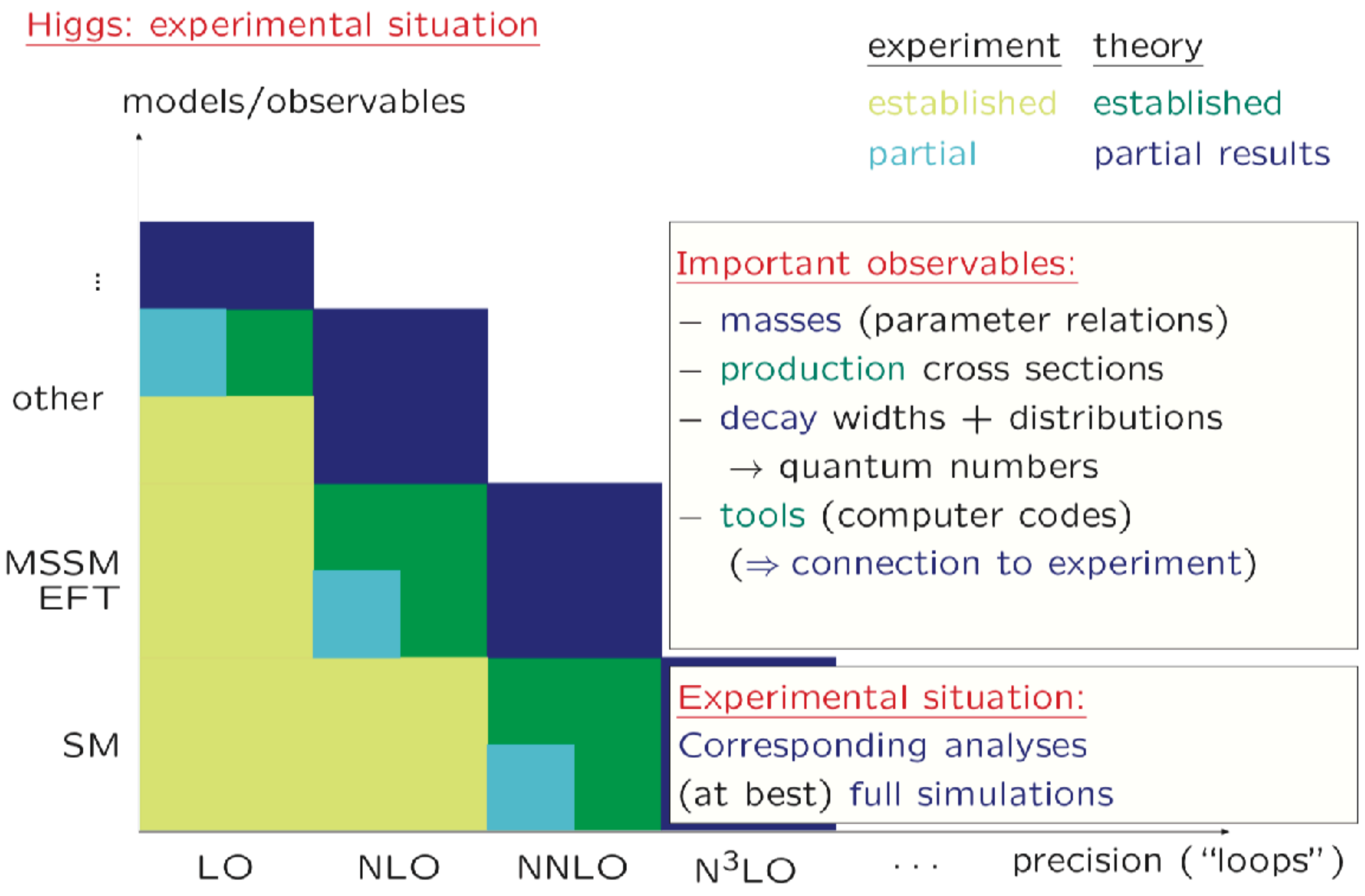
Intrinsic Higgs decay uncertainties, LHCHXSWG



decay	para. m_q	para. α_s	para. M_H
$H \to b\bar{b}$	1.4%	0.4%	–
$H \to c\bar{c}$	4.0%	0.4%	–
$H \to \tau^+\tau^-$	–	–	–
$H \to \mu^+\mu^-$	–	–	–
$H \to gg$	$< 0.2\%$	3.7%	–
$H \to \gamma\gamma$	$< 0.2\%$	–	–
$H \to Z\gamma$	–	–	2.1%
$H \to WW$	–	–	2.6%
$H \to ZZ$	–	–	3.0%

5-10% NLO corrections

Higgs Precision Calculations



Higgs: experimental situation

experiment	theory
established	established
partial	partial results

Important observables:

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- tools (computer codes)
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Experimental situation:
Corresponding analyses
(at best) full simulations

from S. Heinemeyer

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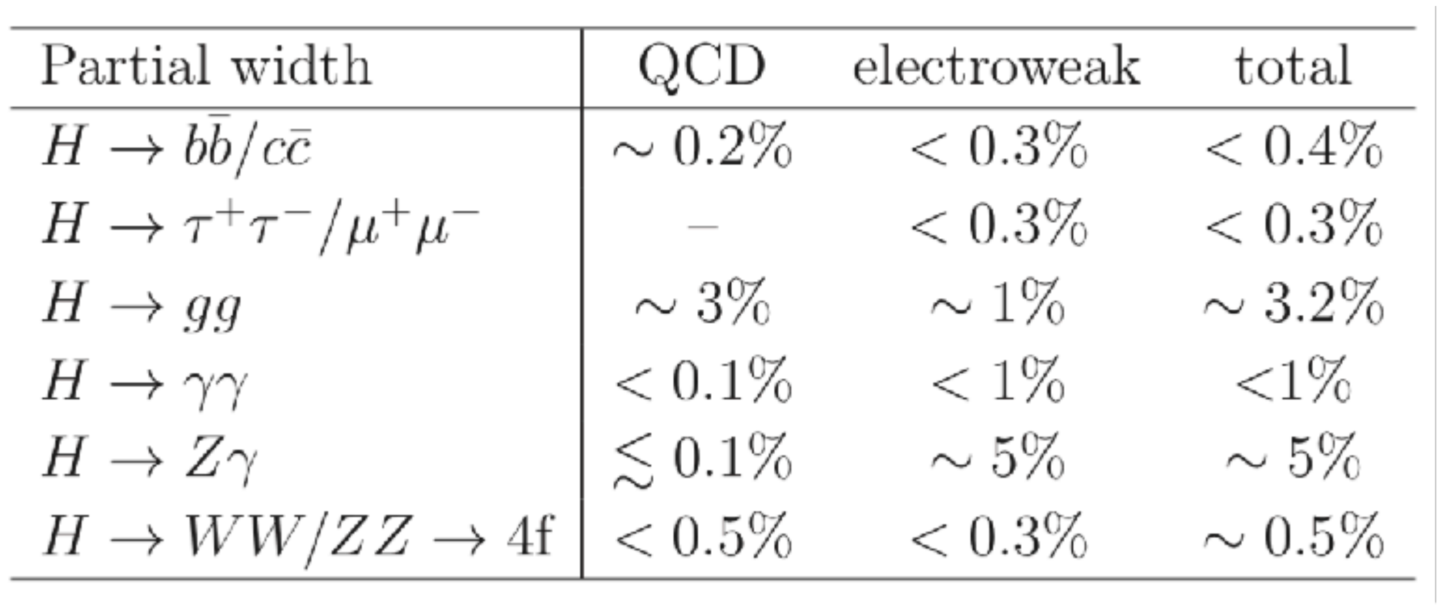
- Full 2-loop for ee → *ZH* available Chen/Guan/He/Li/Liu/Ma, 2209.14953
- Missing NNLO EW corrections *[2→2, 2→3]* : intrinsic uncertainty 1%
- Compared to experimental uncertainty of 0.5-1.0%



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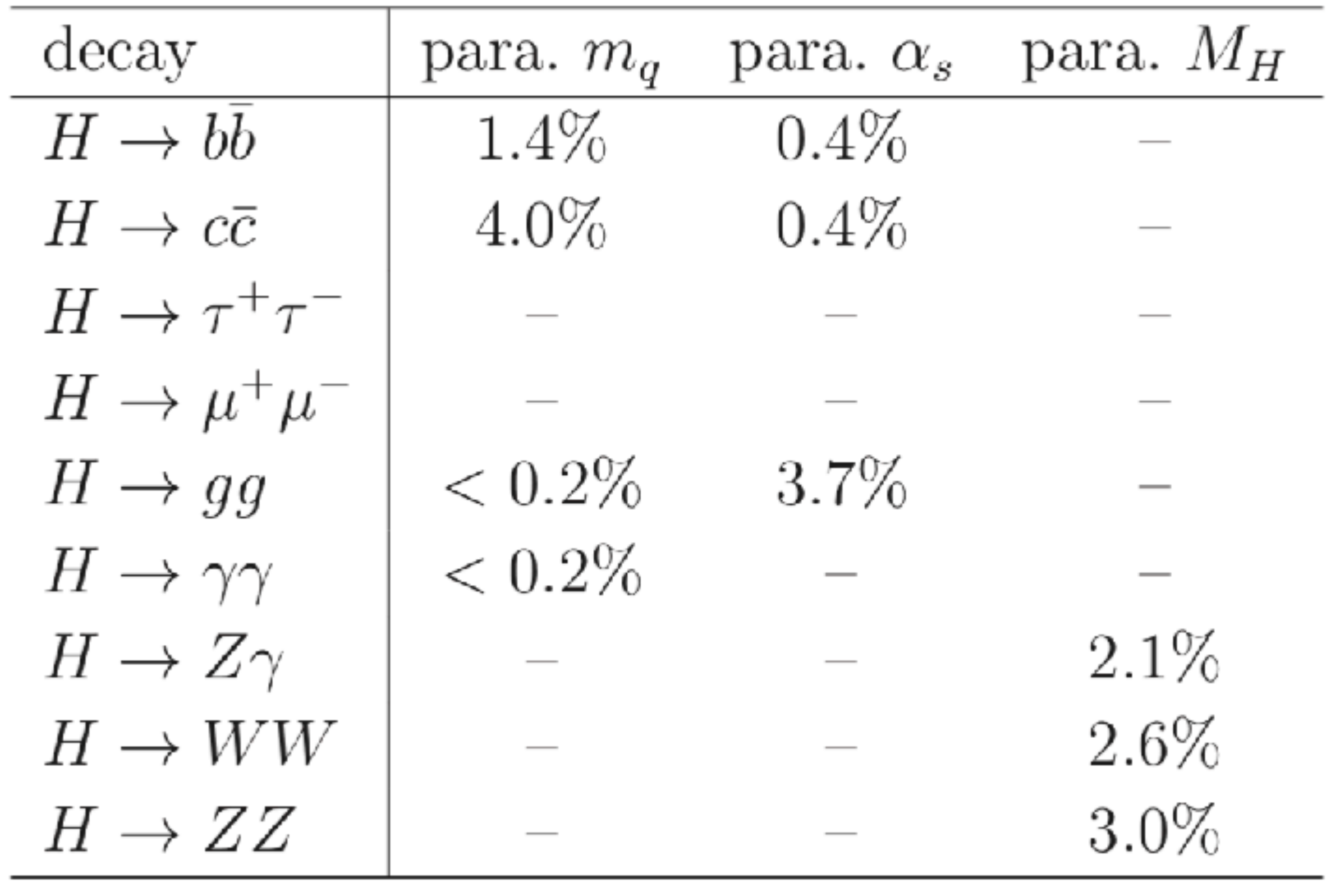
Intrinsic Higgs decay uncertainties, LHCHXSWG



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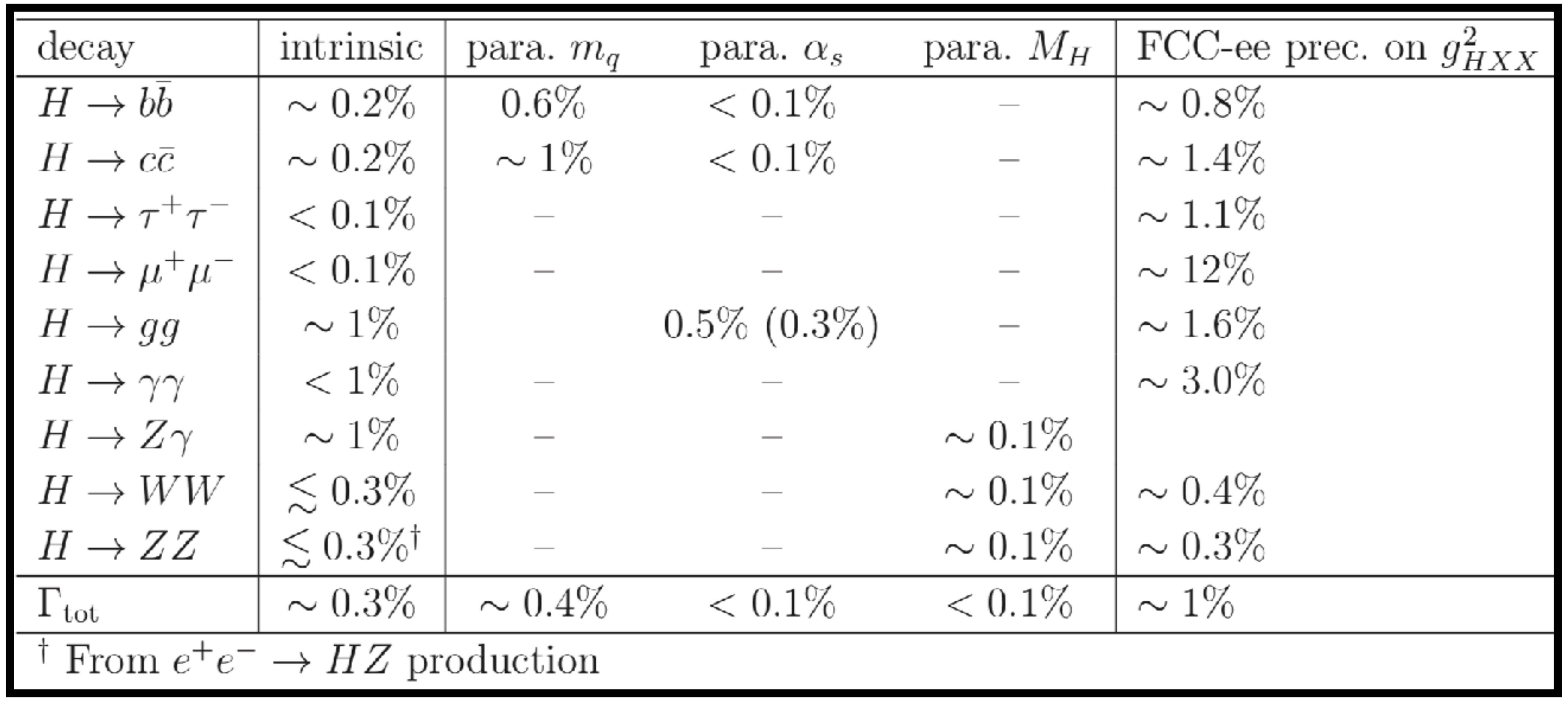
$H \to Z\gamma$	–	–	2.1%
$H \to WW$	–	–	2.6%
$H \to ZZ$	–	–	3.0%

5-10% NLO corrections

NNLO EW
hard task for
VBF !

Higgs Precision Calculations

ILC/FCC-ee projections



decay	intrinsic	para. m_q	para. α_s	para. M_H	FCC-ee prec. on g^2_{HXX}
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	$\sim 0.8\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	$\sim 1.4\%$
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	$\sim 1.1\%$
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	$\sim 12\%$
$H \to gg$	$\sim 1\%$		0.5% (0.3%)	–	$\sim 1.6\%$
$H \to \gamma\gamma$	$< 1\%$	–	–	–	$\sim 3.0\%$
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \to WW$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
$H \to ZZ$	$\lesssim 0.3\%^{\dagger}$	–	–	$\sim 0.1\%$	$\sim 0.3\%$
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

† From $e^+e^- \to HZ$ production

1906.05379

Needed theory effort

- $H \to qq$ ~ N^{4}LO QCD, ≲ $\mathcal{O}(\alpha^2, \alpha\alpha_s)$ ✅
- $H \to gg$ ~ N^{3}LO QCD scale, ≲ $\mathcal{O}(\alpha^2)$ ❌ [N^{4}LO QCD: massless 4-loop]
- $H \to \gamma\gamma$ ≲ $\mathcal{O}(\alpha^2)$ light-fermion dominate ✅
- $H \to Z\gamma$ ≲ $\mathcal{O}(\alpha)$ *NLO EW* smaller than exp. ✅
- $H \to WW, ZZ$ NLO QCD corr., ❌ [non-factorizable NNLO QCD]

Electroweak Precision Physics

$$\sigma^0_{\rm had} = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}], \qquad \text{(from a fit to } \sigma_f(s) \text{ at various values of } s)$$

$$R_\ell = \left[\textstyle\sum_q \sigma_q(M_Z^2)\right]/\sigma_\ell(M_Z^2), \qquad (\ell = e, \mu, \tau)$$

$$R_q = \sigma_q(M_Z^2)/\left[\textstyle\sum_q \sigma_q(M_Z^2)\right], \qquad (q = b, c)$$

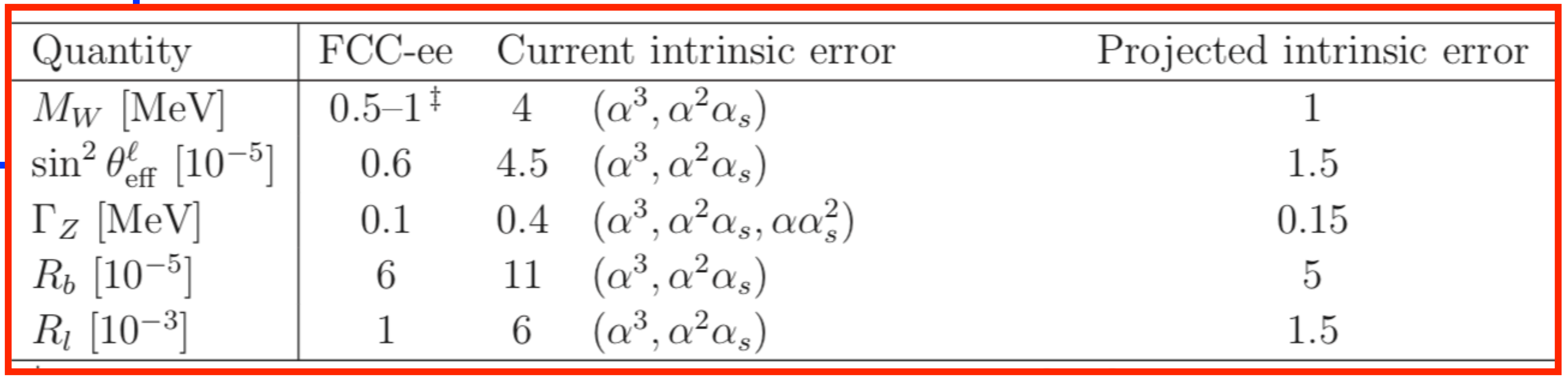
$$A^f_{\rm FB} = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \tfrac{3}{4}\mathcal{A}_e\mathcal{A}_f,$$

$$A^f_{\rm LR} = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e|P_e|.$$

$$\mathcal{A}_f = \frac{1 - 4|Q_f|\sin^2\theta^f_{\rm eff}}{1 - 4|Q_f|\sin^2\theta^f_{\rm eff} + 8(Q_f\sin^2\theta^f_{\rm eff})^2}.$$

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

$\mathcal{O}(\alpha\alpha_s^2)$ complete
$\mathcal{O}(\alpha^2\alpha_s)$ fermionic
$\mathcal{O}(\alpha^3)$ double-fermionic
$\mathcal{O}(\alpha_t\alpha_s^3)$ 4-loop



Quantity	FCC-ee	Current intrinsic error	Projected intrinsic error
M_W [MeV]	0.5–1‡	4 $(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	0.6	4.5 $(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4 $(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b [10^{-5}]	6	11 $(\alpha^3, \alpha^2\alpha_s)$	5
R_l [10^{-3}]	1	6 $(\alpha^3, \alpha^2\alpha_s)$	1.5

Theoretical uncertainties for *WW* threshold don't match exp. precision: 3 GeV uncertainty

Beneke/Falgari/Schwinn/Signer/Zanderighi, 0707.0773; Actis/Beneke/Falgari/Schwinn, 0807.0102; C. Schwinn, in 1905.05078

needed: full 2-loop corr. $e^+e^- \to W^+W^-$ and $W \to ff$, ISR & matching (later); 3-loop Coulomb-enhanced

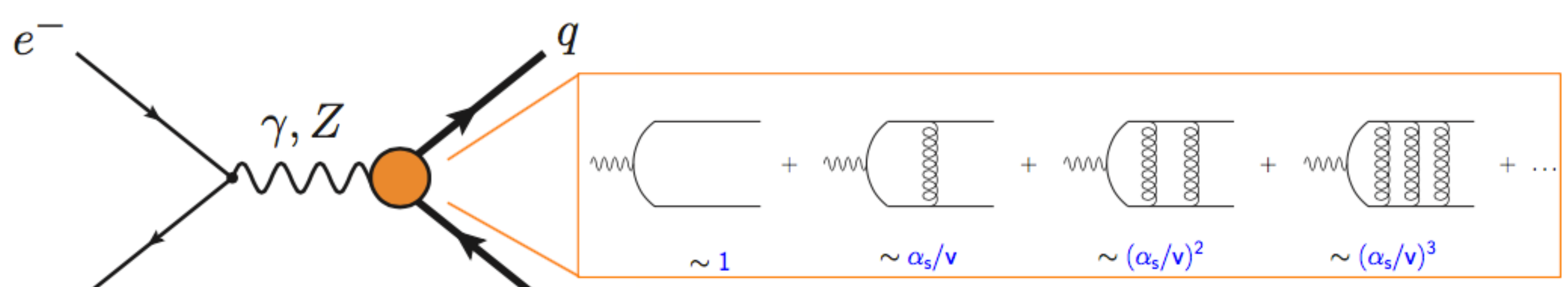
↪ Talk by A. Freitas

New efforts in $e^+e^- \to ff$ (2-loop, logarithmic corr.)

Blümlein/de Freitas/Raab/Schönwald, 1901.08018, 1910.05759, 2003.14283, 2004.04287

The Top Threshold

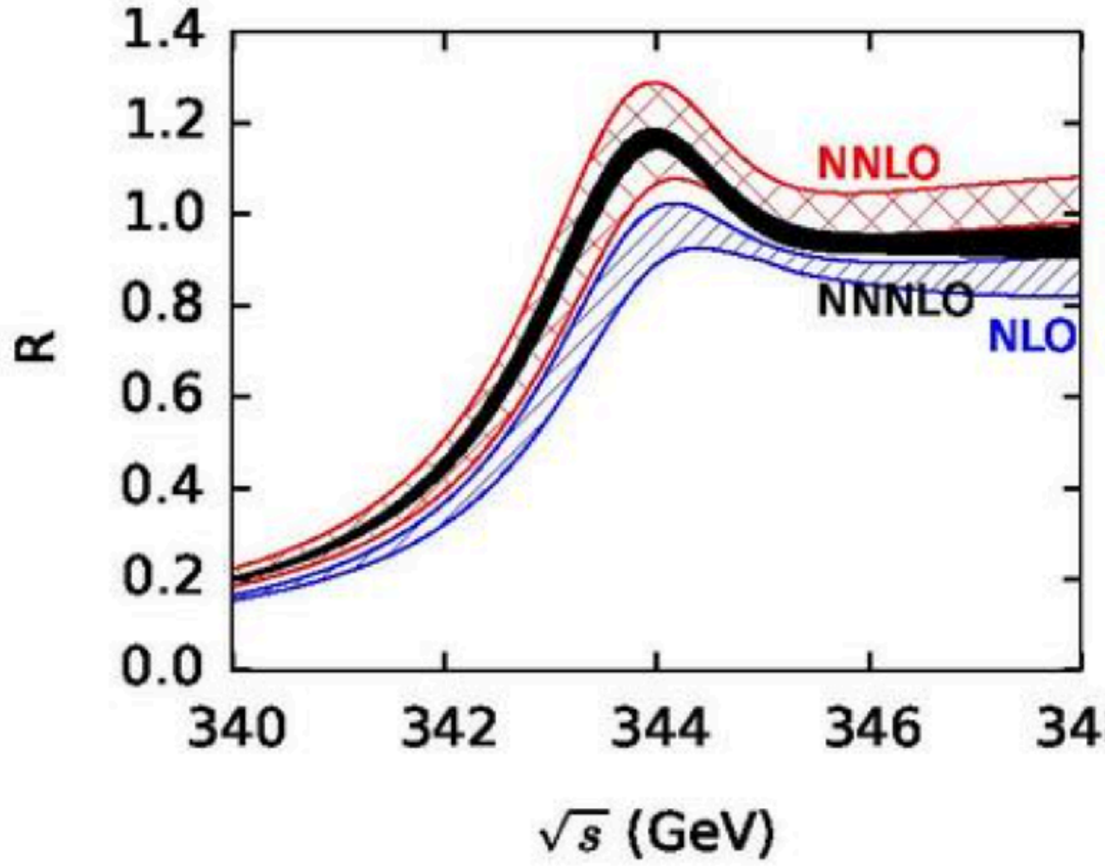
↪ Talk by F. Simon + WG1 HTE



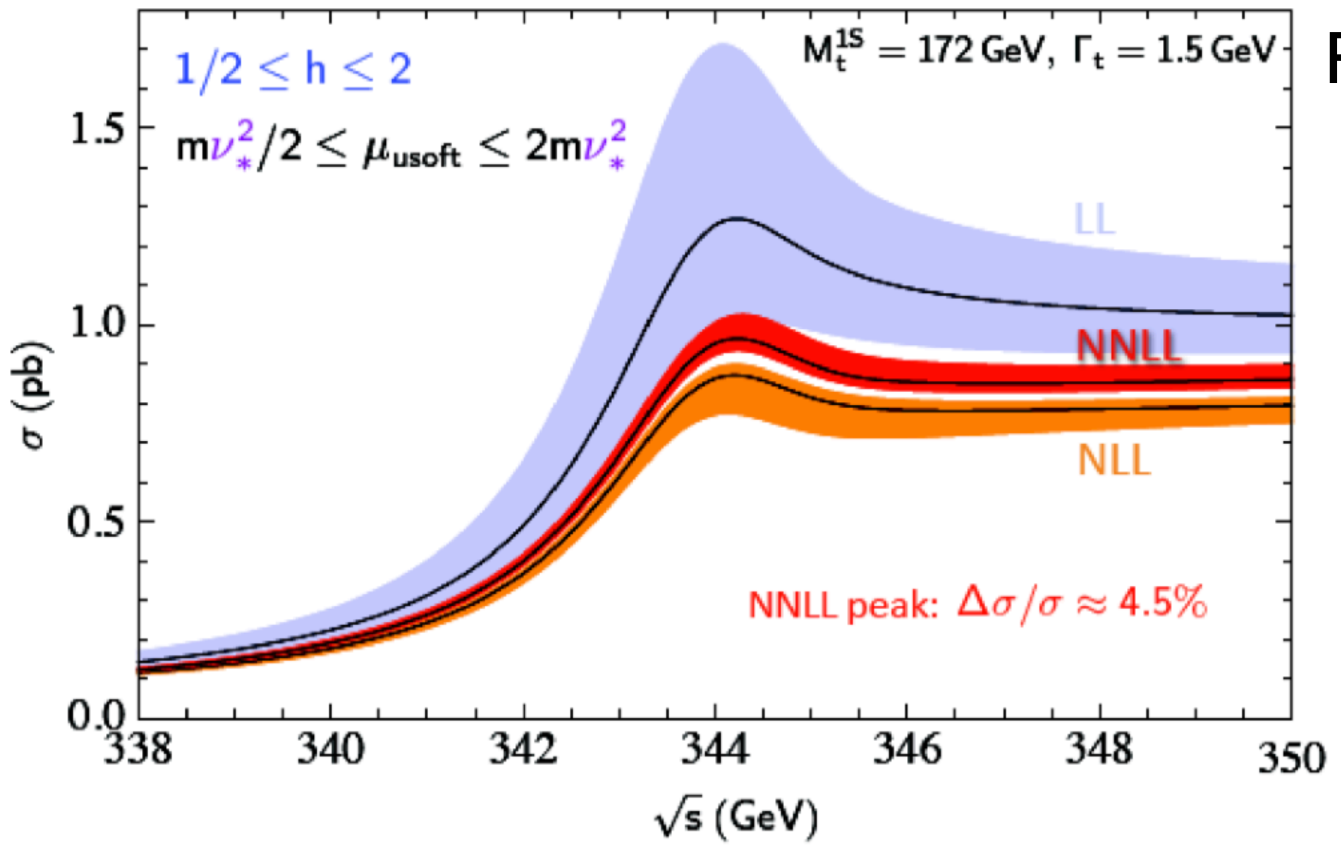
$v \sim \alpha_s \ll 1$

NRQCD NNNLO fixed order
+ α_s logarithms

Kiyo et al., 2005; Beneke et al., 2008-2015

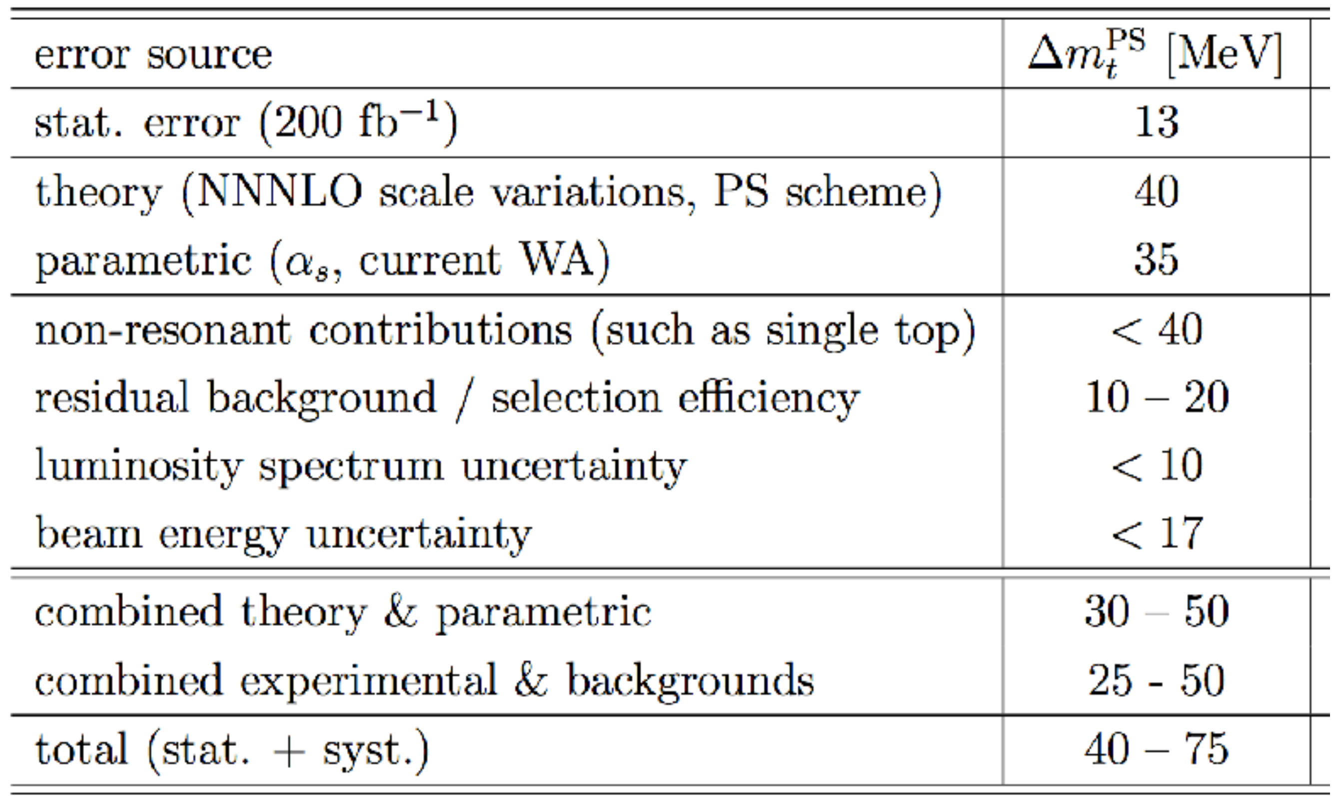


- N3LO NRQCD & NNLL resummation
- Translation IR and MSbar mass under control
- Event selection needs differential predictions
- NLO⊕NLL
  [NNLO⊕NNLL needed]



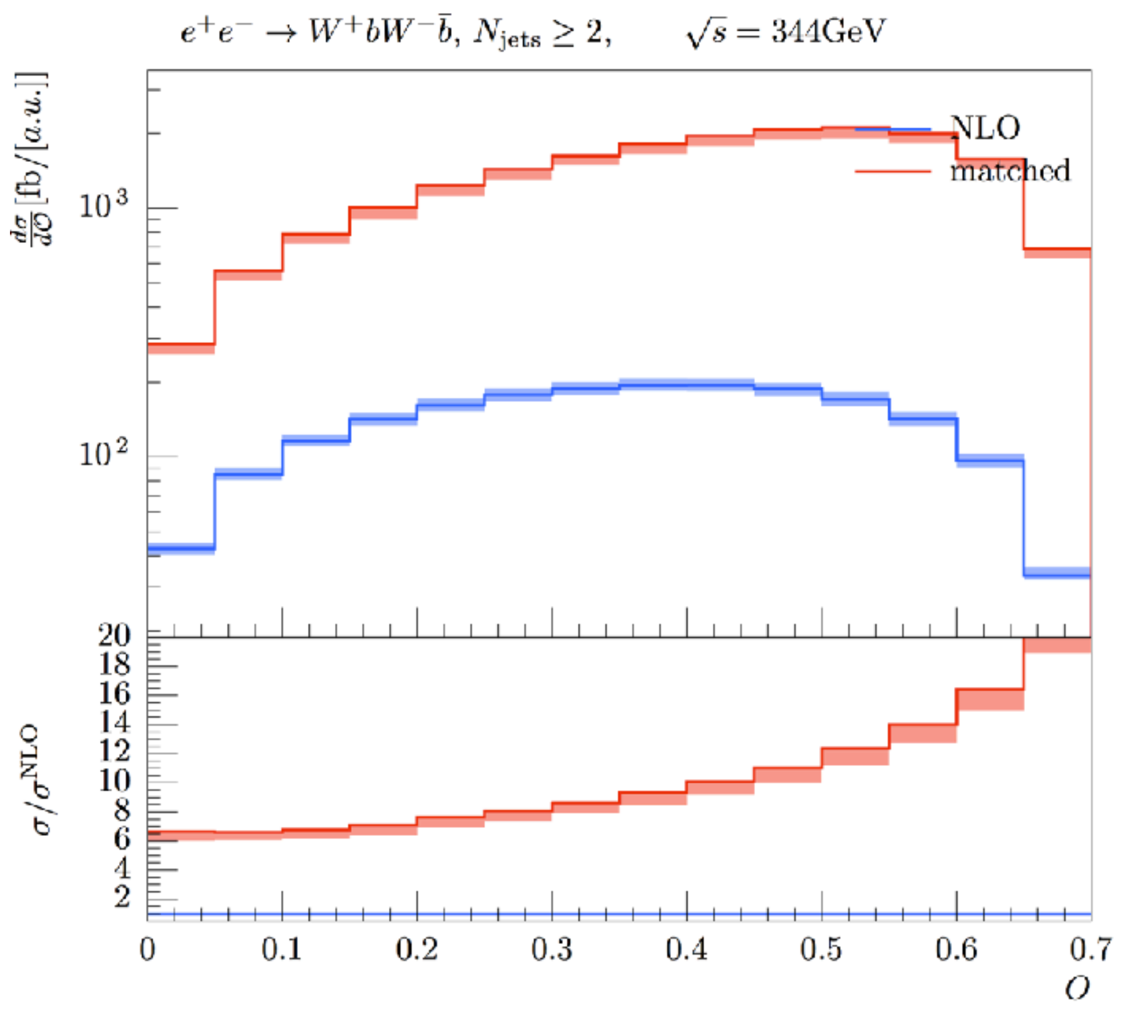
Resummation of
velocity logarithms

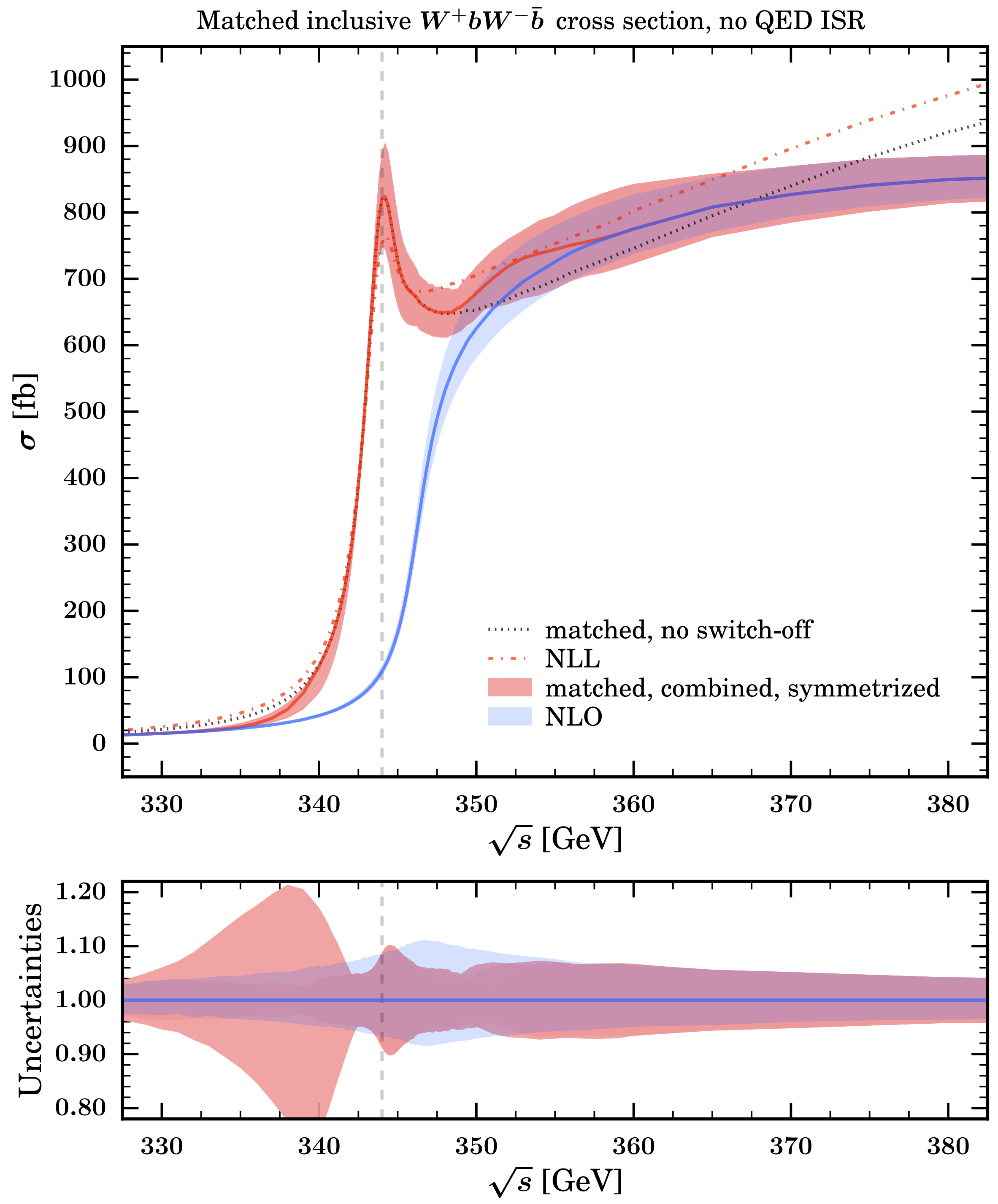
Hoang/Stahlhofen, 2012



error source	$\Delta m_t^{\rm PS}$ [MeV]
stat. error (200 fb^{-1})	13
theory (NNNLO scale variations, PS scheme)	40
parametric (α_s, current WA)	35
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 - 50
total (stat. + syst.)	40 – 75

Marquard/Smirnov/Smirnov/Steinhauser, 1502.01030





Bach ea., 2017

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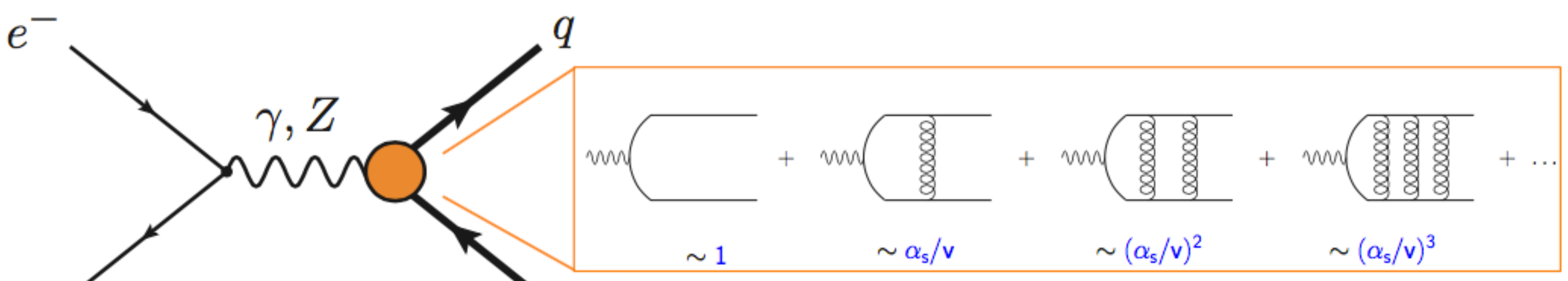
↪ Talk by F. Simon + WG1 HTE



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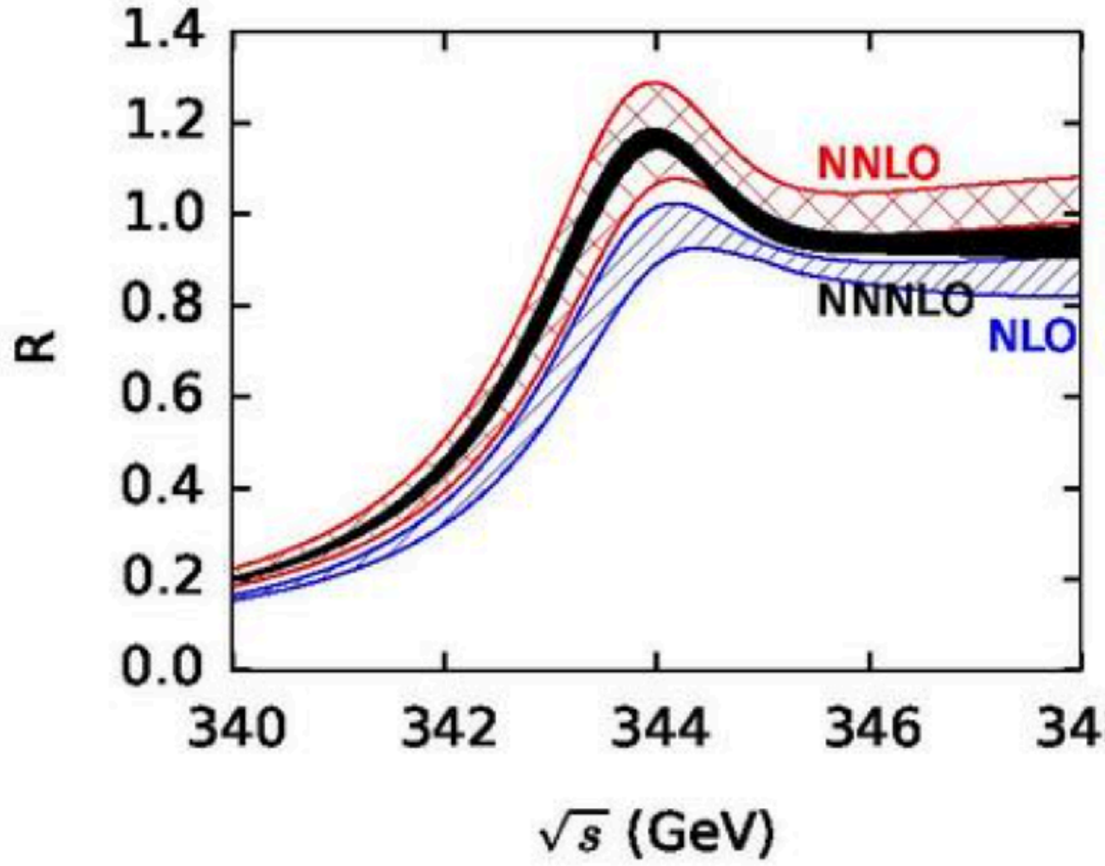
Kiyo et al., 2005; Beneke et al., 2008-2015



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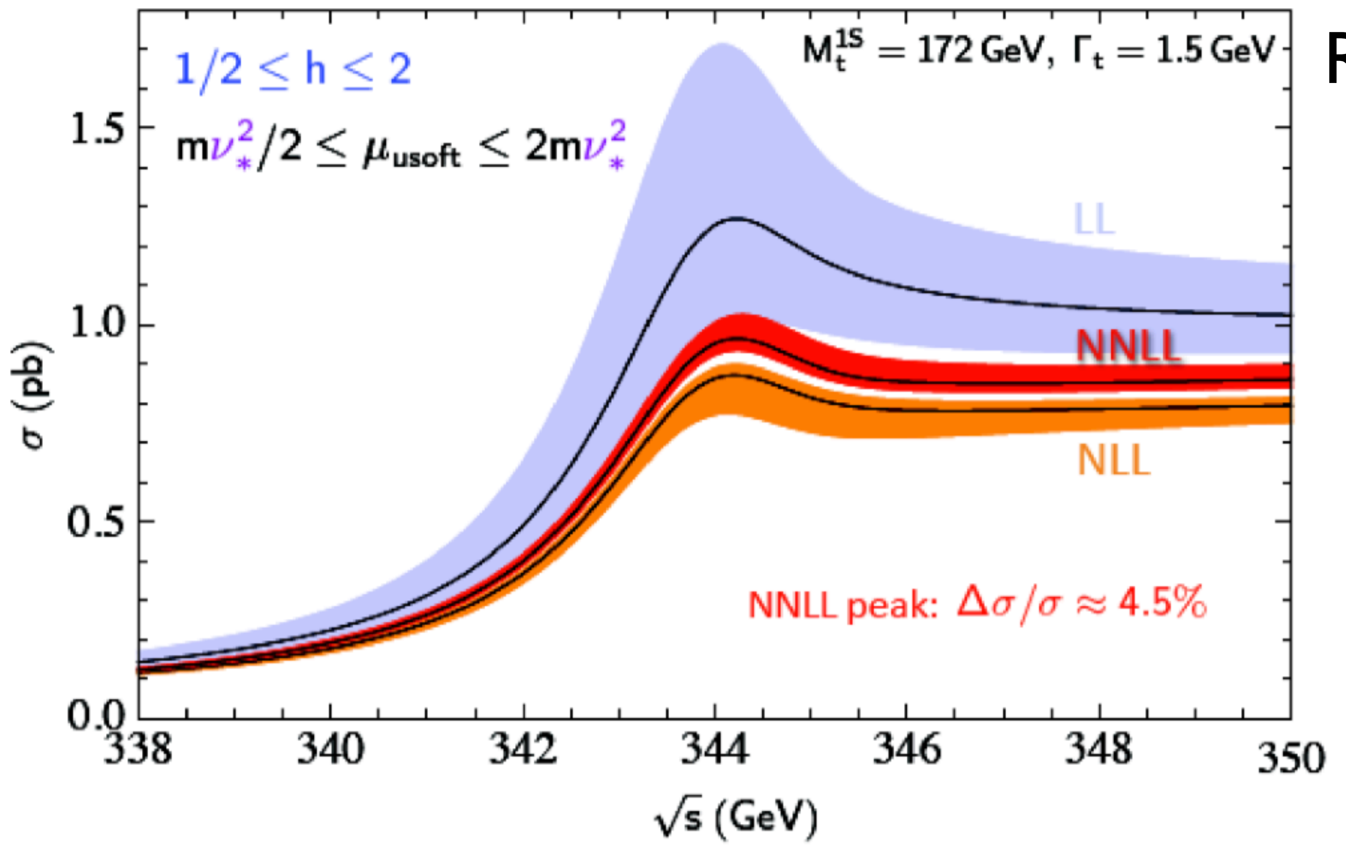

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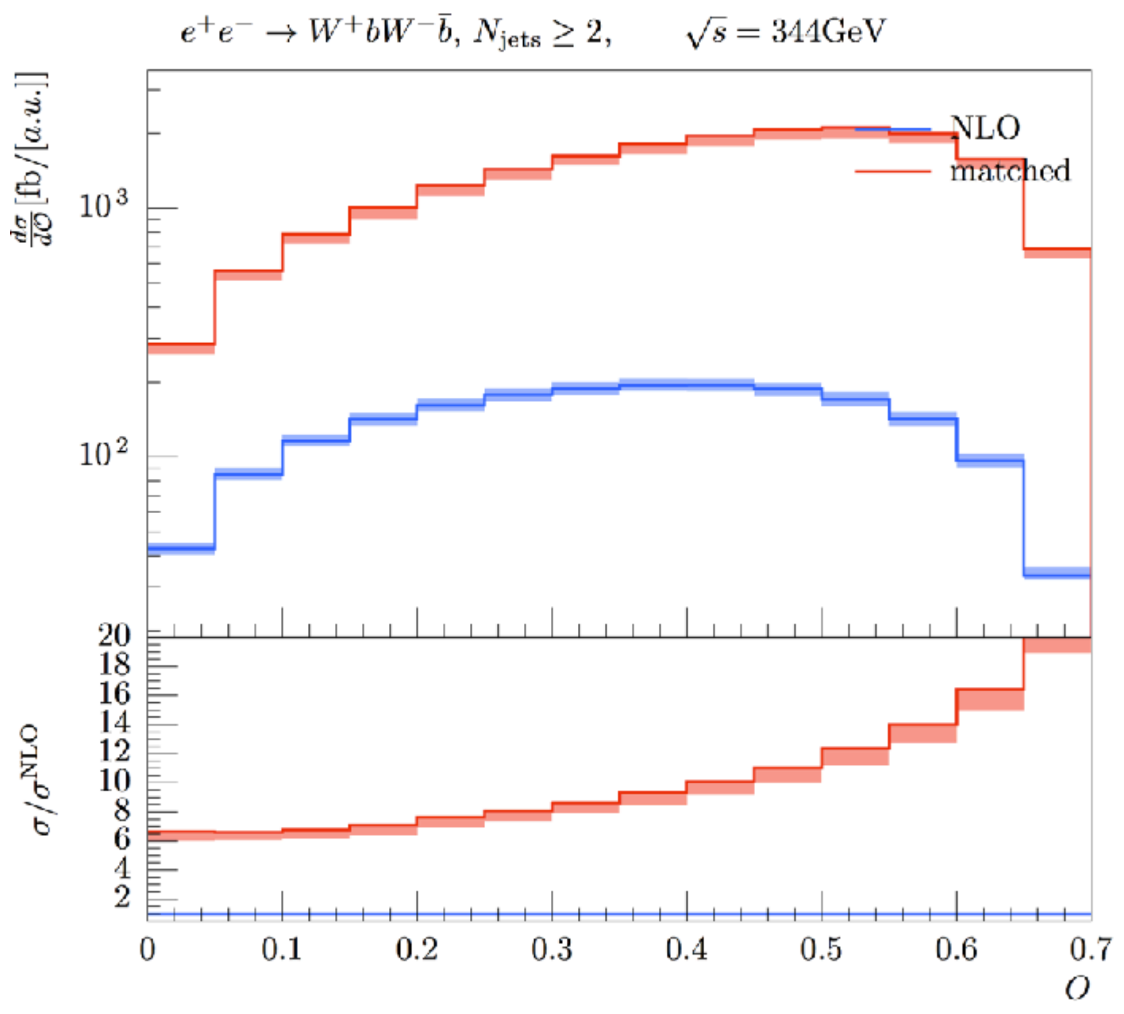
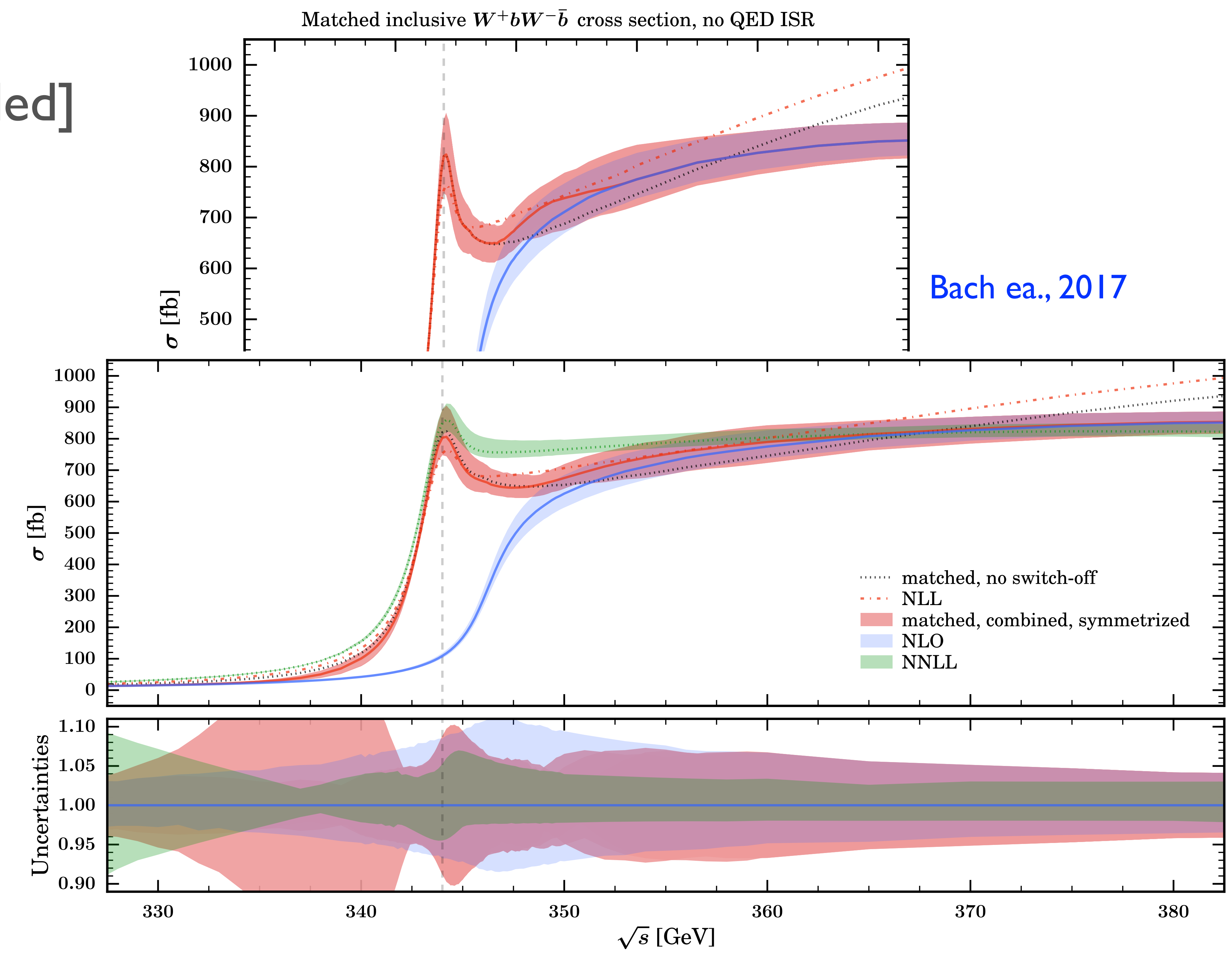
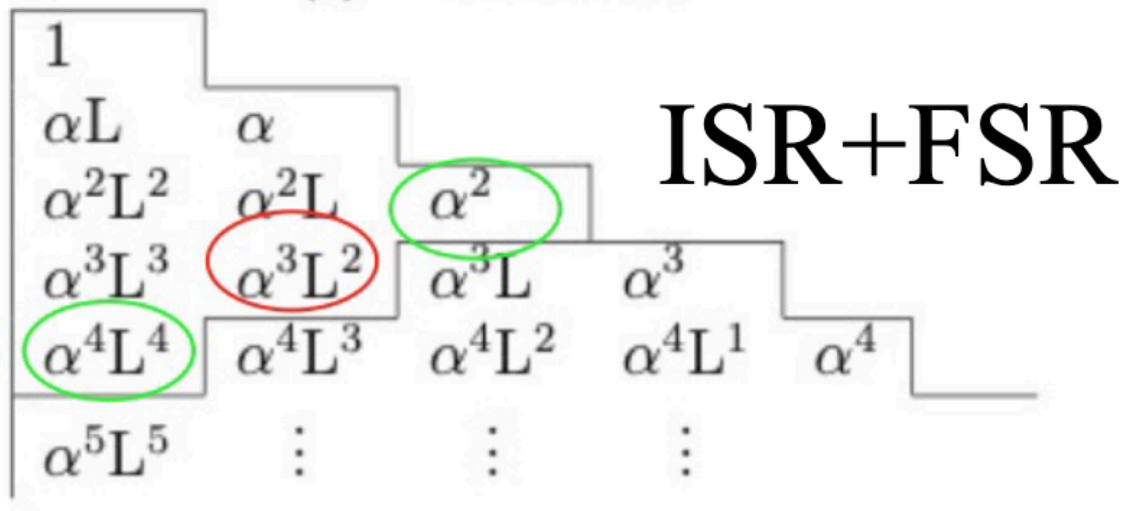




Bach ea., 2017

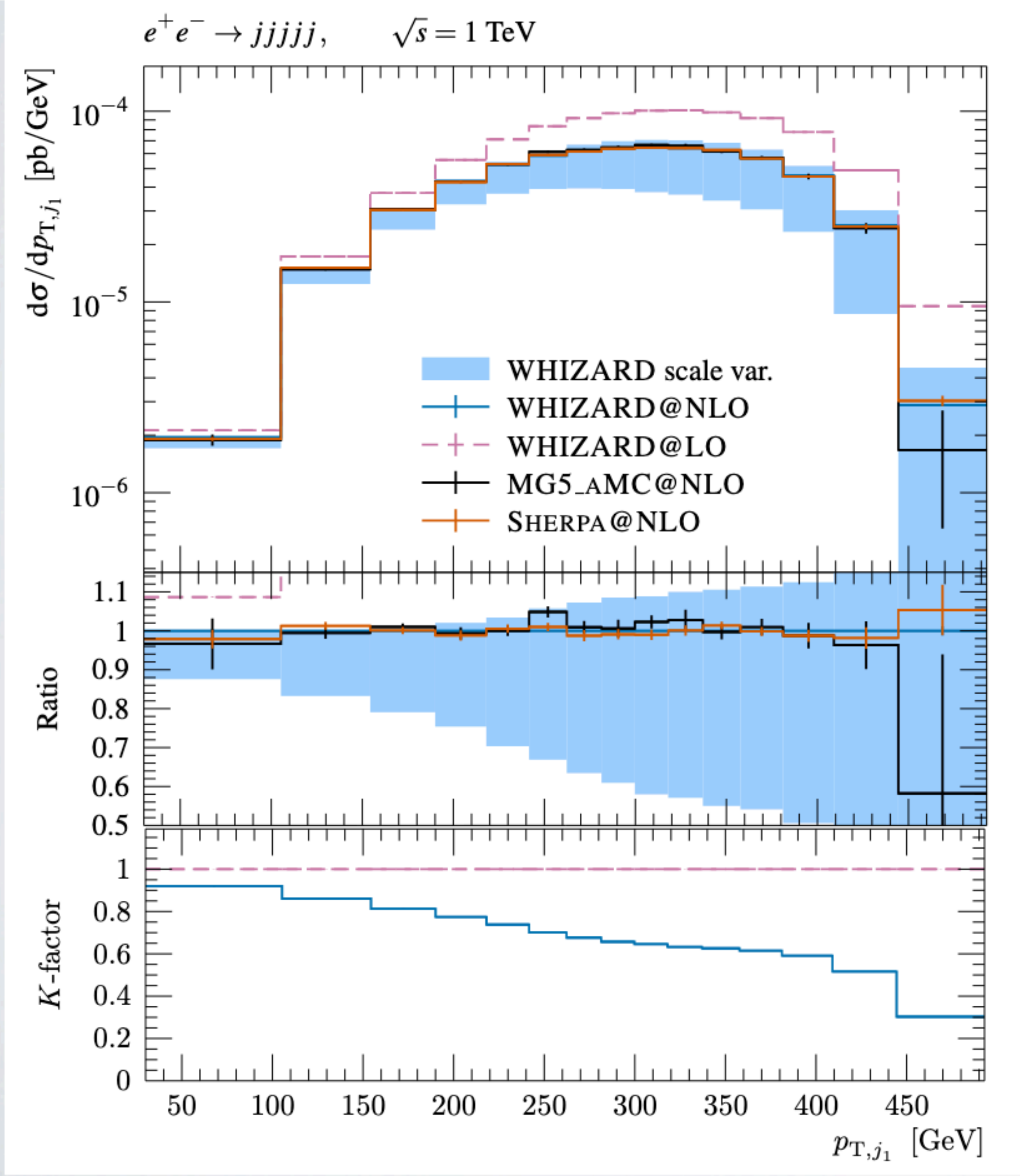
The “Exclusive” Frontier — fN(N)LO and MCs

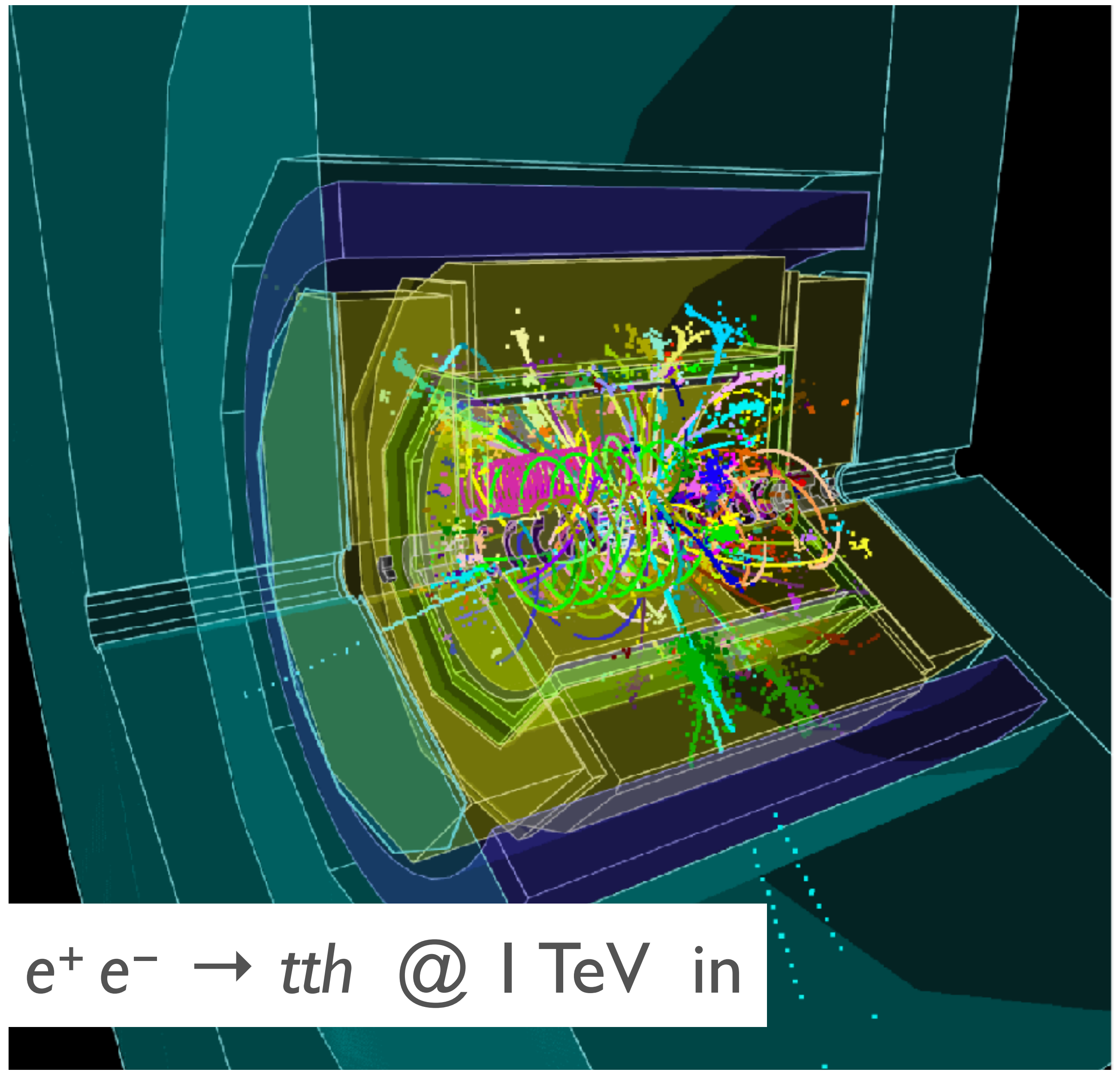
- ☑ [Beam spectra and overlays: “non-perturbative”/classical part of event simulation]
- ☑ Hard matrix elements @ NN(N)LO QCD @ N(N)LO EW
- ☑ QED ISR: correct normalization [inclusive part], ISR photons [exclusive part]
- ☑ QED FSR: interference w/ ISR
- ☑ QED showers: proper matching of exclusive and resummed prescriptions
- ☑ High-energy colliders (CLIC, Plasma, Muon): EW PDFs, EW showers, event selection/definition!?



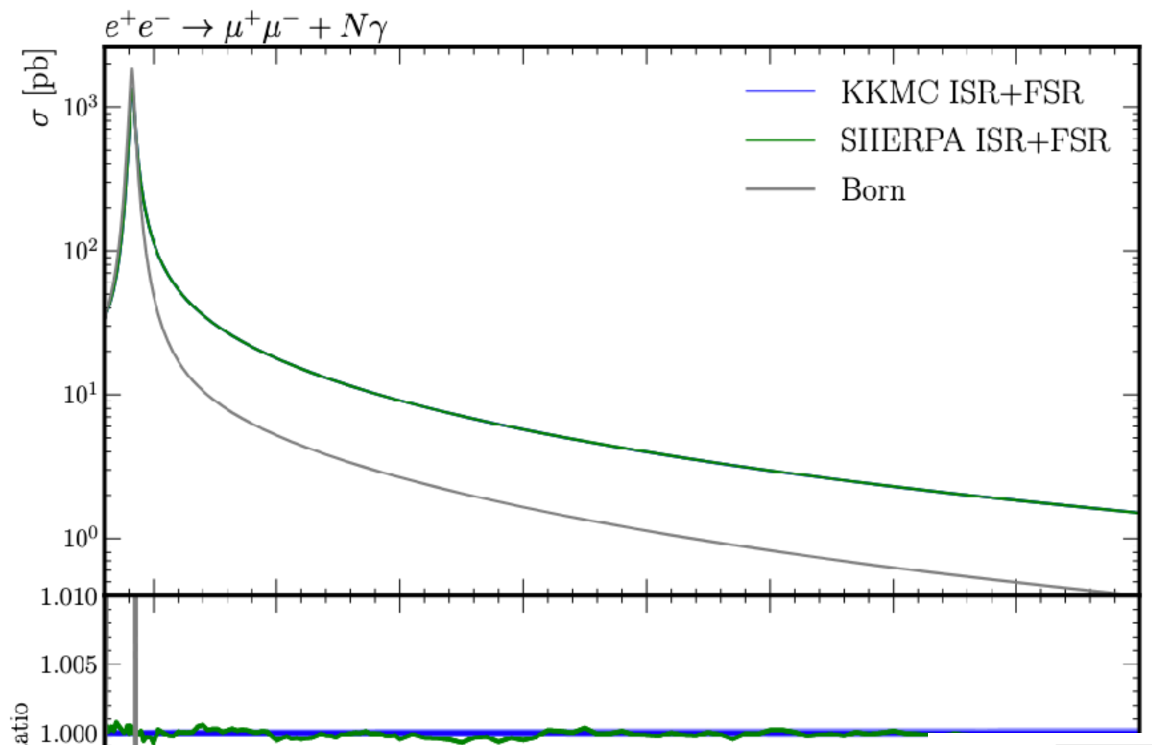
ISR+FSR

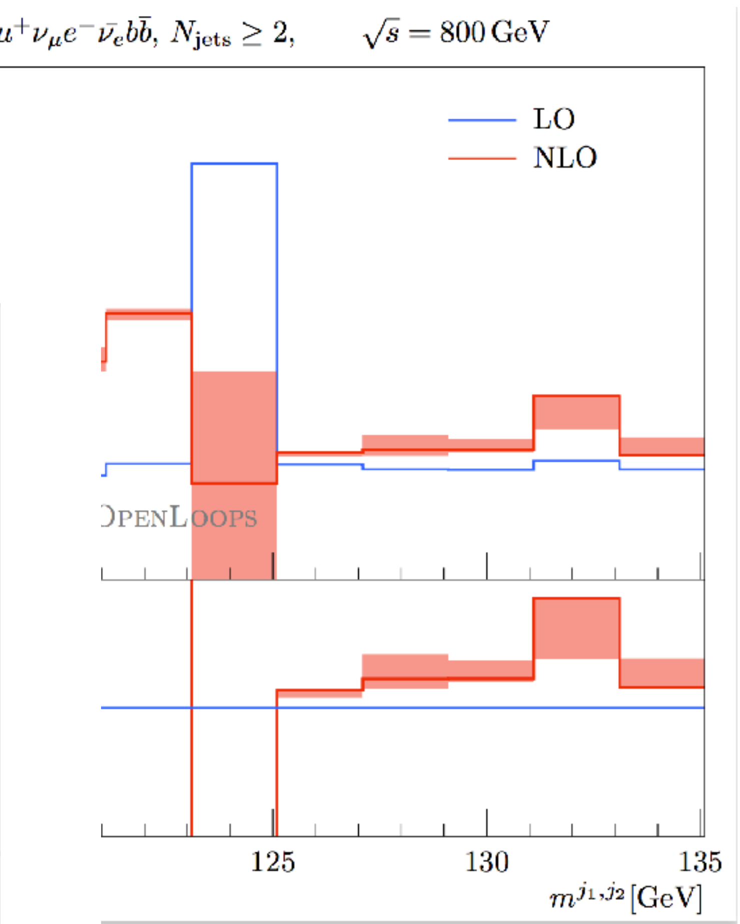
1				
αL	α			
$\alpha^2 L^2$	$\alpha^2 L$	α^2		
$\alpha^3 L^3$	$\alpha^3 L^2$	$\alpha^3 L$	α^3	
$\alpha^4 L^4$	$\alpha^4 L^3$	$\alpha^4 L^2$	$\alpha^4 L^1$	α^4
$\alpha^5 L^5$	$\vdots$	$\vdots$	$\vdots$	

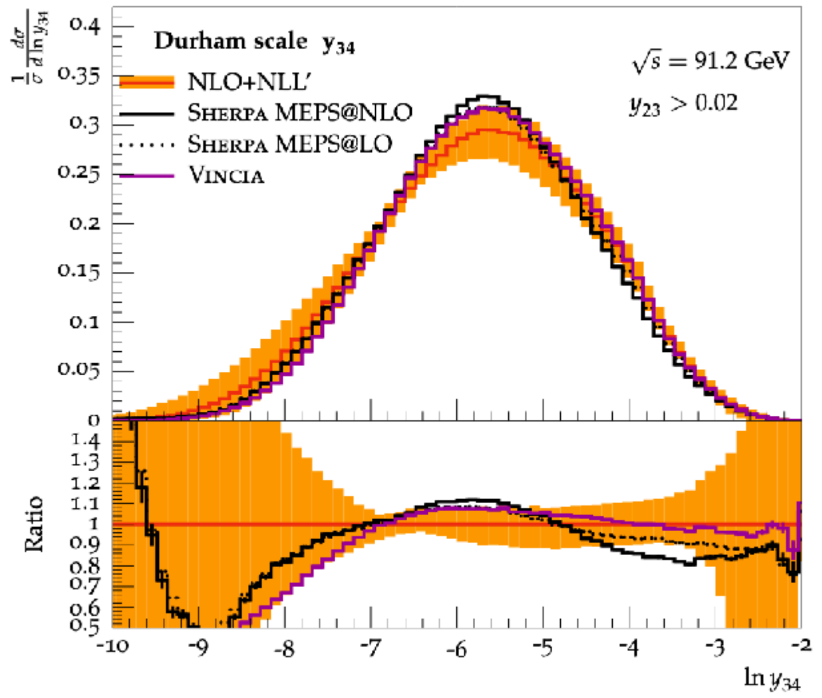




$e^+ e^- \to tth$ @ 1 TeV in

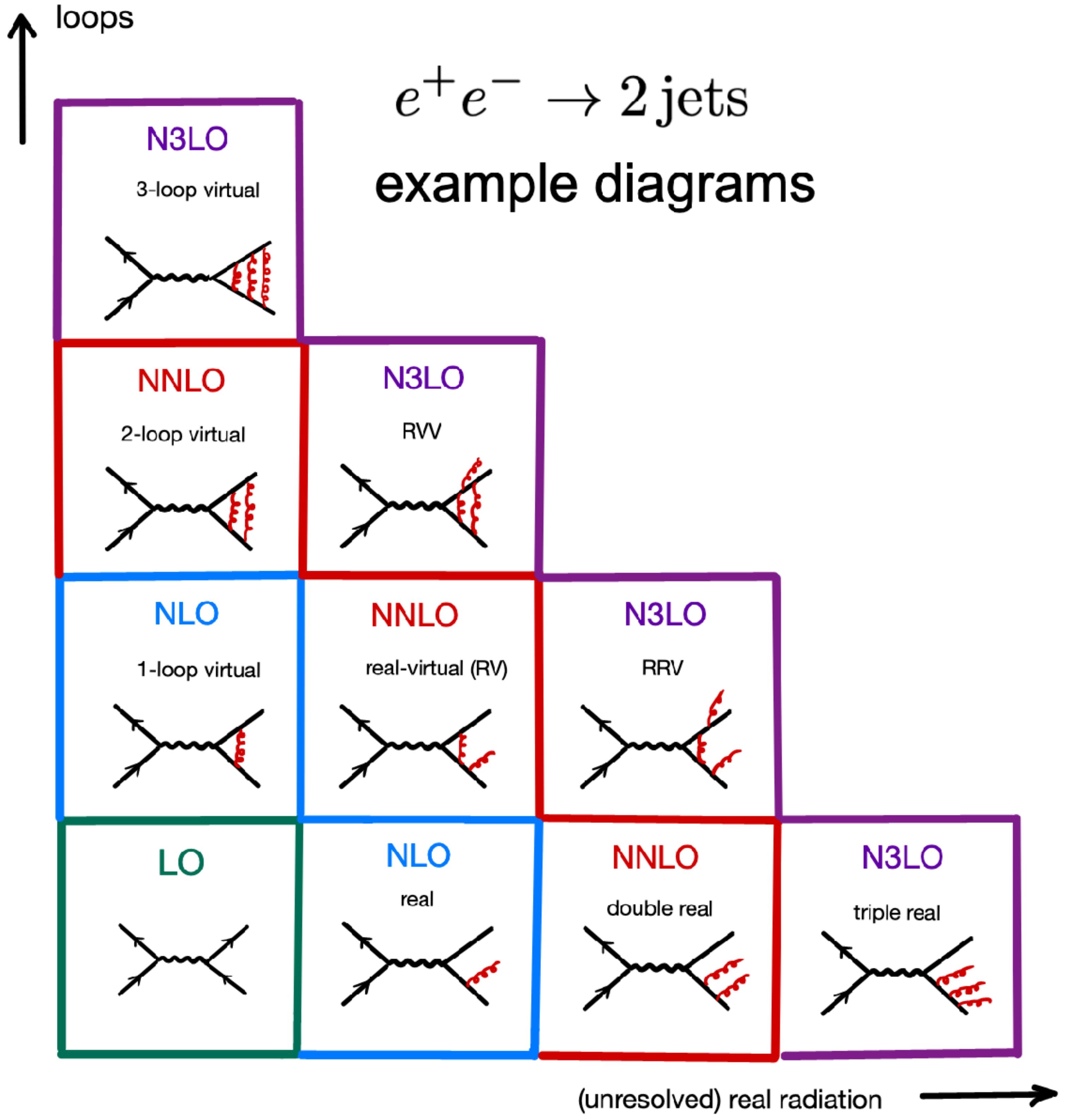






The “Exclusive” Frontier — fN(N)LO, Automation and MCs

- Fixed-order N(N)LO, resummation and matching in MCs
- Determination of efficiencies and systematic uncertainties
- Need e⁺e⁻ → *2f, 3f, 4f, 5f, 6f, [7-10f] @ NLO QCD ⊕ EW* (arbitrary cuts, fully differential)



$e^+e^- \to 2\,\text{jets}$

example diagrams

- NLO QCD ⊕ EW automated: Sherpa, MG5, Whizard
- Caveats and fine-prints

	$\sigma_{\rm LO}$[fb]	$\sigma_{\rm NLO}$[fb]	K
$e^+e^- \to jj$	622.737(8)	639.39(5)	1.027
$e^+e^- \to jjj$	340.6(5)	317.8(5)	0.933
$e^+e^- \to jjjj$	105.0(3)	104.2(4)	0.992
$e^+e^- \to jjjjj$	22.33(5)	24.57(7)	1.100
$e^+e^- \to jjjjjj$	3.583(17)	4.46(4)	1.245
$e^+e^- \to t\bar{t}$	166.37(12)	174.55(20)	1.049
$e^+e^- \to t\bar{t}j$	48.12(5)	53.41(7)	1.110
$e^+e^- \to t\bar{t}jj$	8.592(19)	10.526(21)	1.225
$e^+e^- \to t\bar{t}jjj$	1.035(4)	1.405(5)	1.357

from 2104.11141 & 2208.09438

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\rm incl}_{\rm LO}$ [fb]	$\sigma^{\rm incl}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]
W^+W^-	$4.6591(2)\cdot 10^{2}$	$4.847(7)\cdot 10^{2}$	+4.0(2)
ZZ	$2.5988(1)\cdot 10^{1}$	$2.656(2)\cdot 10^{1}$	+2.19(6)
HZ	$1.3719(1)\cdot 10^{0}$	$1.3512(5)\cdot 10^{0}$	−1.51(4)
HH	$1.60216(7)\cdot 10^{-7}$	$5.66(1)\cdot 10^{-7}$ *	
W^+W^-Z	$3.330(2)\cdot 10^{1}$	$2.568(8)\cdot 10^{1}$	−22.9(2)
W^+W^-H	$1.1253(5)\cdot 10^{0}$	$0.895(2)\cdot 10^{0}$	−20.5(2)
ZZZ	$3.598(2)\cdot 10^{-1}$	$2.68(1)\cdot 10^{-1}$	−25.5(3)
HZZ	$8.199(4)\cdot 10^{-2}$	$6.60(3)\cdot 10^{-2}$	−19.6(3)
HHZ	$3.277(1)\cdot 10^{-2}$	$2.451(5)\cdot 10^{-2}$	−25.2(1)
HHH	$2.9699(6)\cdot 10^{-8}$	$0.86(7)\cdot 10^{-8}$ *	
$W^+W^-W^+W^-$	$1.484(1)\cdot 10^{0}$	$0.993(6)\cdot 10^{0}$	−33.1(4)
W^+W^-ZZ	$1.209(1)\cdot 10^{0}$	$0.699(7)\cdot 10^{0}$	−42.2(6)
W^+W^-HZ	$8.754(8)\cdot 10^{-2}$	$6.05(4)\cdot 10^{-2}$	−30.9(5)
W^+W^-HH	$1.058(1)\cdot 10^{-2}$	$0.655(5)\cdot 10^{-2}$	−38.1(4)
$ZZZZ$	$3.114(2)\cdot 10^{-3}$	$1.799(7)\cdot 10^{-3}$	−42.2(2)
$HZZZ$	$2.693(2)\cdot 10^{-3}$	$1.766(6)\cdot 10^{-3}$	−34.4(2)
$HHZZ$	$9.828(7)\cdot 10^{-4}$	$6.24(2)\cdot 10^{-4}$	−36.5(2)
$HHHZ$	$1.568(1)\cdot 10^{-4}$	$1.165(4)\cdot 10^{-4}$	−25.7(2)

Two major bottlenecks

☐ Virtual integrals with many mass scales / off-shell legs   Abreu ea., Badger ea., Baglio ea., Brønnum-Hansen ea.

☐ IR pole treatment / subtraction   CS, FKS, NS, Stripper, qT/sub-jettiness etc.

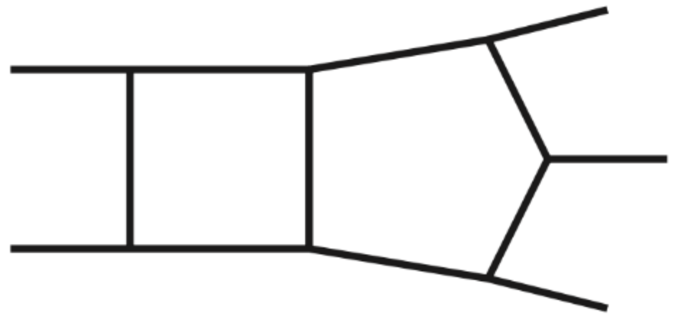
☑ FKS soft/eikonal subtraction sufficient for low-energy machines

☑ NNLO QED (massive, virtuals pending): McMule   [Whizard]

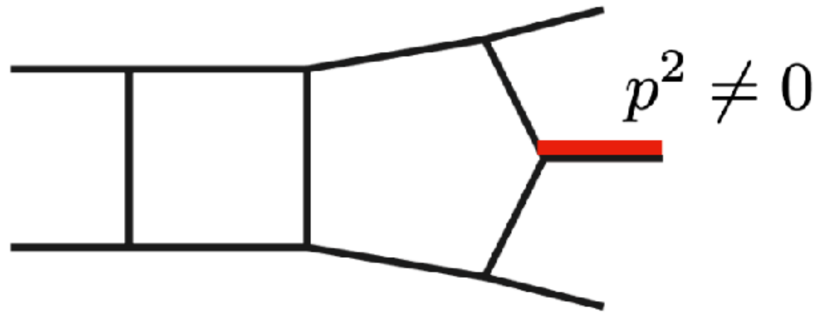
☑ for NNLO EW need for full-fledged soft+collinear NNLO subtraction

Virtual corrections — (N)NNLO master integrals

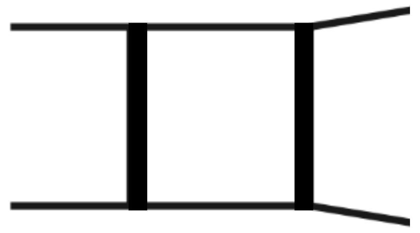
Status: massless 5-point functions



massless 5-point functions, 1 off-shell line



4-point functions w/ massive propagator(s)



G. Heinrich, DESY Theory Workshop talk, 09/22

current frontiers:

- NNLO efficiency
- N3LO coloured
- 2 loops, 4 legs, several mass scales
- 2 loops, 5 legs
- more than 2 loops

☑ Tensor & IntegrationByParts reduction to master integrals

☑ Important tools: `Fire6`, `FireFly`, `LiteRed`, `FiniteFlow`, `Caravel`

☑ Solution analytical via differential equations (DE)

☑ (Semi-)Numerical solution of DE: `DiffExp`, `AMFlow`

☑ Using $pp \to Vjj$ towards $ee \to jjjj$ @ NNLO QCD

☑ For NNLO EW: γ_5 scheme

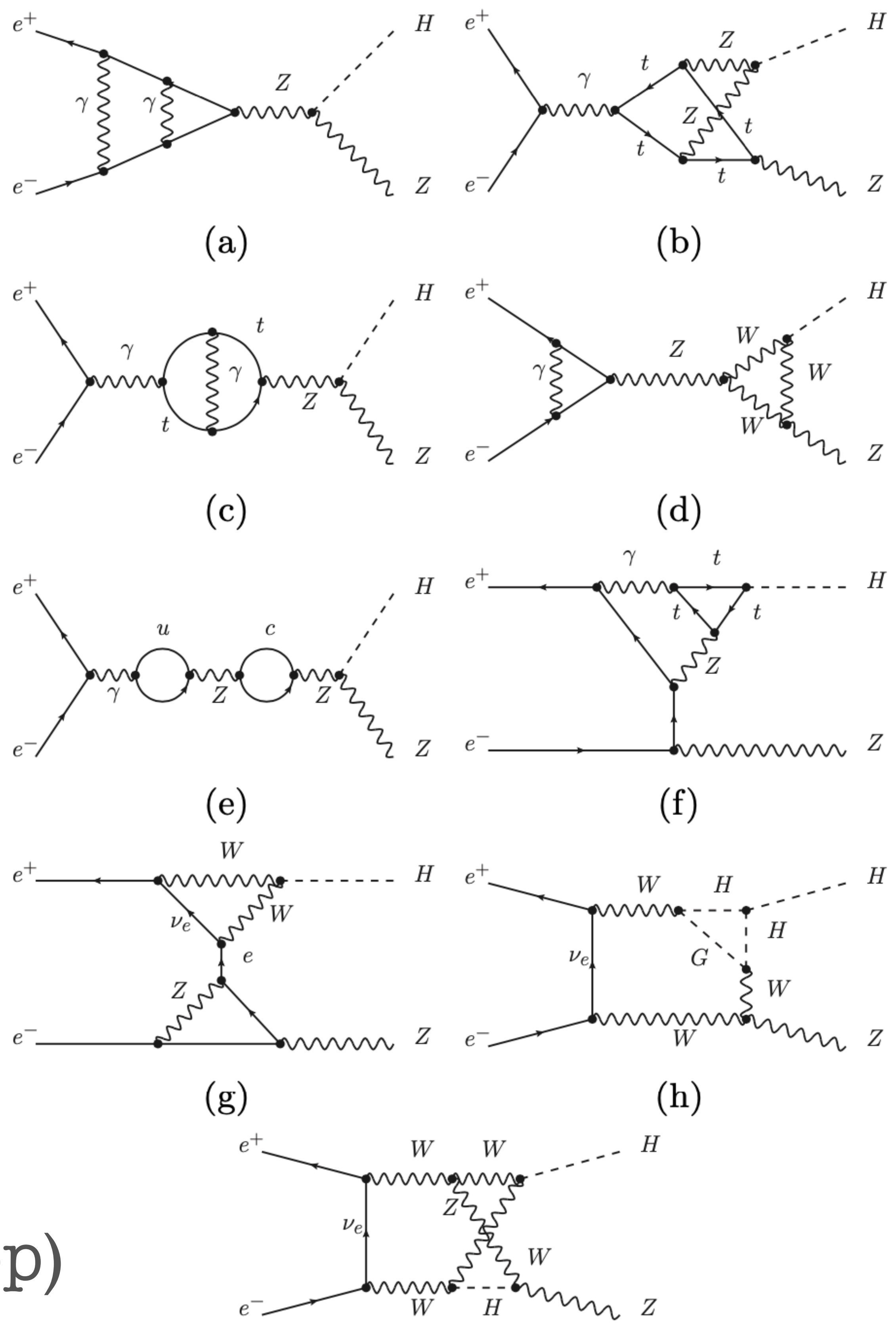
S. Abreu, C. Duhr, J. Gluza, J. Henn, V. Hirschi, D. Kossower, A. von Manteuffel,
E. Panzer, T. Pezaro, V. Sotnikov, S. Weinzierl, M. Zoller amm.

☐ No analytic 2-loop with massive propagators yet:
unknown generalized functions (beyond HPLs)

☐ Cross talk between numerical and analytical methods needed

☐ Ongoing work on automated 2-loop virtuals, `Openloops2loop`

☐ Local unitarity/loop-tree duality: NLO/NNLO at integrand level (αLoop)



QED: ePDFs, Resummation …

QED ISR, inclusive part

- Soft-collinear resummation to all orders Gribov/Lipatov, 1972; Kuraev/Fadin, 1985
- Hard collinear radiation $\mathcal{O}(\alpha^2)$ Kuraev/Fadin, 1985 , $\mathcal{O}(\alpha^3)$ Skrzypek/Jadach, 1992
- LO boundary conditions, collinear evolution @ LL Skrzypek/Jadach, 1992; Cacciari/Deandrea/Montagna/Nicrosini, 1992
- NLO boundary conditions for QED PDFs Frixione, 1909.03886
- NLO QED PDFs, collinear evolution @ NLL Bertone/Cacciari/Frixione/Stagnitto, 1911.12040 + 2207.03265
- Crucial: numerical stability at kinematically peaked limit $z \to 1$

$$d\sigma_{kl}(p_k, p_l) = \sum_{ij=e^+,e^-,\gamma} \int dz_+ dz_- \, \Gamma_{i/k}(z_+, \mu^2, m^2) \, \Gamma_{j/l}(z_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \mathcal{O}\left(\left(\frac{m^2}{s}\right)^p\right)$$

important: fast interpolation grids

$$\mathbb{P}_{\mathrm{S}} = \begin{pmatrix} P_{\Sigma\Sigma} & P_{\Sigma\gamma} \\ P_{\gamma\Sigma} & P_{\gamma\gamma} \end{pmatrix},$$

$$P_{\mathrm{NS}} = P_{e^\pm e^\pm} - P_{e^\pm e^\mp} \equiv P^{\mathrm{V}}_{ee} - P^{\mathrm{V}}_{e\bar{e}}.$$

$$\begin{aligned}
\Gamma_i^{[0]}(z, \mu_0^2) &= \delta_{ie^-}\delta(1-z)\,, \\
\Gamma_{e^-}^{[1]}(z, \mu_0^2) &= \left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-z) - 1\right)\right]_+ + K_{ee}(z)\,, \\
\Gamma_{\gamma}^{[1]}(z, \mu_0^2) &= \frac{1+(1-z)^2}{z}\left(\log\frac{\mu_0^2}{m^2} - 2\log z - 1\right) + K_{\gamma e}(z)\,, \\
\Gamma_{e^+}^{[1]}(z, \mu_0^2) &= 0\,,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathbb{E}_N(t)}{\partial t} &= \frac{b_0\alpha^2(\mu)}{\beta(\alpha(\mu))} \sum_{k=0}^{\infty} \left(\frac{\alpha(\mu)}{2\pi}\right)^k \mathbb{P}_N^{[k]} \, \mathbb{E}_N(t) \\
&= \left[\mathbb{P}_N^{[0]} + \frac{\alpha(\mu)}{2\pi}\left(\mathbb{P}_N^{[1]} - \frac{2\pi b_1}{b_0}\,\mathbb{P}_N^{[0]}\right)\right] \mathbb{E}_N(t) + \mathcal{O}(\alpha^2)
\end{aligned}$$

ePDFs for polarized leptons !?

QED ISR [+FSR], exclusive part

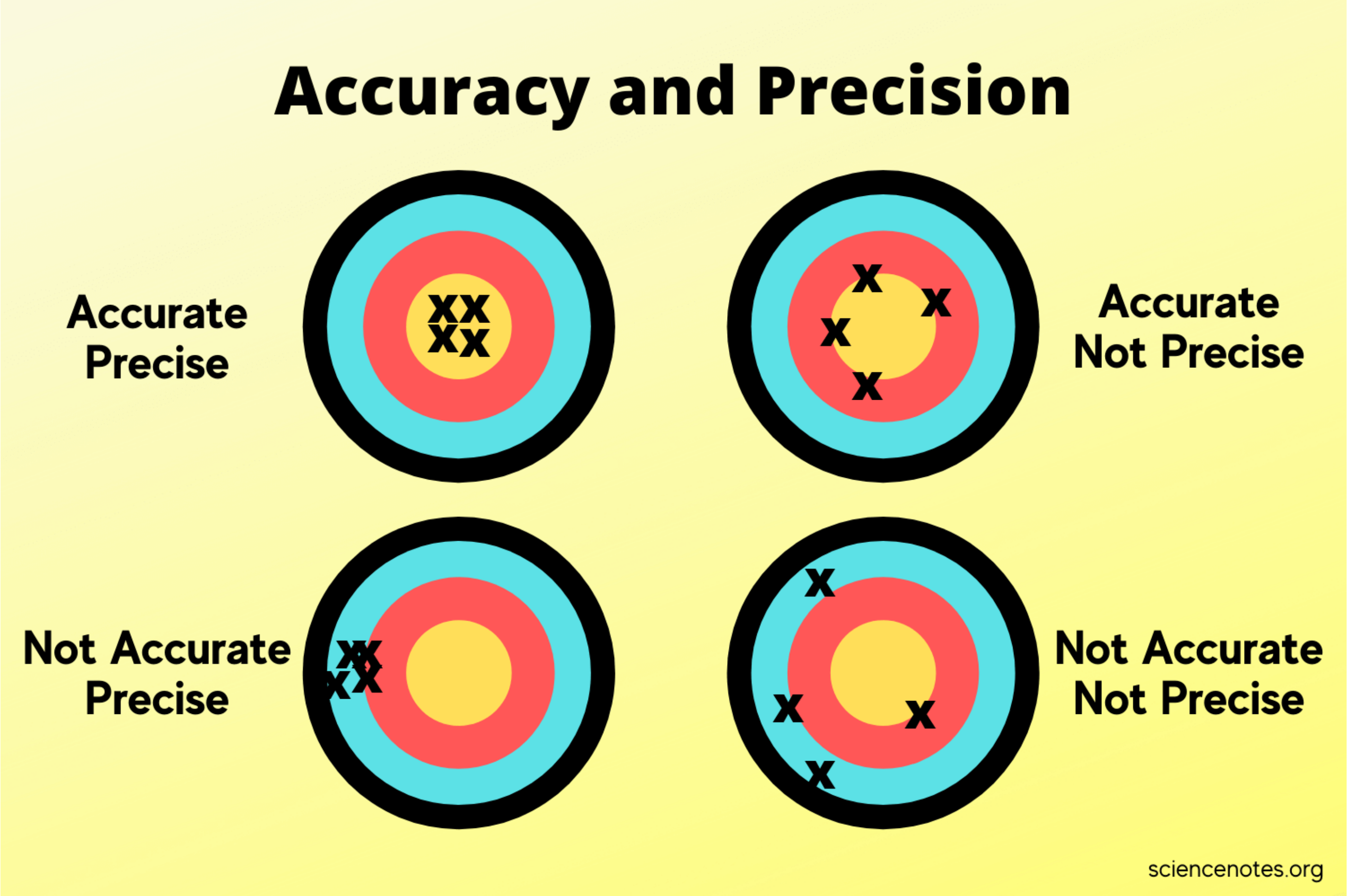
- Exclusive (“coherent”) resummation Yennie/Frautschi/Suura, 1961
- Explicitly matches ME photons Jadach/Ward/Yost, hep-ph/0103163+0104049+0211132+0602197, Piccinini ea.
- Coherent exponentiated EW corrections (CEEX) Jadach/Ward/Wąs, hep-ph/0006359; 1409.4173 ; Krauss/Price/Schönherr, 2203.10948

QED Full Factorization

- Fully factorized QED amplitudes for small/vanishing m_e Laenen et al. 2008.01736

Conclusions & Outlook

- Spectacular experimental Higgs + EW precision program in e^+e^- collisions
- Most measurements allow per-cent down to (sub-) per-mil level precision
- **Hard theoretical work needed to match this precision!**
- *Z* / *WW* threshold: massive 2- and 3-loop 4-point functions needed, leading 4-loop
- Top threshold: N4LO NRQCD maybe not necessary, more differentially: NNLO+NNLL matched possible
- Massive 2- & 3-loop diagrams: PDE, sector decomposition, Mellin methods etc.
- Higgs precision program: production processes NNLO, decays @ 3-loop
- "Exclusive frontier" (I): 2 → 4, 6, (8) NLO SM corrections, NLO $e^\pm$ PDFs
- "Exclusive frontier" (II): Exclusive exponentiation, QED showers & matching
- Tools, tools, tools: community must value and support codes (loops, MC, fits)
- More precise precision goals: maybe Les Houches 2023 !?



Precision is reconciling Loops and Legs

Precision is reconciling Loops and Legs

Getty Villa, Pacific Palisades, Etruscan, 525 BC