Vector Boson Scattering at $e^+e^-$ machines & News from WHIZARD

Jürgen R. Reuter

DESY, Hamburg

Alboteanu/Kilian/JRR, JHEP 0811 (2008) 010;
Beyer/Kilian/Krstonošić/Mönig/JRR/Schmitt/Schröder, EPJC 48 (2006), 353;
JRR/Kilian/Sekulla, 1307.8170; Kilian/JRR/Ohl/Sekulla, 1408.6207 + in prep.
Kilian/Ohl/JRR, EPJ C71 (2011) 1742

CLIC 2015 Workshop, CERN, Jan. 29th, 2015
Motivation

- After discovery of light Higgs boson: what is left to do?
- Mechanism behind generating Higgs vev missing ($\Rightarrow$ Higgs physics)
- Dynamics of EW interactions: $\Rightarrow$ Multiboson Interactions (MBI)
- Anomalous Triple Gauge Couplings: dibosons
- Anomalous Quartic Gauge Couplings: tribosons, VV scattering
- Existing studies assume: $\mathcal{P}(e^-) = 80 - 90\%$, $\mathcal{P}(e^+) = 30 - 60\%$
  - Longitudinal polarization of beams: $(V - A)$ couplings of $W/Z$
  - $e_L$ and $e_R$ different multiplets $\Rightarrow$ access completely different couplings

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F. Gianotti, CLIC-Workshop 2014, CERN

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Exploration of E-frontier $\Rightarrow$ look for heavy objects, including high-mass $V_LV_L$ scattering:
- requires as much integrated luminosity as possible (cross-section goes like $1/s$)
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- After discovery of light Higgs boson: what is left to do?
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**Exploration of E-frontier ⇒ look for heavy objects, including high-mass \( V_LV_L \) scattering:
- requires as much integrated luminosity as possible (cross-section goes like \( 1/s \))**

Extensions of the SM

- Lagrangian of the EW SM (no fermions/QCD here):

\[
\mathcal{L}_{EW} = -\frac{1}{2} \text{tr} [W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2
\]

with building blocks:

\[
D_\mu = \partial_\mu + \frac{i}{2} g \tau^I W_\mu^I + \frac{i}{2} g' B_\mu
\]

\[
W_{\mu\nu} = \frac{i}{2} g \tau^I (\partial_\mu W^I_\nu - \partial_\nu W^I_\mu + g \epsilon_{IJK} W^J_\mu W^K_\nu)
\]

\[
B_{\mu\nu} = \frac{i}{2} g' (\partial_\mu B_\nu - \partial_\nu B_\mu)
\]

- Any EFT has higher-dimensional operators:  

Weinberg, 1979

\[
\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[ \frac{a_i}{\Lambda} O_i^{(5)} + \frac{c_i}{\Lambda^2} O_i^{(6)} + \frac{e_i}{\Lambda^4} O_i^{(8)} \ldots \right]
\]

- without more fundamental theory \(\Rightarrow\) no clue on the scale (neither on the coefficients)
Classification of Operators (I): Dim 6

(always $\nu^2$ subtracted)

- Dimension-6 operators (CP-conserving)
  \[
  O_{WWW} = \text{Tr}[W_{\mu\nu}W^{\nu\rho}W^{\mu}_\rho]
  \]
  \[
  O_W = (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi)
  \]
  \[
  O_B = (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)
  \]
  \[
  O_{\partial \Phi} = \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)
  \]
  \[
  O_{\Phi W} = (\Phi^\dagger \Phi) \text{Tr}[W^{\mu\nu}W_{\mu\nu}]
  \]
  \[
  O_{\Phi B} = (\Phi^\dagger \Phi) B^{\mu\nu} B_{\mu\nu}
  \]

- Dimension-6 operators (CP-violating)
  \[
  O_{\tilde{W}WW} = \Phi^\dagger \tilde{W}_{\mu\nu} W^{\mu\nu} \Phi
  \]
  \[
  O_{\tilde{B}BB} = \Phi^\dagger \tilde{B}_{\mu\nu} B^{\mu\nu} \Phi
  \]
  \[
  O_{\tilde{W}WW} = \text{Tr}[\tilde{W}_{\mu\nu}W^{\nu\rho}W^{\mu}_\rho]
  \]
  \[
  O_{\tilde{W}} = (D_\mu \Phi)^\dagger \tilde{W}^{\mu\nu} (D_\nu \Phi)
  \]

<table>
<thead>
<tr>
<th></th>
<th>ZWW</th>
<th>AWW</th>
<th>HWW</th>
<th>HZZ</th>
<th>HZA</th>
<th>HAA</th>
<th>WWWWW</th>
<th>ZZWW</th>
<th>ZAWW</th>
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<td>$O_{\Phi B}$</td>
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<tr>
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</tbody>
</table>
Classification of Operators (II): Dim 8 (always $v^2$ subtracted)

- Dimension-8 operators (only $D_\mu \Phi$)

$$\mathcal{O}_{S,0} = \left[(D_\mu \Phi)^\dagger D_\nu \Phi\right] \times \left[(D_\mu \Phi)^\dagger D_\nu \Phi\right],$$
$$\mathcal{O}_{S,1} = \left[(D_\mu \Phi)^\dagger D_\mu \Phi\right] \times \left[(D_\nu \Phi)^\dagger D_\nu \Phi\right],$$

- Dimension-8 operators (only field strength/mixed)

$$\mathcal{O}_{T,0} = \text{Tr} \left[W_{\mu\nu} W^{\mu\nu}\right] \cdot \text{Tr} \left[W_{\alpha\beta} W^{\alpha\beta}\right],$$
$$\mathcal{O}_{T,1} = \text{Tr} \left[W_{\alpha\nu} W^{\mu\beta}\right] \cdot \text{Tr} \left[W_{\mu\beta} W^{\alpha\nu}\right],$$
$$\mathcal{O}_{T,2} = \text{Tr} \left[W_{\alpha\mu} W^{\mu\beta}\right] \cdot \text{Tr} \left[W_{\beta\nu} W^{\nu\alpha}\right],$$
$$\mathcal{O}_{T,5} = \text{Tr} \left[W_{\mu\nu} W^{\mu\nu}\right] \cdot B_{\alpha\beta} B^{\alpha\beta},$$
$$\mathcal{O}_{T,6} = \text{Tr} \left[W_{\alpha\nu} W^{\mu\beta}\right] \cdot B_{\mu\beta} B^{\alpha\nu},$$
$$\mathcal{O}_{T,7} = \text{Tr} \left[W_{\alpha\mu} W^{\mu\beta}\right] \cdot B_{\beta\nu} B^{\nu\alpha},$$
$$\mathcal{O}_{T,8} = B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta},$$
$$\mathcal{O}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha}.$$
Classification of Operators (III)

<table>
<thead>
<tr>
<th>$O_{S,0/1}$</th>
<th>WWWW</th>
<th>WWZZ</th>
<th>ZZZZ</th>
<th>WWAZ</th>
<th>WWAA</th>
<th>ZZZA</th>
<th>ZZAA</th>
<th>ZAAA</th>
<th>AAAA</th>
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<td>$O_{M,0/1/6/7}$</td>
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<td>$O_{M,2/3/4/5}$</td>
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<tr>
<td>$O_{T,0/1/2}$</td>
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<td>✓</td>
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<tr>
<td>$O_{T,5/6/7}$</td>
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<tr>
<td>$O_{T,8/9}$</td>
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</tr>
</tbody>
</table>

- Dim. 8 operators generate aQGCs, but not aTGCs
- generate neutral quartics
- Redundancy of the operators:
  - Equations of motion: $D_\mu W^{\mu\nu} = \Phi^\dagger (D^\nu \Phi) - (D^\nu \Phi)^\dagger \Phi + \ldots$
  - Gauge symmetry structure: $[D_\mu, D_\nu] \Phi \propto W_{\mu\nu} \Phi$
  - Integration by parts (up to total derivatives)
  - Leads to relations like:

$$O_B = O_W + \frac{1}{2} O_{WW} - \frac{1}{2} O_{BB}$$
$$O_{BW} = -2 O_W - O_{WW}$$
$$O_{\partial W} = -4 O_{WWW} + \text{gauge-fermion operators}$$
EFT coefficients vs. anomalous couplings

- Switch operator bases (vertex-dep.): Snowmass EW White Paper, 1310.6708

\[
\begin{align*}
\text{WWWW-Vertex:} & \quad \alpha_4 = \frac{f_{S,0}}{\Lambda^4} \frac{v^4}{8} \\
\alpha_4 + 2 \cdot \alpha_5 &= \frac{f_{S,1}}{\Lambda^4} \frac{v^4}{8} \\
\text{WWZZ-Vertex:} & \quad \alpha_4 = \frac{f_{S,0}}{\Lambda^4} \frac{v^4}{16} \\
& \quad \alpha_5 = \frac{f_{S,1}}{\Lambda^4} \frac{v^4}{16} \\
\text{ZZZZ-Vertex:} & \quad \alpha_4 + \alpha_5 = \left( \frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4} \right) \frac{v^4}{16}
\end{align*}
\]

- Full agreement among generators: VBF@NLO, WHIZARD, Madgraph
Vector Boson Scattering at $e^+e^-$ machines

Signal

Irreducible bkgd.

(Partially) reducible bkgd.
Vector Boson Scattering

1 TeV, 1 ab$^{-1}$, full 6$f$ final states, 80% $e_{R}^-$, 60% $e_{L}^+$ polarization, binned likelihood

Contributing channels: $WW \rightarrow WW$, $WW \rightarrow ZZ$, $WZ \rightarrow WZ$, $ZZ \rightarrow ZZ$

<table>
<thead>
<tr>
<th>Process</th>
<th>Subprocess</th>
<th>$\sigma$ [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \rightarrow \nu_e\bar{\nu}_e q\bar{q} q\bar{q}$</td>
<td>$WW \rightarrow WW$</td>
<td>23.19</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu_e\bar{\nu}_e q\bar{q} q\bar{q}$</td>
<td>$WW \rightarrow ZZ$</td>
<td>7.624</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \nu\bar{\nu} q\bar{q} q\bar{q}$</td>
<td>$V \rightarrow VVV$</td>
<td>9.344</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^- q\bar{q} q\bar{q}$</td>
<td>$ZZ \rightarrow ZZ$</td>
<td>132.3</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^- q\bar{q} q\bar{q}$</td>
<td>$ZZ \rightarrow W+\bar{W}$</td>
<td>414.</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow b\bar{b}X$</td>
<td>$e^+e^- \rightarrow t\bar{t}$</td>
<td>331.768</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q} q\bar{q}$</td>
<td>$e^+e^- \rightarrow W+W-$</td>
<td>3560.108</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow q\bar{q} q\bar{q}$</td>
<td>$e^+e^- \rightarrow ZZ$</td>
<td>173.221</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e\nu q\bar{q}$</td>
<td>$e^+e^- \rightarrow e\nu W$</td>
<td>279.588</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow e^+e^- q\bar{q}$</td>
<td>$e^+e^- \rightarrow e^+e^- Z$</td>
<td>134.935</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow X$</td>
<td>$e^+e^- \rightarrow q\bar{q}$</td>
<td>1637.405</td>
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</table>

$SU(2)_c$ conserved case, all channels

<table>
<thead>
<tr>
<th>coupling</th>
<th>$\sigma -$</th>
<th>$\sigma +$</th>
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<tbody>
<tr>
<td>$16\pi^2\alpha_4$</td>
<td>-1.41</td>
<td>1.38</td>
</tr>
<tr>
<td>$16\pi^2\alpha_5$</td>
<td>-1.16</td>
<td>1.09</td>
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</table>

$SU(2)_c$ broken case, all channels

<table>
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<tr>
<th>coupling</th>
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<tbody>
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<td>$16\pi^2\alpha_4$</td>
<td>-2.72</td>
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<td>$16\pi^2\alpha_6$</td>
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<td>$16\pi^2\alpha_7$</td>
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<td>$16\pi^2\alpha_{10}$</td>
<td>-5.55</td>
<td>4.55</td>
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</table>
Vector Boson Scattering: Observables

Study of $WW$ scattering @ 1.6 TeV

Boos/Kilian/He/Mühlleitner/Pukhov/Zerwas, hep-ph/9708310
Vector Boson Scattering: Observables

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Vector Boson Scattering: Observables

Study of $WW$ scattering @ 1.6 TeV  
Boos/Kilian/He/Mühlleitner/Pukhov/Zerwas, hep-ph/9708310
Simplified Models for VBS (and VVV): Resonances

- Resonances in all accessible spin/isospin channels
- Couplings to the Higgs and gauge sectors are unrelated and arbitrary
- Still include anomalous couplings
- Unitarization (later)

New physics in electroweak sector:

- Narrow resonances ⇒ particles (weakly interacting model)
- Wide resonances ⇒ continuum (strongly interacting model)

$SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' 
eq 0$ and fermion masses)

<table>
<thead>
<tr>
<th></th>
<th>$J = 0$</th>
<th>$J = 1$</th>
<th>$J = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = 0$</td>
<td>$\sigma^0$ (Higgs ?)</td>
<td>$\omega^0$ ($\gamma'/Z'$ ?)</td>
<td>$f^0$ (Graviton ?)</td>
</tr>
<tr>
<td>$I = 1$</td>
<td>$\pi^\pm, \pi^0$ (2HDM ?)</td>
<td>$\rho^\pm, \rho^0$ ($W'/Z'$ ?)</td>
<td>$a^\pm, a^0$</td>
</tr>
<tr>
<td>$I = 2$</td>
<td>$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)</td>
<td>—</td>
<td>$t^{\pm\pm}, t^\pm, t^0$</td>
</tr>
</tbody>
</table>

- $I = 0$: resonant in $W^+W^-$ and $ZZ$ scattering
- $I = 1$: resonant in $W^+Z$ and $W^-Z$ scattering
- $I = 2$: resonant in $W^+W^+$ and $W^-W^-$ scattering
Example: a Scalar Resonance  [Not counting $\phi$ with $M = 126$ GeV.]

- Mass $M_{\sigma}$.
- Coupling to the Higgs sector (Higgs and longitudinal $W/Z$):
  \[
g_{L}^{\sigma} (D_{\mu} \Phi)^{\dagger} (D^{\mu} \Phi) \sigma
  \]
- Coupling to the gauge sector (transversal $W/Z$):
  \[
g_{T}^{\sigma} \text{tr} [W^{\mu\nu} W_{\mu\nu}] \sigma
  \]

Possible Origin: 2HDM isosinglet (renormalizable)

\[
g_{L}^{\sigma} = O \left( \frac{1}{M_{\sigma}} \right) \quad \text{[tree]}, \quad g_{T}^{\sigma} = O \left( \frac{1}{4\pi M_{\sigma}} \right) \quad \text{[loop]}
\]

Possible Origin: new strong interactions

\[
g_{L}^{\sigma} = O \left( \frac{1}{M_{\sigma}} \right) \quad \text{[tree]}, \quad g_{T}^{\sigma} = O \left( \frac{1}{M_{\sigma}} \right) \quad \text{[tree]}
\]
Interpretation as limits on resonances

Consider the width to mass ratio, \( f_\sigma = \Gamma_\sigma / M_\sigma \)

**SU(2) conserving scalar singlet**

\[
M_\sigma = v \left( \frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}
\]

**SU(2) broken vector triplet**

needs input from TGC covariance matrix

\[
M_{\rho\pm} = v \left( \frac{12\pi\alpha_4 f_{\rho\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2(\alpha_4^\lambda)^2 / (2c_w^2)} \right)^{\frac{1}{4}}
\]

\( f = 1.0 \) (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

upper/lower limit from \( \lambda_Z \), grey area: magnetic moments

### Final result:

<table>
<thead>
<tr>
<th>Spin</th>
<th>( I = 0 )</th>
<th>( I = 1 )</th>
<th>( I = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.55</td>
<td>–</td>
<td>1.95</td>
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<td>1</td>
<td>–</td>
<td>2.49</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>3.29</td>
<td>–</td>
<td>4.30</td>
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<tr>
<th>Spin</th>
<th>( I = 0 )</th>
<th>( I = 1 )</th>
<th>( I = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.39</td>
<td>1.55</td>
<td>1.95</td>
</tr>
<tr>
<td>1</td>
<td>1.74</td>
<td>2.67</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>3.00</td>
<td>3.01</td>
<td>5.84</td>
</tr>
</tbody>
</table>
Unitarizing $S$ matrices

- Cayley transform of $S$ matrix: \[ S = \frac{1+iK/2}{1-iK/2} \] Heitler, 1941; Schwinger, 1948

- "$K$" matrix: translates to transition operator: \[ T = \frac{K}{1-iK/2} \]

- Works beyond perturbation theory, but allows perturbative expansion

- Diagonalize $S$ matrix (partial waves):
  \[ \mathcal{M}(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) A_\ell(s) P_\ell(\cos \theta) \] ("Power spectrum")

- Complex eigenvalues: \[ t = 2a \quad k = 2a_K \Rightarrow a_K = \frac{a}{1+ia} \]

- Corresponds to stereographic projection:

- Coulomb singularities Bloch/Nordsieck, 1937; Yennie/Frautschi/Suura, 1961
Unitarization Primer

Kilian/JRR/Ohl/Sekulla, 1408.6207

- Unitarization prescription not unique

- Padé (reordering pert. series) introduces artificial poles

- Form factors parameterize close-by new physics (additional parameters)

\[ \frac{1}{\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^n} \]

unphysical form factor scale \( \Lambda_{FF} \) and multipole order \( n \)

- minimal version \((K\text{ or } T\text{ matrix}) \Rightarrow\) just saturation no new parameters, does not rely on pert. expansion, stable against small perturbations

- Additional known features (resonances) should be implemented before unitarization
Unitarity Bound for $\alpha_4$ AQGC

 Bounds for $\alpha_4$

$\ell = 0 : \sqrt{s} \leq \left( \frac{6\pi}{\alpha_4} \right)^{\frac{1}{4}} v \approx \frac{0.5 \text{ TeV}}{\sqrt[4]{\alpha_4}}$

$\ell = 2 : \sqrt{s} \leq \left( \frac{60\pi}{\alpha_4} \right)^{\frac{1}{4}} v \approx \frac{0.9 \text{ TeV}}{\sqrt[4]{\alpha_4}}$

- **Bound depends on coupling $\alpha_4$**

- **Use strongest bound**
Diboson invariant masses

General cuts: $M_{jj} > 500$ GeV; $\Delta \eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$
Diboson invariant masses

General cuts: $M_{jj} > 500 \text{ GeV}; \Delta \eta_{jj} > 2.4; p_T^j > 20 \text{ GeV}; |\eta_j| < 4.5$
$p_T$ and angular distributions

$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj$, $\sqrt{s} = 14$ TeV, $\mathcal{L} = 1000$ fb$^{-1}$

Simulations with WHIZARD →

Not possible to use automated tool due to $s$-channel prescription

$F_{HD} = 30$ TeV$^{-2}$

General cuts: $M_{jj} > 500$ GeV; $\Delta \eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$, $p_T^\ell > 20$ GeV
$p_T$ and angular distributions

$pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj$, $\sqrt{s} = 14$ TeV, $\mathcal{L} = 1000$ fb$^{-1}$

Simulations with WHIZARD

Not possible to use automated tool due to $s$-channel prescription

$F_{S,0} = 480$ TeV$^{-4}$

General cuts: $M_{jj} > 500$ GeV; $\Delta \eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$, $p_T^{\ell} > 20$ GeV
$p_T$ and angular distributions

$pp \rightarrow e^+\mu^+\nu_e\nu_\mu jj$, $\sqrt{s} = 14$ TeV, $\mathcal{L} = 1000$ fb$^{-1}$

Simulations with WHIZARD

Not possible to use automated tool due to $s$-channel prescription

$F_{S,1} = 480$ TeV$^{-4}$

General cuts: $M_{jj} > 500$ GeV; $\Delta\eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\eta_j| < 4.5$, $p_T^\ell > 20$ GeV
And Triple Vector Boson Production?

Yes, the same Feynman graphs (in the SM), but... Tribosons:

- one external $W/Z/\gamma$ is always far off-shell
- Unitarization formalism not available
- different (anom.) couplings contribute (particularly for resonances)

\[
\sigma(e^+e^- \to VVV) \gtrsim \frac{1}{s}
\]
Limits usefulness to subprocess energies in the lower range where cross section of fusion process still small

\[
\sigma_{VBS}(e^+e^- \to \nu\bar{\nu}W^+W^-) \gtrsim \log(s)
\]

$e^+e^- \rightarrow ZZZ$

$e^+e^- \rightarrow WWZ$

$e^+e^- \rightarrow WW\gamma$

$\Rightarrow$ Important physics independent w.r.t. VBS. Don’t just combine results!
Summary/Conclusions: $e^+e^-$ VBS

- Access to (deviations from) EW sector via:
  - via diboson/triboson production and vector boson scattering

- Task: Unify LHC and LEP/ILC/CLIC descriptions (model-independent limit setting $(\alpha_4, \frac{f_{s,0}}{\Lambda^4_{NP}})$)

- Simplified Models: minimally unitarized operators

- Unitarization scheme: no additional structure to the theory (model dependence minimized)

- Sensitivity rises with number of new intermediate states:
  - LHC14 sensitivity limited in pure EW sector: $\sim 1 - X$ TeV (???)
  - ILC1000: $1.5 - 6$ TeV
  - (Tensor) Resonances very interesting Kilian/JRR/Sekulla, in preparation

- All simulations need to be updated (include light Higgs)

- Multi-TeV $e^+e^-$ [+ pol. ?] probably best machine for VBS

- Crucial experimental tasks: hadronic $W/Z$ separation at high energies, separation of longitudinal/transversal final $W/Z$
Advertisement: MBI 2015 @ DESY

2.-4. Sept. 2015, DESY, Hamburg
Switching gears: WHIZARD
WHIZARD in a Nutshell

WHIZARD universal event generator for processes at colliders: \( e^+e^- \), \( pp \), \( p\bar{p} \), \( \gamma\gamma \), \( ep \) etc.

1. O’Mega: Optimized automatic matrix elements for arbitrary elementary processes, supports SM and many BSM extensions
2. Phase-space parameterization module (very efficient PS)
3. VAMP: Generic adaptive Monte Carlo integration and (unweighted) event generation
4. CIRCE1/2: Lepton/[photon] collider beam spectra
5. Intrinsic support or external interfaces for: Feynman rules, beams cascade decays, shower, hadronization, analysis, event file formats, etc.
6. Free-format steering language SINDARIN

WHIZARD 2.2.3 release: Nov. 30, 2014
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**WHIZARD 2.2.4**

release: Febr. 06, 2015

The **WHIZARD team**: F. Bach, B. Chokoufè, W. Kilian, T. Ohl, JRR, M. Sekulla, F. Staub, C. Weiss, DESY summer students

**Web address:** [http://projects.hepforge.org/whizard](http://projects.hepforge.org/whizard)

**Standard Reference:** Kilian/Ohl/JRR, EPJ C71 (2011) 1742, arXiv:0708.4233
WHIZARD 2: Status 2010-15 – Technical Features

- Fortran2003/2008 (gfortran 4.7.4 or newer) and OCaml (for MEs)

- WHIZARD core: separate interface from implementation
  - Complete object orientation
  - Replaceable modules with well-defined interface: matrix-elements, beam structure, phase space, integration, decays, shower, ...
  - Much easier to outsource small(er) projects
  - Much better self checks, regression testing and maintainability

- OpenMP parallelization

- Operation modes: dynamic linking, static linking, library mode, shell mode

- Standard conformance: uses autotools: automake/autoconf/libtool
  Installation: ./configure, make, make check, make install

- Large self test suite: unit tests, feature tests, run tests

- Version control (svn) at HepForge: use of ticket system and bug tracker

- Continuous integration system (jenkins) linked with svn repository
WHIZARD Manual

with distribution and online: http://whizard.hepforge.org/manual
Correlated lepton beam spectra with Circe2

- Guinea-Pig++ event files too short for high lumi simulations
- Fixed width histogramming struggles with steep distributions
- Circe1 too restrictive, assumes
  - factorized beam spectra: \( D_{p_1 p_2}(x_1, x_2) = D_{p_1}(x_1) D_{p_2}(x_2) \)
  - power laws in continuum: \( D(x) = d \cdot \delta(1 - x) + c \cdot x^\alpha(1 - x)^\beta \)
- Circe2 algorithm:
  - Adapt 2D factorized variable width histogram (à la VEGAS) to steep part of distribution
  - smooth the correlated fluctuations with a moderate gaussian filter to suppress artifacts from limited Guinea-Pig++ statistics
  - smooth separately continuum/boundary bins (avoid artificial beam energy spread)

Smoothing \( x_{e^+} = 1 \) boundary bin with Gaussian filters of width 3 and 10 bins, resp.
5 bins reasonable compromise for histograms with 100 bins.
[bins are not equidistant, shrink with power law towards the \( x_{e^-} = 1 \) boundary on RHS!]
Workflow Guinea-Pig++/Circe2/WHIZARD

1. Run Guinea-Pig++ with
   
   do_lumi=7; num_lumi=100000000; num_lumi_eg=100000000; num_lumi_gg=100000000;

   to produce lumi.[eg][eg].out with \((E_1, E_2)\) pairs.

   [Large event numbers, as Guinea-Pig++ will produce only a small fraction!]

2. Run circe2_tool.opt with steering file
   
   
   to produce correlated beam description

3. Run WHIZARD with SINDARIN input:

   beams = e1, E1 => circe2
   $circe2_file = "ilc500.circe"
   $circe2_design = "ILC"
   ?circe_polarized = false

   • Soon also files for polarized beams within distribution
Top quark threshold in $e^+e^-$

- $e^+e^-$ top threshold scan offers best option for $m_t$
- now: analytic LL $ttV$ form factor implemented
- default parameters: $M^{1S} = 172$ GeV, $\Gamma_t = 1.5$ GeV, $\alpha_s(M^{1S}) = 0.1077$
- analytic LL unstable far off-shell: top mass cut $\Delta M_t \leq 30$ GeV
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- Proper NLO/NLL matched implementation
- **TOPPIK code ships with WHIZARD**
- Own model: **SM_tt_threshold**
- Parameters: $w_{top}, m_{lS}, v_{soft}, \text{match}$

![Graph showing the top quark threshold in $e^+e^-$](image)

Matching scale

- $K_{\text{thresh}} \sim 0.7$
- $K_{\text{cont}} \sim 1.5$

Bach/Hoang/JRR/Stahlhofen/Teubner
courtesy to T. Teubner
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---

**“true” NLL threshold**
Possible on short time scale: **v2.2.2**

**“true” NLO continuum**
Needs virtual NLO matrix elements: $\sim 2015$

**“proper” NLL/NLO matching**
At intermediate energy: $\sim 2015$

*together with M Stahlhofen*
Status of NLO automation in WHIZARD

- **BLHA(2) interface** MC / OLP programs
  
  Speckner/JRR/Weiss, 2014  ✓

- **First implementation:** GoSAM  (also FeynArts/FormCalc, OpenLoops)

- **Work flow / Plans**
  
  - Automatic generation of subtraction terms  Speckner, 2012; Kilian/JRR/Weiss, 2014
  
  - proof-of-concept code in WHIZARD 2.2
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  - First examples: $e^+e^- \rightarrow q\bar{q}$ and $e^+e^- \rightarrow \ell^+\ell^-q\bar{q}, \ell\nu q\bar{q}, e^+e^- \rightarrow t\bar{t}$
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  - Plan: PowHeg box formalism for NLO processes (w/ Matching)
  - Release: WHIZARD 3.0
WHIZARD Outlook: upcoming releases 2.3 – 2.4 – 3.0

► New features in production version 2.2

- LHAPDF 6 support, FastJet interface ✓
- ILC TDR beam spectra, CLIC (correlated) spectra (CIRCE1/2) ✓
- Direct Guinea-Pig interface ✓
- LCIO support test phase ✓
- Complete Reweighting of Event Samples (incl. LHEF 2013) ✓
- Process containers: inclusive production samples (e.g. SUSY) ✓
- Automatic generation of decays, depending on the model ✓
- Simplified models for electroweak vector bosons (w/ light Higgs) ✓
- O’Mega Virtual Machine: faster + (much) smaller code Chokoufé/Ohl/JRR ✓

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- (N)LL/(N)LO matched $e^+e^-$ top threshold Bach/Hoang/JRR/Stahlhofen/Teubner
- New syntax/features decays and chains (steering unstable particles):
  $\text{process } higgsstr = e_1, E_1 \Rightarrow (Z \Rightarrow e_2, E_2), (H \Rightarrow b, \bar{b})$
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2nd International WHIZARD Forum 16.-18.3.2015

- SM and BSM physics
- LHC, ILC, CLIC, FCC collider physics
- Matrix elements, models and effective theories
- QED/QCD/Weak Radiation and Merging
- Higher Orders: Automation and Interfacing
- User interfaces, computing and performance & Event formats
BACKUP SLIDES:
Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

\[
O_{JJ}^{(I)} = \frac{1}{\Lambda^2} \text{tr} \left[ J^{(I)} \cdot J^{(I)} \right]
\]

\[
O'_{h,1} = \frac{1}{\Lambda^2} \left( (D\Phi)^\dagger \Phi \right) \cdot \left( h^\dagger (D\Phi) \right) - \frac{v^2}{2} |D\Phi|^2
\]

\[
O'_{hh} = \frac{1}{\Lambda^2} \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right) (D\Phi)^\dagger \cdot (D\Phi)
\]

\[
O'_{h,3} = \frac{1}{\Lambda^2} \frac{1}{3} \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^3
\]
\[ O_{\Phi W} = -\frac{1}{\Lambda^2} \frac{1}{2} (\Phi^\dagger \Phi - v^2/2) \text{tr} [W_{\mu\nu} W^{\mu\nu}] \]

\[ O_B = \frac{1}{\Lambda^2} \frac{i}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi) \]

\[ O_{\Phi B} = -\frac{1}{\Lambda^2} \frac{1}{4} (\Phi^\dagger \Phi - v^2/2) B_{\mu\nu} B^{\mu\nu} \]

\[ O_{Vq} = \frac{1}{\Lambda^2} \overline{q} h (\slash \! \! \! p h) q \]
Integrating out resonances

- Simplest example: scalar singlet $\sigma$:

$$\mathcal{L}_\sigma = -\frac{1}{2} [\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v\sigma \text{tr} [V_\mu V^\mu] - h_\sigma \text{tr} [TV_\mu] \text{tr}[TV^\mu]]$$

- Effective Lagrangian $\mathcal{L}_{\sigma}^{\text{eff}} = \frac{v^2}{8M_\sigma^2} \left[ g_\sigma \text{tr} [V_\mu V^\mu] + h_\sigma \text{tr} [TV_\mu] \text{tr}[TV^\mu] \right]^2$

- leads to anomalous quartic couplings (aQGCs)

$$\alpha_5 = g_\sigma^2 \left( \frac{v^2}{8M_\sigma^2} \right) \quad \alpha_7 = 2g_\sigma h_\sigma \left( \frac{v^2}{8M_\sigma^2} \right) \quad \alpha_{10} = 2h_\sigma^2 \left( \frac{v^2}{8M_\sigma^2} \right)$$
Unitary Description of EW interactions

- Five possible cases:
  - Amplitude perturbative, close to zero, small imag. part (SM)
  - Amplitude rises, gets imag. part, strongly interacting regime (presence of at least one dim. 8 operator)
  - Amplitude approaches maximum absolute value asymptotically
  - Turn over: new resonance
  - New inelastic channels open: eff. form factor, extra channels observable in multi-vector boson processes
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- Interpretation of EFT operator coefficients changes: formally still low-energy coefficients of Taylor expansion ⇒ threshold parameters
- Complete description necessary (only) beyond threshold
Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section:

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$$

Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im} [\mathcal{M}_{ii}(t = 0)] / s \quad t = -s(1 - \cos \theta)/2$$

Partial wave amplitudes:

$$\mathcal{M}(s, t, u) = 32\pi \sum_\ell (2\ell + 1) A_\ell(s) P_\ell(\cos \theta)$$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_\ell \frac{32\pi (2\ell + 1)}{s} |A_\ell|^2 \overset{!}{=} \sum_\ell \frac{32\pi (2\ell + 1)}{s} \text{Im} [A_\ell] \quad \Rightarrow \quad |A_\ell|^2 = \text{Im} [A_\ell]$$

Argand circle

$$|A(s) - i/2| = 1/2$$

Resonance:

$$A(s) = \frac{-M\Gamma_{el}}{s - M^2 + iM\Gamma_{tot}}$$

Counterclockwise circle, radius $$\frac{x_{el}}{2}$$

Pole at $$s = M^2 - iM\Gamma_{tot}$$
Unitarization Prescriptions

**K-matrix unitarization prescription** Gupta, 1950; Berger/Chanowitz, 1991

- Hermitian $K$-matrix interpreted as incompletely calculated approximation to true amplitude
- $\Rightarrow$ Unitary $S$, $T$ as a non-perturbative completion of this approximation
- Insert pert. expansion into expansion:

$$a = \frac{a_K}{1-ia_K} \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \text{Re}a_0^{(2)} + \ldots}{1-i(a_0^{(1)} + \text{Re}a_0^{(2)} + \ldots)}$$

- Prescription does a partial resummation of perturbative series
- Example Dyson resummation:

$$a^{(0)}_K(s) = \frac{\lambda s - m^2}{s - m^2} \to a^{(0)}(s) = \frac{\lambda}{s - m^2 - i\lambda}$$

**Drawbacks of (original) K-matrix:**

- Needs to construct self-adjoint $K$-matrix as intermediate step
- Problem if $S$-matrix is not diagonal, or ...
  
  there are non-perturbative contributions

**T-matrix unitarization**

- $a_0$ complex approximation to eigenvalue of true $T$ matrix
- use again pseudo-stereographic projection (intersection of Argand circle with line $a_0 i$)

- Results in:

$$a = \frac{\text{Re}a_0}{1-ia_0^*} \Rightarrow a^{(n)} = \frac{a_0^{(1)} + \text{Re}a_0^{(2)} + \ldots}{1-i(a_0^{(1)} + \text{Re}a_0^{(2)} - \text{Im}a_0^{(2)} + \ldots)}$$
Form Factor

\[
\text{Form Factor} = \frac{1}{\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^n}
\]

- Use Form Factor to suppress breaking of unitarity
- Can be generally used for arbitrary anomalous operator
- Need "Fine Tuning"

\[n\] Chosen to prevent breaking of Unitarity

\[\Lambda_{FF}\] Calculate highest possible value that satisfy real Unitarity bound (0th partial wave)

Parameters
K-Matrix

K-Matrix Unitarisation

\[ \mathcal{A}_K(s) = \frac{1}{\text{Re}(\frac{1}{\mathcal{A}(s)}) - i} \]

\[ = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} \quad \text{if} \quad \mathcal{A}(s) \in \mathbb{R} \]

- Projection of elastic amplitudes onto Argand-Circle
- At high energies the amplitude saturises
- Is usable for complex amplitudes
- Not dependent on additional parameters
Alternative Unitarization Prescriptions

- **Comparison of $T$-matrix and (original) $K$-matrix:**
  - $T$-matrix does not rely on perturbation theory
  - Special treatment for non-normal $T$ matrices (eigenvalues having imaginary parts larger than $i$; Riesz-Dunford operator calculus)
    1. $T$ matrix description leads to point on the Argand circle
    2. For real $a \Rightarrow$ (original) $K$-matrix case
    3. $a_0$ on Argand circle $\Rightarrow$ left invariant

- **Thales circle construction:**

  ![Thales Circle Diagram](image)

  - Defined via $\left| a - \frac{aK}{2} \right| = \frac{aK}{2} \Rightarrow a = \frac{1}{\text{Re}(\frac{1}{a_0})} - i$
  - avoids non-normal matrices, but not single-valued around $a = 0$
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- avoids non-normal matrices, but not single-valued around $a = 0$
ILC Results: Triboson production

\[ e^+ e^- \rightarrow W W Z / Z Z Z, \text{ dep. on } (\alpha_4 + \alpha_6), (\alpha_5 + \alpha_7), \alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10}) \]

Polarization populates longitudinal modes, suppresses SM bkgd.

Simulation with WHIZARD Kilian/Ohl/JRR

1 TeV, 1 ab\(^{-1}\), full 6-fermion final states, SIMDET fast simulation

Observables: \( M^2_{WW}, M^2_{WZ}, \angle(e^-, Z) \)

A) unpol., B) 80% \( e^-_R \), C) 80% \( e^-_R \), 60% \( e^+_L \)

32 % hadronic decays

Durham jet algorithm

Bkgd. \( t \bar{t} \rightarrow 6 \) jets

Veto against \( E^2_{\text{mis}} + p^2_{\perp,\text{mis}} \)

No angular correlations yet
ILC Results: Triboson production

\[ e^+ e^- \rightarrow WWZ/ZZZ, \text{ dep. on } (\alpha_4 + \alpha_6), (\alpha_5 + \alpha_7), \alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10}) \]

Polarization populates longitudinal modes, suppresses SM bkgd.

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<tr>
<th>( 16\pi^2 \times )</th>
<th>WWZ</th>
<th>ZZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>no pol.</td>
<td>( e^- ) pol.</td>
<td>both pol.</td>
</tr>
<tr>
<td>( \Delta \alpha_4^+ )</td>
<td>9.79</td>
<td>4.21</td>
</tr>
<tr>
<td>( \Delta \alpha_4^- )</td>
<td>-4.40</td>
<td>-3.34</td>
</tr>
<tr>
<td>( \Delta \alpha_5^+ )</td>
<td>3.05</td>
<td>2.69</td>
</tr>
<tr>
<td>( \Delta \alpha_5^- )</td>
<td>-7.10</td>
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The WHIZARD Event Generator – Release 2.2

1.0 Project started around 1999: Studies for electroweak multi-particle processes at TESLA (W, Higgs, Z)

1.5 Event samples for LC studies at SLAC

1.9 Full SM w/ QCD, beam properties, SUSY/BSM, event formats

2.1 QCD shower+matching, FeynRules support, internal density-matrix formalism (cascade decays), SINDARIN as user interface, OpenMP, ...

2.2 Major refactoring, event reweighting, inclusive processes and selective decay chains (production version)

Plan Improve $e^+e^-$ support; NLO + matching; improve user interface ⇒ adapt to specific needs of user groups
Beams and hard matrix elements

- **Hadron Colliders structured beams**
  - LHAPDF interface, most prominent PDFs directly included
  - QCD ISR and FSR (2 diff. own implementations, interface to PYTHIA)
  - Matching/merging matrix elements/showers
  - Underlying event/multiple interactions (proof of principle)

- **Hadronic events/hadronic decays + hadronic (QED) FSR (ext.)**

- **Lepton Colliders structured beams**
  - Beam structure (CIRCE1/2 module) more later
  - Arbitrarily polarized beams (density matrices)
  - QED ISR (Skrzypek/Jadach, Kuraev/Fadin, incl. $p_T$ distributions [caveat!])
  - [Photon collider spectra (CIRCE2 module)]

- **Hard matrix elements:**
  - Particle spins: $0, \frac{1}{2}, 1, \frac{3}{2}, 2$
  - Lorentz structures: hugh set of hard-coded structures
  - Fully general Lorentz structures foreseen for 2.3.0
  - Color structures: $3, \bar{3}, 8, [6]$
  - Color flow formalism
  - General color structures $6, 10, \epsilon_{ijk} \phi^i \phi^j \phi^k$

Stelzer/Willenbrock, 2003; Kilian/Ohl/JRR/Speckner, 2011
WHIZARD – Overview over Physics Models

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new models easily: FeynRules interface [Christensen/Duhr/Fuks/JRR/Speckner, 1010.3251]

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JRR et al. 1408.6207  
Talk LCWS14 EW session 7.10.
Analytic Parton Shower

- Analytic Parton Shower:
  - no shower veto: shower history is exactly known
  - allows reweighting and maybe more reliable error estimate

- new algorithm for initial state QCD radiation

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- Improvement/Tuning/Merging with higher-order matrix elements