(Electroweak) Vector Boson Scattering at the LHC after the Higgs discovery

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HELMHOLTZ GEMEINSCHAFT

DESY

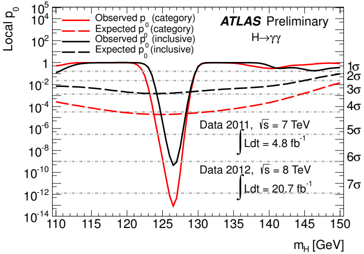
PHYSICS AT THE TERA SCALE Helmholtz Alliance

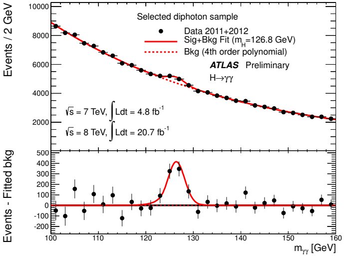
Alboteanu/Kilian/JRR, **JHEP 0811** (2008) 010;
Beyer/Kilian/Krstonošić/Mönig/JRR/Schmitt/Schröder, **EPJC 48** (2006), 353;
JRR/Kilian/Sekulla, 1307.8170; Kilian/JRR/Ohl/Sekulla, 1408.6207 (**PRD**) + in prep.

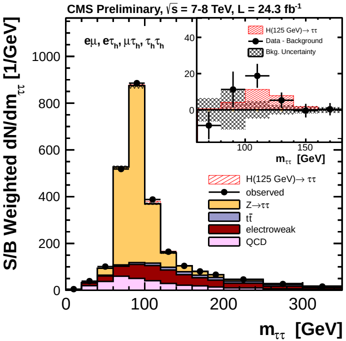
Seminar, University of Helsinki, Feb. 17th, 2015

Standard Model Triumph:

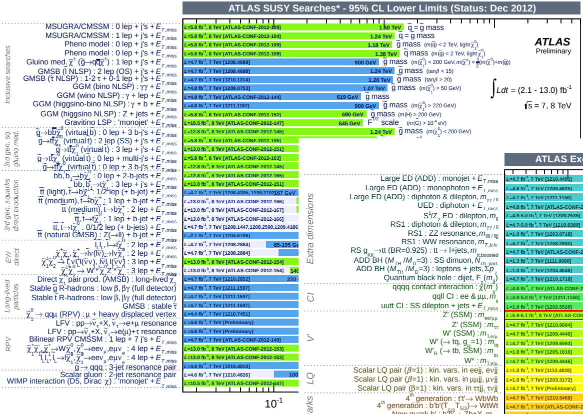
- 2012: Discovery of a Higgs boson



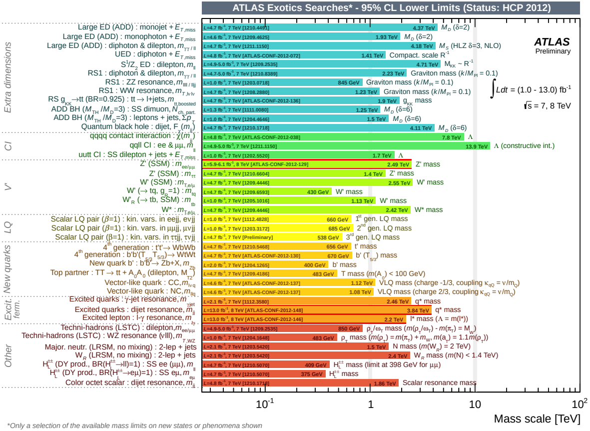




No evidence beyond SM ... and what now?



*Only a selection of the available mass limits on new states or phenomena shown.
All limits quoted are observed minus 1σ theoretical signal cross section uncertainty.



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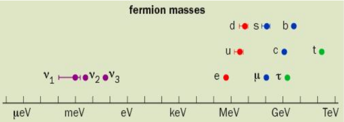
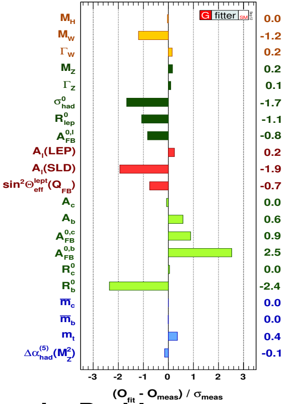
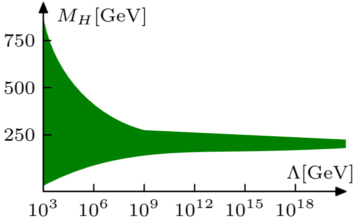
Doubts on the Standardmodel

- describes microcosm (too good?)
- 28 free parameters



- Higgs ?, form of Higgs potential ?





Hierarchy Problem

chiral symmetry: $\delta m_f \propto v \ln(\Lambda^2/v^2)$

no symmetry for quantum corrections to Higgs mass

$$\delta M_H^2 \propto \Lambda^2 \sim M_{\text{Planck}}^2 = (10^{19})^2 \,\text{GeV}^2$$

$(125\ \text{GeV})^2$ = (1000000000000000000000000000000000020000 –

1000000000000000000000000000000000000000) GeV^2

Extensions of the SM

- Lagrangian of the EW SM (no fermions/QCD here):

$$\mathcal{L}_{EW} = -\frac{1}{2}\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \boxed{(D_\mu\Phi)^\dagger(D^\mu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2}$$

with building blocks:

$$\begin{aligned} D_\mu &= \partial_\mu + \frac{i}{2} g\tau^I W^I_\mu + \frac{i}{2} g' B_\mu \\ W_{\mu\nu} &= \frac{i}{2} g\tau^I (\partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g\epsilon_{IJK} W^J_\mu W^K_\nu) \\ B_{\mu\nu} &= \frac{i}{2} g' (\partial_\mu B_\nu - \partial_\nu B_\mu) \end{aligned}$$

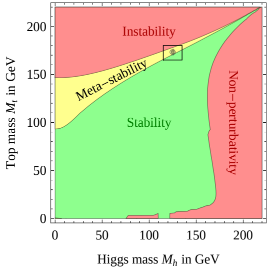
- Any EFT has higher-dimensional operators: Weinberg, 1979

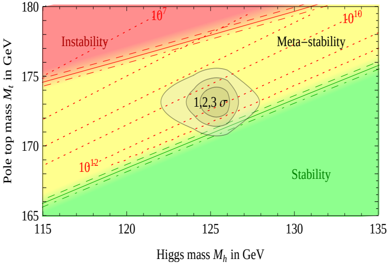
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[\frac{a_i}{\Lambda}\mathcal{O}_i^{(5)} + \frac{c_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \frac{e_i}{\Lambda^4}\mathcal{O}_i^{(8)} \cdots\right]$$

- without more fundamental theory ⇒ no clue on the scale (neither on the coefficients)

Electroweak vacuum stability

- Recent analysis: Metastable vacuum with lifetime longer than the age of the universe   Degrassi et al., arXiv:1205.6497





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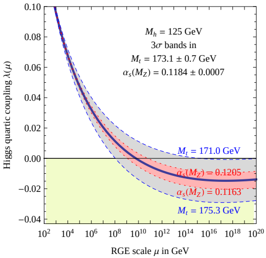

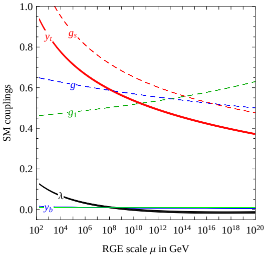

- Could the Higgs field ever have fallen in the correct vacuum? Hertzberg, arXiv:1210.3624

- Importance of higher terms in Higgs potential (gravity etc.) ?

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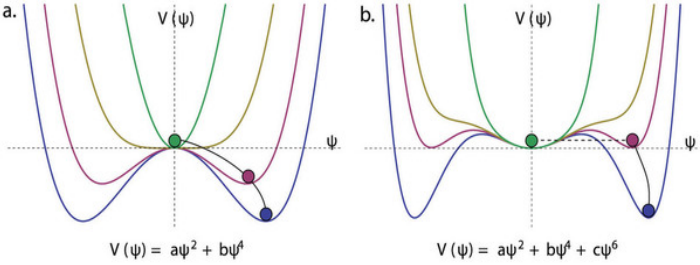

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Motivation

- Light (SM-like) Higgs boson found (clear from EWPO):
  Mediator of EW Symmetry Breaking [EWSB] boson found
- Mechanism of EWSB still poorly understood:
  - single Higgs field vs. Higgs sector
  - Higgs potential: stable vs. metastable vs. unstable !?
  - Higgs self-coupling vs. Higgs field scattering (longitudinal Ws)
- Dynamics of EW interactions: $\Longrightarrow$ Multiboson Interactions (MBI)
  - Anomalous Triple Gauge Couplings: dibosons
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- Higgs suppression makes VBS a prime candidate for BSM searches
- Hot topic: SM@LHC, LHCEWWG, Snowmass 13, MBI Workshops
- F. Gianotti, CLIC-Workshop 2014, CERN

Exploration of E-frontier →look for heavy objects, including high-mass $V_L V_L$ scattering:
❑ requires as much integrated luminosity as possible (cross-section goes like 1/s)

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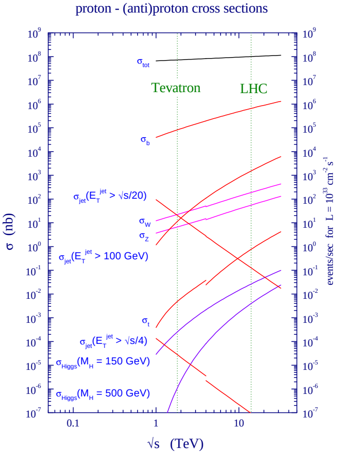
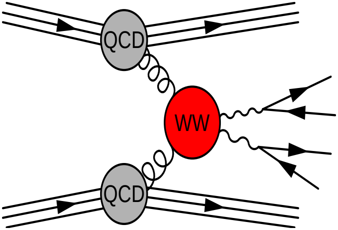
- Not really new: Elementary Particle Physics And Future Facilities. Proceedings, 1982 DPF Summer Study, Snowmass, USA, June 28 - July 16, 1982

The Challenge of LHC

Partonic subprocesses: qq, qg, gg

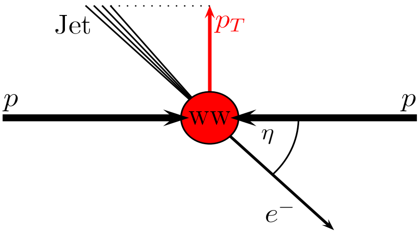
No fixed partonic energy



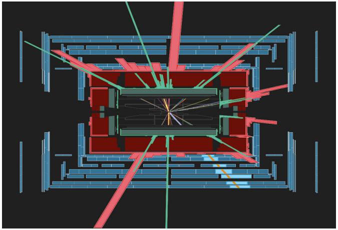


$R = \sigma\mathcal{L} \qquad \mathcal{L} = 10^{34}\,\text{cm}^{-1}\text{s}^{-1}$

High rates for t, W/Z, H, $\Rightarrow$ **large SM backgrounds**



The Challenge of LHC



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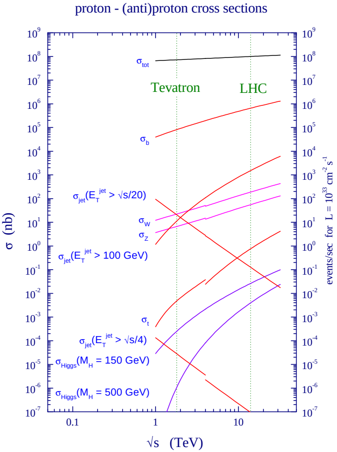

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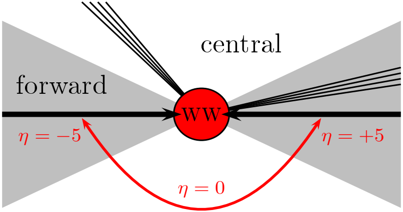
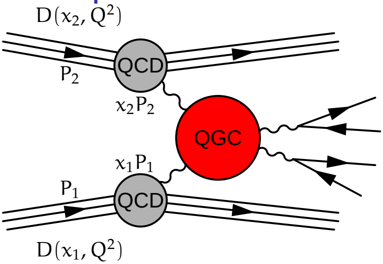


The importance of Vector Boson Scattering



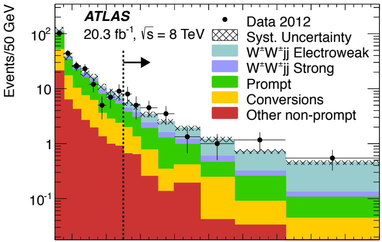
$$pp \to jj(ZZ/WW) \to jj\ell^-\ell^+\nu_\ell\bar{\nu}_\ell$$

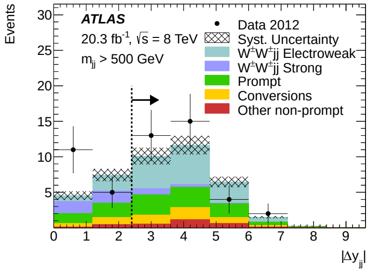
$$\sigma \approx 40\,\text{fb}$$

Background:

- $t\bar{t} \to WbWb$, $\sigma \approx 52$ pb
- Single t, misrec. jet: $\sigma \approx 4.8$ pb
- QCD: $\sigma \approx 0.21$ pb

- W^+W^+ scattering first seen in nature ATLAS, PRL 113 (2014) 141803

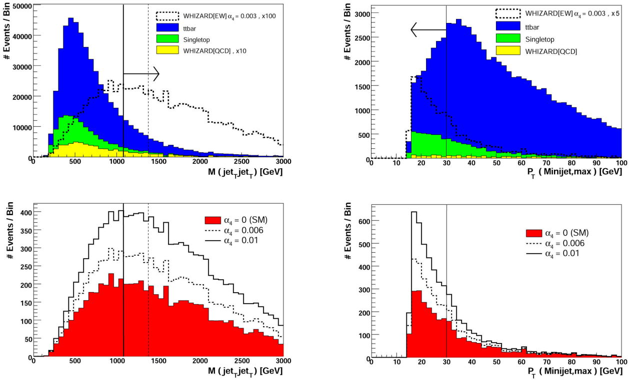




Tagging and Cuts:

- $\ell\ell jj$-Tag, $\eta_{tag}^{min} < \eta_\ell < \eta_{tag}^{max}$, b-Veto
- $|\Delta\eta_{jj}| > 4.4$, $M_{jj} > 1080$ GeV
- Minijet-Veto: $p_{T,j} < 30$ GeV
- $E_j > 600, 400$ GeV, $p_{T,j}^1 > 60, 24$ GeV

Improves $S/\sqrt{B}$ from 3.3 to 29.7



Model-Independent Way – Effective Field Theories

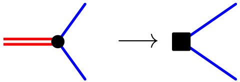
How to obtain higher-dimensional operators from first principles?

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[\frac{a_i}{\Lambda} \mathcal{O}_i^{(5)} + \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{e_i}{\Lambda^4} \mathcal{O}_i^{(8)} \cdots \right]$$

Low-energy effective theory $\Rightarrow$ integrating out heavy degrees of freedom (DOF), set up Power Counting

Toy model: Two interacting scalar fields φ, Φ

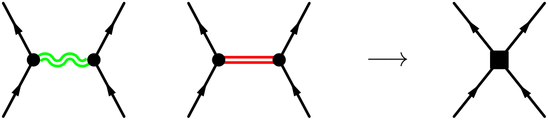
$$\Phi' = \Phi + \frac{\lambda}{M^2} \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \Rightarrow$$



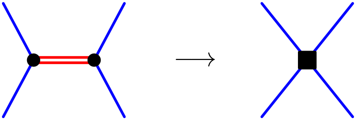
$$\frac{1}{2}(\partial\Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda \varphi^2 \Phi = -\frac{1}{2} \Phi' (M^2 + \partial^2) \Phi' + \frac{\lambda^2}{2M^2} \varphi^2 \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 .$$

Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

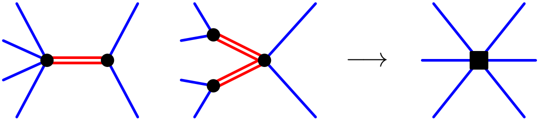


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{\Lambda^2} \mathrm{tr}\left[J^{(I)} \cdot J^{(I)} \right]$$

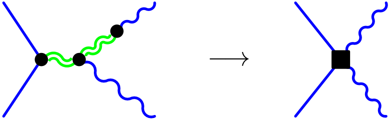


$$\mathcal{O}'_{h,1} = \frac{1}{\Lambda^2} \left((D\Phi)^\dagger \Phi \right) \cdot \left(h^\dagger (D^\Phi) \right) - \frac{v^2}{2} |D\Phi|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{\Lambda^2} (\Phi^\dagger \Phi - v^2/2) \, (D\Phi)^\dagger \cdot (D\Phi)$$



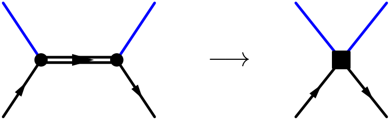
$$\mathcal{O}'_{h,3} = \frac{1}{\Lambda^2} \frac{1}{3} (\Phi^\dagger \Phi - v^2/2)^3$$



$$\mathcal{O}_{\Phi W} = -\frac{1}{\Lambda^2}\frac{1}{2}(\Phi^\dagger\Phi - v^2/2)\text{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

$$\mathcal{O}_B = \frac{1}{\Lambda^2}\frac{i}{2}(D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi)$$

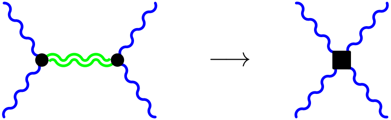
$$\mathcal{O}_{\Phi B} = -\frac{1}{\Lambda^2}\frac{1}{4}(\Phi^\dagger\Phi - v^2/2)B_{\mu\nu}B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{\Lambda^2}\overline{q}h(\not{D}h)q$$

Effective Dim. 8 Operators

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993



$$\mathcal{O}_\lambda = \frac{i}{\Lambda^4}\mathrm{tr}\left[W_{\mu\nu} \times W^{\nu\rho}(\Phi^\dagger \frac{\vec{\sigma}}{2}\left[D_\rho, D^\mu\right]\Phi)\right.$$

$$\mathcal{O}_\kappa = (D^\mu\Phi)^\dagger(D^\nu\Phi)(\Phi^\dagger\left[D_\mu, D_\nu\right]\Phi)$$

- operators linked through e.o.m.
- SM: 59 independent operators (1 fermion gen.)
  Buchmüller/Wyler, 1986; Grzadkowski/Iskrzynski/Misiak/Rosiek, 2010
- Renormalization mixes operators
- Beware of power counting

Classification of Operators (I): Dim 6

(always v^2 subtracted)

- Dimension-6 operators (CP-conserving)

$$\mathcal{O}_{WWW} = \mathrm{Tr}[W_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}] \qquad \mathcal{O}_{\partial\Phi} = \partial_\mu \left(\Phi^\dagger \Phi\right) \partial^\mu \left(\Phi^\dagger \Phi\right)$$

$$\mathcal{O}_{W} = (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi) \qquad \mathcal{O}_{\Phi W} = \left(\Phi^\dagger \Phi\right) \mathrm{Tr}[W^{\mu\nu} W_{\mu\nu}]$$

$$\mathcal{O}_{B} = (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi) \qquad \mathcal{O}_{\Phi B} = \left(\Phi^\dagger \Phi\right) B^{\mu\nu} B_{\mu\nu}$$

- Dimension-6 operators (CP-violating)

$$\mathcal{O}_{\widetilde{W}W} = \Phi^\dagger \widetilde{W}_{\mu\nu} W^{\mu\nu} \Phi \qquad \mathcal{O}_{\widetilde{W}WW} = \mathrm{Tr}[\widetilde{W}_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_{\tilde{B}B} = \Phi^\dagger \tilde{B}_{\mu\nu} B^{\mu\nu} \Phi \qquad \mathcal{O}_{\widetilde{W}} = (D_\mu \Phi)^\dagger \widetilde{W}^{\mu\nu} (D_\nu \Phi)$$

	ZWW	AWW	HWW	HZZ	HZA	HAA	WWWW	ZZWW	ZAWW	AAWW
$\mathcal{O}_{WWW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{W}$	✓	✓	✓	✓	✓		✓	✓	✓	
$\mathcal{O}_{B}$	✓	✓		✓	✓					
$\mathcal{O}_{\Phi d}$			✓	✓						
$\mathcal{O}_{\Phi W}$			✓	✓	✓	✓				
$\mathcal{O}_{\Phi B}$				✓	✓	✓				
$\mathcal{O}_{\tilde{W}WW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{\tilde{W}}$	✓	✓	✓	✓	✓					
$\mathcal{O}_{\tilde{W}W}$			✓	✓	✓	✓				
$\mathcal{O}_{\tilde{B}B}$				✓	✓	✓				

Classification of Operators (II): Dim 8 (always v^2 subtracted)

- Dimension-8 operators (only $D_\mu\Phi$)

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] ,$$
$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] ,$$

- Dimension-8 operators (only field strength/mixed)

$$\mathcal{O}_{T,0} = \text{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \cdot \text{Tr}\left[W_{\alpha\beta}W^{\alpha\beta}\right] ,$$
$$\mathcal{O}_{T,1} = \text{Tr}\left[W_{\alpha\nu}W^{\mu\beta}\right] \cdot \text{Tr}\left[W_{\mu\beta}W^{\alpha\nu}\right] ,$$
$$\mathcal{O}_{T,2} = \text{Tr}\left[W_{\alpha\mu}W^{\mu\beta}\right] \cdot \text{Tr}\left[W_{\beta\nu}W^{\nu\alpha}\right] ,$$
$$\mathcal{O}_{T,5} = \text{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \cdot B_{\alpha\beta}B^{\alpha\beta} ,$$
$$\mathcal{O}_{T,6} = \text{Tr}\left[W_{\alpha\nu}W^{\mu\beta}\right] \cdot B_{\mu\beta}B^{\alpha\nu} ,$$
$$\mathcal{O}_{T,7} = \text{Tr}\left[W_{\alpha\mu}W^{\mu\beta}\right] \cdot B_{\beta\nu}B^{\nu\alpha} ,$$
$$\mathcal{O}_{T,8} = B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}$$
$$\mathcal{O}_{T,9} = B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha} .$$

$$\mathcal{O}_{M,0} = \text{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right]$$
$$\mathcal{O}_{M,1} = \text{Tr}\left[W_{\mu\nu}W^{\nu\beta}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right]$$
$$\mathcal{O}_{M,2} = \left[B_{\mu\nu}B^{\mu\nu}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] ,$$
$$\mathcal{O}_{M,3} = \left[B_{\mu\nu}B^{\nu\beta}\right] \cdot \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}D^\mu\Phi\right] \cdot B^{\beta\nu} ,$$
$$\mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}D^\nu\Phi\right] \cdot B^{\beta\mu} ,$$
$$\mathcal{O}_{M,6} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}W^{\beta\nu}D^\mu\Phi\right] ,$$
$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger W_{\beta\nu}W^{\beta\mu}D^\nu\Phi\right] ,$$

Classification of Operators (III)

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0/1}$	✓	✓	✓						
$\mathcal{O}_{M,0/1/6/7}$	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{M,2/3/4/5}$		✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{T,0/1/2}$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,8/9}$			✓			✓	✓	✓	✓

- Dim. 8 operators generate aQGCs, but not aTGCs
- generate neutral quartics
- Redundancy of the operators:
  - Equations of motion: $D_\mu W^{\mu\nu} = \Phi^\dagger(D^\nu\Phi) - (D^\nu\Phi)^\dagger\Phi + \ldots$
  - Gauge symmetry structure: $[D_\mu, D_\nu]\,\Phi \propto W_{\mu\nu}\Phi$
  - Integration by parts (up to total derivatives)
  - Leads to relations like:

$$\begin{aligned}
\mathcal{O}_B &= \mathcal{O}_{\tilde{W}} + \frac{1}{2}\mathcal{O}_{WW} - \frac{1}{2}\mathcal{O}_{BB} \\
\mathcal{O}_{BW} &= -2\,\mathcal{O}_W - \mathcal{O}_{WW} \\
\mathcal{O}_{\partial W} &= -4\,\mathcal{O}_{WWW} + \text{gauge-fermion operators}
\end{aligned}$$

Unique way of operator assignment?

- Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- VVV and VBS to access the highest possible energies

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- VVV and VBS to access the highest possible energies
- Answer: NO UNIQUE WAY!
- But: at e^+e^- machines, gauge-fermion operators can be rotated away
- At LHC this is not possible! Buchalla et al., 1302.6481
- There is no common operator basis for V + jets, VV, VVV and VBS at LHC
- Incoherent sum of channels at LHC prevent eliminating operators!
- Similar to B physics: observables process [decay] specific

EFT coefficients *vs.* anomalous couplings

- Switch operator bases (vertex-dep.): Snowmass EW White Paper, 1310.6708

WWWW-Vertex:

$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{8}$$

$$\alpha_4 + 2\cdot\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{8}$$

WWZZ-Vertex:

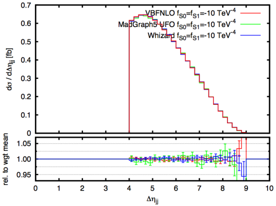
$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{16}$$

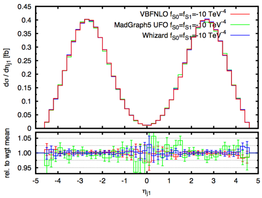
$$\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{16}$$

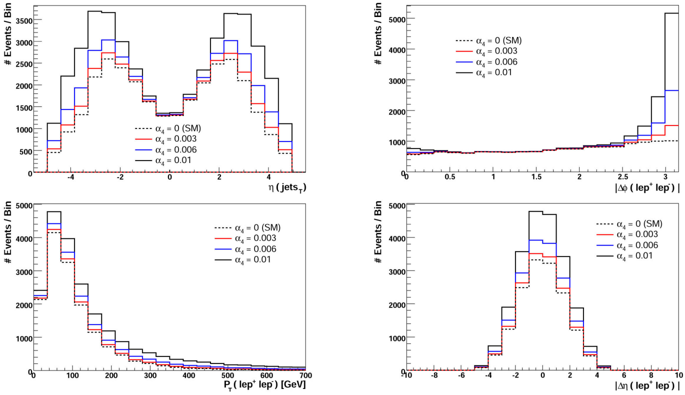
ZZZZ-Vertex:

$$\alpha_4 + \alpha_5 = \left(\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4}\right)\frac{v^4}{16}$$

- Full agreement among generators: VBF@NLO, WHIZARD, Madgraph







Results: (1σ Sensitivity to αs)

Coupl.	ILC ($1\,\mathrm{ab}^{-1}$)	LHC ($100\,\mathrm{fb}^{-1}$)
α_4	0.0088	0.00160
α_5	0.0071	0.00098

Limits for Λ [TeV]:

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Simplified Models for VBS (and VVV): Resonances

- Resonances in all accessible spin/isospin channels
- Couplings to the Higgs and gauge sectors are unrelated and arbitrary
- Still include anomalous couplings
- Unitarization (later)

New physics in electroweak sector:

- Narrow resonances $\Rightarrow$ particles (weakly interacting model)
- Wide resonances $\Rightarrow$ continuum (strongly interacting model)

$SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' \neq 0$ and fermion masses)

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 (Higgs ?)	ω^0 $(\gamma'/Z'\ ?)$	f^0 (Graviton ?)
$I = 1$	$\pi^\pm, \pi^0$ (2HDM ?)	$\rho^\pm, \rho^0$ $(W'/Z'\ ?)$	$a^\pm, a^0$
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

- $I = 0$: resonant in W^+W^- and ZZ scattering
- $I = 1$: resonant in W^+Z and W^-Z scattering
- $I = 2$: resonant in W^+W^+ and W^-W^- scattering

Resonances, Example: Scalar [Not counting ϕ with $M = 126$ GeV.]

Scalar Resonance (Mass M_σ)

- Coupling to Higgs sector (Higgs/longitudinal W/Z): $g_L^\sigma (D_\mu \Phi)^\dagger (D^\mu \Phi)\, \sigma$
- Coupling to gauge sector (transversal W/Z): $g_T^\sigma \mathrm{tr}\left[\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right]\, \sigma$
- Possible Origin: 2HDM isosinglet (renormalizable) [LHM, SUSY, Twin Higgs]

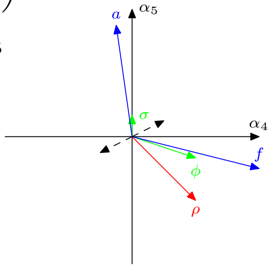
$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \qquad g_T^\sigma = O\left(\frac{1}{4\pi M_\sigma}\right) \quad \text{[loop]}$$

- Possible Origin: new strong interactions [Composite Higgs]

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}, \qquad g_T^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad \text{[tree]}$$

- $\Rightarrow$ anomalous quartic couplings (aQGCs) $\Delta\alpha_{4/5}$

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2 M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}\left(\frac{v^2}{M^2}\right)$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$



Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section: $\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$

Optical Theorem (Unitarity of the S(cattering) Matrix):

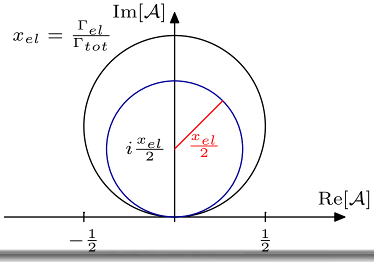
$$\sigma_{\text{tot}} = \text{Im}\left[\mathcal{M}_{ii}(t=0)\right]/s \qquad t = -s(1-\cos\theta)/2$$

Partial wave amplitudes:

$$\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1)\mathcal{A}_\ell(s) P_\ell(\cos\theta) \qquad \text{(“Power spectrum”)}$$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_\ell \frac{32\pi(2\ell+1)}{s}|\mathcal{A}_\ell|^2 \overset{!}{=} \sum_\ell \frac{32\pi(2\ell+1)}{s}\text{Im}\left[\mathcal{A}_\ell\right] \quad \Rightarrow \quad \boxed{|\mathcal{A}_\ell|^2 = \text{Im}\left[\mathcal{A}_\ell\right]}$$



Argand circle $\boxed{\left|\mathcal{A}(s) - \frac{i}{2}\right| = \frac{1}{2}}$

Resonance: $\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s - M^2 + iM\Gamma_{\text{tot}}}$

Counterclockwise circle, radius $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{\text{tot}}$

Unitarity in the EW sector: SM

- Project out isospin eigenamplitudes — Lee/Quigg/Thacker, 1973

$$\mathcal{A}_\ell(s) = \frac{1}{32\pi} \int_{-s}^{0} \frac{dt}{s} \mathcal{A}(s,t,u) P_\ell(1+2t/s) \qquad \cos\theta = 1 + 2t/s$$

Remember Legendre polynomials: $P_0(s) = 1$ $\quad P_1(s) = \cos\theta$ $\quad P_2(s) = (3\cos^2\theta - 1)/2$

- SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I,\text{spin}=J}$):

$$\mathcal{A}_{I=0} = 2\frac{s}{v^2} P_0(s) \qquad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2} P_1(s) \qquad \mathcal{A}_{I=2} = -\frac{s}{v^2} P_0(s)$$

$$\boxed{\mathcal{A}_{0,0} = \frac{s}{16\pi v^2}} \qquad \boxed{\mathcal{A}_{1,1} = \frac{s}{96\pi v^2}} \qquad \boxed{\mathcal{A}_{2,0} = -\frac{s}{32\pi v^2}}$$

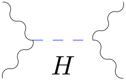
exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$$I = 0: \qquad E \sim \sqrt{8\pi} v = 1.2 \text{ TeV}$$

$$I = 1: \qquad E \sim \sqrt{48\pi} v = 3.5 \text{ TeV}$$

$$I = 2: \qquad E \sim \sqrt{16\pi} v = 1.7 \text{ TeV}$$

Higgs exchange:



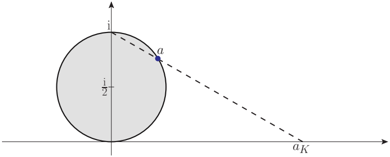
$$\mathcal{A}(s,t,u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2}$$

Unitarity: $M_H \lesssim \sqrt{8\pi} v \sim 1.2$ TeV

Unitarization S matrices

Kilian/JRR/Ohl/Sekulla, 1408.6207

- Unitarization prescription not unique
- Padé (reordering pert. series) introduces artificial poles
- Form factors parameterize close-by new physics (add. parameters)
- minimal version (K or T matrix) $\Rightarrow$ just saturation no new parameters, does not rely on pert. expansion, stable against small perturbations
  - Cayley transform of S matrix: $S = \frac{\mathbf{1}+\mathrm{i}K/2}{\mathbf{1}-\mathrm{i}K/2}$ Heitler, 1941; Schwinger, 1948
  - "K" matrix: translates to transition operator: $T = \frac{K}{\mathbf{1}-\mathrm{i}K/2}$
  - Corresponds to stereographic projection:



  - Coulomb singularities Bloch/Nordsieck, 1937; Yennie/Frautschi/Suura, 1961
- Additional known features (resonances) should be implemented before unitarization

Cut-Off Method (a.k.a. "Event Clipping")

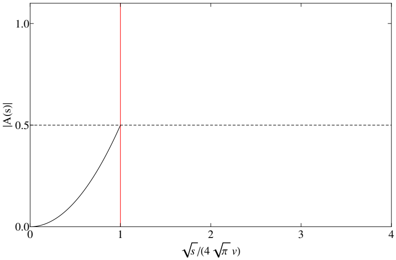
Cut-Off function

$$\Theta\left(\Lambda_C^2 - s\right)$$

- Naive prevention of Unitarity violation
- No continuous transition at Λ_C
- Ignore any interesting physics above Unitary bound
- Artifical construction

Cut-Off energy Λ_C

Λ_C equates unitarity bounds (often 0th partial wave)



Form Factor

Parameters

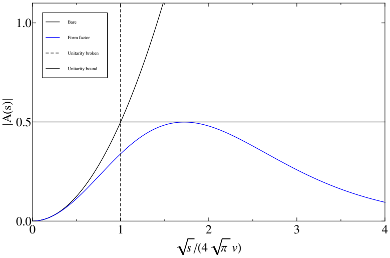
n Chosen to prevent breaking of Unitarity

Λ_{FF} Calculate highest possible value that satisfy real Unitarity bound ($0th$ partial wave)

Form Factor

$$\frac{1}{\left(1+\frac{s}{\Lambda_{FF}^2}\right)^n}$$

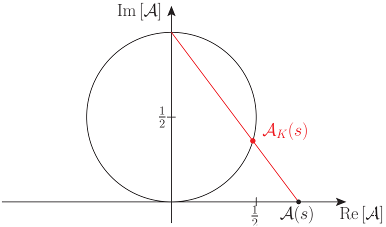
- Use Form Factor to suppress breaking of unitarity
- Can be generally used for arbitrary anomalous operator
- Needs "Fine Tuning"



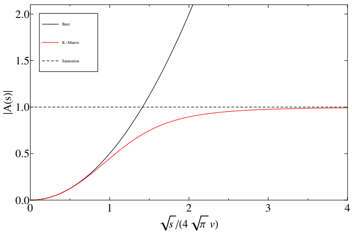
K-Matrix

K-Matrix Unitarisation

$$\mathcal{A}_K(s) = \frac{1}{\mathrm{Re}(\frac{1}{\mathcal{A}(s)}) - \mathrm{i}} = \frac{\mathcal{A}(s)}{1 - \mathrm{i}\mathcal{A}(s)} \quad \text{if } \mathcal{A}(s) \in \mathbb{R}$$

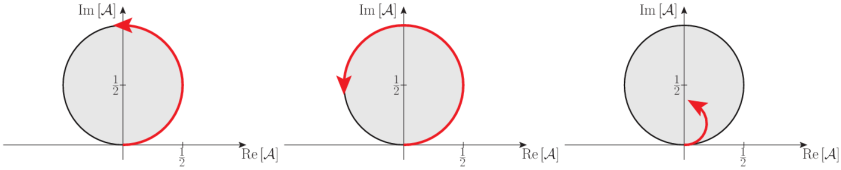


- Projection of elastic amplitudes onto Argand-Circle
- At high energies the amplitude saturizes
- Is usable for complex amplitudes
- Doesn't depend on additional parameters



Unitary Description of EW interactions

- Five possible cases:
  - Amplitude perturbative, close to zero, small imag. part (SM)
  - Amplitude rises, gets imag. part, strongly interacting regime (presence of at least one dim. 8 operator)
  - Amplitude approaches maximum absolute value asymptotically
  - Turn over: new resonance
  - New inelastic channels open: eff. form factor, extra channels observable in multi-vector boson processes



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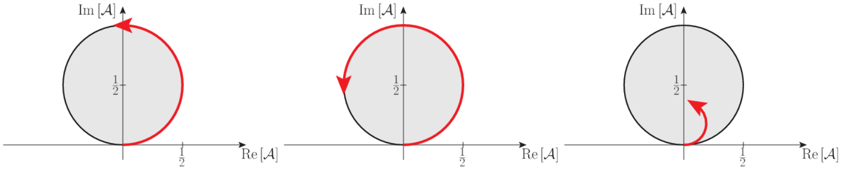


- Interpretation of EFT operator coefficients changes: formally still low-energy coefficients of Taylor expansion $\Rightarrow$ threshold parameters
- Complete description necessary (only) beyond threshold

Unitarity Bound for α_4 AQGC

Bounds for α_4

$$\ell = 0 : \sqrt{s} \leq \left(\frac{6\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.5 \text{ TeV}}{\sqrt[4]{\alpha_4}}$$

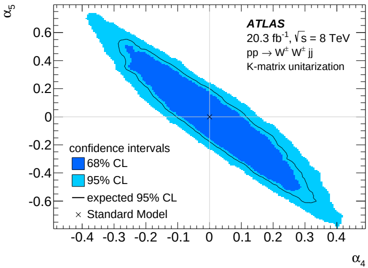
$$\ell = 2 : \sqrt{s} \leq \left(\frac{60\pi}{\alpha_4}\right)^{\frac{1}{4}} v \approx \frac{0.9 \text{ TeV}}{\sqrt[4]{\alpha_4}}$$

α_4 AQGC contribution to

$WW \to ZZ$

$$\mathcal{A}(s,t,u) = 4\alpha_4 \frac{t^2 + u^2}{v^4}$$

- Bound depends on coupling α_4 use strongest bound
- First (unitarized) LHC limits: ATLAS, PRL 113 (2014) 141803



The Multi-Purpose Generator WHIZARD

WHIZARD universal event generator for colliders: e^+e^-, pp, $p\bar{p}$, $\gamma\gamma$, ep etc.

1. O'Mega: Optimized automatic matrix elements for arbitrary elementary processes, supports SM and many BSM extensions
2. Phase-space parameterization module (very efficient PS)
3. VAMP: Generic adaptive Monte Carlo integration and (unweighted) event generation
4. CIRCE1/2: Lepton/[photon] collider beam spectra
5. Collective support for: Feynman rules, beams cascade decays, shower, hadronization, analysis, event file formats, etc.
6. Free-format steering language SINDARIN

WHIZARD 2.2.4 release: Febr. 06, 2015

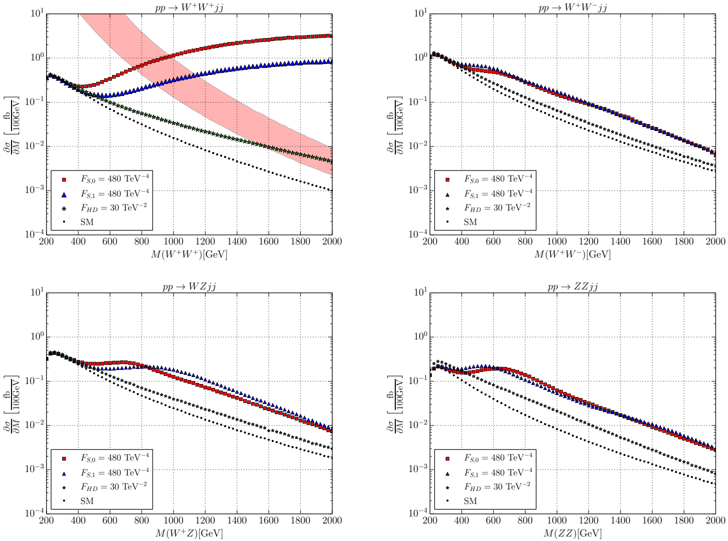
The WHIZARD team: F. Bach, B. Chokoufé, W. Kilian, T. Ohl, JRR, M. Sekulla, F. Staub, C. Weiss, DESY summer students

Web address: `http://projects.hepforge.org/whizard`

Standard Reference: Kilian/Ohl/JRR, EPJ **C71** (2011) 1742, arXiv:0708.4233

Diboson invariant masses

Kilian/JRR/Ohl/Sekulla, 1408.6207



General cuts: $M_{jj} > 500$ GeV; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$

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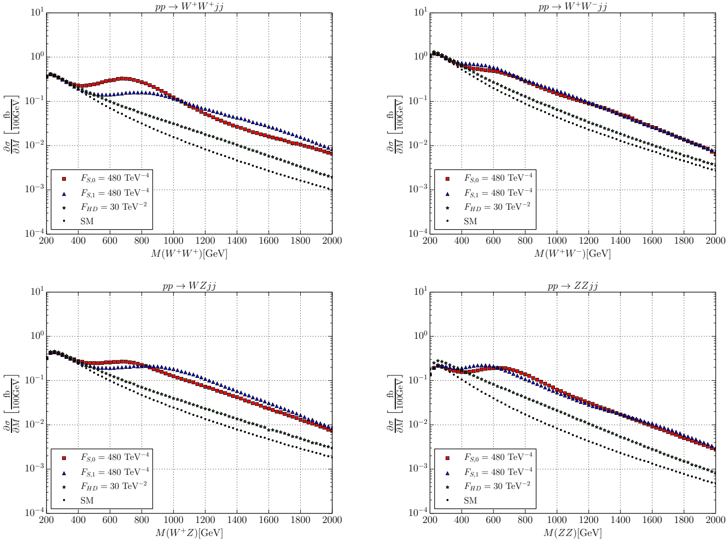
Kilian/JRR/Ohl/Sekulla, 1408.6207



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p_T and angular distributions

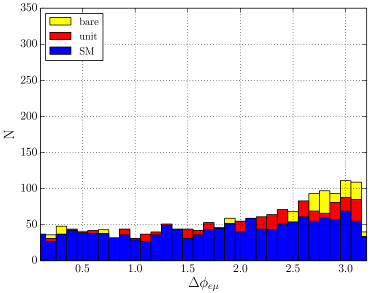
Kilian/JRR/Ohl/Sekulla, 1408.6207

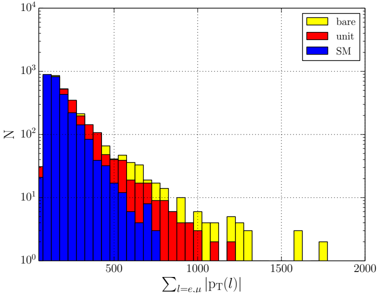
$pp \to e^+\mu^+\nu_e\nu_\mu jj$, $\sqrt{s} = 14$ TeV, $\mathcal{L} = 1000\,\text{fb}^{-1}$

Simulations with WHIZARD $\to$

Not possible to use automated tool due to s-channel prescription

$F_{HD} = 30\ \text{TeV}^{-2}$





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p_T and angular distributions Kilian/JRR/Ohl/Sekulla, 1408.6207

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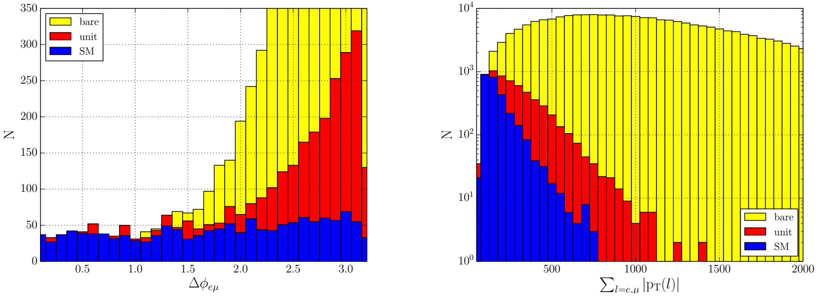
Simulations with WHIZARD $\to$

Not possible to use automated tool due to s-channel prescription

$F_{S,0} = 480\ \text{TeV}^{-4}$



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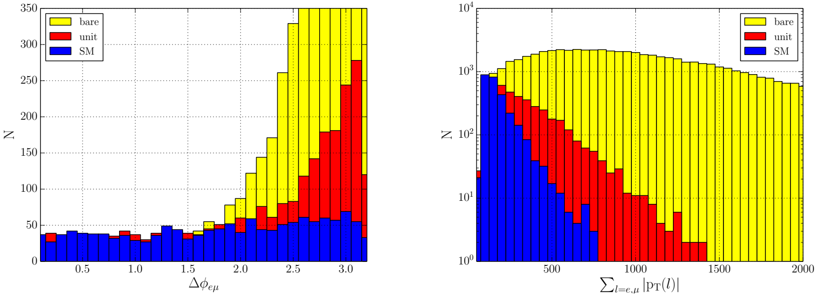
p_T and angular distributions Kilian/JRR/Ohl/Sekulla, 1408.6207

$pp \to e^+\mu^+\nu_e\nu_\mu jj$, $\sqrt{s} = 14$ TeV, $\mathcal{L} = 1000\,\text{fb}^{-1}$

Simulations with WHIZARD $\longrightarrow$

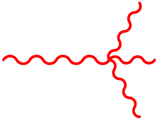
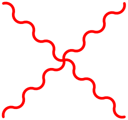
Not possible to use automated tool due to s-channel prescription

$F_{S,1} = 480$ TeV^{-4}



General cuts: $M_{jj} > 500$ GeV; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$, $p_T^\ell > 20$ GeV

And Triple Vector Boson Production?

relate  to  ??

Yes, the same Feynman graphs (in the SM), but. . . Tribosons:

- one external $W/Z/\gamma$ is always far off-shell
- Unitarization formalism not available
- different (anom.) couplings contribute (particularly for resonances)

$$\sigma(e^+e^- \to VVV) \underset{\sim}{\propto} \frac{1}{s}$$

Limits usefulness to subprocess energies in the lower range where cross section of fusion process still small

$$\sigma_{\text{VBS}}(e^+e^- \to \nu\bar{\nu}W^+W^-) \underset{\sim}{\propto} \log(s)$$

$e^+e^- \to ZZZ$
$\to WWZ$

ZH, $\hookrightarrow WW$, $\hookrightarrow ZZ$ — Present in spectrum

$\to WW\gamma$ Complementary (and present at lower energies)

$\Rightarrow$ Important physics independent w.r.t. VBS. Don't just combine results!

Unitarization Prescriptions

- **K-matrix unitarization prescription** Heitler, 1941; Schwinger, 1949; Gupta, 1950
  - Hermitian K-matrix interpreted as incompletely calculated approximation to true amplitude
  - $\Rightarrow$ Unitary S, T as a non-perturbativ completion of this approximation
  - Insert pert. expansion into expansion:

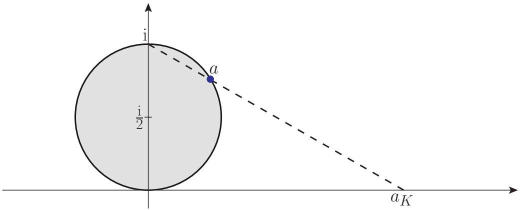
    $a = \frac{a_K}{1-\mathrm{i}a_K} \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \mathrm{Re}a_0^{(2)} + \ldots}{1 - \mathrm{i}(a_0^{(1)} + \mathrm{Re}a_0^{(2)} + \ldots)}$
  - Prescription does a partial resummation of perturbative series
  - Example Dyson resummation: $a_K^{(0)}(s) = \frac{\lambda}{s-m^2} \longrightarrow a^{(0)}(s) = \frac{\lambda}{s-m^2-\mathrm{i}\lambda}$
- **Drawbacks of (original) K-matrix:**
  - Needs to construct self-adjoint K-matrix as intermediate step
  - Problem if S-matrix is not diagonal, or ...

    there are non-perturbative contributions
- **T-matrix unitarization**
  - a_0 complex approximation to eigenvalue of true T matrix
  - use again pseudo-stereographic projection (intersection of Argand circle with line $\overline{a_0\ i}$)
  - Results in: $a = \frac{\mathrm{Re}a_0}{1-\mathrm{i}a_0^*} \quad \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \mathrm{Re}a_0^{(2)} + \ldots}{1 - \mathrm{i}(a_0^{(1)} + \mathrm{Re}a_0^{(2)} - \mathrm{iIm}a_0^{(2)} + \ldots)}$

Alternative Unitarization Methods

Kilian/JRR/Ohl/Sekulla, 1408.6207

- **Comparison of T-matrix and (original) K-matrix:**
  - T-matrix does not rely on perturbation theory
  - Special treatment for non-normal T matrices (eigenvalues having imaginary parts larger than i; Riesz-Dunford operator calculus)
  1. T matrix description leads to point on the Argand circle
  2. For real a $\Rightarrow$ (original) K-matrix case
  3. a_0 on Argand circle $\Rightarrow$ left invariant
- **Thales circle construction:**



  - Defined via $\left|a - \frac{a_K}{2}\right| = \frac{a_K}{2} \;\Rightarrow\; a = \frac{1}{\mathrm{Re}\left(\frac{1}{a_0}\right) - \mathrm{i}}$
  - avoids non-normal matrices, but not single-valued around $a = 0$

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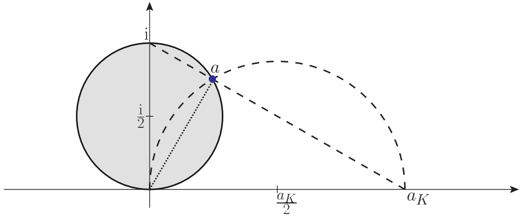
Kilian/JRR/Ohl/Sekulla, 1408.6207

- **Comparison of T-matrix and (original) K-matrix:**
  - T-matrix does not rely on perturbation theory
  - Special treatment for non-normal T matrices (eigenvalues having imaginary parts larger than i; Riesz-Dunford operator calculus)

  1. T matrix description leads to point on the Argand circle
  2. For real $a \Rightarrow$ (original) K-matrix case
  3. a_0 on Argand circle $\Rightarrow$ left invariant

- **Thales circle construction:**



  - Defined via $\left|a - \frac{a_K}{2}\right| = \frac{a_K}{2} \Rightarrow a = \frac{1}{\mathrm{Re}\left(\frac{1}{a_0}\right) - \mathrm{i}}$
  - avoids non-normal matrices, but not single-valued around $a = 0$

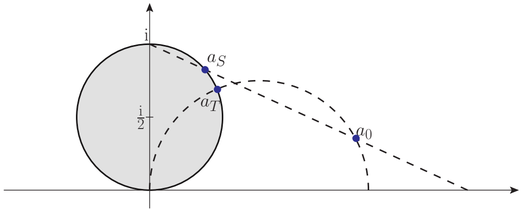
Alternative Unitarization Methods

Kilian/JRR/Ohl/Sekulla, 1408.6207

- **Comparison of T-matrix and (original) K-matrix:**
  - T-matrix does not rely on perturbation theory
  - Special treatment for non-normal T matrices (eigenvalues having imaginary parts larger than i; Riesz-Dunford operator calculus)

  1. T matrix description leads to point on the Argand circle
  2. For real a $\Rightarrow$ (original) K-matrix case
  3. a_0 on Argand circle $\Rightarrow$ left invariant

- **Thales circle construction:**



- Defined via $\left|a - \frac{a_K}{2}\right| = \frac{a_K}{2} \quad \Rightarrow \quad a = \frac{1}{\mathrm{Re}\left(\frac{1}{a_0}\right) - \mathrm{i}}$
- avoids non-normal matrices, but not single-valued around $a = 0$

Summary/Conclusions/Outlook

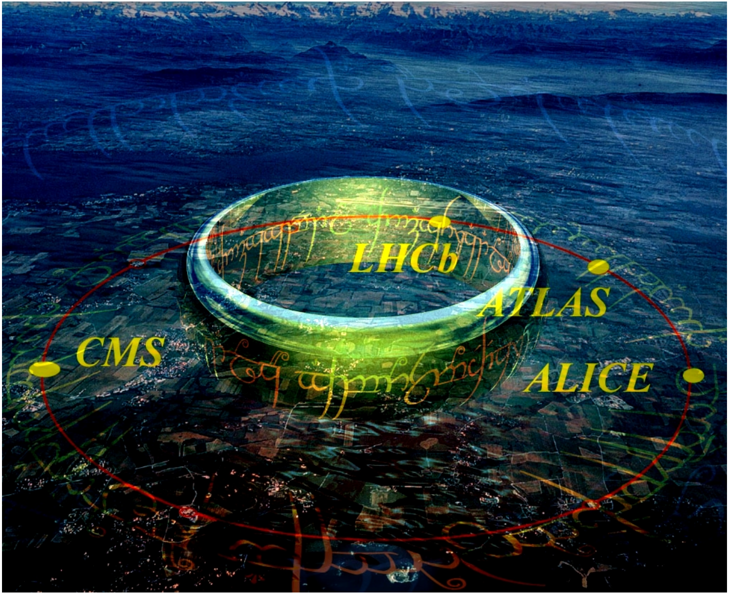
- Access to (deviations from) EW sector via:
  - via diboson/triboson production and vector boson scattering
- Photon-induced processes: better sensitivity, but higher constraints!
- Task: Unify LHC and LEP/ILC/CLIC descriptions
  (model-independent limit setting $(\alpha_4, \frac{f_{S,0}}{\Lambda_{NP}^4})$
- Simplified Models: minimally unitarized operators
- Unitarization scheme: no additional structure to the theory
  (model dependence minimized)
- Sensitivity rises with number of new intermediate states:
  - LHC14 sensitivity limited in pure EW sector: $\sim 1 - X$ TeV (???)
  - ILC1000 : $1.5 - 6$ TeV
  - (Tensor) Resonances very interesting Kilian/JRR/Sekulla, in preparation
  - Multi-TeV e^+e^- [+ pol. ?] probably best machine for VBS (100 TeV pp ??)
- Most simulations need to be updated (include light Higgs)
- Crucial: Discrimination between longitudinal and transversal modes!

Advertisement: MBI 2015 @ DESY

2.-4. Sept. 2015, DESY, Hamburg

One Ring to Find them ... One Ring to Rule them Out

One Ring to Find them ... One Ring to Rule them Out



Electroweak Chiral Lagrangian

Originally for heavy Higgses or Higgsless models

$$\mathcal{L}_\chi \ni \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v}\sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2}\sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \dots \qquad \alpha_i^{(n)} = \frac{v^{n-4}}{\Lambda^{n-4}} C_i^n$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \mathrm{tr}\,[\mathbf{T}\mathbf{V}_\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \mathrm{tr}\,[\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}] \qquad \mathcal{L}_6 = \mathrm{tr}\,[\mathbf{V}_\mu\mathbf{V}_\nu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_2 = i\mathrm{tr}\,[\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu]] \qquad \mathcal{L}_7 = \mathrm{tr}\,[\mathbf{V}_\mu\mathbf{V}^\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}_\nu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_3 = i\mathrm{tr}\,[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu]] \qquad \mathcal{L}_8 = \tfrac{1}{4}\mathrm{tr}\,[\mathbf{T}\mathbf{W}_{\mu\nu}]\,\mathrm{tr}\,[\mathbf{T}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_4 = \mathrm{tr}\,[\mathbf{V}_\mu\mathbf{V}_\nu]\,\mathrm{tr}\,[\mathbf{V}^\mu\mathbf{V}^\nu] \qquad \mathcal{L}_9 = \tfrac{i}{2}\mathrm{tr}\,[\mathbf{T}\mathbf{W}_{\mu\nu}]\,\mathrm{tr}\,[\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu]]$$

$$\mathcal{L}_5 = \mathrm{tr}\,[\mathbf{V}_\mu\mathbf{V}^\mu]\,\mathrm{tr}\,[\mathbf{V}_\nu\mathbf{V}^\nu] \qquad \mathcal{L}_{10} = \tfrac{1}{2}\left(\mathrm{tr}\,[\mathbf{T}\mathbf{V}_\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\mu]\right)^2$$

BACKUP SLIDES

Electroweak Chiral Lagrangian

Originally for heavy Higgses or Higgsless models

$$\mathcal{L}_\chi \ni \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \ldots \qquad \alpha_i^{(n)} = \frac{v^{n-4}}{\Lambda^{n-4}} C_i^n$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \mathrm{tr}\,[\mathbf{T V}_\mu]\,\mathrm{tr}\,[\mathbf{T V}^\mu]$$

$$\mathcal{L}_1 = \mathrm{tr}\,[\mathbf{B}_{\mu\nu} \mathbf{W}^{\mu\nu}] \qquad \mathcal{L}_6 = \mathrm{tr}\,[\mathbf{V}_\mu \mathbf{V}_\nu]\,\mathrm{tr}\,[\mathbf{T V}^\mu]\,\mathrm{tr}\,[\mathbf{T V}^\nu]$$

$$\mathcal{L}_2 = i\mathrm{tr}\,[\mathbf{B}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \mathcal{L}_7 = \mathrm{tr}\,[\mathbf{V}_\mu \mathbf{V}^\mu]\,\mathrm{tr}\,[\mathbf{T V}_\nu]\,\mathrm{tr}\,[\mathbf{T V}^\nu]$$

$$\mathcal{L}_3 = i\mathrm{tr}\,[\mathbf{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \mathcal{L}_8 = \frac{1}{4}\mathrm{tr}\,[\mathbf{T W}_{\mu\nu}]\,\mathrm{tr}\,[\mathbf{T W}^{\mu\nu}]$$

$$\mathcal{L}_4 = \mathrm{tr}\,[\mathbf{V}_\mu \mathbf{V}_\nu]\,\mathrm{tr}\,[\mathbf{V}^\mu \mathbf{V}^\nu] \qquad \mathcal{L}_9 = \frac{i}{2}\mathrm{tr}\,[\mathbf{T W}_{\mu\nu}]\,\mathrm{tr}\,[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_5 = \mathrm{tr}\,[\mathbf{V}_\mu \mathbf{V}^\mu]\,\mathrm{tr}\,[\mathbf{V}_\nu \mathbf{V}^\nu] \qquad \mathcal{L}_{10} = \frac{1}{2}\left(\mathrm{tr}\,[\mathbf{T V}_\mu]\,\mathrm{tr}\,[\mathbf{T V}^\mu]\right)^2$$

Indirect info on new physics in $\beta_1, \alpha_i, \ldots$ (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1 \qquad \alpha_1 = 0.0026 \pm 0.0020$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}} \qquad \beta_1 = -0.00062 \pm 0.00043$$

$$\Delta U = -16\pi\alpha_8 \qquad \alpha_8 = -0.0044 \pm 0.0026$$

Isospin decomposition

- Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + \alpha_4 \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right] + \alpha_5\left(\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right)^2$$

- Leads to the following amplitudes: $s = (p_1 + p2)^2 \quad t = (p1 - p3)^2 \quad u = (p1 - p4)^2$

$$\boxed{\mathcal{A}(s,t,u) =:} \qquad \mathcal{A}(w^+w^- \to zz) = \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(w^+z \to w^+z) = \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2+u^2}{v^4}$$

$$\mathcal{A}(w^+w^- \to w^+w^-) = -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5)\frac{s^2+t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4}$$

$$\mathcal{A}(w^+w^+ \to w^+w^+) = -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4\left(\alpha_4 + 2\alpha_5\right)\frac{t^2+u^2}{v^4}$$

$$\mathcal{A}(zz \to zz) = 8\left(\alpha_4 + \alpha_5\right)\frac{s^2+t^2+u^2}{v^4}$$

- (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\mathcal{A}(I=0) = 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$
$$\mathcal{A}(I=1) = \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t)$$
$$\mathcal{A}(I=2) = \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$

"Comparison" Form Factor vs. K-Matrix

- Which Unitarisation scheme provides the best description?

→ All of them:
Unitarisation schemes are an arbitrary way to guarantee Unitarity

Form Factor

- Suppression of amplitude to get below Unitarity bound

MC Generate less events than possible

K-Matrix

- Saturation of amplitude to achieve Unitarity

MC Generate maximal possible number of events

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = \mathrm{ie}\left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\,\nu} W_{\nu\rho}^+ A^{\rho\mu}\right]$$
$$+ \mathrm{ie}\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\,\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{QGC} = e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W^-_\mu W^+_\nu - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W^+_\nu \right] + e^2 \frac{c_w}{s_w} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W^-_\mu W^+_\nu + W^+_\mu W^-_\nu \right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W^+_\nu \right] + e^2 \frac{c_w^2}{s_w^2} \left[g_1^{ZZ} Z^\mu Z^\nu W^-_\mu W^+_\nu - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W^+_\nu \right] + \frac{e^2}{2 s_w^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W^-_\mu W^+_\nu - g_2^{WW} \left(W^{-\mu} W^+_\mu \right)^2 \right] + \frac{e^2}{4 s_w^2 c_w^4} h^{ZZ} (Z^\mu Z_\mu)^2$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$ $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2}\alpha_3 \qquad \Delta g_1^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2 (\alpha_9 - \alpha_8) - g^2 (\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2 \left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10}) \right]$$

SM Lagrangian

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] \qquad W^{\pm},\, Z$$

$$+ (\partial_\mu \phi)^\dagger \partial^\mu \phi - V(\phi) \qquad h$$

$$+ \frac{v^2}{4}\text{tr}\left[(\mathbf{D}_\mu \Sigma)^\dagger (\mathbf{D}^\mu \Sigma)\right] \qquad w^{\pm},\, z$$

$$- \frac{g_h v}{2}\text{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] h$$

Vector Bosons

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + \mathrm{i}g\left[\mathbf{W}_\mu, \mathbf{W}_\nu\right]$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$$

$$\mathbf{W}_\mu = W^a_\mu \frac{\tau^a}{2} \qquad \mathbf{B}_\mu = B_\mu \frac{\tau^3}{2}$$

$$\mathbf{D}_\mu = \partial_\mu + \mathrm{i}g\mathbf{W}_\mu - \mathrm{i}g'\mathbf{B}_\mu$$

Higgs Sector

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$\Sigma = \exp\left[-\frac{\mathrm{i}}{v} w^a \tau^a\right]$$

$$\mathbf{V}_\mu = \Sigma\,(\mathbf{D}_\mu \Sigma)$$

Unitary Gauge

- Goldstone bosons are absorbed by vector bosons as longitudinal degrees of freedom
- $w^a \equiv 0 \to \Sigma \equiv 1$
- $\mathbf{D}_\mu = \partial_\mu - \mathbf{V}_\mu = \partial_\mu + \frac{\mathrm{i}g}{2}\left(\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_w} Z\tau^3\right)$

$$\mathcal{L}_{\text{min}} = -\frac{1}{2}\operatorname{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2}\operatorname{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \underbrace{(\partial_\mu\phi)^\dagger\,\partial^\mu\phi - \frac{v^2}{4}\operatorname{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] - \frac{g_h v}{2}\operatorname{tr}\left[\mathbf{V}^\mu\mathbf{V}_\mu\right] h}_{\underset{g_h=1}{\widehat{=}}(\mathbf{D}_\mu\phi)^\dagger\mathbf{D}^\mu\phi} - V(\phi)$$

- Coincides with known SM parametrisation

Vector Resonances

$$\begin{aligned}\mathcal{L}_\rho = & -\frac{1}{8}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{\Delta M_\rho^2}{8}\left(\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\right)^2 + \mathrm{i}\frac{\mu_\rho}{2} g\,\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{W}^{\mu\nu}\boldsymbol{\rho}_\nu\right] \\ & + \mathrm{i}\frac{\mu'_\rho}{2} g'\,\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{B}^{\mu\nu}\boldsymbol{\rho}_\nu\right] + \mathrm{i}\frac{g_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right] + \mathrm{i}\frac{h_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right] \\ & + \frac{g' v^2 k_\rho}{2M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{B}^{\nu\mu},\mathbf{V}_\nu]\right] + \frac{g v^2 k'_\rho}{4M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{T},\mathbf{V}_\nu]\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^{\nu\mu}\right] \\ & + \frac{g v^2 k''_\rho}{4M_\rho^2}\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\mathrm{tr}\left[[\mathbf{T},\mathbf{V}_\nu]\mathbf{W}^{\nu\mu}\right] + \mathrm{i}\frac{\ell_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right] \\ & + \mathrm{i}\frac{\ell'_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{B}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right] + \mathrm{i}\frac{\ell''_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right]\end{aligned}$$

all $\alpha_i \sim 1/M_\rho^4$, except for $\beta_1 \sim \Delta\rho \sim T \sim h_\rho^2/M_\rho^2$

4-fermion contact interaction $j_\mu j^\mu \sim 1/M_\rho^2$ (eff. T and U parameter)

vector coupling $j_\mu V^\mu \sim 1/M_\rho^2$ (eff. S parameter)
Mismatch: measured fermionic vs. bosonic coupling g Nyffeler/Schenk, 2000; Kilian/JR, 2003

Effects on Triple Gauge Couplings

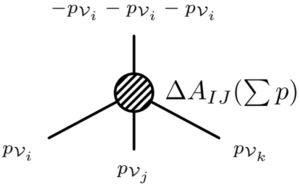
- $\mathcal{O}(1/M^2)$: Renormalization of ZWW coupling
- $\mathcal{O}(1/M^4)$: shifts in Δg_1^Z, $\Delta\kappa^\gamma$, $\Delta\kappa^Z$, λ^γ, λ^Z

Effects on Quartic Gauge Couplings

- $\mathcal{O}(1/M^4)$, orthogonal (in α_4–α_5 space) to scalar case

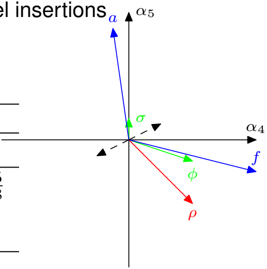
Implementation of Unitarization

- Explicit "time arrow" in WHIZARD



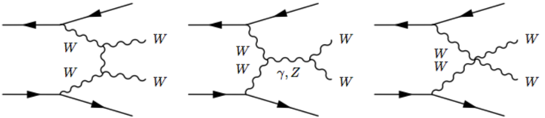
  - trace back pairs of momenta at quartic vertices to external legs
  - guarantee for only s-channel insertions

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2 M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}(\frac{v^2}{M^2})$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$

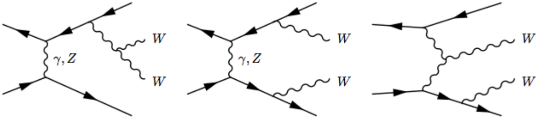


Vector Boson Scattering at e^+e^- machines

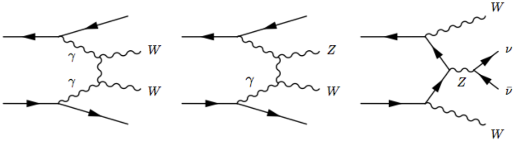
Signal



Irreducible bkgd.



(Partially) reducible bkgd.



Vector Boson Scattering

Beyer et al.,hep-ph/0604048

1 TeV, $1\,\mathrm{ab}^{-1}$, full $6f$ final states, 80 % e^-_R, 60 % e^+_L polarization, binned likelihood

Contributing channels: $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$

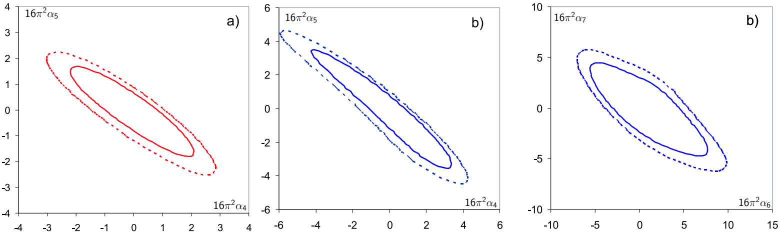
Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e\bar{\nu}_e q\bar{q}q\bar{q}$	$WW \to WW$	23.19
$e^+e^- \to \nu_e\bar{\nu}_e q\bar{q}q\bar{q}$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar{\nu}q\bar{q}q\bar{q}$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar{q}q\bar{q}$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar{q}q\bar{q}$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar{q}q\bar{q}$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar{b}X$	$e^+e^- \to t\bar{t}$	331.768
$e^+e^- \to q\bar{q}q\bar{q}$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar{q}q\bar{q}$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar{q}$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar{q}$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar{q}$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55



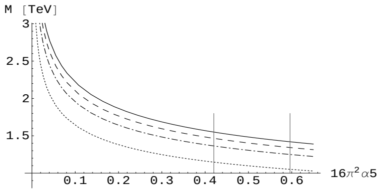
Interpretation as limits on resonances

Beyer et al.,hep-ph/0604048

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma / M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

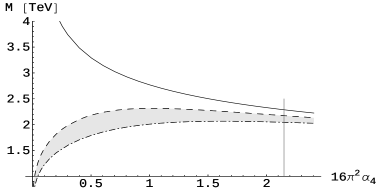


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2(\alpha_4^\lambda)^2/(2c_w^2)} \right)^{\frac{1}{4}}$$



upper/lower limit from λ_Z, grey area: magnetic moments

Final result:

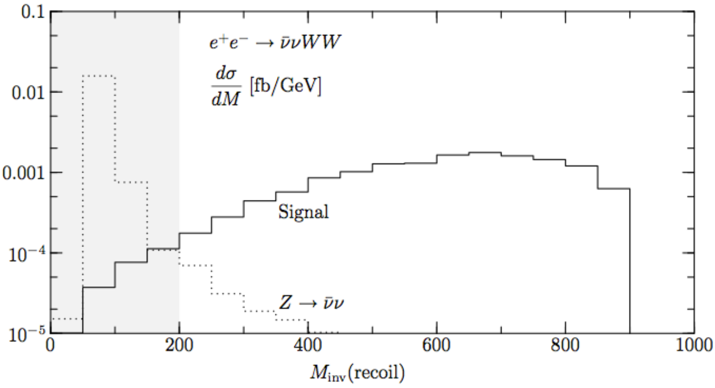
Spin	$I=0$	$I=1$	$I=2$
0	1.55	–	1.95
1	–	2.49	–
2	3.29	–	4.30

Spin	$I=0$	$I=1$	$I=2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Vector Boson Scattering: Observables

Study of WW scattering @ 1.6 TeV

Boos/Kilian/He/Mühlleitner/Pukhov/Zerwas, hep-ph/9708310



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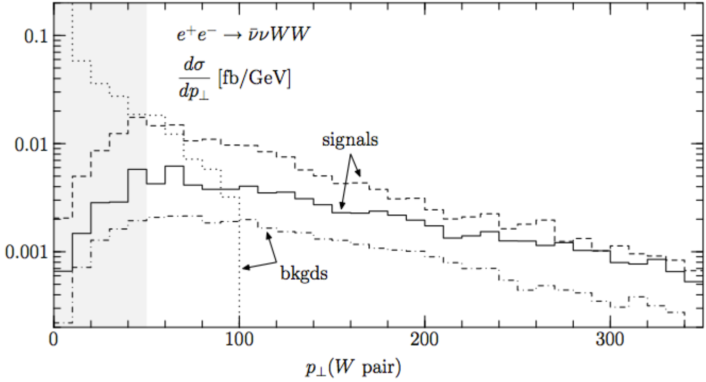

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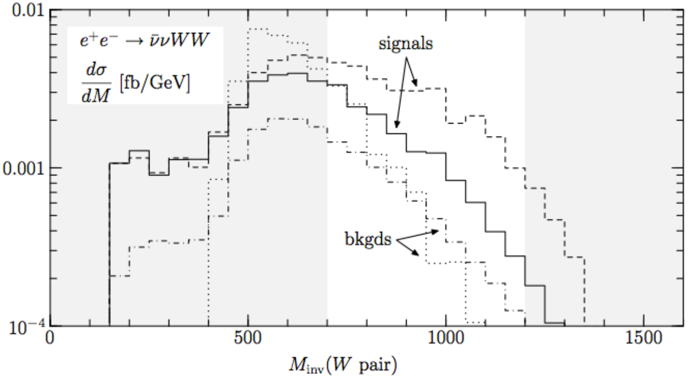
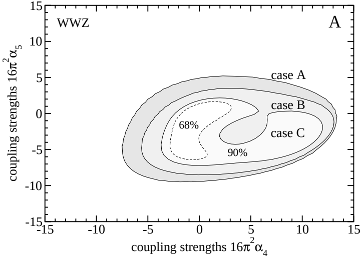


ILC Results: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\mathrm{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

$16\pi^2\times$	WWZ no pol.	WWZ e^- pol.	WWZ both pol.	ZZZ no pol.	best
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E^2_{\mathrm{mis}} + p^2_{\perp,\mathrm{mis}}$

No angular correlations yet

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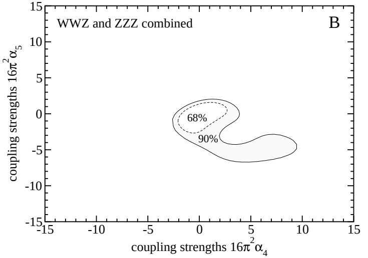

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Veto against $E^2_{\rm mis} + p^2_{\perp,\rm mis}$

No angular correlations yet

The Effective W approximation

- $M_{\mathcal{V}}$, $\hat{t}_i$ small corrections, $\mathcal{V}$ nearly onshell:

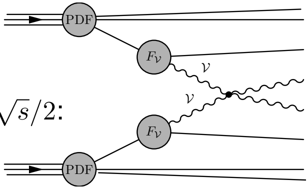
$$\sigma(q_1 q_2 \to q_1' q_2' \mathcal{V}_1' \mathcal{V}_2') \approx \sum_{\lambda_1,\lambda_2} \int dx_1\, dx_2\, F^{\lambda_1}_{q_1 \to q_1' \mathcal{V}_1}(x_1)\, F^{\lambda_2}_{q_2 \to q_2' \mathcal{V}_2}(x_2)\, \sigma^{\lambda_1 \lambda_2}_{\mathcal{V}_1 \mathcal{V}_2 \to \mathcal{V}_1' \mathcal{V}_2'}(x_1 x_2 s)$$

- In addition to Weizsäcker-Williams: longitudinal polarisation

$$F^{+}_{q \to q' \mathcal{V}}(x) = \frac{(V-A)^2 + (V+A)^2 (1-x)^2}{16\pi^2 \; x} \left[\ln\left(\frac{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}}{(1-x) m^2_{\mathcal{V}}} \right) - \frac{p^2_{\perp,\max}}{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}} \right]$$

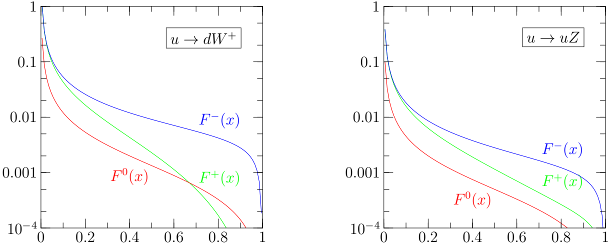
$$F^{-}_{q \to q' \mathcal{V}}(x) = \frac{(V+A)^2 + (V-A)^2 (1-x)^2}{16\pi^2 \; x} \left[\ln\left(\frac{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}}{(1-x) m^2_{\mathcal{V}}} \right) - \frac{p^2_{\perp,\max}}{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}} \right]$$

$$F^{0}_{q \to q' \mathcal{V}}(x) = \frac{V^2 + A^2}{8\pi^2} \, \frac{2(1-x)}{x} \, \frac{p^2_{\perp,\max}}{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}}$$



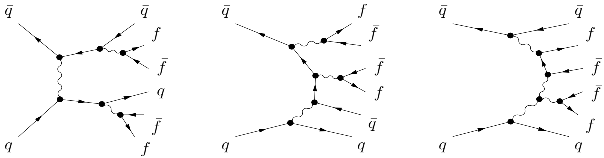
- Dominant contribution from small $\mathcal{V}$ virtualities
- Transverse momentum cutoff $p_{\perp,\max} \leq (1-x)\sqrt{s}/2$:
  - longitudinal pol.: finite for $p_{\perp,\max} \to \infty$
  - Transversal pol.: logarithmic singularity

- EWA structure functions: W (left) and Z (right)



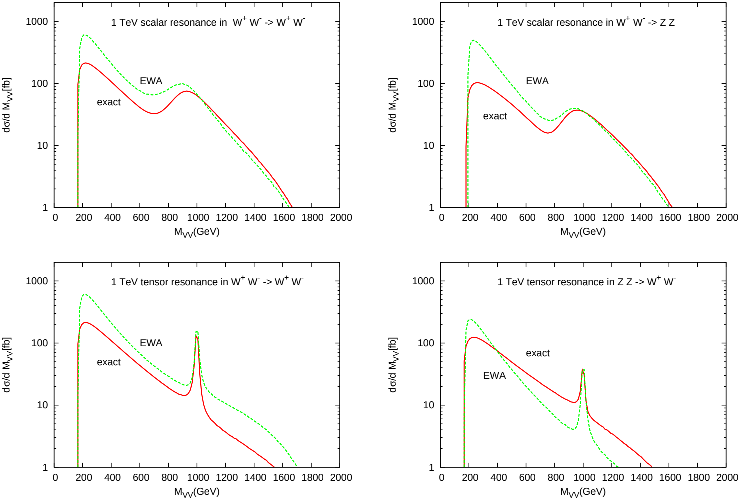
  – Emission from u, $\sqrt{s} = 2$ TeV
  – preferred at high energy: transversal emission

- Problem: Irreducible background to weak-boson scattering



  – Double ISR/FSR
  – t-channel like diagrams

- Coulomb-singularity (peak): cut on $p_{T,V} \gtrsim 30$ GeV



- Effective W approx. vs. WHIZARD full matrix elements
- Shapes/normalization of distributions heavily affected
- **EWA: Sideband subtraction completely screwed up!**

Discrimination of Longitudinal Modes

- Most important for separating/measuring scattering of longitudinal modes
- No known (working) method up to now!

Different Selection Criteria:

- General selection criteria
  - exactly 2 leptons within detector acceptance,
  - 2 tag jets with $2 < |\eta_j| < 5$ and opposite directions, but no b-tag
  - $M_{j_1 l_2}, M_{j_2 l_1} >$ 200 GeV
  - $M_{jj} >$ 400 GeV
  - $\Delta R_{jl} >$ 0.4
  - $p_T^{l_1}, p_T^{l_2} >$ 40 GeV
  - $|\eta_{l_1}|, |\eta_{l_2}| <$ 1.5 $\Delta\phi_{ll} >$ 2.5
  - $M_{ll} >$ 200 GeV
- Proposal of new variable

  Doroba/Kalinowski/Kuczmarski/Pokorski/Rosiek/Szleper/Tkaczyk, 1201.2768

$$R_{p_T} = (p_T^{l_1} \cdot p_T^{l_2}) \; / \; (p_T^{j_1} \cdot p_T^{j_2})$$

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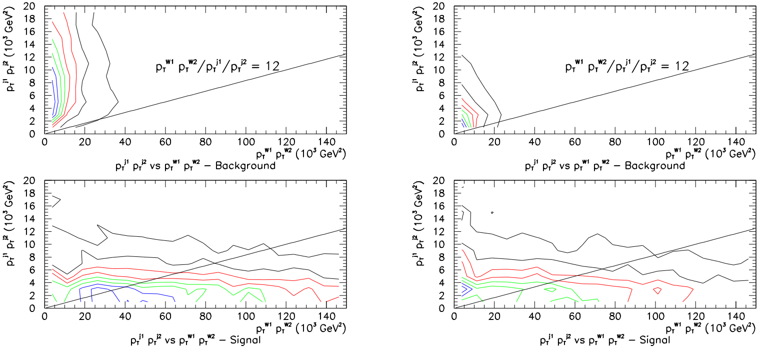
  Doroba/Kalinowski/Kuczmarski/Pokorski/Rosiek/Szleper/Tkaczyk, 1201.2768

$$R_{p_T} = (p_T^{l_1} \cdot p_T^{l_2}) \;/\; (p_T^{j_1} \cdot p_T^{j_2})$$

- Works well for $W^\pm W^\pm$, not feasible for W^+W^-

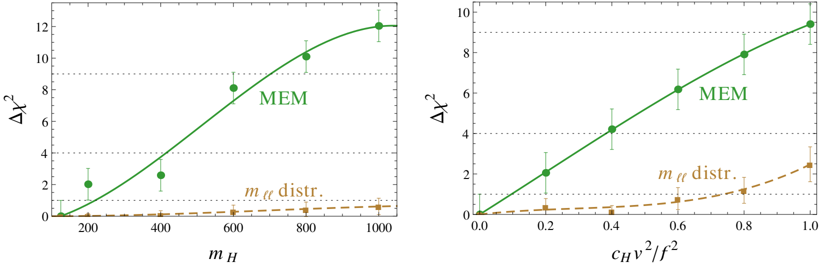


- Might allow to relax jet vetoes: gain for high pile-up!

Matrix Element Method (MEM)

Kondo, 1988; Dalitz/Goldstein, 1992; CDF; D∅; Freitas/Gainer, arXiv:1212.3598

- Construct a likelihood from the squared matrix elements
- Marginalize over invisible particles
- Case study for $pp \to jjW^+W^+$ Freitas/Gainor, 2012
- Up to now only compared to dilepton mass: $m_{\ell\ell}$



- Important possibility for gain of sensitivity