

New Physics in Transversal Bosons in VBS



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based on work with:

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HELMHOLTZ
RESEARCH FOR GRAND CHALLENGES



EPJC78(18),11.931 [1807.02512]

PRD93(16),3. 036004 [1511.00022]

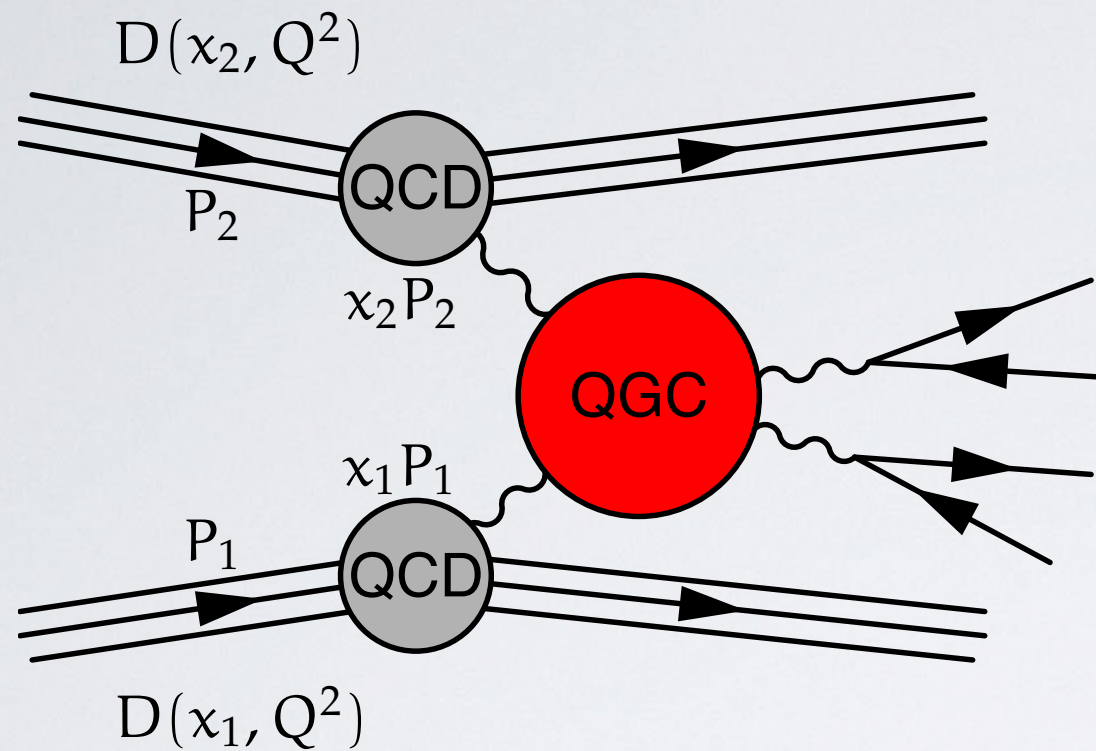
PRD91(15) 096007 [1408.6207]



Anatomy of Vector Boson Scattering (VBS)

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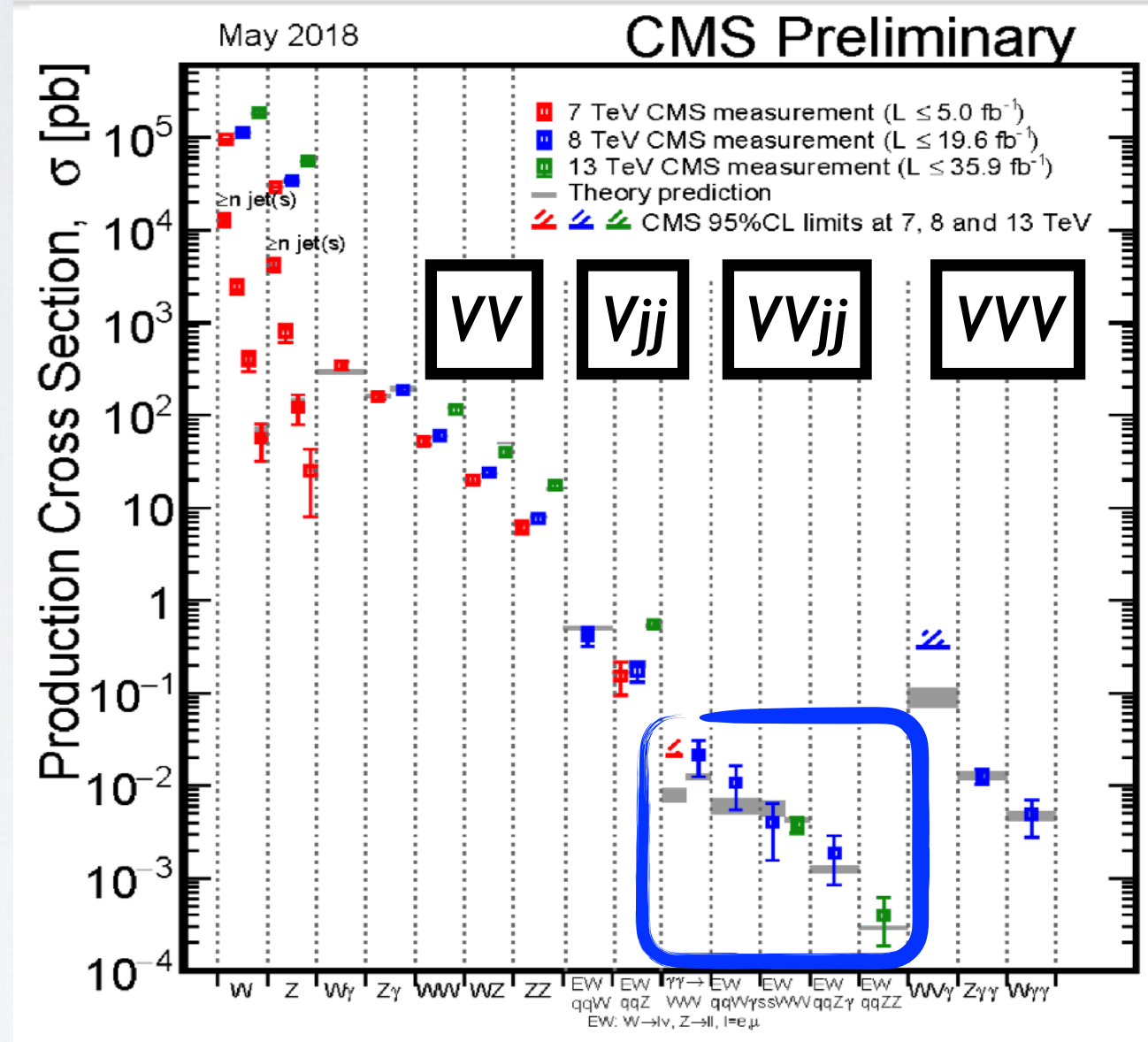
$$pp \rightarrow WWjj \rightarrow \ell\ell\nu\nu jj$$



Fiducial phase space volume:

- $lljj$ tag
- $m_{jj} > 500$ GeV (“jet recoil”)
- $|\Delta y_{jj}| > 2.4$ (“rapidity distance”)
- Cuts on E_j , p_T^j
- No / little central jet activity

Smallest accessible SM cross sections



Subtle cancellation of amplitudes in SM

Dim-8 operators
for MBI physics

Longitudinal operators

Mixed operators

$$\begin{aligned}
 \mathcal{O}_{M,0} &= \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot [(D_\beta \Phi)^\dagger D^\beta \Phi] \\
 \mathcal{O}_{M,1} &= \text{Tr} [W_{\mu\nu} W^{\nu\beta}] \cdot [(D_\beta \Phi)^\dagger D^\mu \Phi] \\
 \mathcal{O}_{M,2} &= [B_{\mu\nu} B^{\mu\nu}] \cdot [(D_\beta \Phi)^\dagger D^\beta \Phi] \\
 \mathcal{O}_{M,3} &= [B_{\mu\nu} B^{\nu\beta}] \cdot [(D_\beta \Phi)^\dagger D^\mu \Phi] \\
 \mathcal{O}_{M,4} &= [(D_\mu \Phi)^\dagger W_{\beta\nu} D^\mu \Phi] \cdot B^{\beta\nu} \\
 \mathcal{O}_{M,5} &= [(D_\mu \Phi)^\dagger W_{\beta\nu} D^\nu \Phi] \cdot B^{\beta\mu} \\
 \mathcal{O}_{M,6} &= [(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\nu} D^\mu \Phi] \\
 \mathcal{O}_{M,7} &= [(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\mu} D^\nu \Phi]
 \end{aligned}$$

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 \mathcal{O}_{S,0} &= [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi] \\
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Transversal operators

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 \mathcal{O}_{T,0} &= \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot \text{Tr} [W_{\alpha\beta} W^{\alpha\beta}] \\
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	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0/1}$	✓	✓	✓						
$\mathcal{O}_{M,0/1/6/7}$	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{M,2/3/4/5}$		✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{T,0/1/2}$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,8/9}$			✓			✓	✓	✓	✓

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Energy rise of operators lead to unitarity violation

Unitarity violation cancels between operators in UV-complete Theory

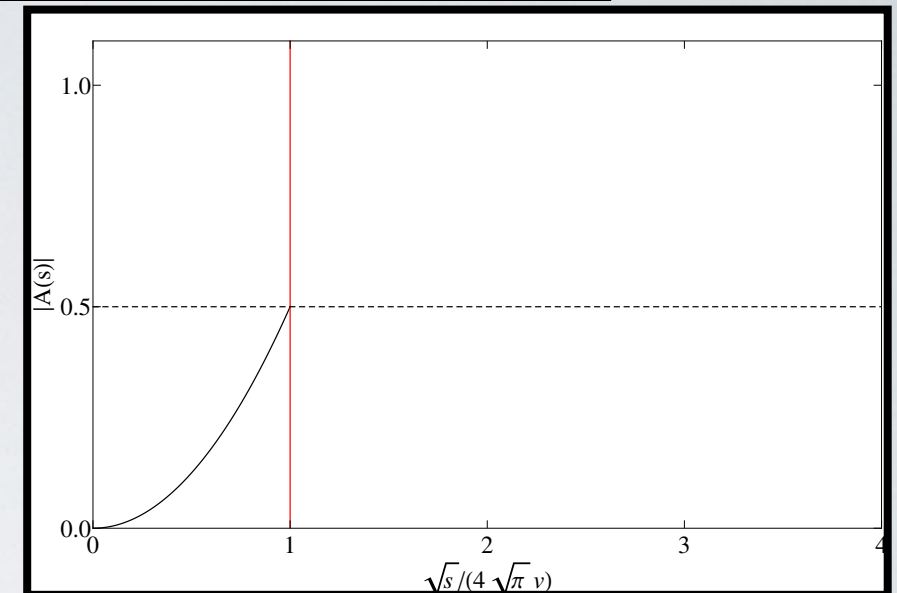
$\mathcal{O}_{S,0/1}$	✓	✓	✓						
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$\mathcal{O}_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,8/9}$			✓			✓	✓	✓	✓

Procedures to treat unitarity violations

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Cut-off (a.k.a. “Event clipping”) $\theta(\Lambda_C^2 - s)$

unitarity bound (0th partial wave) at Λ_C
no continuous transition beyond
Effect on BDT training not clear



Procedures to treat unitarity violations

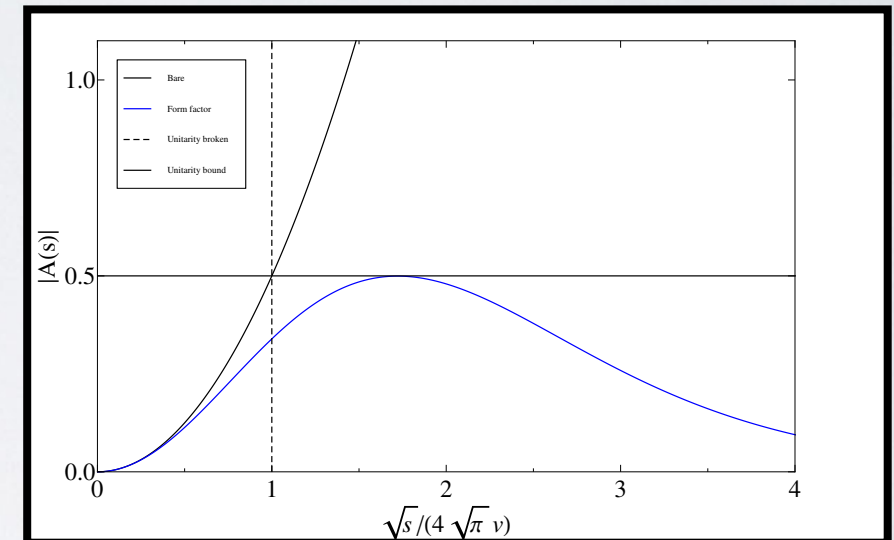
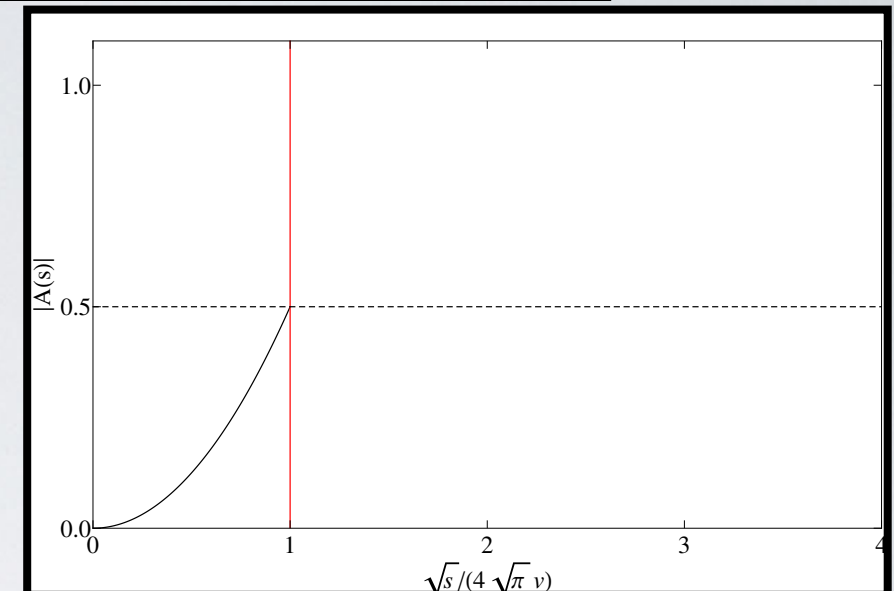
4 / 15

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Form factor $\frac{1}{\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^n}$

Applicable for arbitrary operators, tuning in 2
parameters: n damps unitarity violation, Λ_{FF}
highest value to satisfy 0th partial wave



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4 / 15

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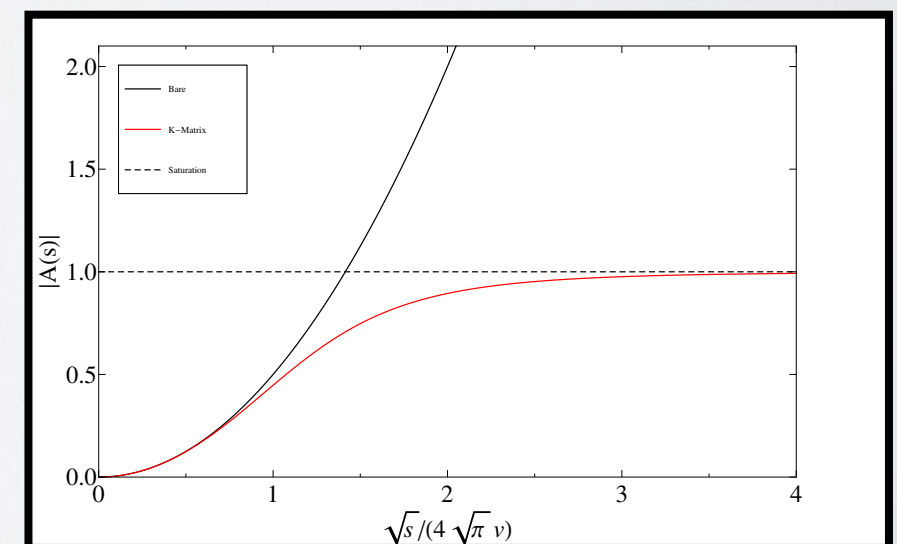
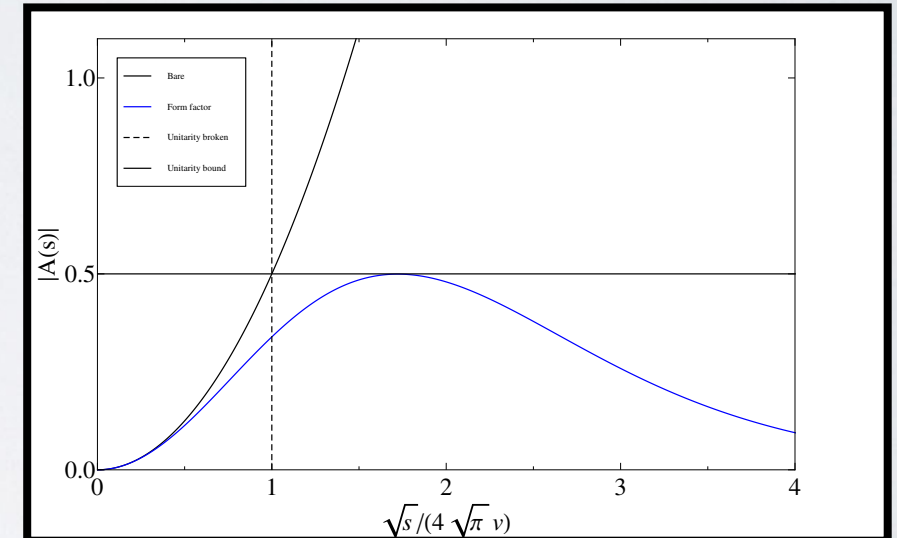
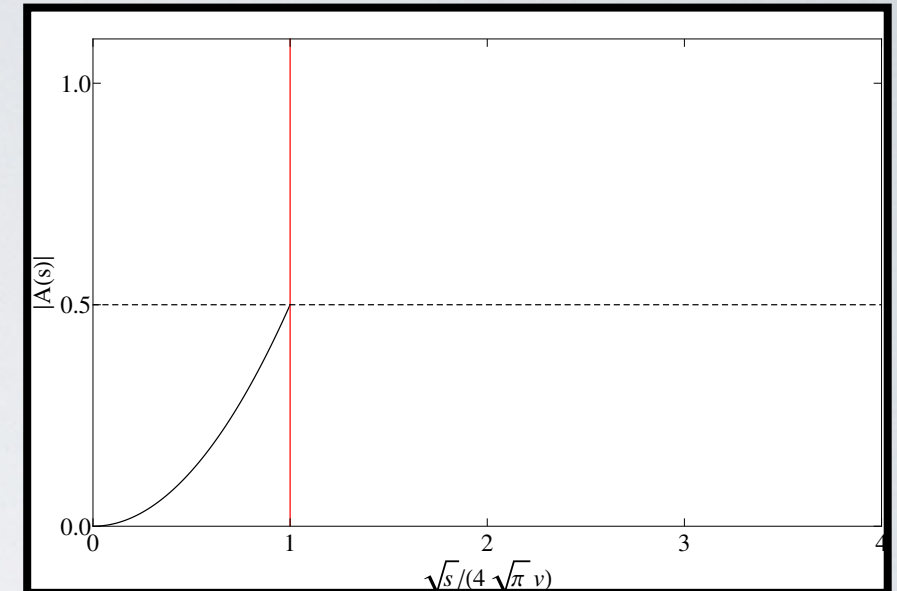
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K-/T-matrix saturation $a = \frac{1}{\text{Re}\left(\frac{1}{a_0}\right) - i}$

saturates amplitude [projection to unitarity circle],
also for complex ampl., **no additional parameters**



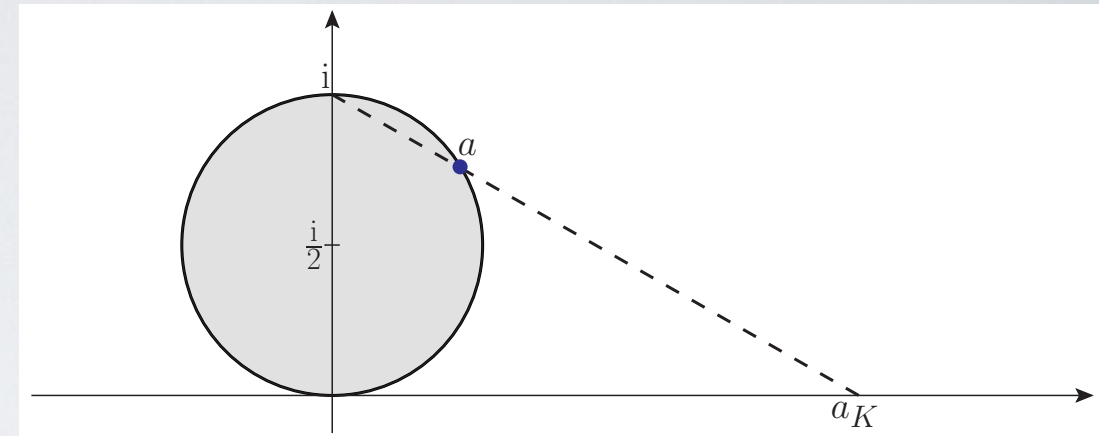
Alboreanu/Kilian/JRR, 0806.4145

Kilian/Ohl/JRR/Sekulla, 1408.6207

- **K-matrix:** Cayley transform of S-matrix
- Stereographic projection to Argand circle

Heitler, 1941; Schwinger, 1949; Gupta, 1950

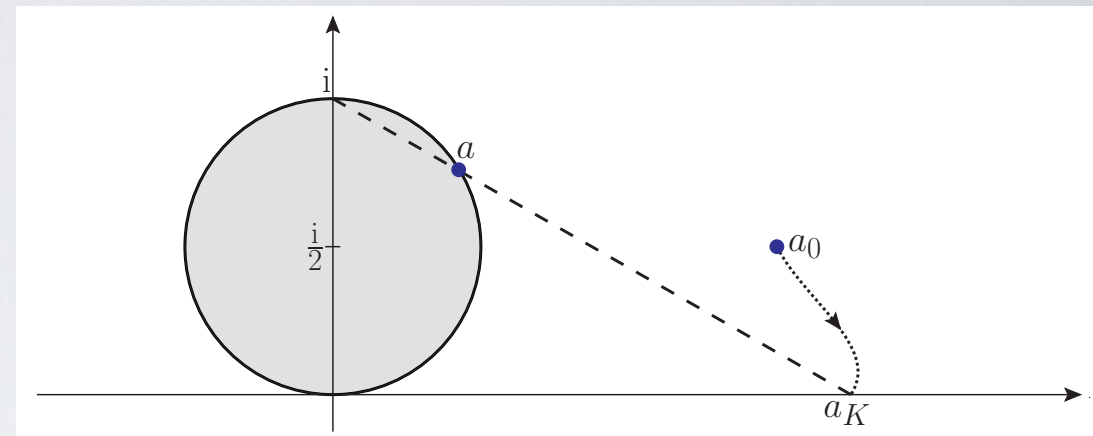
$$S = \frac{1+iK/2}{1-iK/2} \quad a_K(s) = \frac{a(s)}{1-ia(s)}$$



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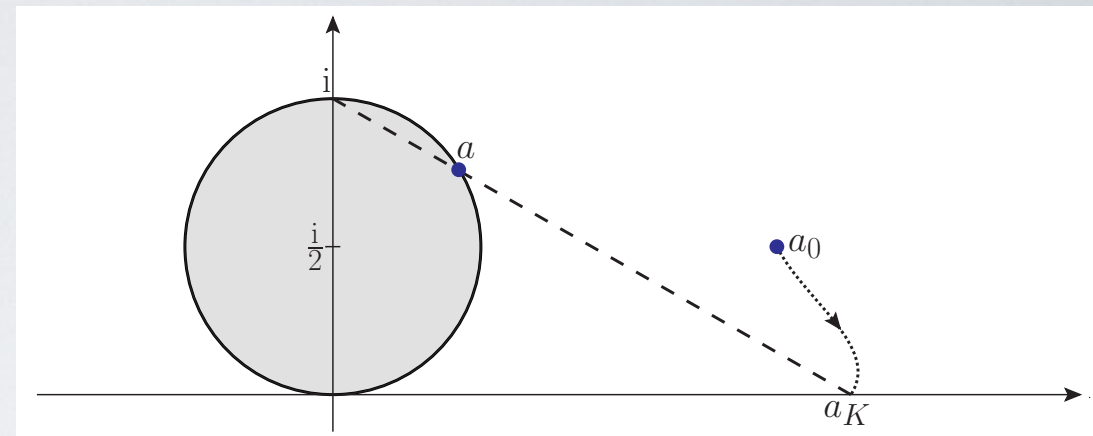


- Stereographic projection to Argand circle
- Formalism does a partial resummation of perturbative series
- need to construct (orig.) K-matrix as self-adjoint intermediate operator
Problems, if S-matrix non-diagonal, presence of non-perturbative contrib.

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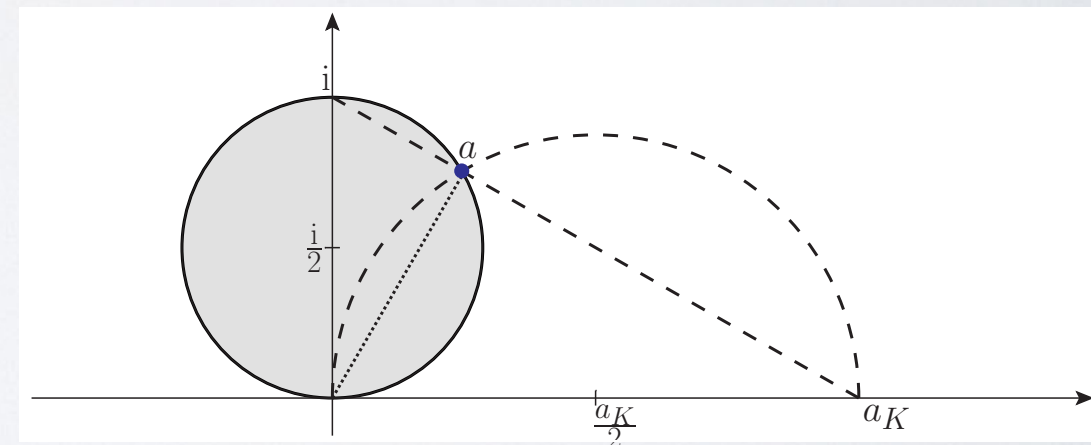


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Kilian/Ohl/JRR/Sekulla, 1408.6207

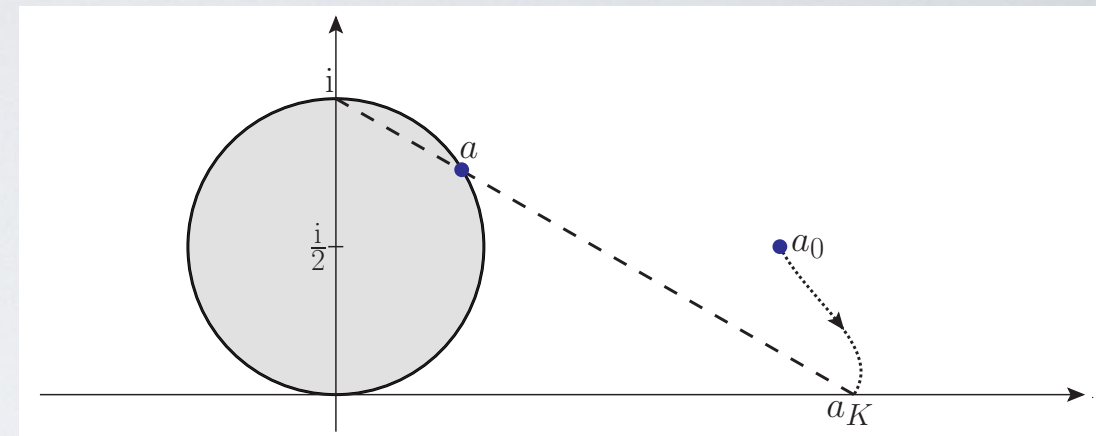
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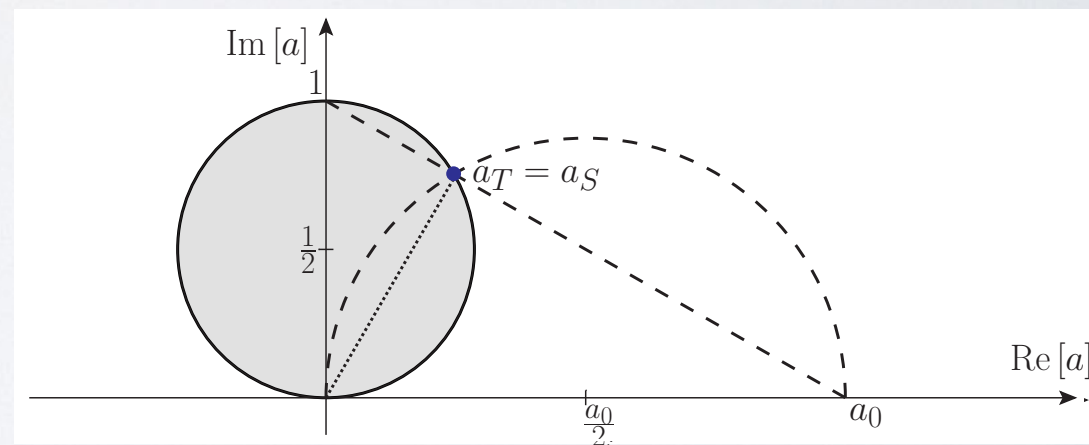
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Kilian/Ohl/JRR/Sekulla, 1408.6207

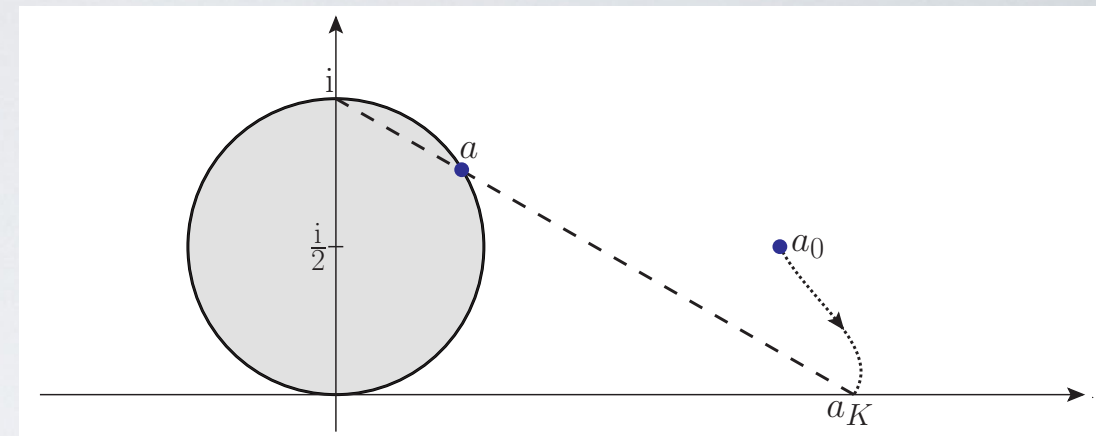


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- Points on Argand circle left invariant
- Does not rely on perturbation theory
- Applicable for amplitudes with imaginary parts (models with resonances)

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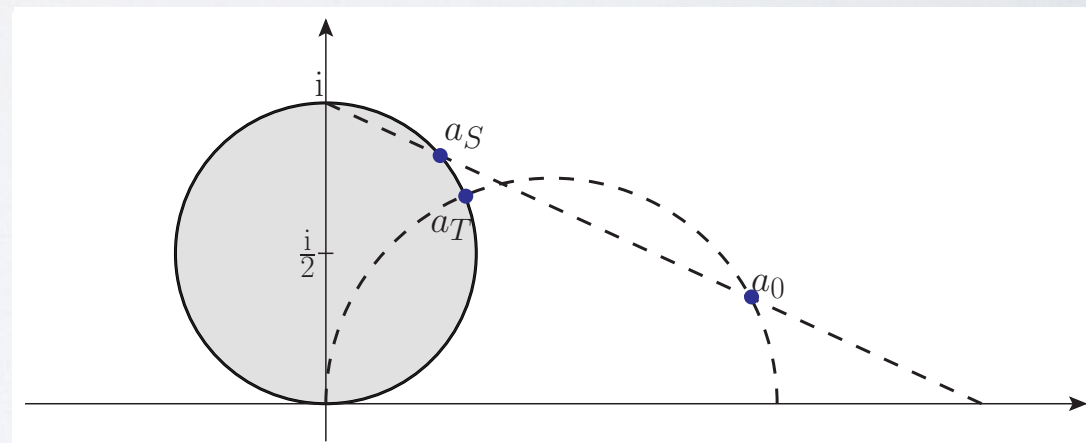
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Kilian/Ohl/JRR/Sekulla, 1408.6207



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- Independent Amplitude Method (IAM) [Truong, 1988; Dobado/Herrero/Truong, 1990]
 - Padé Method [Padé, 1890; Basdevant/Lee, 1970]
 - N/D method [Chew/Mandelstam, 1960]
 - Focus on correct descriptions of certain explicit (known) resonance channels
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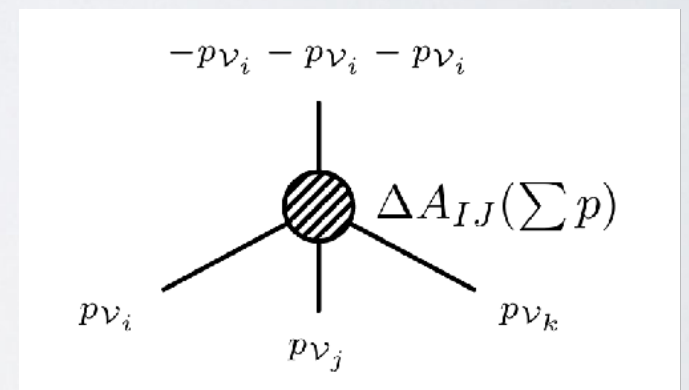
Unitarization of operators

- Clebsch-Gordan decomposition into spin–isospin eigenamplitudes
- Amplitudes should be modified only in s–channel configurations

$$\mathcal{A}(I = 0) = 3\mathcal{A}(s, t, u) + \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)$$

$$\mathcal{A}(I = 1) = \mathcal{A}(t, s, u) - \mathcal{A}(u, s, t)$$

$$\mathcal{A}(I = 2) = \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)$$



- Evaluate modified Feynman rules off-shell
- Scale that is used for the diboson system in s-channel setups: $\sqrt{\hat{s}_{VV}}$

Unitarization of [transverse] operators

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> Use spin-isospin eigenamplitudes **exclusive in helicities**:

$$\mathcal{A}_0(s, t, u; \boldsymbol{\lambda})$$

> Can be obtained by using **Wigner's d-functions** [Wigner, 1931]

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

$$\mathcal{A}_{IJ}(s; \boldsymbol{\lambda}) = \int_{-s}^0 \frac{dt}{s} A_I(s, t, u; \boldsymbol{\lambda}) \cdot d_{\lambda, \lambda'}^J \left[\arccos \left(1 + 2 \frac{t}{s} \right) \right]$$

$$\lambda = \lambda_1 - \lambda_2 \quad \lambda' = \lambda_3 - \lambda_4$$

> **Extract all partial waves:**

$$A_{ij}(s; \boldsymbol{\lambda}) / (g^4 s^2) = (c_0 F_{T_0} + c_1 F_{T_1} + c_2 F_{T_2})$$

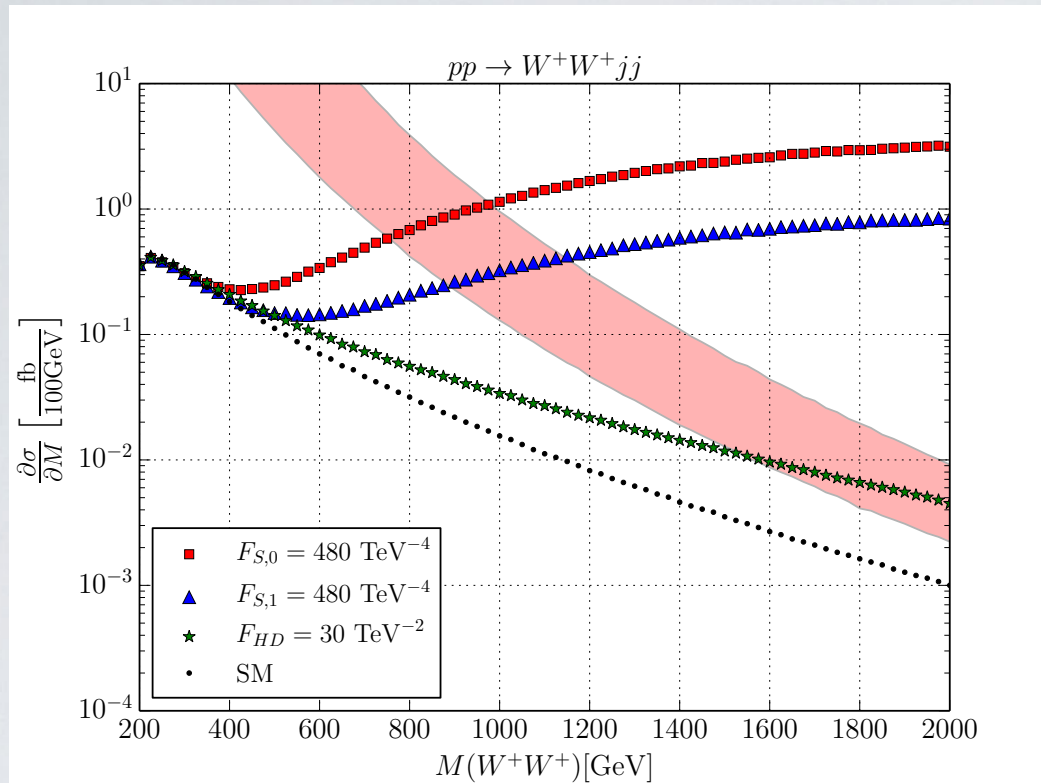
Braß/Fleper/Kilian/JRR/Sekulla,
1807.02512

i \ j	0			1			2			λ			
0	-6	-2	$-\frac{5}{2}$	0	0	0	0	0	0	+	+	+	+
	0	0	0	0	0	0	$-\frac{2}{3\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	+	-	+	-
	0	0	0	0	0	0	$-\frac{2}{3\sqrt{2}}$	$-\frac{4}{5\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}$	+	-	-	+
	$-\frac{22}{3}$	$-\frac{14}{3}$	$-\frac{11}{6}$	0	0	0	$-\frac{2}{15}$	$-\frac{4}{15}$	$-\frac{1}{30}$	+	+	-	-
1	0	0	0	0	0	0	0	0	0	+	+	+	+
	0	0	0	0	0	0	$\frac{2}{3\sqrt{2}}$	$-\frac{1}{5\sqrt{2}}$	0	+	-	+	-
	0	0	0	0	0	0	$-\frac{2}{3\sqrt{2}}$	$\frac{1}{5\sqrt{2}}$	0	+	-	-	+
	0	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	0	0	0	+	+	-	-
2	0	-2	-1	0	0	0	0	0	0	+	+	+	+
	0	0	0	0	0	0	$-\frac{2}{3\sqrt{2}}$	$-\frac{1}{5\sqrt{2}}$	$-\frac{1}{5\sqrt{2}}$	+	-	+	-
	0	0	0	0	0	0	$-\frac{2}{3\sqrt{2}}$	$-\frac{1}{5\sqrt{2}}$	$-\frac{1}{5\sqrt{2}}$	+	-	-	+
	$-\frac{4}{3}$	$-\frac{8}{3}$	$-\frac{1}{3}$	0	0	0	$-\frac{2}{15}$	$-\frac{1}{15}$	$-\frac{1}{30}$	+	+	-	-
	c_0	c_1	c_2	c_0	c_1	c_2	c_0	c_1	c_2				

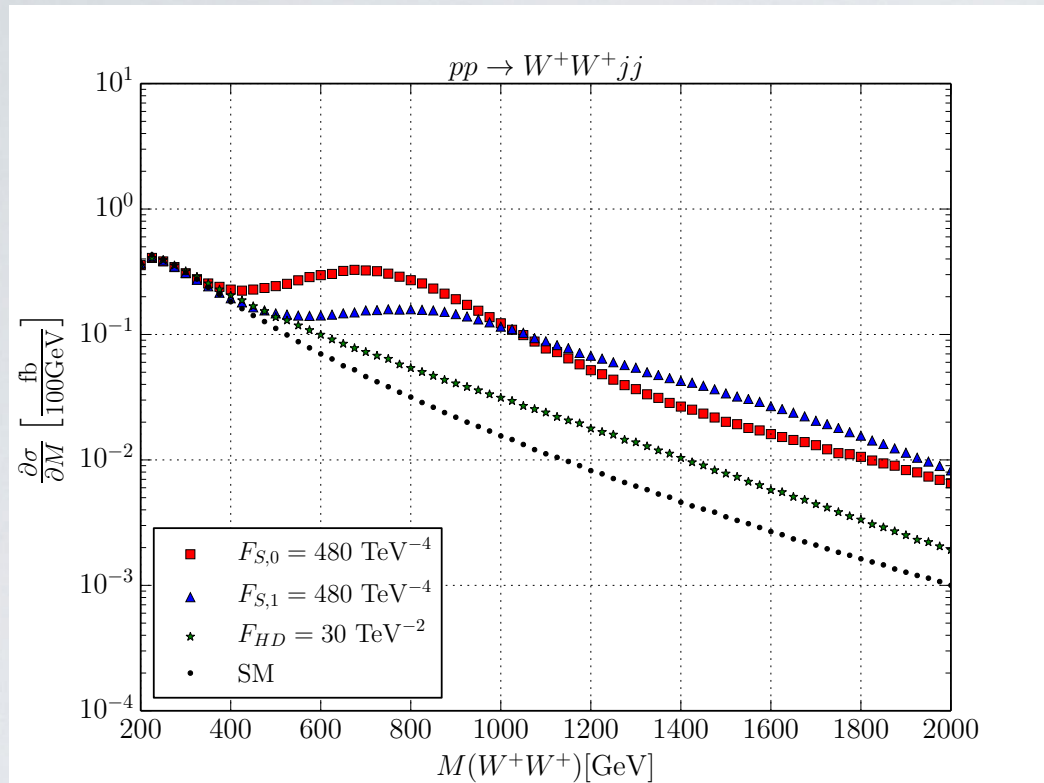
> Corrections for off-shell vectors: important [Perez/Sekulla/Zeppenfeld, 1807.02707]

> Implementation in WHIZARD takes leading corrections into account

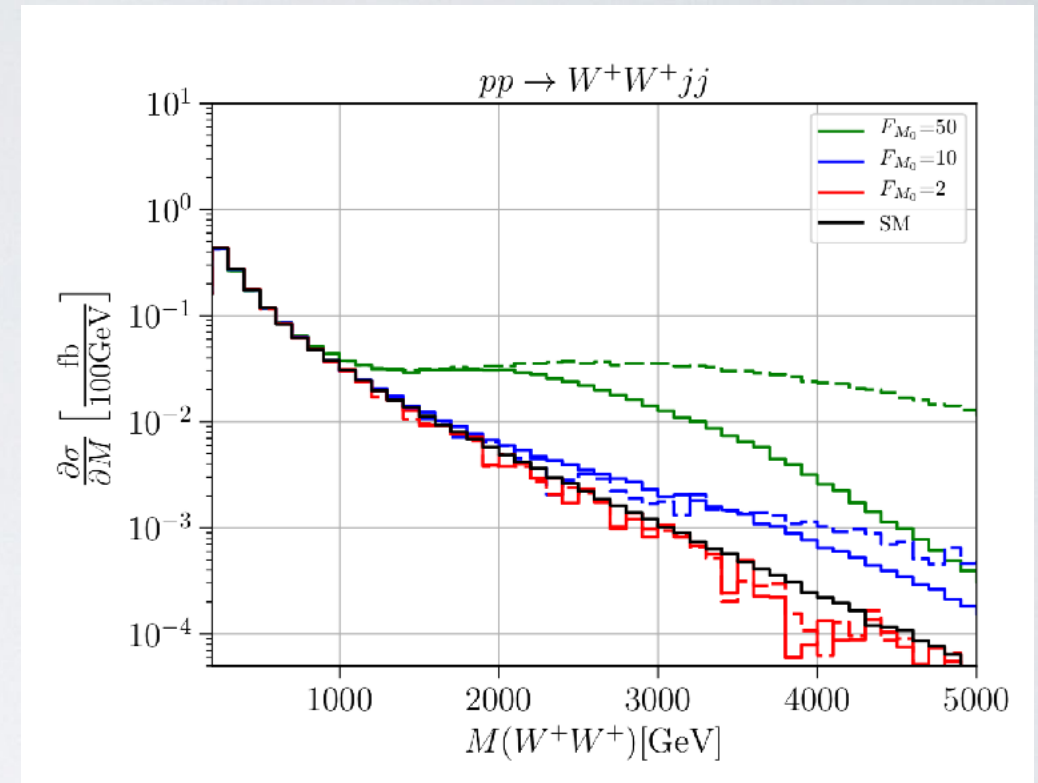
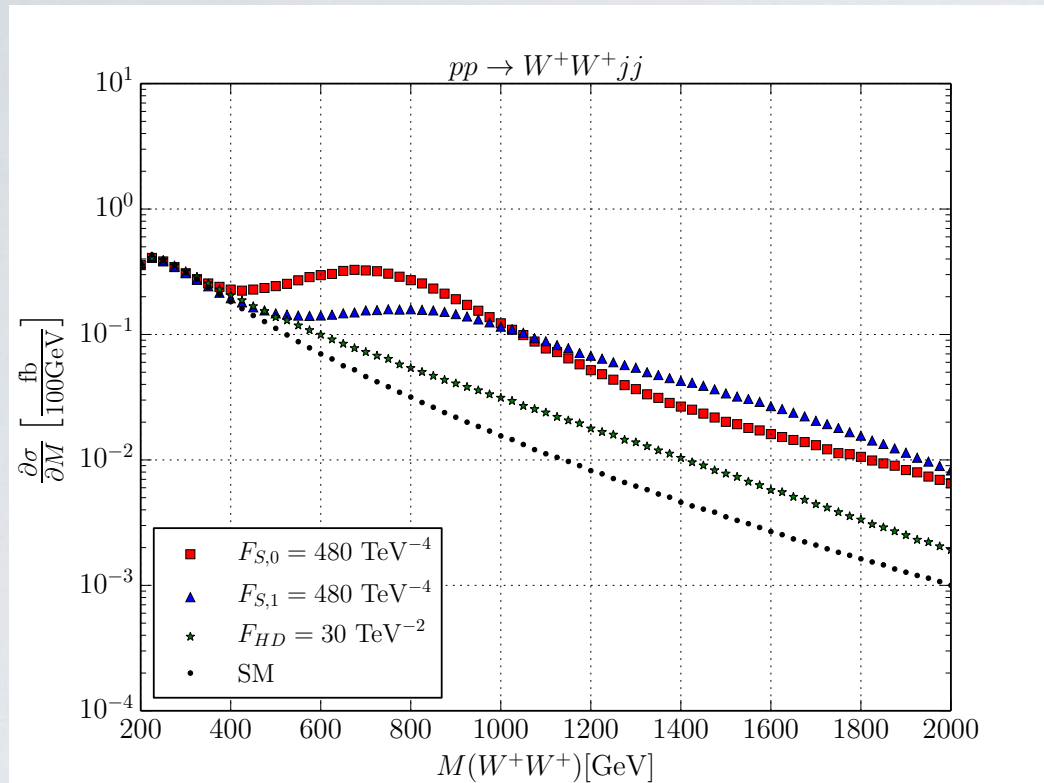




General cuts: $M_{jj} > 500 \text{ GeV}$; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20 \text{ GeV}$; $|\Delta\eta_j| < 4.5$



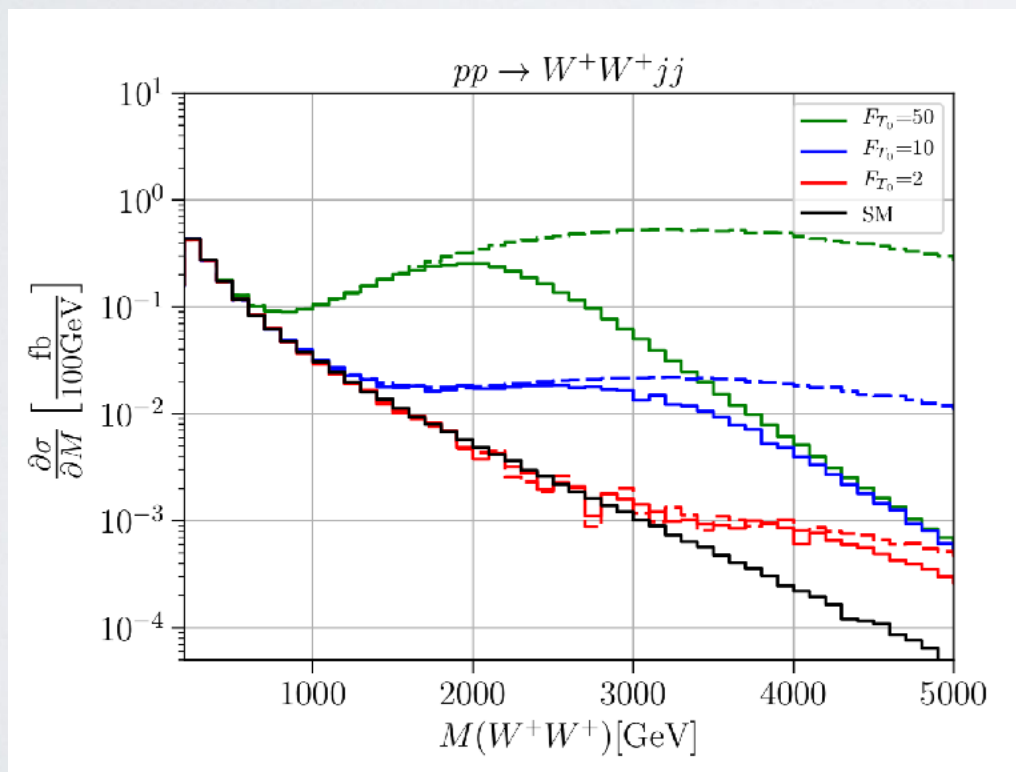
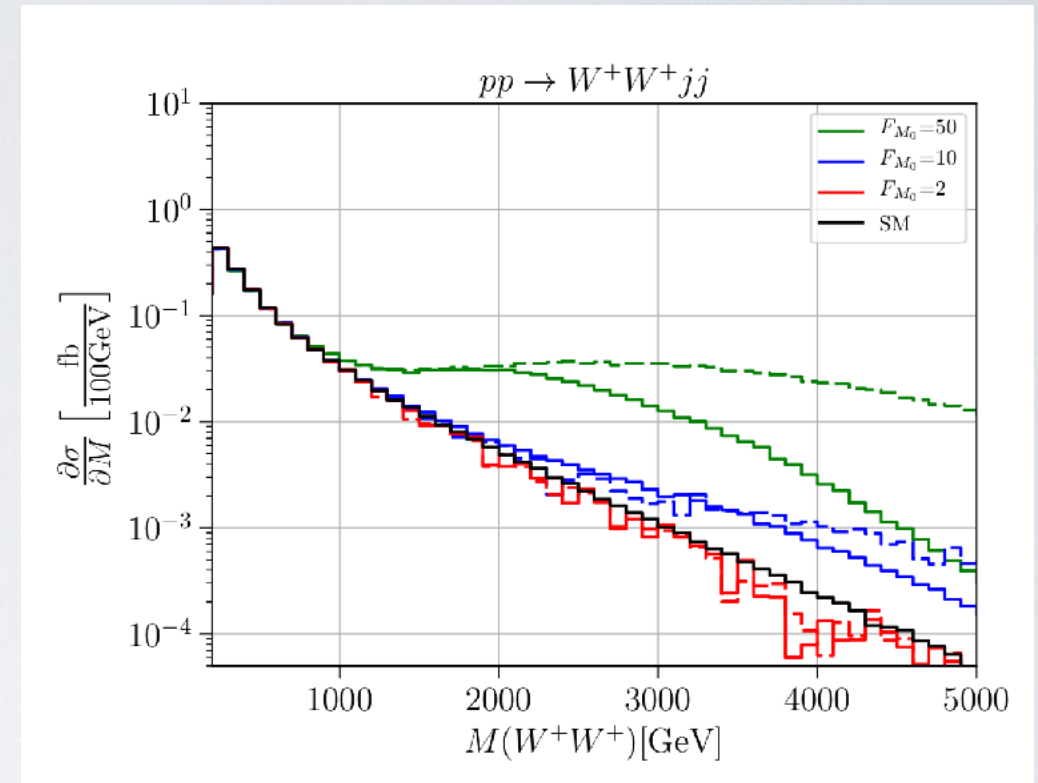
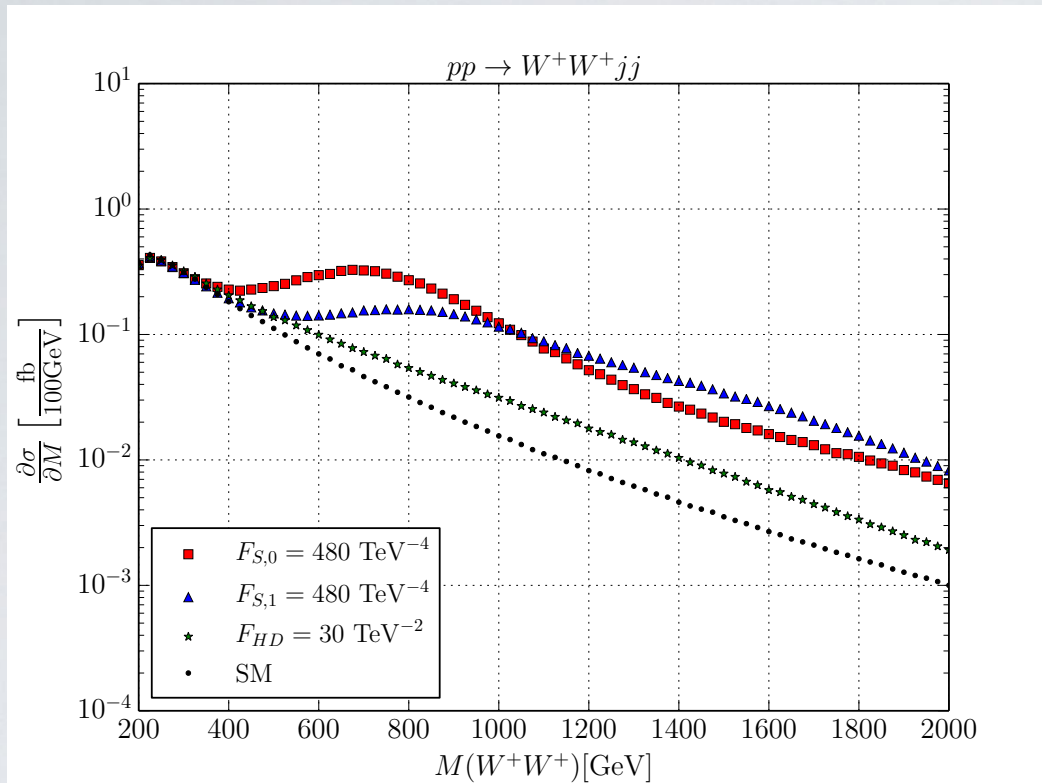
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VBS diboson spectra

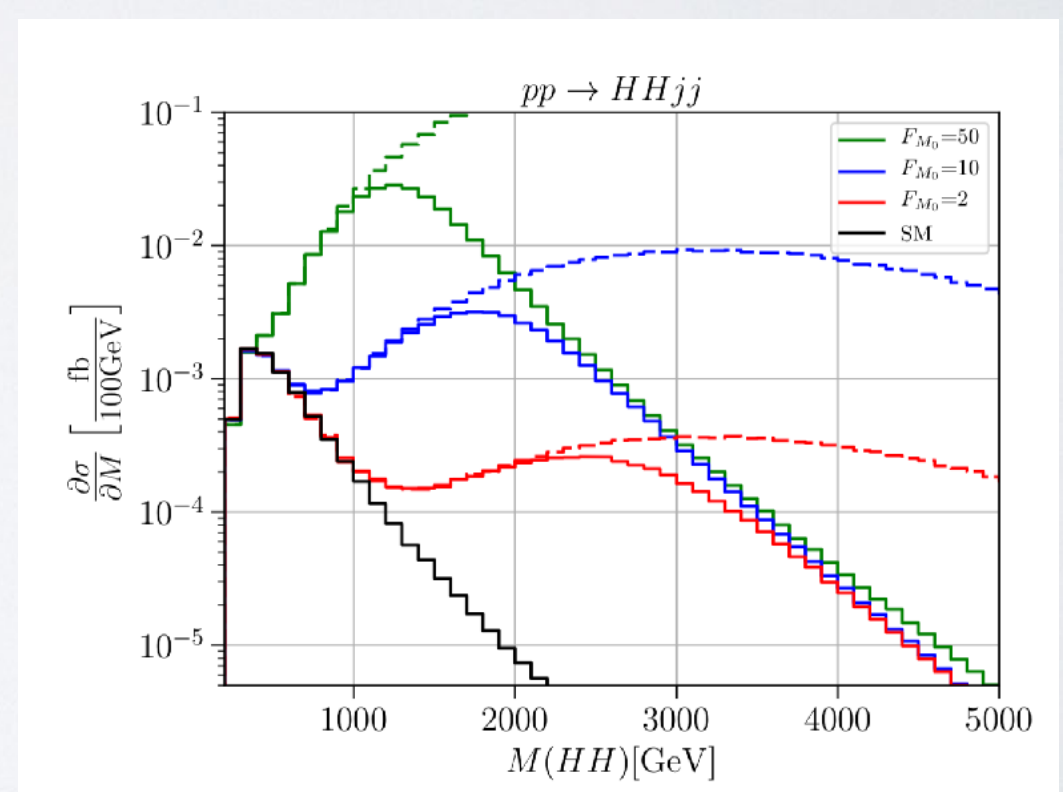
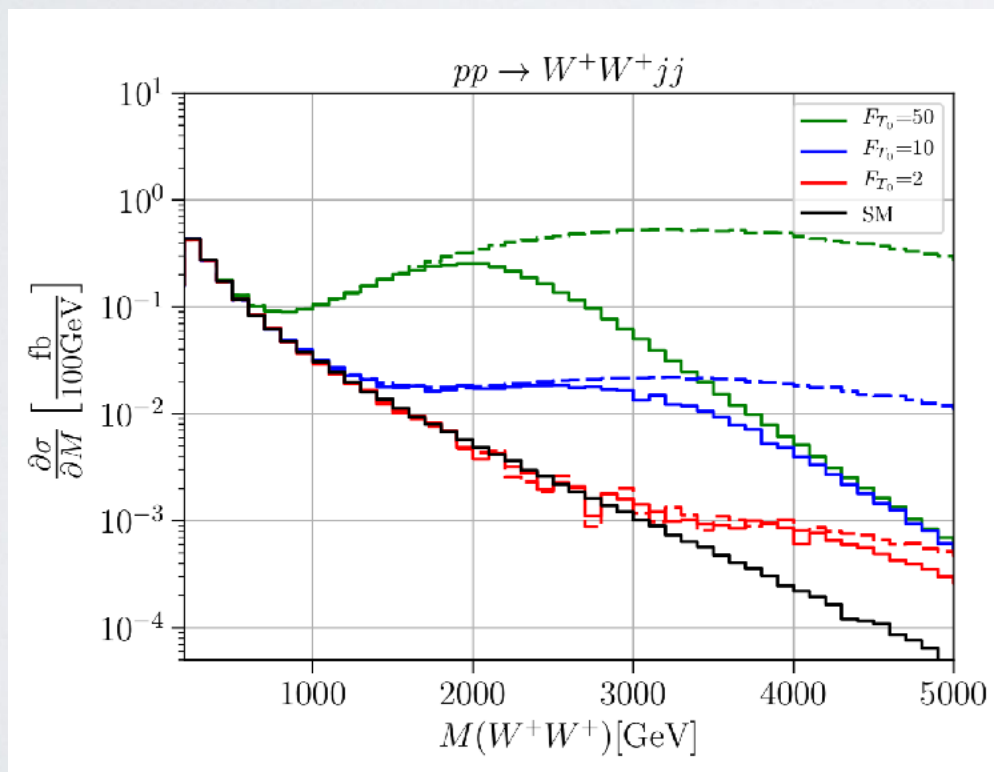
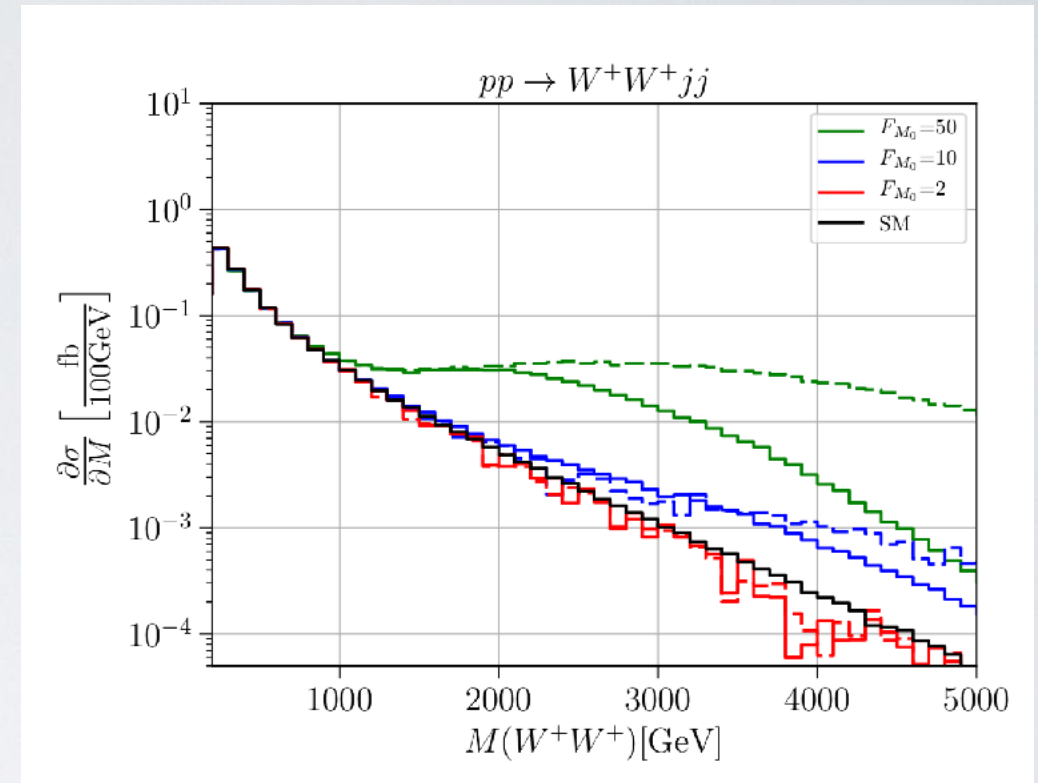
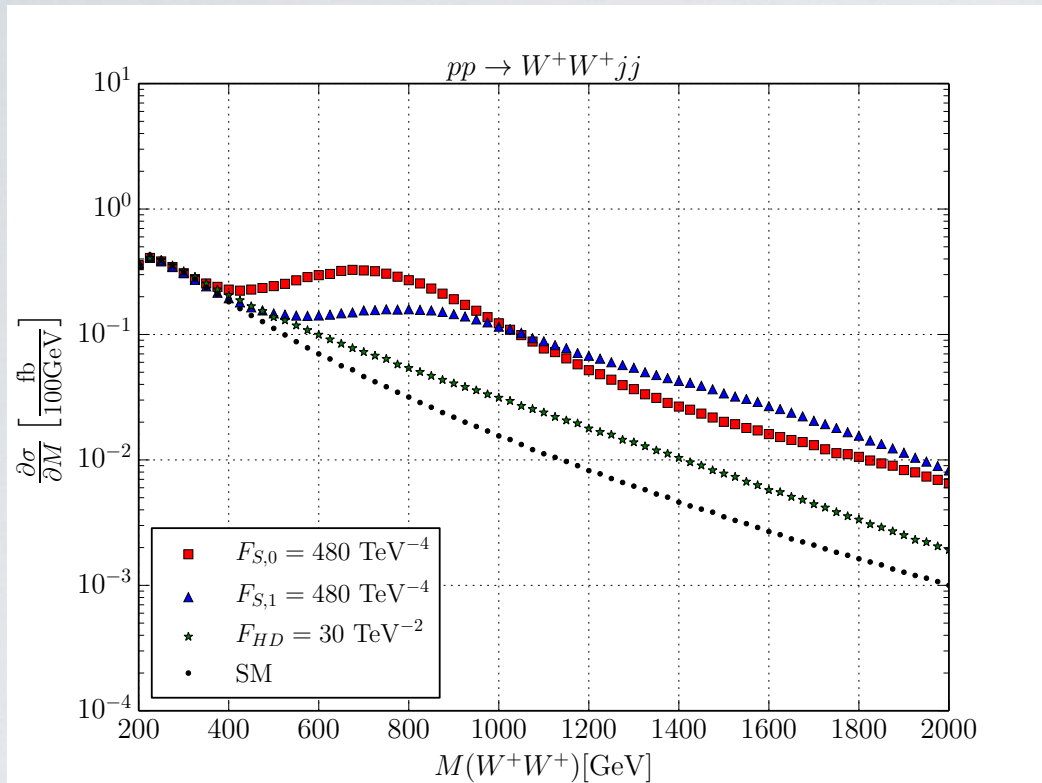
8 / 15



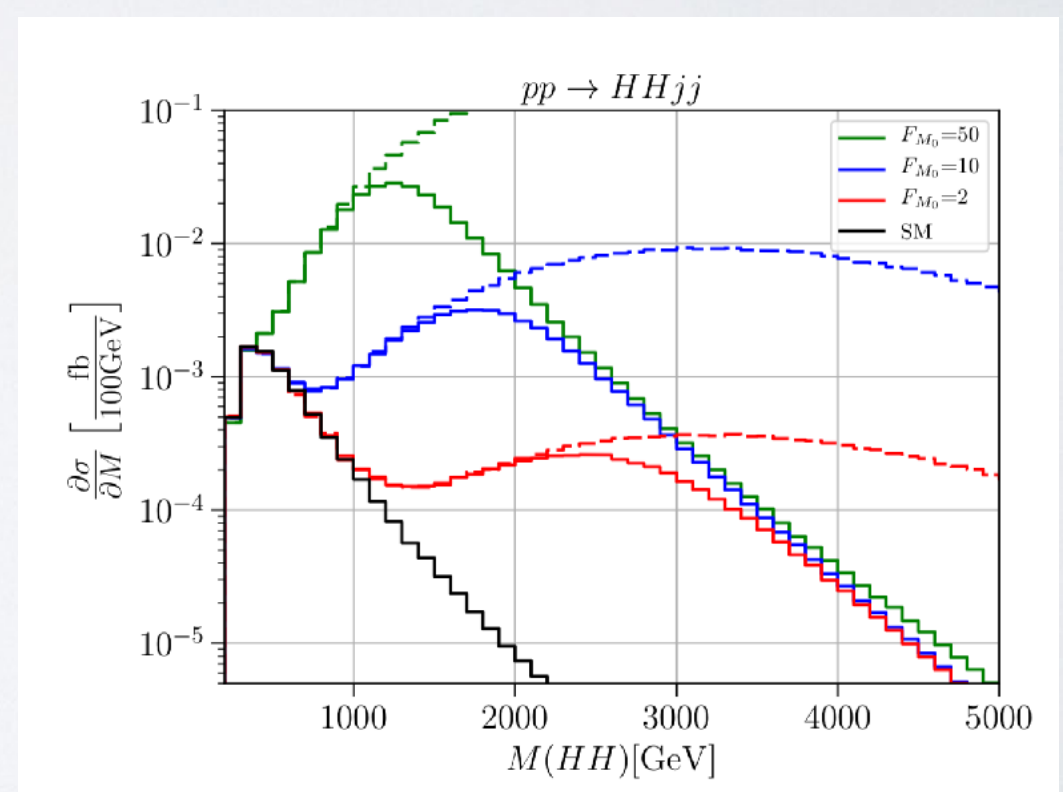
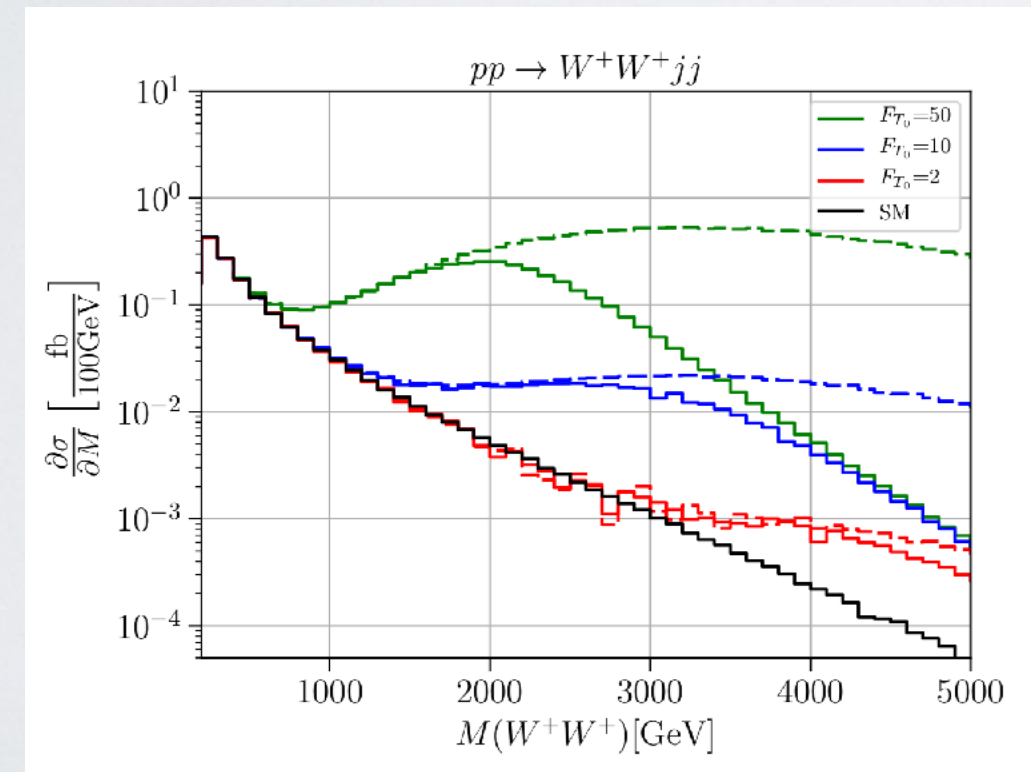
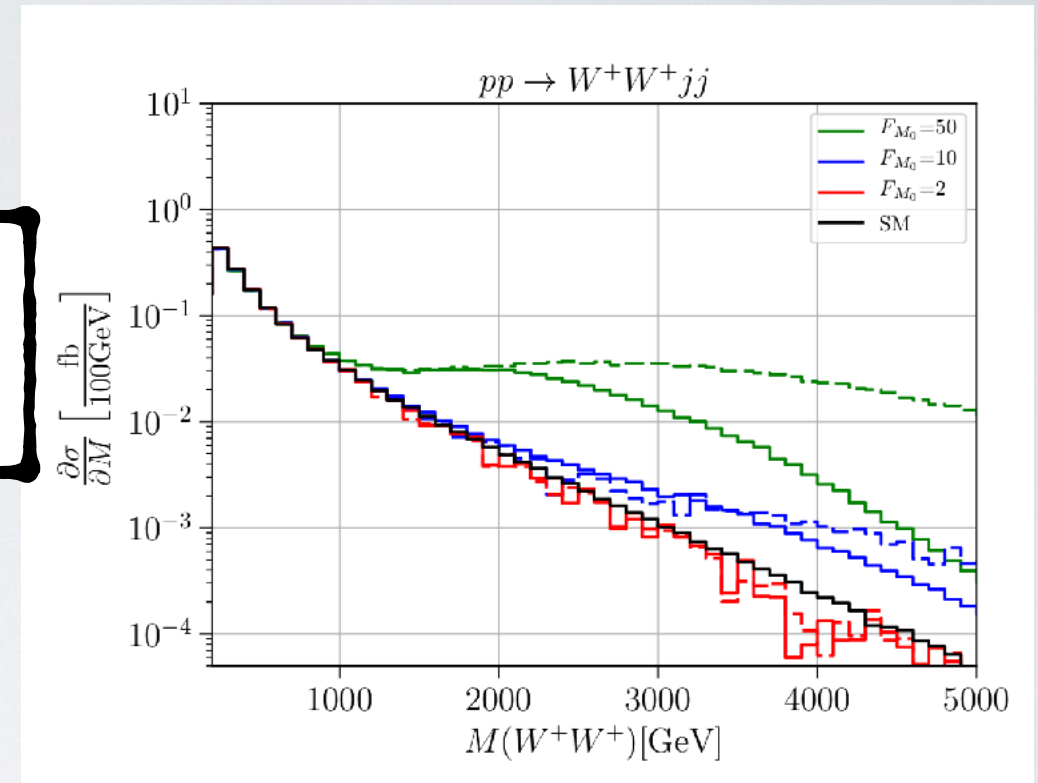
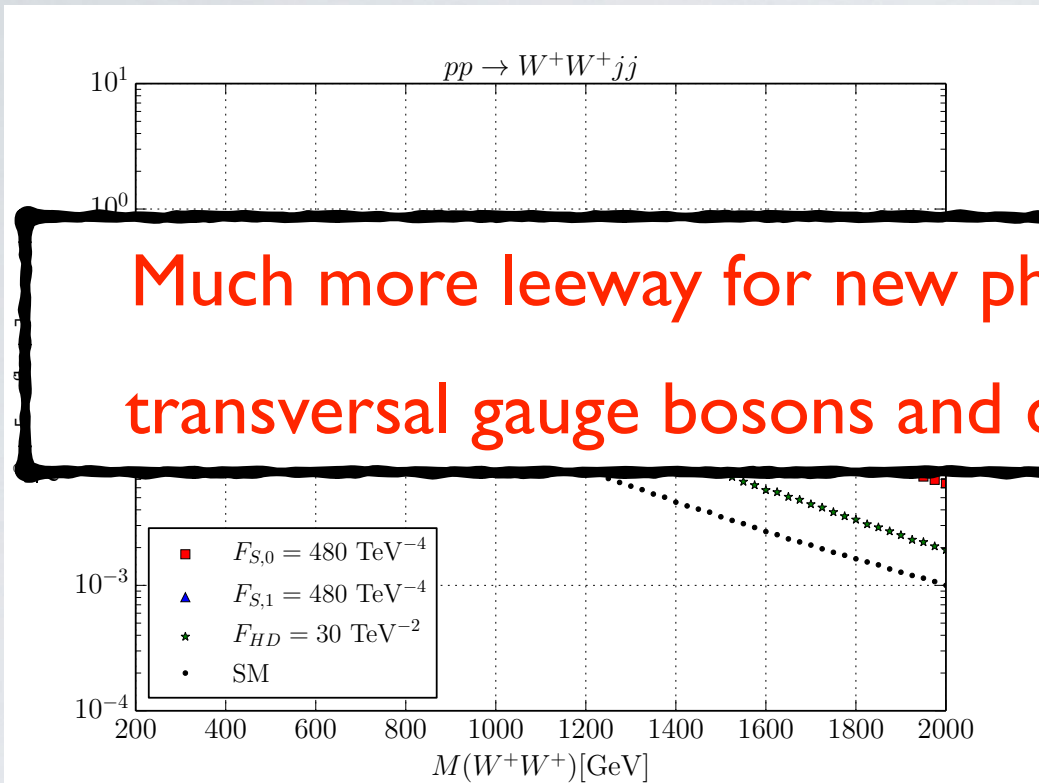
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VBS diboson spectra

8 / 15



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❑ **Resonances in direct reach** (not clear: strongly interacting models [e.g. σ resonance])

❑ **Estimate of operator coefficients** (difficult for strongly coupled models)

$$\mathcal{A}_{\text{SM}} \times \mathcal{A}_{\text{dim-6}} \gtrsim |\mathcal{A}_{\text{dim-6}}|^2 \quad \mathcal{A}_{\text{SM}} \times \mathcal{A}_{\text{dim-8}} \gtrsim |\mathcal{A}_{\text{dim-8}}|^2 \quad \mathcal{A}_{\text{SM}} \times \mathcal{A}_{\text{dim-6}} \gtrsim \mathcal{A}_{\text{SM}} \times \mathcal{A}_{\text{dim-8}}$$

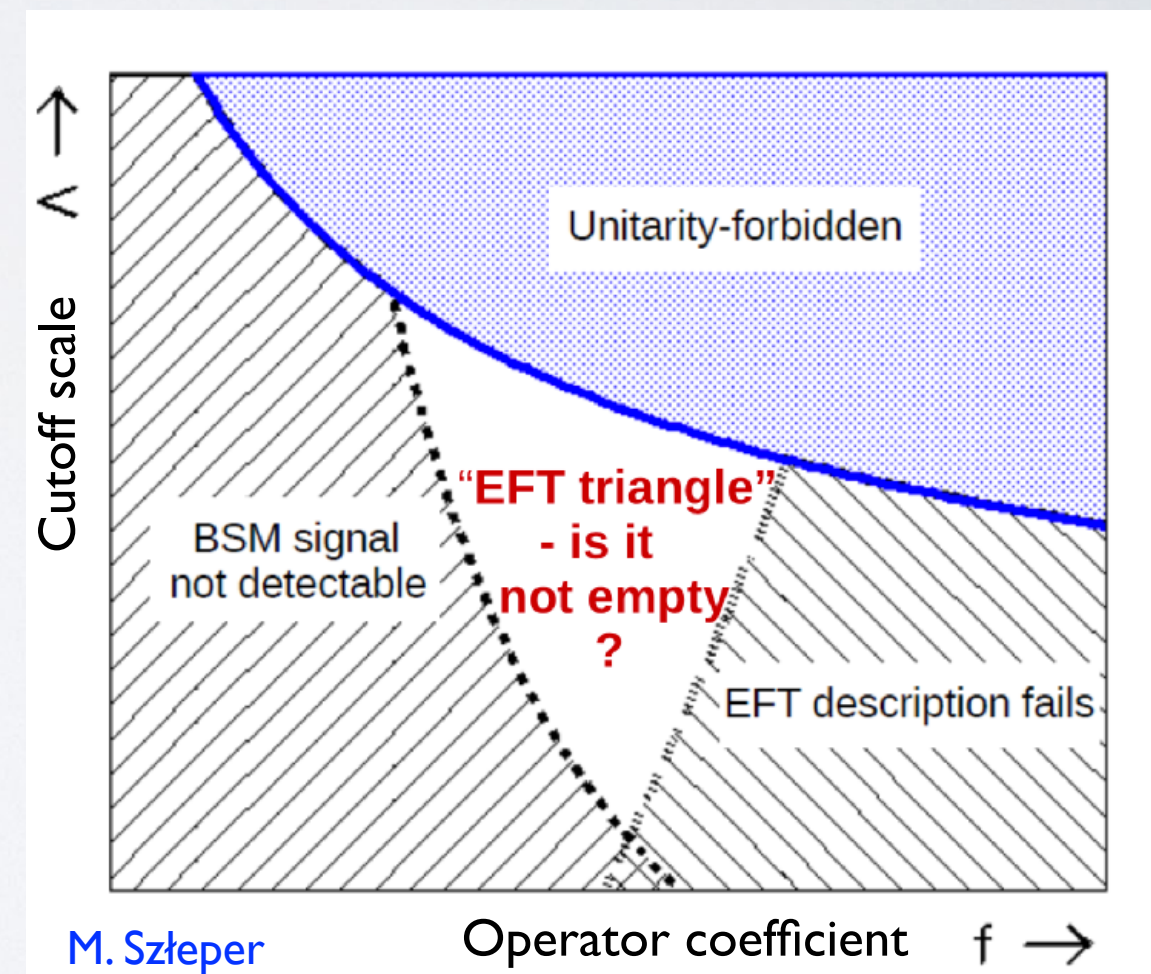
❑ **Partial wave unitarity:** gives guidance on maximally possible event numbers

❑ **Positivity constraints on operator coefficients**

❑ **Size of coefficients:** dichotomy between validity and detectability

❑ **EFT better/best[?] suited in intensity frontier** [example: HEFT @ $\mathcal{O}(100 \text{ GeV})$]

❑ **EFT borderline in energy frontier physics**



Simplified signal models: generic resonances

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🔧 Rise of amplitude: is Taylor expansion below a resonance

Courtesy: Jorge de Blas

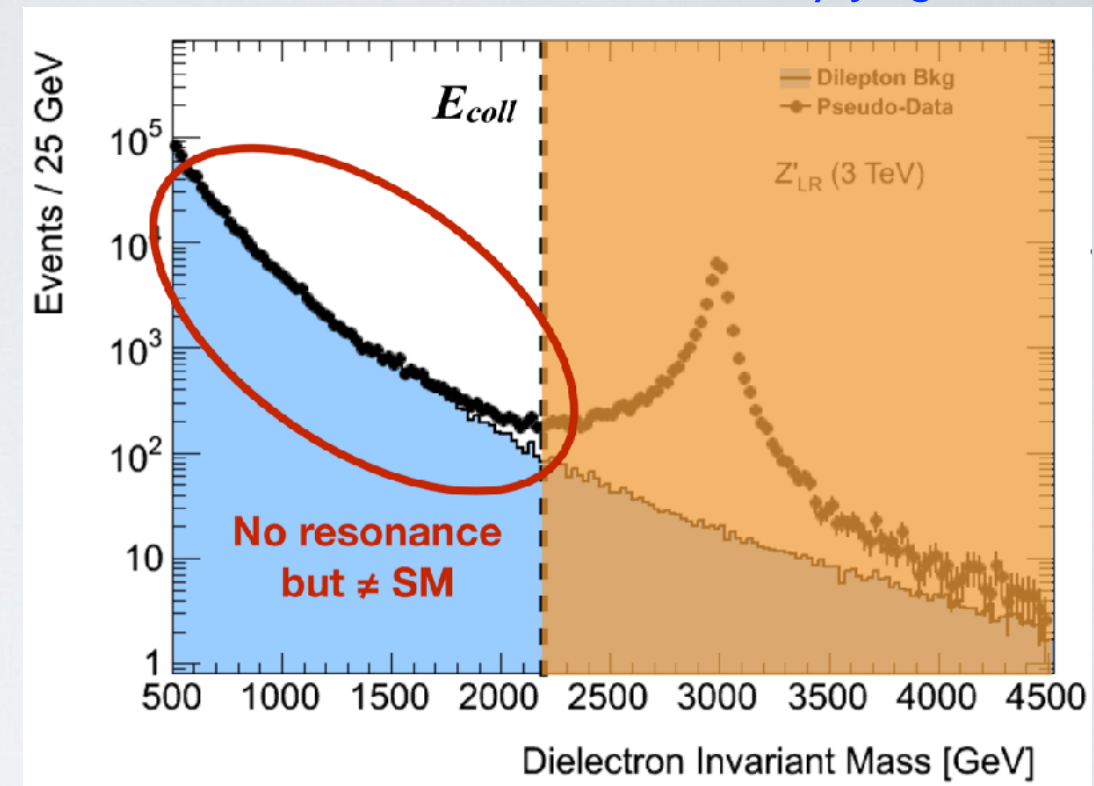
🔧 Resonances might be in direct reach of LHC

🔧 EFT framework EW-restored regime:

$$SU(2)_L \times SU(2)_R, SU(2)_L \times U(1)_Y \text{ gauged}$$

🔧 Include EFT operators in addition (more resonances, continuum contribution)

🔧 Apply T -matrix unitarization beyond resonance (“UV-incomplete” model)



Spins 0, 2 considered, Spin 1 has (partially) different physics (mixing with W/Z)

	isoscalar	isotensor
scalar	σ^0	$\phi_t^{--}, \phi_t^-, \phi_t^0, \phi_t^+, \phi_t^{++}$ $\phi_v^-, \phi_v^0, \phi_v^+$ ϕ_s^0
tensor	f^0	$\left(X_t^{--}, X_t^-, X_t^0, X_t^+, X_t^{++} \right)$ X_v^-, X_v^0, X_v^+ X_s^0
...	...	...

$$32\pi\Gamma/M^5$$

	σ	ϕ	f	X
$F_{S,0}$	$\frac{1}{2}$	2	15	5
$F_{S,1}$	—	$-\frac{1}{2}$	-5	-35

Translation into Wilson coefficients
below resonance

Start with **Fierz-Pauli Lagrangian** for symmetric tensor

$$\mathcal{L}_{\text{FP}} = \frac{1}{2} \partial_\alpha f_{\mu\nu} \partial^\alpha f^{\mu\nu} - \frac{1}{2} m^2 f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} \partial_\alpha f^\mu{}_\mu \partial^\alpha f^\nu{}_\nu + \frac{1}{2} m^2 f^\mu{}_\mu f^\nu{}_\nu \\ - \partial^\alpha f_{\alpha\mu} \partial_\beta f^{\beta\mu} - f^\alpha{}_\alpha \partial^\mu \partial^\nu f_{\mu\nu} + f_{\mu\nu} J_f^{\mu\nu}$$

- Symmetric tensor $f_{\mu\nu}$
- On-shell conditions: $10 \rightarrow 5$ components
- Tracelessness: $f^\mu{}_\mu = 0$
- Transversality: $\partial_\mu f^{\mu\nu} = 0$

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- Fierz-Pauli propagator has bad high-energy behavior**
- Use Stückelberg formalism to make off-shell high-energy behavior explicit**
- Introduce compensator fields $\Rightarrow$ no propagators with momentum factors
- Crucial for MCs

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$$\begin{aligned} \mathcal{L} = & \frac{1}{2} f_{f\mu\nu} (-\partial^2 - m_f^2) f_f^{\mu\nu} + \frac{1}{2} f_f^\mu{}_\mu \left(-\frac{1}{2} (-\partial^2 - m_f^2) \right) f_f^\nu{}_\nu \\ & + \frac{1}{2} A_{f\mu} (\partial^2 + m_f^2) A_f^\mu + \frac{1}{2} \sigma_f (-\partial^2 - m_f^2) \sigma_f \\ & + \left(f_{\mu\nu} - \frac{1}{\sqrt{6}} \sigma_f g_{\mu\nu} \right) J_f^{\mu\nu} \\ & - \left(\frac{1}{\sqrt{2} m_f} (A_{f\mu} \partial_\nu + A_{f\nu} \partial_\mu) - \frac{\sqrt{2}}{\sqrt{3} m_f^2} \sigma_f \partial_\mu \partial_\nu \right) J_f^{\mu\nu} \end{aligned}$$

- $f^{\mu\nu}$: on-shell $f^{\mu\nu}$

- ϕ : $\partial_\mu \partial_\nu f^{\mu\nu}$

- A^μ : $\partial_\nu f^{\mu\nu}$

- σ : $f^\mu{}_\mu$

Gauge fixing: $\sigma = -\phi$

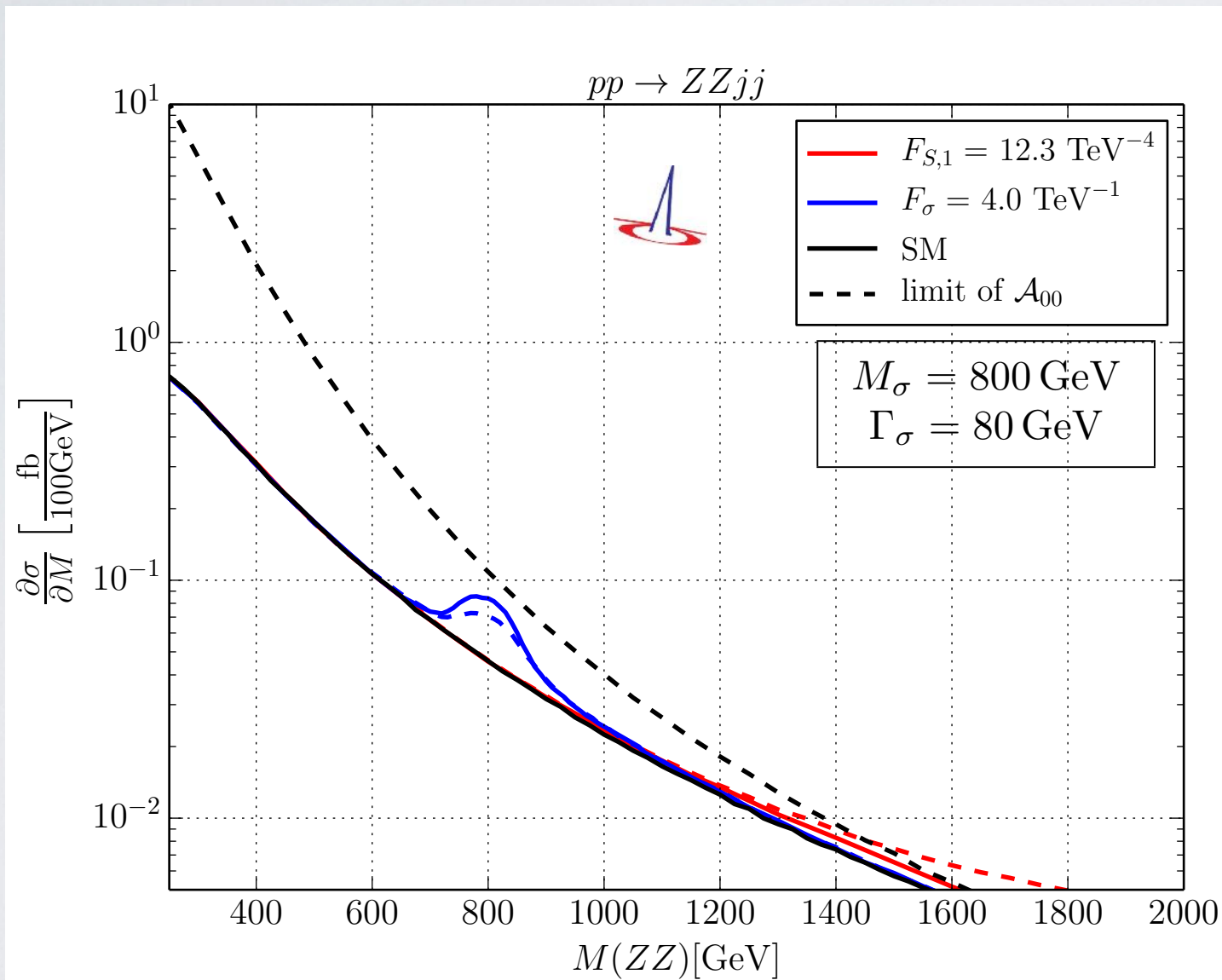
Comparison: Simplified Models & SMEFT

12 / 15

Kilian/Ohl/JRR/Sekulla: 1511.00022

Brass/Fleper/Kilian/JRR/Sekulla: 1807.02512

Black dashed line:
saturation of $\mathcal{A}_{22}(W^+W^+)/\mathcal{A}_{00}(ZZ)$



- EFT fails at resonance
- aQGC describe rise of resonance
- Unitarization applied
- Tensor resonances better visible than scalars

$$M_{jj} > 500 \text{ GeV}; \quad \Delta\eta_{jj} > 2.4; \quad p_T^j > 20 \text{ GeV}; \quad |\Delta\eta_j| < 4.5$$

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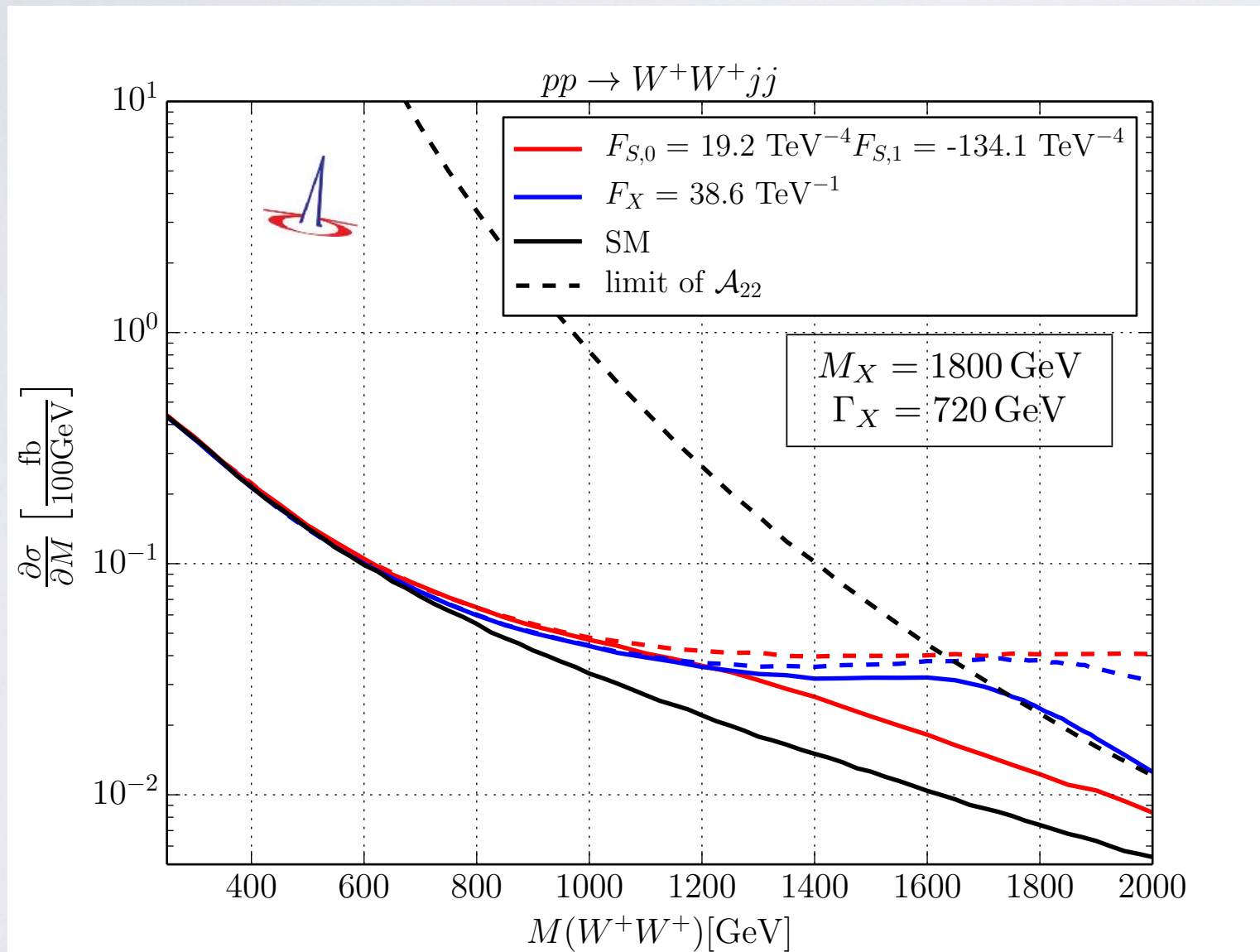
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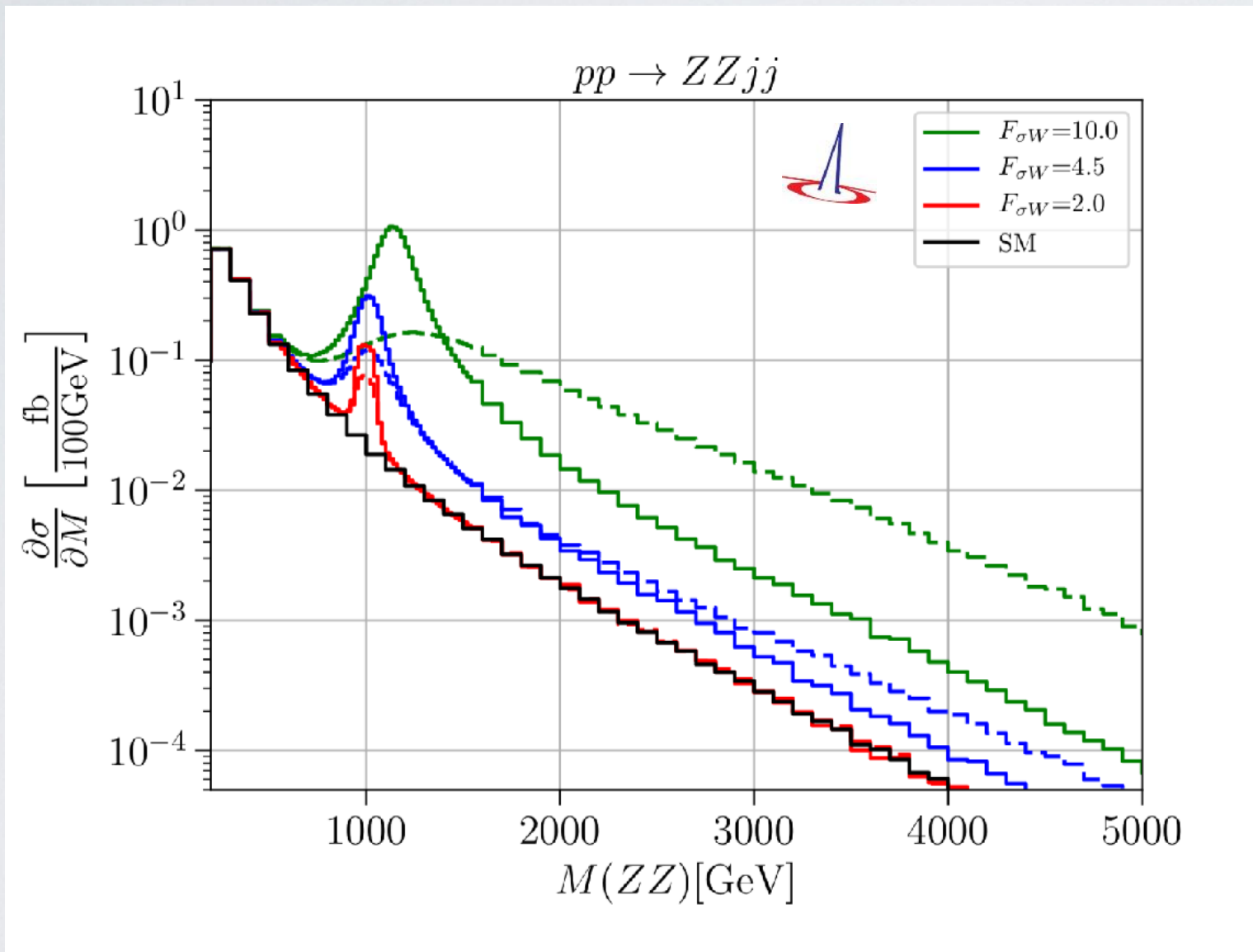
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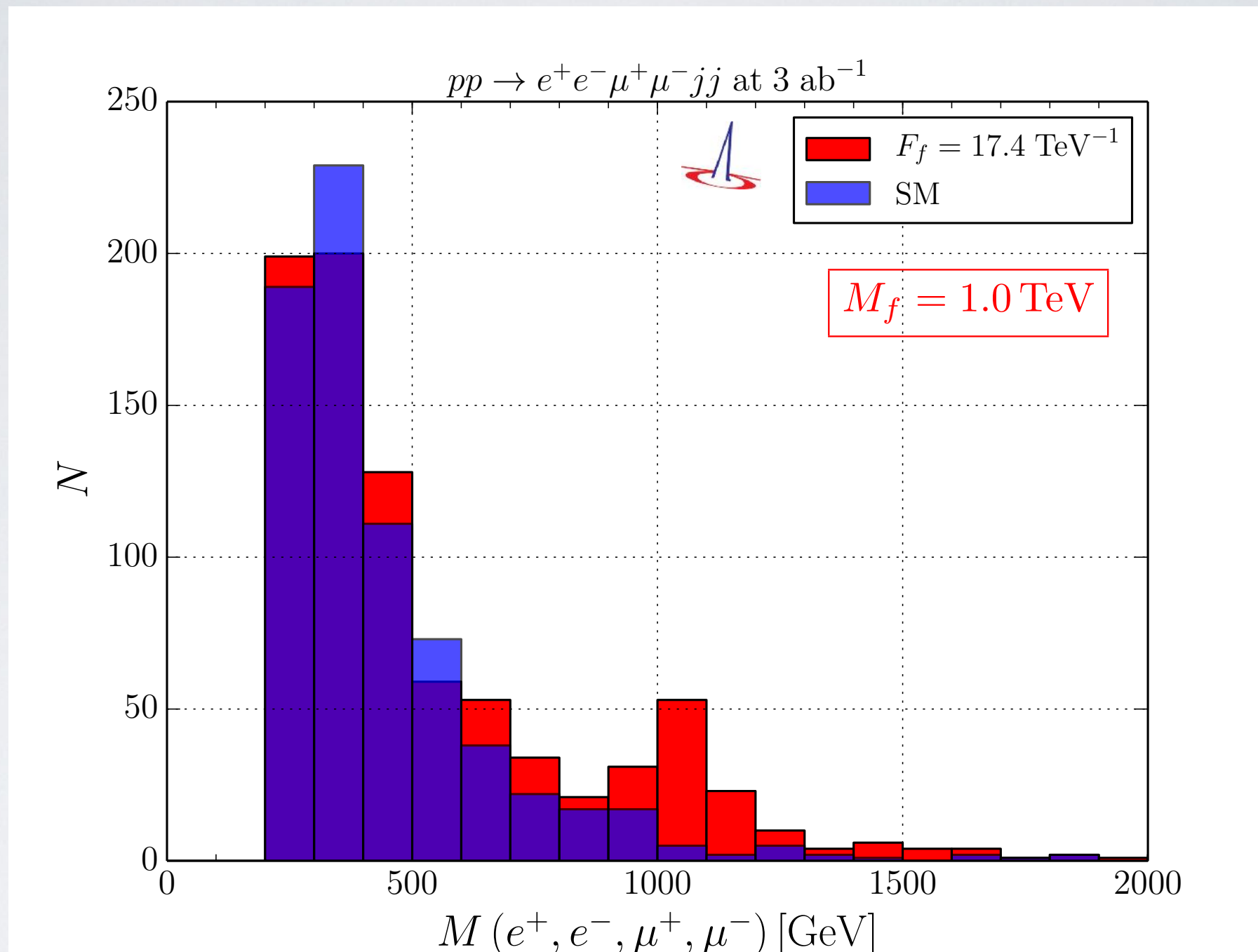
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Triple [multiple] Vector Boson Production ?

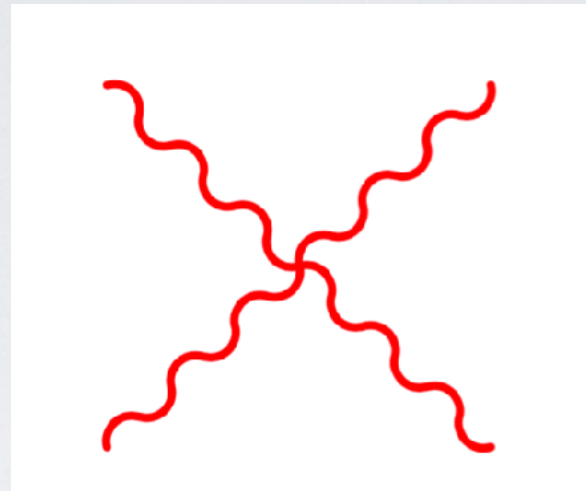
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ATLAS data: 1903.10415

Relate



to



?

Decay channel	Significance	
	Observed	Expected
WWW combined	3.3 σ	2.4 σ
WWW $\rightarrow \ell\nu\ell\nu qq$	4.3 σ	1.7 σ
WWW $\rightarrow \ell\nu\ell\nu\ell\nu$	1.0 σ	2.0 σ
WVZ combined	2.9 σ	2.0 σ
WVZ $\rightarrow \ell\nu qq\ell\ell$	–	1.0 σ
WVZ $\rightarrow \ell\nu\ell\nu\ell\ell/qq\ell\ell\ell\ell$	3.5 σ	1.8 σ
VVV combined	4.0 σ	3.1 σ

- ▶ Yes, same Feynman rule as in VBS, but ...
- ▶ one external $W/Z/\gamma$ always far off-shell
- ▶ Unitarization: work in progress (needs $2 \rightarrow 3$ unitarizations, inelastic channels) [Bahl/Kilian/Kreher/JRR, w.i.p.]
- ▶ Different Wilson coefficients dominate (particularly for resonances)
- ▶ Important physics (partially) independent from VBS (“different fiducial vol.”)

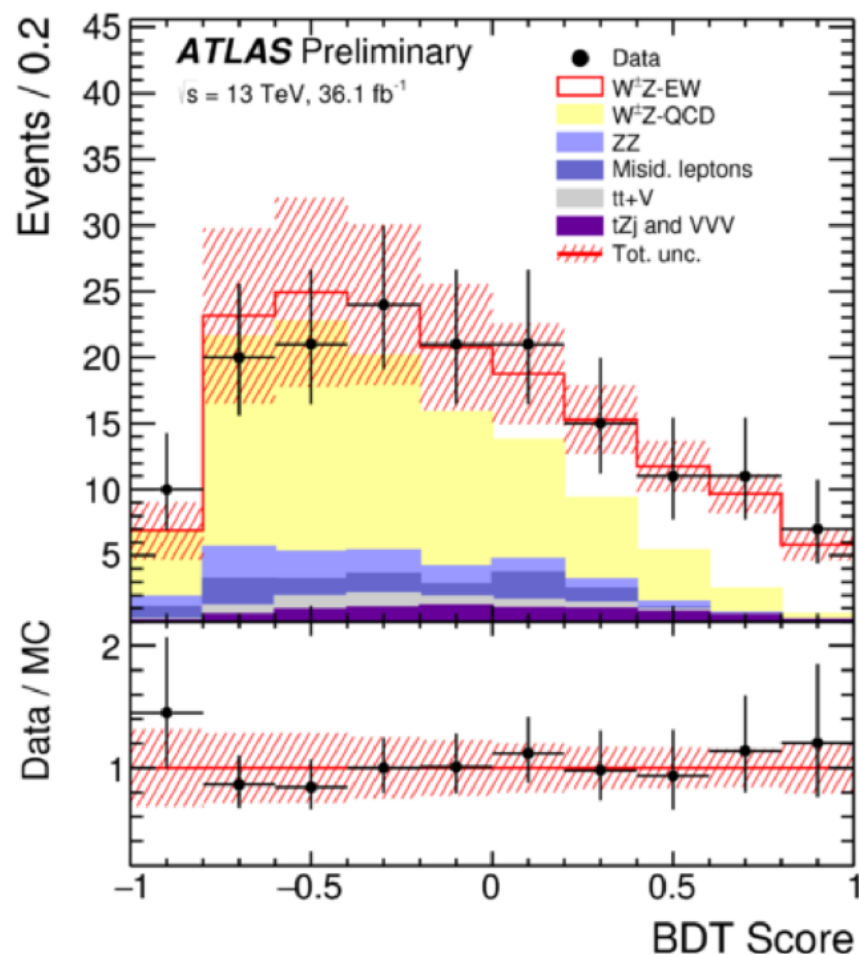
[CMS: downward fluctuation]

- ✦ Vector boson scattering one of the flagship measurements of Runs II/III/IV
- ✦ EFT provides **well-defined (and very limited) framework** for SM deviations
- ✦ There is not really a true model-independent parameterization!
- ✦ **Unitarization for theoretically sane description**
- ✦ T -matrix unitarization: no new parameters, yields maximal (bin-wise) event counts
- ✦ **Unitarization bounds: more space for new physics in di-Higgs / transverse op.**
- ✦ **Simplified models: generic electroweak resonances**
- ✦ **Huge challenge: separation of different helicity fractions**
- ✦ Interesting kinematically different constraints from tri-boson production

BACKUP SLIDES

VBS measured in many different channels

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Post-fit background normalisations

$$\mu_{WZ\text{-}QCD} = 0.60 \pm 0.25$$

$$\mu_{ttV} = 1.18 \pm 0.19$$

$$\mu_{ZZ} = 1.34 \pm 0.29$$

$$pp \rightarrow WZjj \rightarrow l\nu lljj$$

1812.09740

$$pp \rightarrow WZjj \rightarrow (l/\nu)(l/\nu)jjjj$$

1905.07714

WZjj-EW measured signal strength:

$$\mu_{EW} = 1.77 \pm 0.41(\text{stat.}) \pm 0.17(\text{syst.}) = 1.77 \pm 0.45$$

Observed sign.: 5.6σ (3.3σ expected)

Corresponding fid. cross section:

$$\begin{aligned} \sigma_{WZ^{\pm}jj \rightarrow l\nu l\ell jj}^{\text{fid., EW}} &= 0.57^{+0.15}_{-0.14} \text{ fb} \\ &= 0.57^{+0.14}_{-0.13} (\text{stat.})^{+0.05}_{-0.04} (\text{syst.})^{+0.04}_{-0.03} (\text{th.}) \text{ fb} \end{aligned}$$

Philip Chang,
plenary; Usama
Hussain, parallel
24.5.

$$pp \rightarrow WZjj \rightarrow lljj + X$$

1901.04060

$$\sigma_{WZjj}^{\text{fid}} = 3.18^{+0.57}_{-0.52} (\text{stat})^{+0.43}_{-0.36} (\text{syst}) \text{ fb} = 3.18^{+0.71}_{-0.63} \text{ fb}$$

Observed (expected) of EW WZ 1.9σ (2.7σ)

$$pp \rightarrow W^+W^+jj \rightarrow l\nu l\nu jj$$

PRL 120, 081801 (2018)

$$\sigma_{\text{fid}} = 3.83 \pm 0.66 (\text{stat}) \pm 0.35 (\text{syst}) \text{ fb}$$

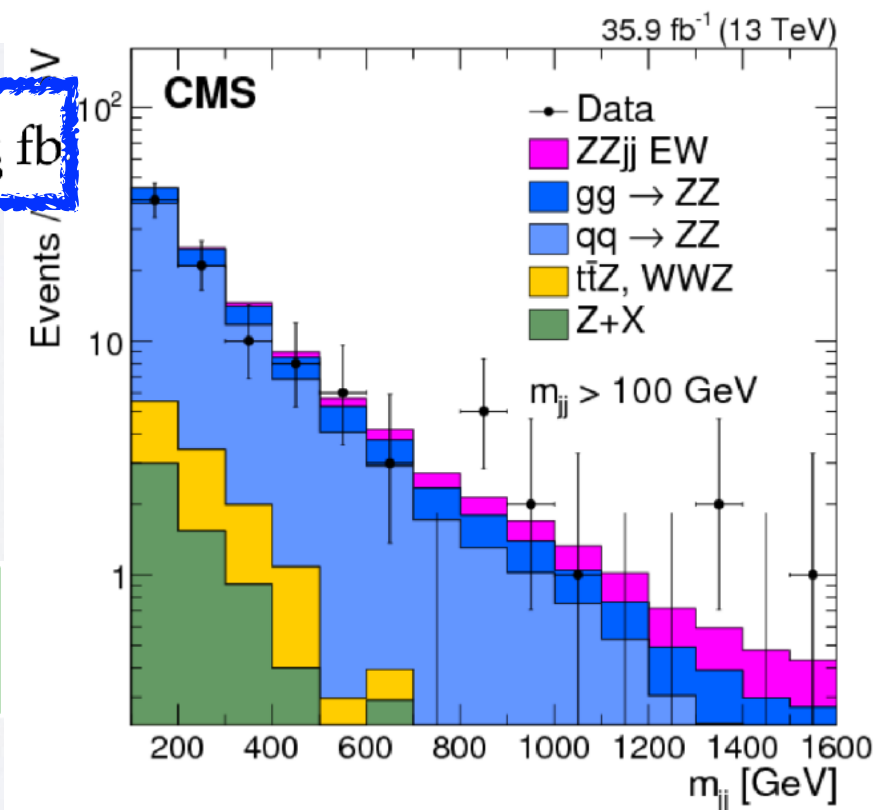
Observed (expected) of 5.5σ (5.7σ)

$$pp \rightarrow ZZjj \rightarrow lllljj$$

PLB 774(2017) 682

$$\mu = \sigma_{\text{obs}}/\sigma_{\text{th.}} = 1.39^{+0.72}_{-0.57} (\text{stat})^{+0.46}_{-0.31} (\text{syst.})$$

Observed (expected) of 2.7σ (1.6σ)



Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im} [\mathcal{M}_{ii}(t = 0)] / s \quad t = -s(1 - \cos \theta)/2$$

Partial wave amplitudes:

$$\mathcal{M}(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) \mathcal{A}_{\ell}(s) P_{\ell}(\cos \theta) \quad (\text{"Power spectrum"})$$

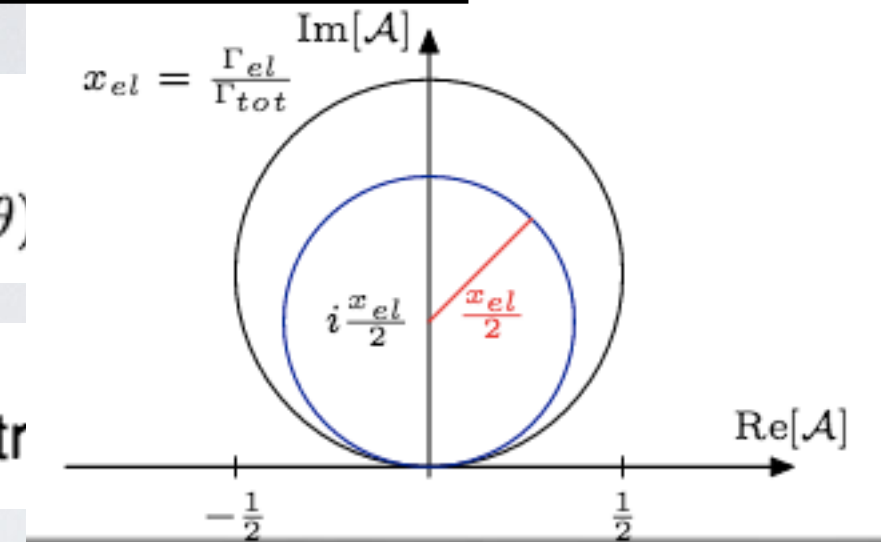
Unitarity in vector boson scattering

18 / 15

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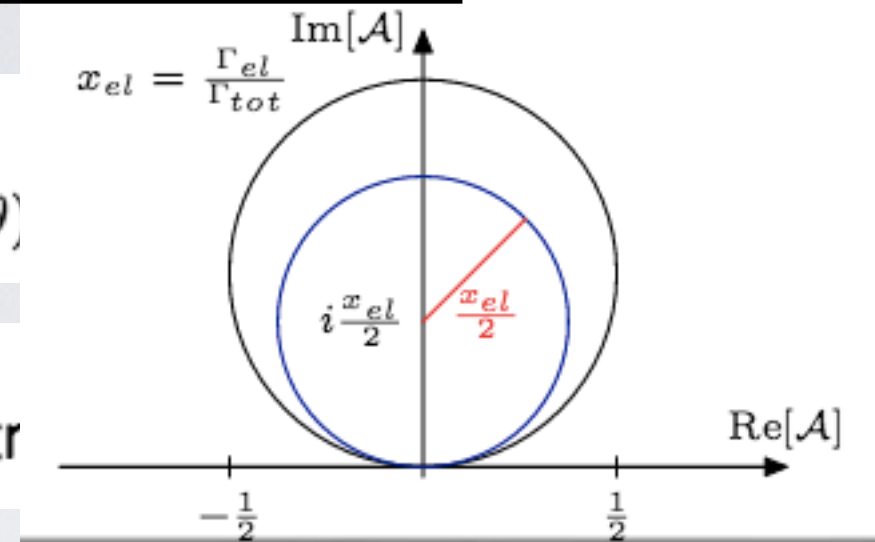
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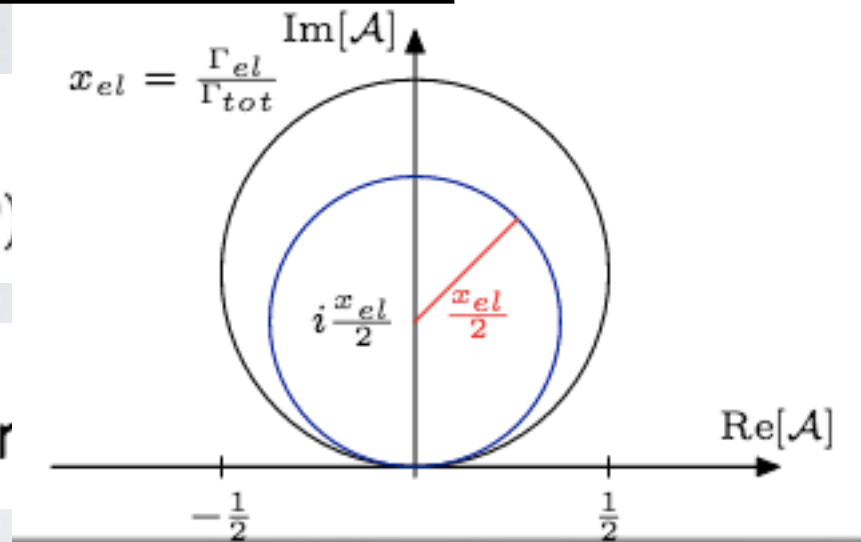
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SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I, \text{spin}=J}$):

$$\mathcal{A}_{I=0} = 2 \frac{s}{v^2} P_0(s) \quad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2} P_1(s) \quad \mathcal{A}_{I=2} = -\frac{s}{v^2} P_0(s)$$



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Lee/Quigg/Thacker, 1973

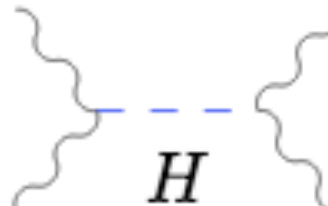
exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$$I = 0 : \quad E \sim \sqrt{8\pi} v = 1.2 \text{ TeV}$$

$$I = 1 : \quad E \sim \sqrt{48\pi} v = 3.5 \text{ TeV}$$

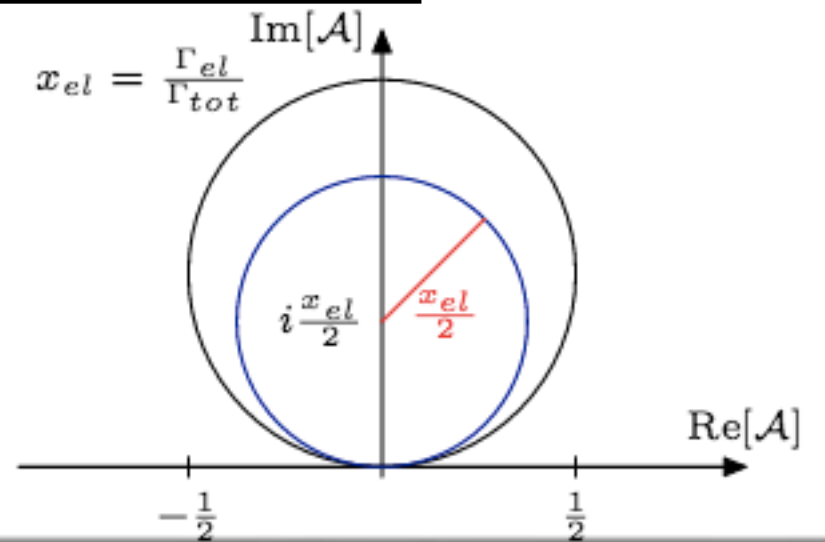
$$I = 2 : \quad E \sim \sqrt{16\pi} v = 1.7 \text{ TeV}$$

Higgs exchange:

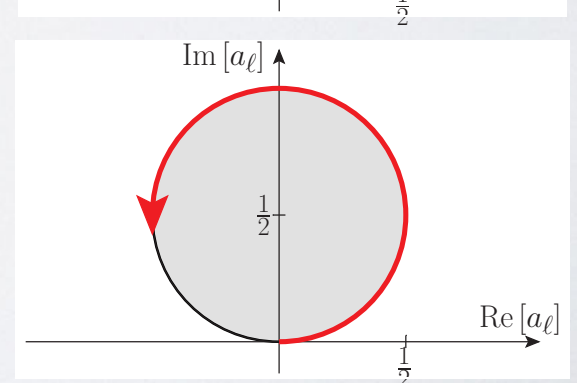
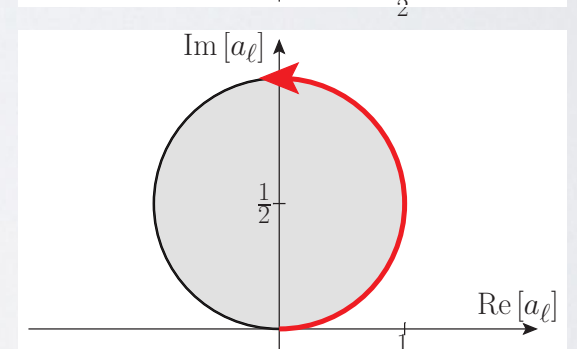
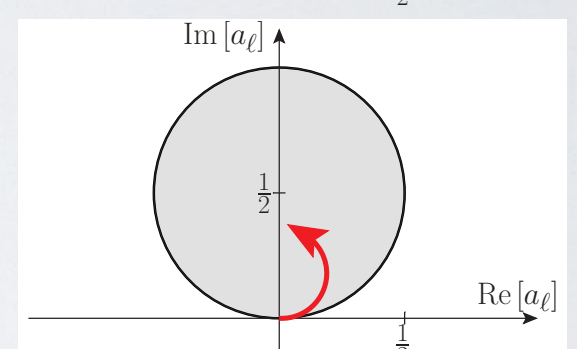
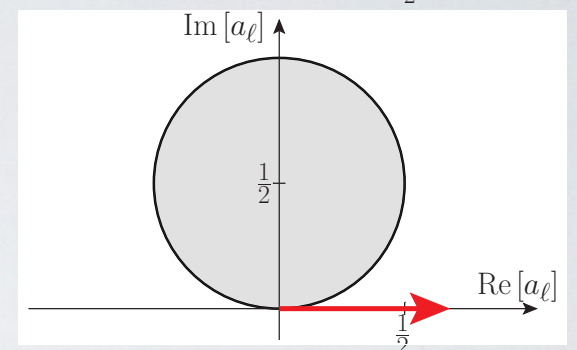
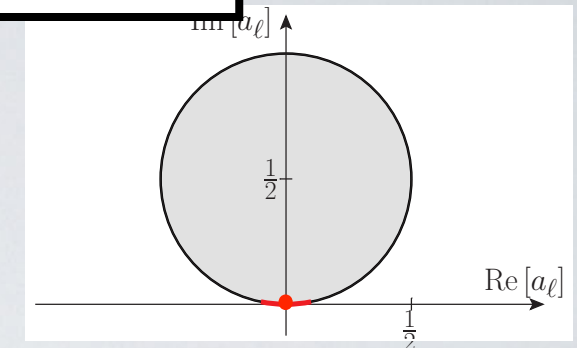


$$\mathcal{A}(s, t, u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2}$$

Unitarity: $M_H \lesssim \sqrt{8\pi} v \sim 1.2 \text{ TeV}$



1. **SM or weakly coupled physics (e.g. 2HDM):** amplitude remains close to origin
2. **Rising amplitude (at least one dim-8 operator):** rise beyond unitarity circle [unphys.], strongly interacting regime
3. **Inelastic channel opens (form-factor description):** new channels open out, multi-boson final states
4. **Saturation of amplitude:** maximal amplitude, strongly interacting continuum, K-/T-matrix unitarization
5. **New resonance:** amplitude turns over



$$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj \quad \sqrt{s} = 14 \text{ TeV} \quad \mathcal{L} = 1 \text{ ab}^{-1}$$

using K-matrix unitarization

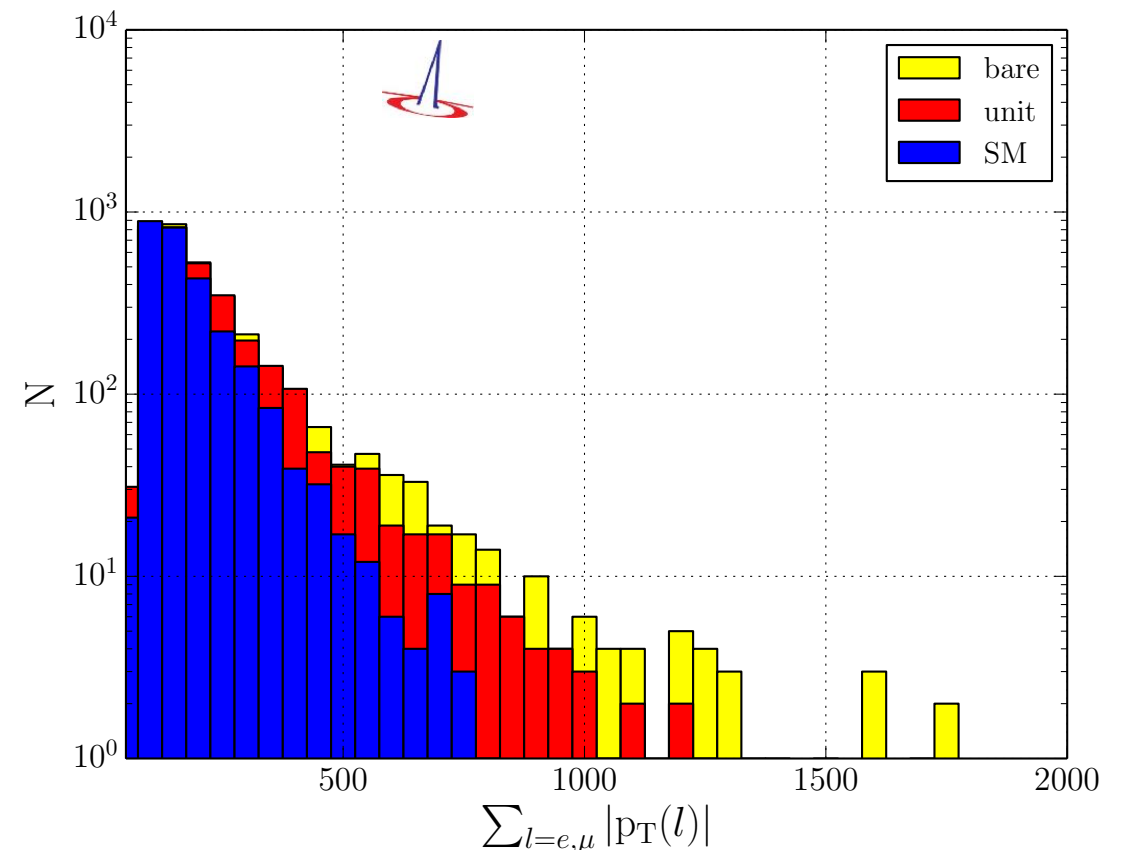
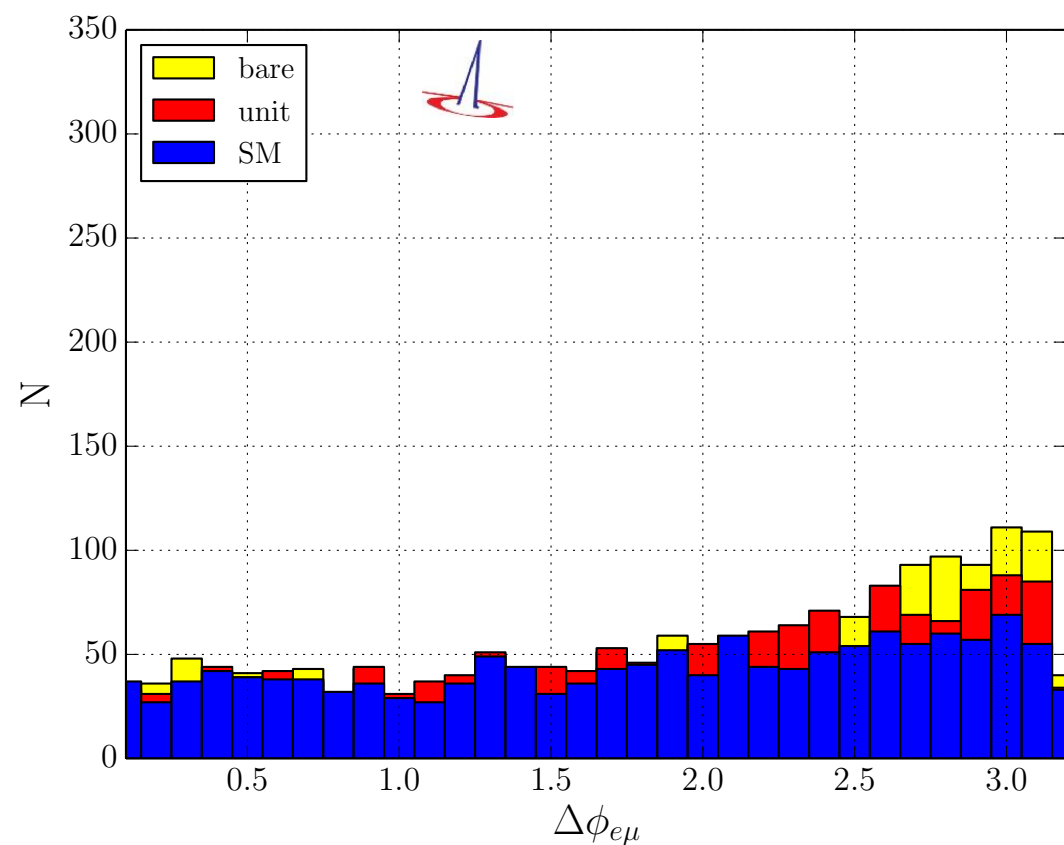
Differential spectra in VBS

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$$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj \quad \sqrt{s} = 14 \text{ TeV} \quad \mathcal{L} = 1 \text{ ab}^{-1}$$

using K-matrix unitarization

$$\mathcal{L}_{HD} = F_{HD} \text{tr} \left[\mathbf{H}^\dagger \mathbf{H} - \frac{v^2}{4} \right] \cdot \text{tr} \left[(\mathbf{D}_\mu \mathbf{H})^\dagger \mathbf{D}_\mu \mathbf{H} \right] \quad F_{HD} = 30 \text{ TeV}^{-2}$$



(now) exaggerated Wilson coefficients

$$M_{jj} > 500 \text{ GeV}; \quad \Delta\eta_{jj} > 2.4; \quad p_T^j > 20 \text{ GeV}; \quad |\Delta\eta_j| < 4.5; \quad p_T^\ell > 20 \text{ GeV}$$

Differential spectra in VBS

20 / 15

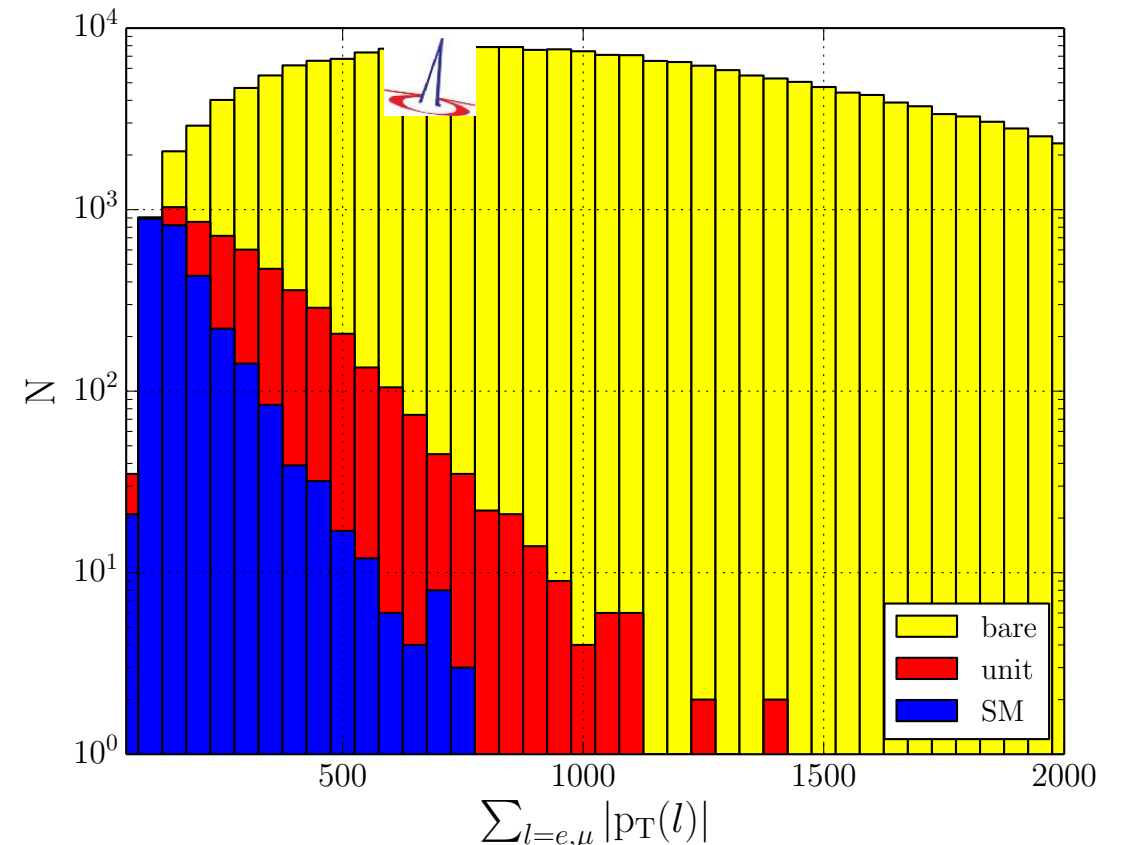
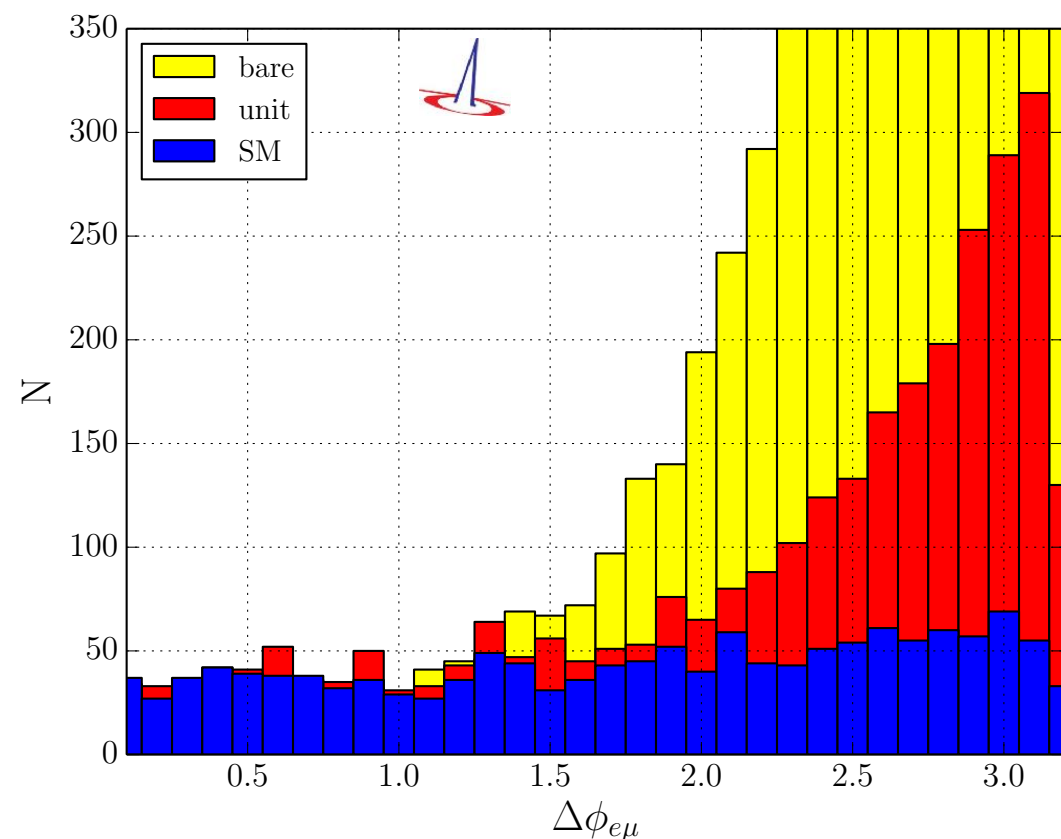
$$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj \quad \sqrt{s} = 14 \text{ TeV} \quad \mathcal{L} = 1 \text{ ab}^{-1}$$

using K-matrix unitarization

$$\mathcal{L}_{S,0} = F_{S,0} \frac{v^4}{16} \text{Tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \text{Tr} [\mathbf{V}^\mu \mathbf{V}^\nu]$$

$$\mathcal{L}_{S,1} = F_{S,1} \frac{v^4}{16} \text{Tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \text{Tr} [\mathbf{V}_\nu \mathbf{V}^\nu]$$

$$F_{S,0} = 480 \text{ TeV}^{-4}$$



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$$M_{jj} > 500 \text{ GeV}; \quad \Delta\eta_{jj} > 2.4; \quad p_T^j > 20 \text{ GeV}; \quad |\Delta\eta_j| < 4.5; \quad p_T^\ell > 20 \text{ GeV}$$

Differential spectra in VBS

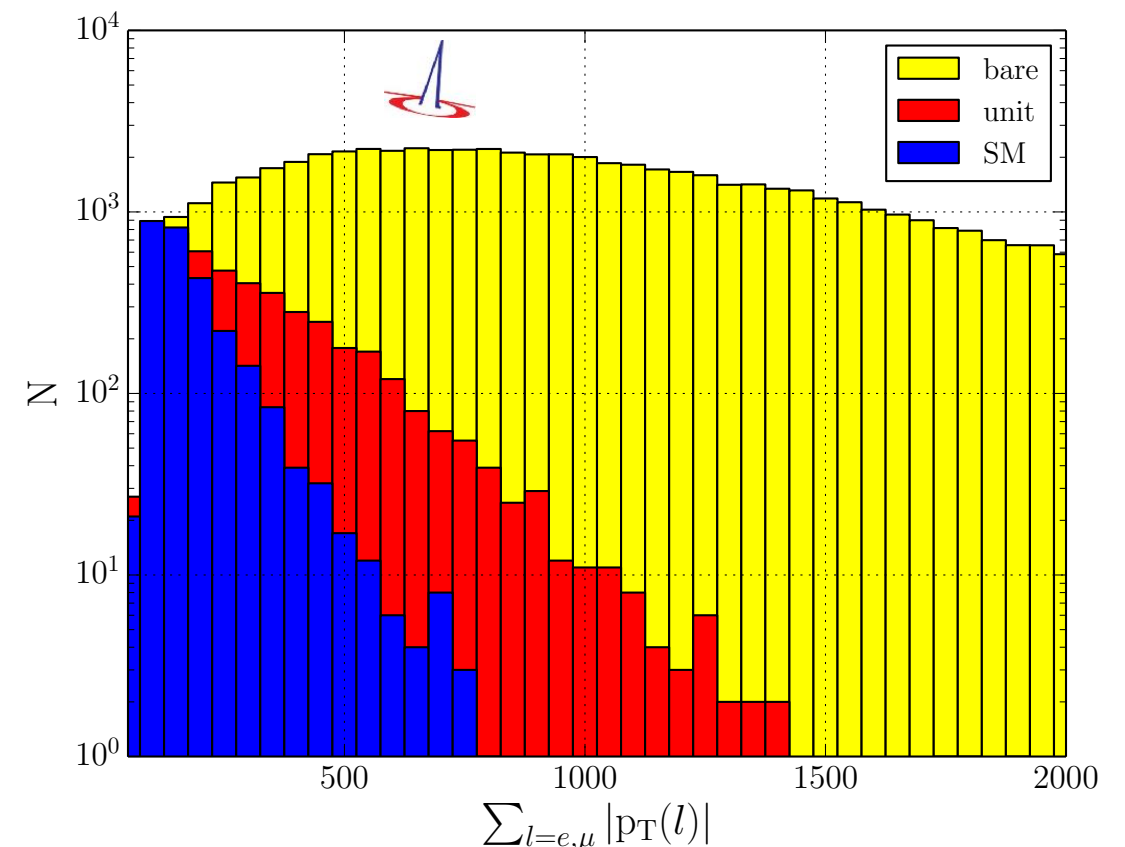
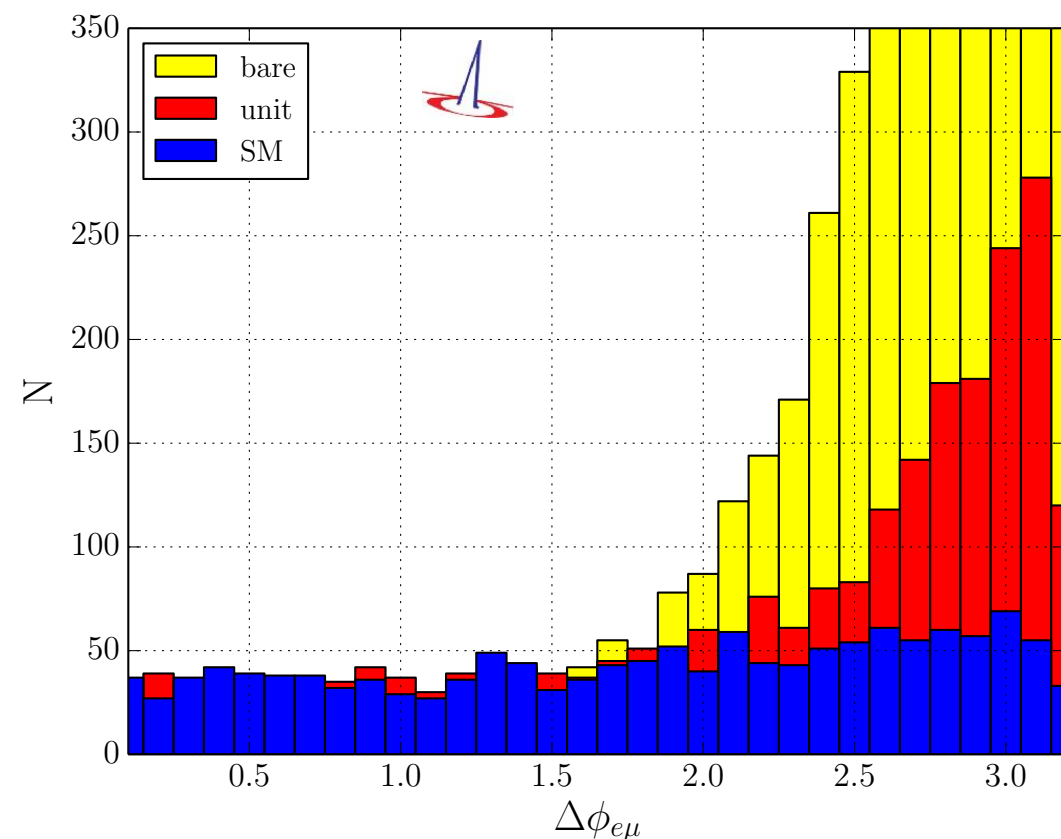
$$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj \quad \sqrt{s} = 14 \text{ TeV} \quad \mathcal{L} = 1 \text{ ab}^{-1}$$

using K-matrix unitarization

$$\mathcal{L}_{S,0} = F_{S,0} \frac{v^4}{16} \text{Tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \text{Tr}[\mathbf{V}^\mu \mathbf{V}^\nu]$$

$$\mathcal{L}_{S,1} = F_{S,1} \frac{v^4}{16} \text{Tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \text{Tr}[\mathbf{V}_\nu \mathbf{V}^\nu]$$

$$F_{S,1} = 480 \text{ TeV}^{-4}$$



(now) exaggerated Wilson coefficients

$$M_{jj} > 500 \text{ GeV}; \quad \Delta\eta_{jj} > 2.4; \quad p_T^j > 20 \text{ GeV}; \quad |\Delta\eta_j| < 4.5; \quad p_T^\ell > 20 \text{ GeV}$$