Resonances and Unitarity in Weak Boson Scattering at the LHC

Jürgen Reuter

DESY Hamburg

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Doubts on the Standardmodel

	Measurement	Fit	$|O^{meas}-O^{fit}|/\sigma^{meas}$
			0 1 2 3
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768	
m_Z [GeV]	91.1875 ± 0.0021	91.1875	
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957	
σ_{had}^0 [nb]	41.540 ± 0.037	41.477	
R_l	20.767 ± 0.025	20.744	
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645	
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481	
R_b	0.21629 ± 0.00066	0.21586	
R_c	0.1721 ± 0.0030	0.1722	
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038	
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742	
A_b	0.923 ± 0.020	0.935	
A_c	0.670 ± 0.027	0.668	
A_l(SLD)	0.1513 ± 0.0021	0.1481	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	
m_W [GeV]	80.398 ± 0.025	80.374	
Γ_W [GeV]	2.140 ± 0.060	2.091	
m_t [GeV]	170.9 ± 1.8	171.3	

– describes microcosm   (too good?)

– 28 free parameters

– Higgs ?, form of Higgs potential ?

Hierarchy Problem

chiral symmetry: $\delta m_f \propto v \ln(\Lambda^2/v^2)$

no symmetry for quantum corrections to Higgs mass

$$\delta M_H^2 \propto \Lambda^2 \sim M_{\text{Planck}}^2 = (10^{19})^2\, \text{GeV}^2$$

20000 GeV^2 = (1000000000000000000000000000000020000 –

1000000000000000000000000000000000000) GeV^2

Open Questions

- Unification of all interactions (?)

- Baryon asymmetrie $\Delta N_B - \Delta N_{\bar{B}} \sim 10^{-9}$
  missing CP violation

- Flavour: three generations

- Tiny neutrino masses: $m_\nu \sim \frac{v^2}{M}$

- Dark Matter:
  - ► stable
  - ► only weakly interacting
  - ► $m_{DM} \sim 100\,\text{GeV}$

- Quantum theory of gravity

- Cosmic inflation

- Cosmological constant
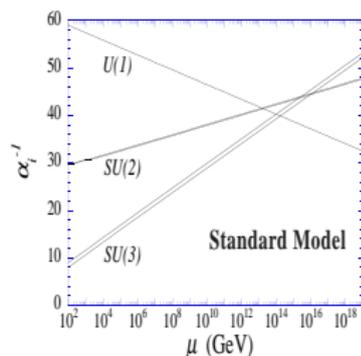
Ideas for New Physics since 1970

Model-Independent Description of the EW sector

► The "IT" boson is observed ... Higgs ?

► Aim: describe any physics beyond the SM as generically as possible

► Implement what we know about the SM

► Implements $SU(2)_L \times U(1)_Y$ gauge invariance

► Building blocks (including longitudinal modes):

$$\psi \text{ (SM fermions)}, \quad W^a_\mu \ (a = 1, 2, 3), \quad B_\mu, \quad \Sigma = \exp\left[\frac{-i}{v} w^a \tau^a\right]$$

► Minimal Lagrangian including gauge interactions

$$\mathcal{L}_{\text{min}} = \sum_\psi \overline{\psi}(i\displaystyle{\not}D)\psi - \frac{1}{2g^2}\text{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2g'^2}\text{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \frac{v^2}{4}\text{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right]$$

The Fundamental Building Blocks

- $\mathbf{V} = \Sigma(\mathbf{D}\Sigma)^\dagger$ (longitudinal vectors), $\mathbf{T} = \Sigma\tau^3\Sigma^\dagger$ (neutral component)

- Unitary gauge (no Goldstones): $\mathbf{w} \equiv 0$, i.e., $\Sigma \equiv 1$.

$$\mathbf{V} \longrightarrow -\frac{\mathrm{i}g}{2} \left[\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_\mathrm{w}}Z\tau^3 \right]$$

$$\mathbf{T} \longrightarrow \tau^3$$

- Gaugeless limit (only Goldstones) $(g, g' \to 0)$:

$$\mathbf{V} \longrightarrow \frac{\mathrm{i}}{v}\left\{ \sqrt{2}\partial w^+\tau^+ + \sqrt{2}\partial w^-\tau^- + \partial z\tau^3 \right\} + O(v^{-2})$$

$$\mathbf{T} \longrightarrow \tau^3 + 2\sqrt{2}\frac{\mathrm{i}}{v}\left(w^+\tau^+ - w^-\tau^-\right) + O(v^{-2})$$

So $\mathbf{T}$ projects out the neutral part:

$$\mathrm{tr}\left[\mathbf{TV}\right] = \frac{2\mathrm{i}}{v}\left[\partial z + \frac{\mathrm{i}}{v}\left(w^+\partial w^- - w^-\partial w^+\right) \right] + O(v^{-3})$$

Electroweak Chiral Lagrangian

Complete Lagrangian contains infinitely many parameters

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{min}} - \sum_{\psi} \overline{\psi}_L \Sigma M \psi_R + \beta_1 \mathcal{L}_0' + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \dots$$

$$\mathcal{L}_0' = \frac{v^2}{4} \operatorname{tr} [\mathbf{T}\mathbf{V}_\mu] \operatorname{tr} [\mathbf{T}\mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \operatorname{tr} [\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}] \qquad \qquad \mathcal{L}_6 = \operatorname{tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \operatorname{tr} [\mathbf{T}\mathbf{V}^\mu] \operatorname{tr} [\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_2 = \operatorname{itr} [\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \qquad \mathcal{L}_7 = \operatorname{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \operatorname{tr} [\mathbf{T}\mathbf{V}_\nu] \operatorname{tr} [\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_3 = \operatorname{itr} [\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \qquad \mathcal{L}_8 = \frac{1}{4}\operatorname{tr} [\mathbf{T}\mathbf{W}_{\mu\nu}] \operatorname{tr} [\mathbf{T}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_4 = \operatorname{tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \operatorname{tr} [\mathbf{V}^\mu \mathbf{V}^\nu] \qquad \qquad \mathcal{L}_9 = \frac{i}{2}\operatorname{tr} [\mathbf{T}\mathbf{W}_{\mu\nu}] \operatorname{tr} [\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_5 = \operatorname{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \operatorname{tr} [\mathbf{V}_\nu \mathbf{V}^\nu] \qquad \qquad \mathcal{L}_{10} = \frac{1}{2} \left(\operatorname{tr} [\mathbf{T}\mathbf{V}_\mu] \operatorname{tr} [\mathbf{T}\mathbf{V}^\mu] \right)^2$$

Indirect info on new physics in $\beta_1, \alpha_i, \dots$ (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1 \qquad \qquad \alpha_1 = 0.0026 \pm 0.0020$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}} \qquad \qquad \beta_1 = -0.00062 \pm 0.00043$$

$$\Delta U = -16\pi\alpha_8 \qquad \qquad \alpha_8 = -0.0044 \pm 0.0026$$

Anomalous triple and quartic gauge couplings

$$
\mathcal{L}_{TGC} = \mathrm{i}e\left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ A^{\rho\mu}\right]
$$
$$
+ \mathrm{i}e\frac{c_{\mathrm{w}}}{s_{\mathrm{w}}}\left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]
$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_{\mathrm{w}}^2 - s_{\mathrm{w}}^2}$ $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$
\Delta g_1^\gamma = 0 \qquad\qquad\qquad\qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)
$$
$$
\Delta g_1^Z = \delta_Z + \frac{g^2}{c_{\mathrm{w}}^2}\alpha_3 \qquad\qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)
$$

$$
\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad\qquad\qquad\qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_{\mathrm{w}}^4}(\alpha_5 + \alpha_7)
$$
$$
\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_{\mathrm{w}}^2}\alpha_3 \qquad\qquad \Delta g_1^{WW} = 2c_{\mathrm{w}}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4
$$
$$
\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_{\mathrm{w}}^4}(\alpha_4 + \alpha_6) \qquad\qquad \Delta g_2^{WW} = 2c_{\mathrm{w}}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)
$$
$$
h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]
$$

Anomalous triple and quartic gauge couplings

$$
\begin{aligned}
\mathcal{L}_{QGC} = {} & e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+ \right] \\
& + e^2 \frac{c_{\mathrm{w}}}{s_{\mathrm{w}}} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^- \right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+ \right] \\
& + e^2 \frac{c_{\mathrm{w}}^2}{s_{\mathrm{w}}^2} \left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+ \right] \\
& + \frac{e^2}{2 s_{\mathrm{w}}^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW} \left(W^{-\mu} W_\mu^+ \right)^2 \right] + \frac{e^2}{4 s_{\mathrm{w}}^2 c_{\mathrm{w}}^2} h^{ZZ} (Z^\mu Z_\mu)^2
\end{aligned}
$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_{\mathrm{w}}^2 - s_{\mathrm{w}}^2}$ $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad\qquad\qquad\qquad \Delta \kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$
$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_{\mathrm{w}}^2} \alpha_3 \qquad\qquad \Delta \kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad\qquad\qquad\qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_{\mathrm{w}}^4}(\alpha_5 + \alpha_7)$$
$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_{\mathrm{w}}^2} \alpha_3 \qquad\qquad \Delta g_1^{WW} = 2 c_{\mathrm{w}}^2 \Delta g_1^{\gamma Z} + 2 g^2(\alpha_9 - \alpha_8) + g^2 \alpha_4$$
$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_{\mathrm{w}}^4}(\alpha_4 + \alpha_6) \qquad\qquad \Delta g_2^{WW} = 2 c_{\mathrm{w}}^2 \Delta g_1^{\gamma Z} + 2 g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$
$$h^{ZZ} = g^2 \left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10}) \right]$$

Parameters and Scales, Resonances

α_i measurable at ILC

- $\alpha_i \ll 1$ (LEP)
- $\alpha_i \gtrsim 1/16\pi^2 \approx 0.006$ (renormalize divergencies, $16\pi^2 \alpha_i \gtrsim 1$)

Translation of parameters into new physics scale Λ: $\alpha_i = v^2/\Lambda^2$

- Operator normalization is arbitrary
- Power counting can be intricate

To be specific: consider resonances that couple to EWSB sector
Resonance mass gives detectable shift in the α_i

- Narrow resonances $\Rightarrow$ particles
- Wide resonances $\Rightarrow$ continuum

$\beta_1 \ll 1 \Rightarrow SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge
$g' \neq 0$ and fermion masses)

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 (Higgs ?)	ω^0 (γ'/Z' ?)	f^0 (Graviton ?)
$I = 1$	$\pi^\pm, \pi^0$ (2HDM ?)	$\rho^\pm, \rho^0$ (W'/Z' ?)	$a^\pm, a^0$
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

accounts for weakly and strongly interacting models

Model-Independent Way – Effective Field Theories

How to <u>clearly</u> separate effects of heavy degrees of freedom?

Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j, J] = \int \mathcal{D}[\Phi]\, \mathcal{D}[\varphi]\ \exp\left[i \int dx \left(\tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}\Phi(\Box + M^2)\Phi - \lambda\varphi^2\Phi - \ldots + J\Phi + j\varphi \right)\right]$$

Low-energy effective theory $\Rightarrow$ integrating out heavy degrees of freedom (DOF) in path integrals, set up Power Counting

Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2}\left(1 + \frac{\partial^2}{M^2}\right)^{-1}\varphi^2 \Rightarrow$$

$$\frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\varphi^2\Phi = -\frac{1}{2}\Phi'(M^2 + \partial^2)\Phi' + \frac{\lambda^2}{2M^2}\varphi^2\left(1 + \frac{\partial^2}{M^2}\right)^{-1}\varphi^2.$$

Effective Dim. 6 Operators
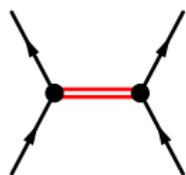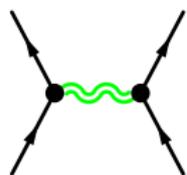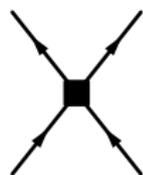
$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{F^2}\operatorname{tr}\left[[]\, J^{(I)} \cdot J^{(I)}\right]$$
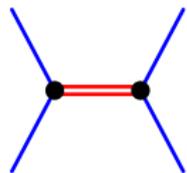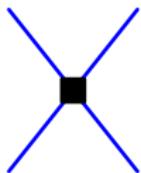
$$\mathcal{O}'_{h,1} = \frac{1}{F^2}\left((Dh)^\dagger h\right) \cdot \left(h^\dagger (D^h)\right) - \frac{v^2}{2}|Dh|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{F^2}(h^\dagger h - v^2/2)\,(Dh)^\dagger \cdot (Dh)$$
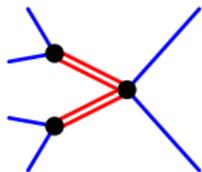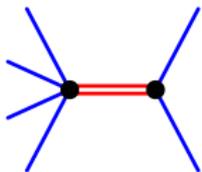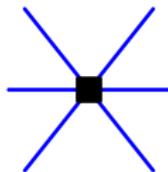
$$\mathcal{O}'_{h,3} = \frac{1}{F^2}\frac{1}{3}(h^\dagger h - v^2/2)^3$$
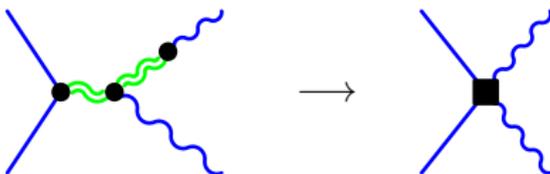
$$\mathcal{O}'_{WW} = -\frac{1}{F^2}\frac{1}{2}(h^\dagger h - v^2/2)\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

$$\mathcal{O}_B = \frac{1}{F^2}\frac{i}{2}(D_\mu h)^\dagger(D_\nu h)B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{F^2}\frac{1}{4}(h^\dagger h - v^2/2)B_{\mu\nu}B^{\mu\nu}$$
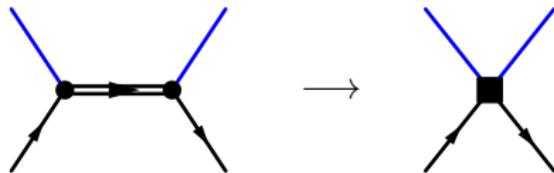
$$\mathcal{O}_{Vq} = \frac{1}{F^2}\bar{q}h(\slashed{D}h)q$$

Oblique Corrections: S, T, U

$$Z_L \longrightarrow Z_L \qquad \longrightarrow \qquad Z_L \; Z_L \qquad \Delta T \sim \Delta \rho \sim \Delta M_Z^2 Z \cdot Z$$

$$Z_T \longrightarrow Z_T \qquad \longrightarrow \qquad Z_T \; Z_T \qquad \Delta S \sim W^0{}_{\mu\nu} B^{\mu\nu}, \Delta U \sim W^0{}_{\mu\nu} W^{0\mu\nu}$$

$\diamond$ All low-energy effects order v^2/F^2 (Wilson coefficients)

$\diamond$ Low-energy observables with low-energy input G_F, α, M_Z affected by **non-oblique** contributions:

$$G_F = \frac{1}{v} \longrightarrow \frac{1}{v}\left(1 - \alpha\Delta T + \delta\right), \qquad \boxed{\delta \equiv -\frac{v^2}{4}f_{JJ}^{(3)}}$$

$S_{\text{eff}} = \Delta S$

$T_{\text{eff}} = \Delta T - \frac{1}{\alpha}\delta$

$U_{\text{eff}} = [\Delta U = 0] + \frac{4s_w^2}{\alpha}\delta$

▶ non-oblique flavour-dependent corrections ⇒ enforce flavour-dependent EW fit

Integrating out resonances

Consider leading order effects of resonances on EW sector:

$$\mathcal{L}_\Phi = z \left[\Phi \left(M_\Phi^2 + DD \right) \Phi + 2\Phi J \right] \qquad \Rightarrow \qquad \mathcal{L}_\Phi^{\text{eff}} = -\frac{z}{M^2} JJ + \frac{z}{M^4} J(DD)J + \mathcal{O}(M^{-6})$$

- Simplest example: scalar singlet σ:

$$\mathcal{L}_\sigma = -\frac{1}{2} \left[\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \text{tr} \left[\mathbf{V}_\mu \mathbf{V}^\mu \right] - h_\sigma \text{tr} \left[\mathbf{T} \mathbf{V}_\mu \right] \text{tr} \left[\mathbf{T} \mathbf{V}^\mu \right] \right]$$

- Effective Lagrangian

$$\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2} \left[g_\sigma \text{tr} \left[\mathbf{V}_\mu \mathbf{V}^\mu \right] + h_\sigma \text{tr} \left[\mathbf{T} \mathbf{V}_\mu \right] \text{tr} \left[\mathbf{T} \mathbf{V}^\mu \right] \right]^2$$

- leads to anomalous quartic couplings

$$\alpha_5 = g_\sigma^2 \left(\frac{v^2}{8M_\sigma^2} \right) \qquad \alpha_7 = 2g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2} \right) \qquad \alpha_{10} = 2h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2} \right)$$

- Special case: SM Higgs with $g_\sigma = 1$ and $h_\sigma = 0$

Coupl. strengths, Anomal. Couplings, Power Counting

Scalar resonance width ($M_\sigma \gg M_W, M_Z$):

$$\Gamma_\sigma = \frac{g_\sigma^2 + \frac{1}{2}(g_\sigma^2 + 2h_\sigma^2)^2}{16\pi}\left(\frac{M_\sigma^3}{v^2}\right) + \Gamma(\text{non} - WW, ZZ)$$

Largest allowed coupling for a broad continuum: $\Gamma \sim M \gg \Gamma(\text{non} - WW, ZZ) \sim 0$
translates to bounds for effective Lagrangian (e.g. scalar with no isospin violation):

$$\alpha_5 \leq \frac{4\pi}{3}\left(\frac{v^4}{M_\sigma^4}\right) \approx \frac{0.015}{(M_\sigma \text{ in TeV})^4} \quad \Rightarrow \quad 16\pi^2\alpha_5 \leq \frac{2.42}{(M_\sigma \text{ in TeV})^4}$$

> Scalar: $\Gamma \sim g^2 M^3$, $\alpha \sim g^2/M^2$ $\Rightarrow$ $\alpha_{\text{max}} \sim 1/M^4$
> Vector: $\Gamma \sim g^2 M$, $\alpha \sim g^2/M^2$ $\Rightarrow$ $\alpha_{\text{max}} \sim 1/M^2$
> Tensor: $\Gamma \sim g^2 M^3$, $\alpha \sim g^2/M^2$ $\Rightarrow$ $\alpha_{\text{max}} \sim 1/M^4$

Vector triplet (simplified)

$$\mathcal{L}_\rho = -\frac{1}{8}\text{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\text{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{ig_\rho v^2}{2}\text{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right]$$

$1/M^2$ term renormalizes kinetic energy (i.e. v), hence unobservable:

$$\mathcal{L}_\rho^{\text{eff}} = \frac{g_\rho^2 v^4}{4M_\rho^2}\text{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right] + \mathcal{O}(1/M_\rho^4)$$

Vector Resonances

$$\mathcal{L}_\rho = -\frac{1}{8}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{\Delta M_\rho^2}{8}\left(\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\right)^2 + \mathrm{i}\frac{\mu_\rho}{2}g\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{W}^{\mu\nu}\boldsymbol{\rho}_\nu\right]$$

$$+ \mathrm{i}\frac{\mu_\rho'}{2}g'\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{B}^{\mu\nu}\boldsymbol{\rho}_\nu\right] + \mathrm{i}\frac{g_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right] + \mathrm{i}\frac{h_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]$$

$$+ \frac{g'v^2 k_\rho}{2M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{B}^{\nu\mu},\mathbf{V}_\nu]\right] + \frac{gv^2 k_\rho'}{4M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{T},\mathbf{V}_\nu]\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^{\nu\mu}\right]$$

$$+ \frac{gv^2 k_\rho''}{4M_\rho^2}\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\mathrm{tr}\left[[\mathbf{T},\mathbf{V}_\nu]\mathbf{W}^{\nu\mu}\right] + \mathrm{i}\frac{\ell_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right]$$

$$+ \mathrm{i}\frac{\ell_\rho'}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{B}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right] + \mathrm{i}\frac{\ell_\rho''}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right]$$

all $\alpha_i \sim 1/M_\rho^4$, except for $\beta_1 \sim \Delta\rho \sim T \sim h_\rho^2/M_\rho^2$

4-fermion contact interaction $j_\mu j^\mu \sim 1/M_\rho^2$ (eff. T and U parameter)

vector coupling $j_\mu V^\mu \sim 1/M_\rho^2$ (eff. S parameter)
Mismatch: measured fermionic vs. bosonic coupling g Nyffeler/Schenk, 2000; Kilian/JR, 2003

Effects on Triple Gauge Couplings

- $\mathcal{O}(1/M^2)$: Renormalization of ZWW coupling
- $\mathcal{O}(1/M^4)$: shifts in Δg_1^Z, $\Delta\kappa^\gamma$, $\Delta\kappa^Z$, λ^γ, λ^Z

Effects on Quartic Gauge Couplings

- $\mathcal{O}(1/M^4)$, orthogonal (in α_4–α_5 space) to scalar case

WHIZARD
Kilian/Ohl/JRR: DESY/Freiburg/Siegen/Würzburg, hep-ph/0102195, 0708.4233

- ▶ Multi-Purpose event generator for collider and astroparticle physics

- ▶ Acronym: **W**, **HI**ggs, **Z**, **A**nd **R**espective **D**ecays (deprecated)

  - ▶ Fast adaptive multi-channel Monte-Carlo integration

  - ▶ Very efficient phase space and event generation

  - ▶ Optimized/-al matrix elements

  - ▶ Recent version: 2.1.1   (18.09.2012)
    http://projects.hepforge.org/whizard und
    http://whizard.event-generator.org

  - ▶ Parton shower ($k^\perp$-ordered and analytical)   Kilian/JRR/Schmidt/Wiesler, 1112.1039

  - ▶ Underlying Event: preliminary (for 2.1)

  - ▶ Arbitrary processes: matrix element generator (O'Mega)

  - ▶ 2.0 Features: ME/PS matching, cascades, versatile new steering syntax, WHIZARD as shared library

- ▶ Interface to FeynRules   Christensen/Duhr/Fuks/JRR/Speckner, 1010.3215

- ▶ Versatile input language: SINDARIN

WHIZARD

Kilian/Ohl/JRR: DESY/Freiburg/Siegen/Würzburg, hep-ph/0102195, 0708.4233

- Multi-Purpose event generator for collider and astroparticle physics

- Focus: LHC, ILC, CLIC, SM, QCD, BSM

MODEL TYPE	with CKM matrix	trivial CKM
QED with e, μ, τ, γ	–	QED
QCD with d, u, s, c, b, t, g	–	QCD
Standard model	SM_CKM	SM
SM with anomalous couplings	SM_ac_CKM	SM_ac
SM with anomalous top couplings	—	SM_top
SM with K matrix	—	SM_KM
MSSM	MSSM_CKM	MSSM
MSSM with Gravitinos	—	MSSM_Grav
NMSSM	NMSSM_CKM	NMSSM
extended SUSY models	—	PSSSM
Littlest Higgs	—	Littlest
Littlest Higgs with ungauged $U(1)$	—	Littlest_Eta
Littlest Higgs with T parity	—	Littlest_Tpar
Simplest Little Higgs (anomaly free)	—	Simplest
Simplest Little Higgs (universal)	—	Simplest_univ
UED	—	UED
3-Site Higgsless Model	—	Threeshl
Noncommutative SM (inoff.)	—	NCSM
SM with Z'	—	Zprime
SM with Gravitino and Photino	—	GravTest
Augmentable SM template	—	Template

easy to
implement new models

- Interface to FeynRules   Christensen/Duhr/Fuks/JRR/Speckner, 1010.3215

- Versatile input language: SINDARIN

Anomalous Gauge Couplings at LHC

ILC: Beyer/Kilian/Krstonošić/Mönig/JR/Schröder/Schmidt, 2006
LHC: Mertens, 2006; Kilian/Kobel/Mader/JR/Schumacher

Anomalous quartic gauge couplings, by chiral EW Lagrangian:

$$\mathcal{L}_4 = \alpha_4 \frac{g^2}{2} \left\{ \left[(W^+ W^+)(W^- W^-) + (W^+ W^-)^2 \right] + \frac{2}{c_W^2} (W^+ Z)(W^- Z) + \frac{1}{2 c_W^4} (ZZ)^2 \right\}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^2}{2} \left\{ (W^+ W^-)^2 + \frac{2}{c_W^2} (W^+ W^-)(ZZ) + \frac{1}{2 c_W^4} (ZZ)^2 \right\}$$

(all leptons, incl. τ):
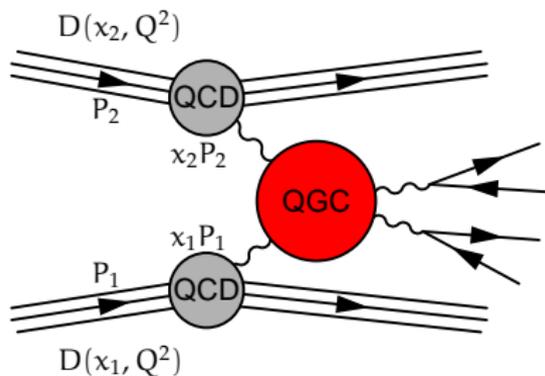
$$pp \to jj(ZZ/WW) \to jj\ell^- \ell^+ \nu_\ell \bar{\nu}_\ell$$

$$\sigma \approx 40 \text{ fb}$$

Background:
- $t\bar{t} \to WbWb$, $\sigma \approx 52$ pb
- Single t, misrec. jet: $\sigma \approx 4.8$ pb
- QCD: $\sigma \approx 0.21$ pb

Tagging and Cuts:

- $\ell\ell jj$-Tag, $\eta_{tag}^{min} < \eta_\ell < \eta_{tag}^{max}$, b-Veto
- $|\Delta\eta_{jj}| > 4.4$, $M_{jj} > 1080 \text{ GeV}$
- Minijet-Veto: $p_{T,j} < 30 \text{ GeV}$
- $E_j > 600, 400 \text{ GeV}$, $p_{T,j}^1 > 60, 24 \text{ GeV}$

Improves $S/\sqrt{B}$ from 3.3 to 29.7
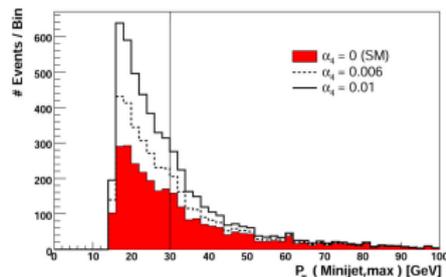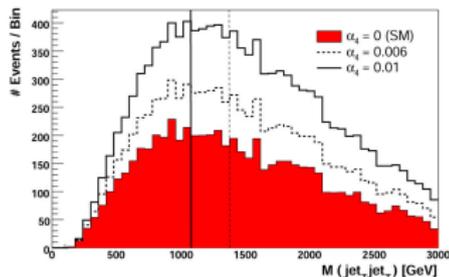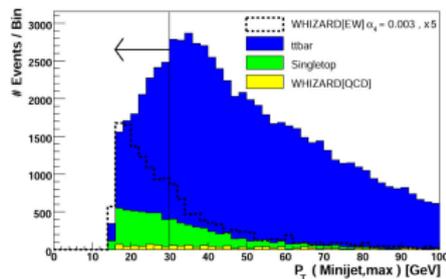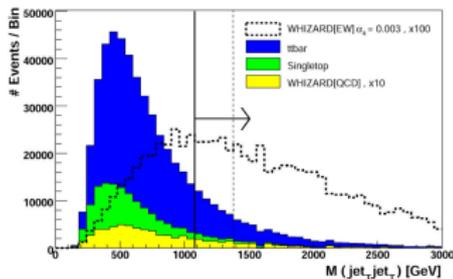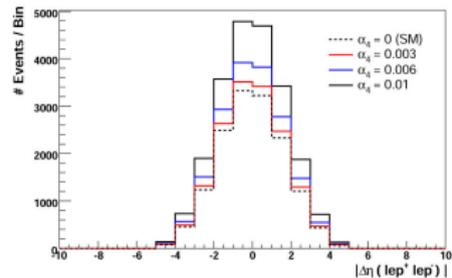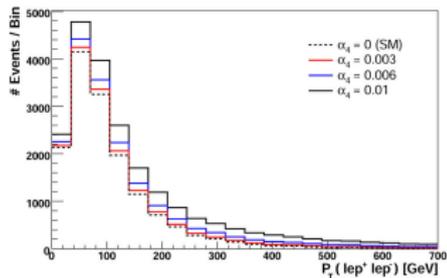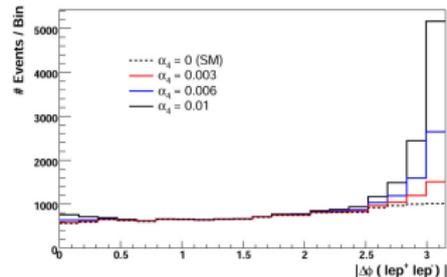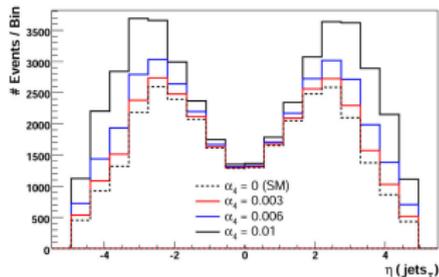
Results: (1σ Sensitivity to αs)

Coupl.	ILC ($1\,\mathrm{ab}^{-1}$)	LHC ($100\,\mathrm{fb}^{-1}$)
α_4	0.0088	0.00160
α_5	0.0071	0.00098

Limits for Λ [TeV]:

Spin	$I=0$	$I=1$	$I=2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Isospin decomposition

- Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4} \mathrm{tr} \left[\mathbf{V}_\mu \mathbf{V}^\mu \right] + \alpha_4 \mathrm{tr} \left[\mathbf{V}_\mu \mathbf{V}_\nu \right] \mathrm{tr} \left[\mathbf{V}^\mu \mathbf{V}^\nu \right] + \alpha_5 \left(\mathrm{tr} \left[\mathbf{V}_\mu \mathbf{V}^\mu \right] \right)^2$$

- Leads to the following amplitudes: $s = (p1 + p2)^2 \quad t = (p1 - p3)^2 \quad u = (p1 - p4)^2$

$$\boxed{\mathcal{A}(s,t,u) =:} \quad \mathcal{A}(w^+ w^- \to zz) = \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2 + u^2}{v^4}$$

$$\mathcal{A}(w^+ z \to w^+ z) = \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2 + u^2}{v^4}$$

$$\mathcal{A}(w^+ w^- \to w^+ w^-) = -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5) \frac{s^2 + t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4}$$

$$\mathcal{A}(w^+ w^+ \to w^+ w^+) = -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5) \frac{t^2 + u^2}{v^4}$$

$$\mathcal{A}(zz \to zz) = 8(\alpha_4 + \alpha_5) \frac{s^2 + t^2 + u^2}{v^4}$$

- (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\mathcal{A}(I = 0) = 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$
$$\mathcal{A}(I = 1) = \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t)$$
$$\mathcal{A}(I = 2) = \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)$$

Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section: $\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2$

Optical Theorem   (Unitarity of the S(cattering) Matrix):
$$\sigma_{\text{tot}} = \text{Im}\left[\mathcal{M}_{ii}(t=0)\right]/s \qquad t = -s(1-\cos\theta)/2$$

Partial wave amplitudes: $\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1)\mathcal{A}_\ell(s)P_\ell(\cos\theta)$

Assuming only elastic scattering:
$$\sigma_{\text{tot}} = \sum_\ell \frac{32\pi(2\ell+1)}{s}|\mathcal{A}_\ell|^2 \overset{!}{=} \sum_\ell \frac{32\pi(2\ell+1)}{s}\text{Im}\left[\mathcal{A}_\ell\right] \quad \Rightarrow \quad \boxed{|\mathcal{A}_\ell|^2 = \text{Im}\left[\mathcal{A}_\ell\right]}$$
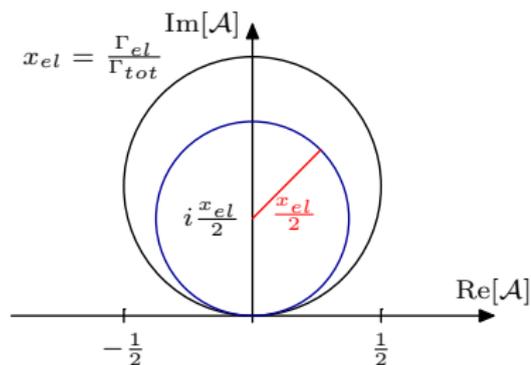
Argand circle $\boxed{\left|\mathcal{A}(s) - \frac{i}{2}\right| = \frac{1}{2}}$

Resonance: $\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s - M^2 + iM\Gamma_{\text{tot}}}$

Counterclockwise circle, radius $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{\text{tot}}$

Unitarity in the EW sector: SM

▶ Project out isospin eigenamplitudes   Lee,Quigg,Thacker, 1973

$$\mathcal{A}_\ell(s) = \frac{1}{32\pi} \int_{-s}^{0} \frac{dt}{s} \mathcal{A}(s,t,u) P_\ell(1 + 2t/s) \qquad \cos\theta = 1 + 2t/s$$

Remember Legendre polynomials: $P_0(s) = 1$ $P_1(s) = \cos\theta$ $P_2(s) = (3\cos^2\theta - 1)/2$

▶ SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I,\text{spin}=J}$):

$$\mathcal{A}_{I=0} = 2\frac{s}{v^2} P_0(s) \qquad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2} P_1(s) \qquad \mathcal{A}_{I=2} = -\frac{s}{v^2} P_0(s)$$

$$\boxed{\mathcal{A}_{0,0} = \frac{s}{16\pi v^2}} \qquad \boxed{\mathcal{A}_{1,1} = \frac{s}{96\pi v^2}} \qquad \boxed{\mathcal{A}_{2,0} = -\frac{s}{32\pi v^2}}$$

exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$I = 0:$ $E \sim \sqrt{8\pi}v = 1.2\,\text{TeV}$

$I = 1:$ $E \sim \sqrt{48\pi}v = 3.5\,\text{TeV}$

$I = 2:$ $E \sim \sqrt{16\pi}v = 1.7\,\text{TeV}$

Higgs exchange:
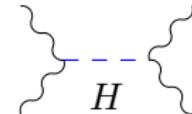
$$\mathcal{A}(s,t,u) = -\frac{M_H^2}{v^2}\frac{s}{s - M_H^2}$$

Unitarity: $M_H \lesssim \sqrt{8\pi}v \sim 1.2\,\text{TeV}$

K-Matrix Unitarization and friends
K-Matrix unitarization

$$\mathcal{A}_K(s) = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} = \mathcal{A}(s)\frac{1 + i\mathcal{A}(s)}{1 + \mathcal{A}(s)^2}$$

Unitarization by infinitely heavy and
wide resonance

▶ Low-energy theorem (LET): $\frac{s}{v^2}$

▶ K-Matrix amplitude:
$|\mathcal{A}(s)|^2 = \frac{s^2}{s^2 + v^4} \overset{s\to\infty}{\to} 1$

▶ Poles $\pm iv$: M_0, Γ large

Padé unitarization
separates higher chiral orders
$$\mathcal{A}_P(s) = \frac{\mathcal{A}^{(0)}(s)^2}{\mathcal{A}^{(0)}(s) - \mathcal{A}^{(1)}(s) - i\mathcal{A}^{(0)}(s)^2}$$
each partial wave dominated
by single resonance

"Naive" Unitarization
Extreme case:

$$\mathcal{A}_N(s) = e^{i\mathcal{A}(s)}\sin\mathcal{A}(s)$$

Infinitely many resonances
becoming denser for $s \to \infty$

BSM Unitarized Resonances: e.g. Scalar Singlet

Assumptions:

- LHC is able to detect a resonance in the EW sector
- Further resonances might exist, but out of reach or not detectable
- Describe 1st resonance by correct amplitude
- Use K-matrix unitarization to define a consistent model

Example: Scalar Singlet

- $\mathcal{L}_\sigma = -\frac{1}{2}\sigma \left(M_\sigma^2 + \partial^2 \right) \sigma + \frac{g_\sigma v}{2}\sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu \right]$

- Feynman rules: $\sigma w^+ w^- : -\frac{2ig_\sigma}{v}(k_+ \cdot k_-)$ $\sigma zz : -\frac{2ig_\sigma}{v}(k_1 \cdot k_2)$

- Amplitude (s-channel exchange): $\boxed{\mathcal{A}^\sigma(s,t,u) = \frac{g_\sigma^2}{v^2}\frac{s^2}{s - M^2}}$

- Isospin eigenamplitudes:

$$\mathcal{A}_0^\sigma(s,t,u) = \frac{g_\sigma^2}{v^2}\left(3\frac{s^2}{s-M^2} + \frac{t^2}{t-M^2} + \frac{u^2}{u-M^2} \right)$$

$$\mathcal{A}_1^\sigma(s,t,u) = \frac{g_\sigma^2}{v^2}\left(\frac{t^2}{t-M^2} - \frac{u^2}{u-M^2} \right)$$

$$\mathcal{A}_2^\sigma(s,t,u) = \frac{g_\sigma^2}{v^2}\left(\frac{t^2}{t-M^2} + \frac{u^2}{u-M^2} \right)$$

Unitarizing the scalar singlet

$$\mathcal{A}_{00}^{\sigma}(s) = 3\frac{g_{\sigma}^2}{v^2}\frac{s^2}{s-M^2} + 2\frac{g_{\sigma}^2}{v^2}\mathcal{S}_0(s) \qquad \mathcal{A}_{02}^{\sigma}(s) = 2\frac{g_{\sigma}^2}{v^2}\mathcal{S}_2(s) \quad = A_{22}^{\sigma}(s)$$

$$\mathcal{A}_{11}^{\sigma}(s) = 2\frac{g_{\sigma}^2}{v^2}\mathcal{S}_1(s) \qquad\qquad\qquad \mathcal{A}_{13}^{\sigma}(s) = 2\frac{g_{\sigma}^2}{v^2}\mathcal{S}_3(s)$$

$$\mathcal{A}_{20}^{\sigma}(s) = 2\frac{g_{\sigma}^2}{v^2}\mathcal{S}_0(s)$$

- S-wave coefficients no longer polynomial, e.g.:
$$\mathcal{S}_0(s) = M^2 - \frac{s}{2} + \frac{M^4}{s}\log\frac{s}{s+M^2}$$

- s-channel pole must be explicitly subtracted:

$$A_{IJ}(s) = A_{IJ}^{(0)}(s) + F_{IJ}(s) + \frac{G_{IJ}(s)}{s-M^2},$$

– $F_{IJ}(s)$ is finite
– $G_{IJ}(s) \propto s$ (vector), $\propto s^2$ (scalar, tensor)

$$\hat{A}_{IJ}(s) = \frac{A_{IJ}(s)}{1 - \frac{i}{32\pi}A_{IJ}(s)} = A_{IJ}^{(0)}(s) + 32\pi i\Delta A_{IJ}(s),$$

$$\boxed{\Delta A_{IJ}(s) = 32\pi i\left(1 + \frac{i}{32\pi}A_{IJ}^{(0)}(s) + \frac{s-M^2}{\frac{i}{32\pi}G_{IJ}(s) - (s-M^2)\left[1 - \frac{i}{32\pi}(A_{IJ}^{(0)}(s) + F_{IJ}(s))\right]}\right)}$$

Implementation and Taxonomy of Resonances

▶ Explicit "time arrow" in WHIZARD

$$-p_{\mathcal{V}_i} - p_{\mathcal{V}_i} - p_{\mathcal{V}_i}$$

$$\Delta A_{IJ}(\sum p)$$

$$p_{\mathcal{V}_i} \qquad p_{\mathcal{V}_j} \qquad p_{\mathcal{V}_k}$$
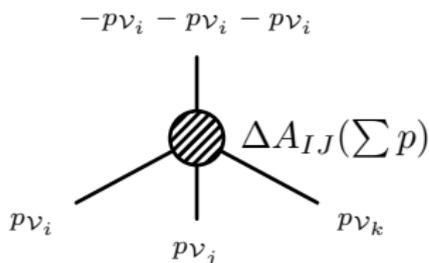
– trace back pairs of momenta at quartic vertices to external legs

– guarantee for only s-channel insertions

▶ Consider the following resonances:

$$\mathcal{L}_\sigma = -\tfrac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \sigma j_\sigma$$

$$\mathcal{L}_\phi = -\tfrac{1}{2}\left[\tfrac{1}{2}\operatorname{tr}\left[\phi\left(M_\sigma^2 + \partial^2\right)\phi\right] + \operatorname{tr}\left[\phi\mathbf{j}_\phi\right]\right]$$

$$\mathcal{L}_\rho = \tfrac{1}{2}\Big[\tfrac{M_\rho^2}{2}\operatorname{tr}\left[\rho_\mu\rho^\mu\right]$$
$$\qquad -\tfrac{1}{4}\operatorname{tr}\left[\rho_{\mu\nu}\rho^{\mu\nu}\right] + \operatorname{tr}\left[\mathbf{j}_\rho^\mu\rho_\mu\right]\Big]$$

$$\mathcal{L}_f = \mathcal{L}_{\mathrm{kin}} - \tfrac{M_f^2}{2}f_{\mu\nu}f^{\mu\nu} + f_{\mu\nu}j_f^{\mu\nu}$$

$$\mathcal{L}_a = \mathcal{L}_{\mathrm{kin}} - \tfrac{M_t^2}{4}\operatorname{tr}\left[\mathbf{t}_{\mu\nu}\mathbf{t}^{\mu\nu}\right] + \tfrac{1}{2}\operatorname{tr}\left[\mathbf{t}_{\mu\nu}\mathbf{j}_a^{\mu\nu}\right]$$

$$j_\sigma = \tfrac{g_\sigma v}{2}\operatorname{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]$$

$$\mathbf{j}_\phi = -\tfrac{g_\phi v}{2}\left(\mathbf{V}_\mu\otimes\mathbf{V}^\mu - \tfrac{\tau^{aa}}{6}\operatorname{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\right)$$

$$\mathbf{j}_\rho^\mu = \mathrm{i}g_\rho v^2\mathbf{V}^\mu$$

$$j_f^{\mu\nu} = -\tfrac{g_f v}{2}\left(\operatorname{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] - \tfrac{g^{\mu\nu}}{4}\operatorname{tr}\left[\mathbf{V}_\rho\mathbf{V}^\rho\right]\right)$$

$$\mathbf{j}_a^{\mu\nu} = -\tfrac{g_a v}{2}\Big[\tfrac{1}{2}\left(\mathbf{V}^\mu\otimes\mathbf{V}^\nu + \mathbf{V}^\nu\otimes\mathbf{V}^\mu\right) - \tfrac{g^{\mu\nu}}{4}\mathbf{V}_\rho\otimes\mathbf{V}^\rho$$
$$\qquad -\tfrac{\tau^{aa}}{6}\operatorname{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] + \tfrac{g^{\mu\nu}\tau^{aa}}{24}\operatorname{tr}\left[\mathbf{V}_\rho\mathbf{V}^\rho\right]\Big]$$

Taxonomy of resonances/Loops

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2 M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}\left(\frac{v^2}{M^2}\right)$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$
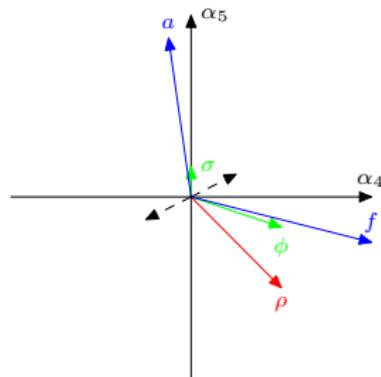
▶ Loop corrections to LET can be switched on/off:
(μ renormalization scale)

$$A_C^{\text{1-loop}}(s,t,u) = \frac{1}{16\pi^2}\left[\left(\frac{1}{2}\ln\frac{\mu^2}{|s|} + 8C_5\right)\frac{s^2}{v^4} + \left(\frac{t(s+2t)}{6v^4}\ln\frac{\mu^2}{|t|} + 4C_4\frac{t^2}{v^4}\right) + (t \leftrightarrow u)\right],$$

▶ Finite scheme-dep. matching coefficients/NLO counterterms
     (e.g. heavy Higgs regulator $\mu = M_H$ Dawson/Willenbrock, 1989)

$$C_4 = -\frac{1}{18} \approx -0.056, \quad C_5 = \frac{9\pi}{16\sqrt{3}} - \frac{37}{36} \approx -0.0075.$$

$$\alpha_4^{(1)} = \frac{1}{16\pi^2}\left(C_4 - \frac{1}{12}\ln\frac{\mu^2}{\mu_0^2}\right)$$

$$\alpha_5^{(1)} = \frac{1}{16\pi^2}\left(C_5 - \frac{1}{24}\ln\frac{\mu^2}{\mu_0^2}\right)$$

$|A_{\text{LET,1-loop}} - 1|$, angular dependence

LET, 1-loop

θ [rad]
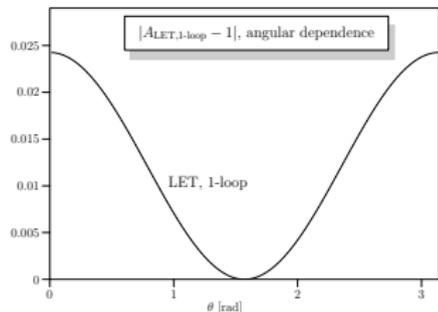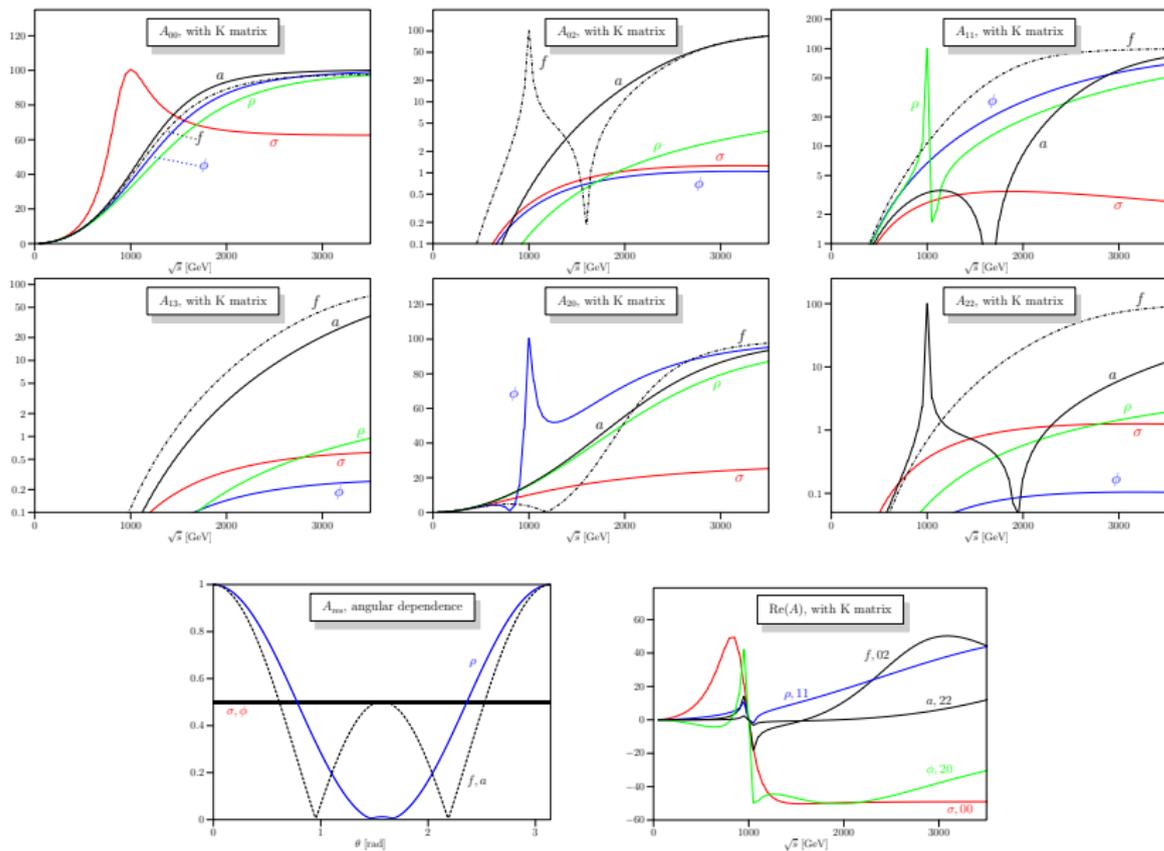
Eigenamplitudes

Eigenamplitudes
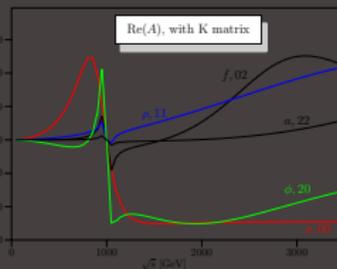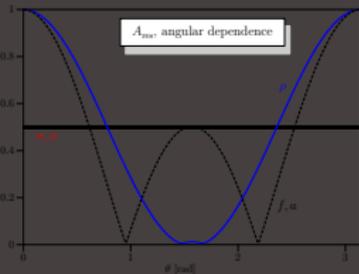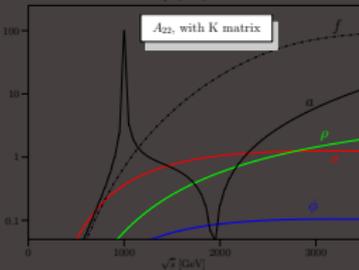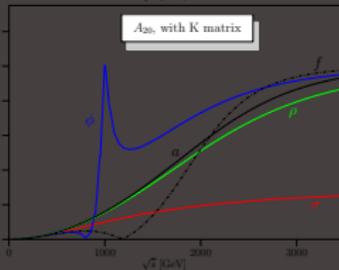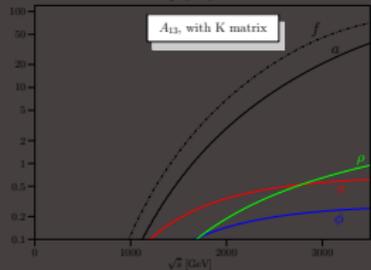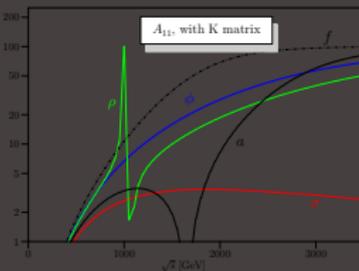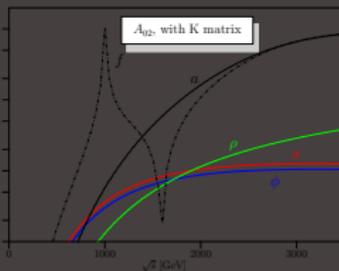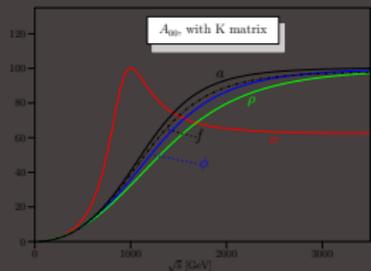
"Partonic" cross sections (I)
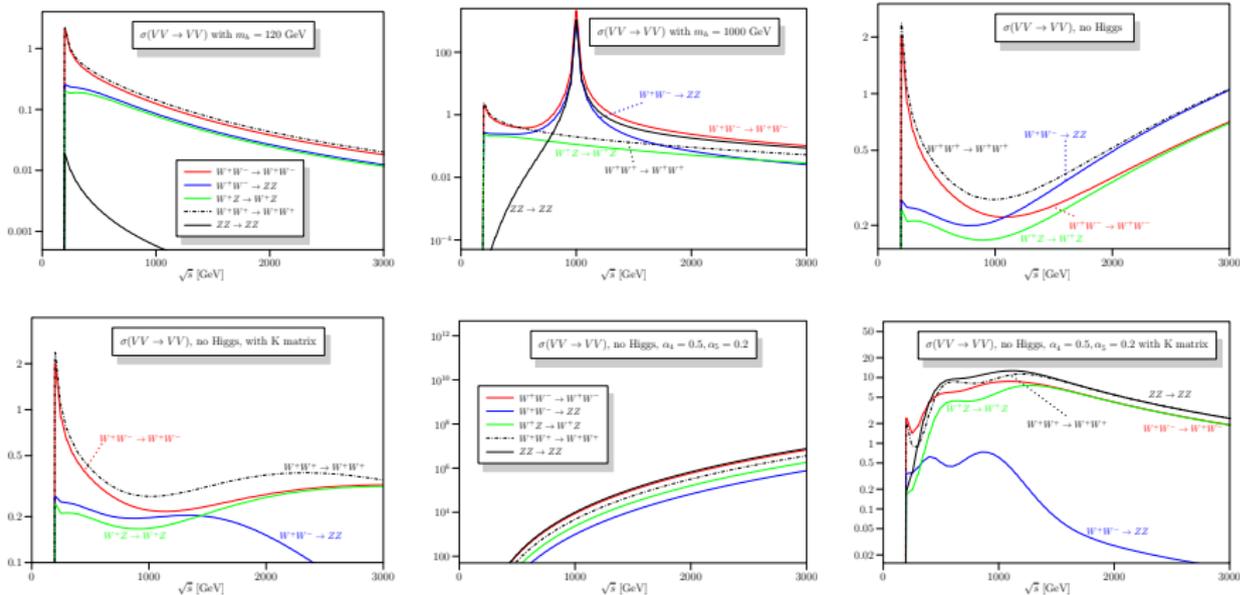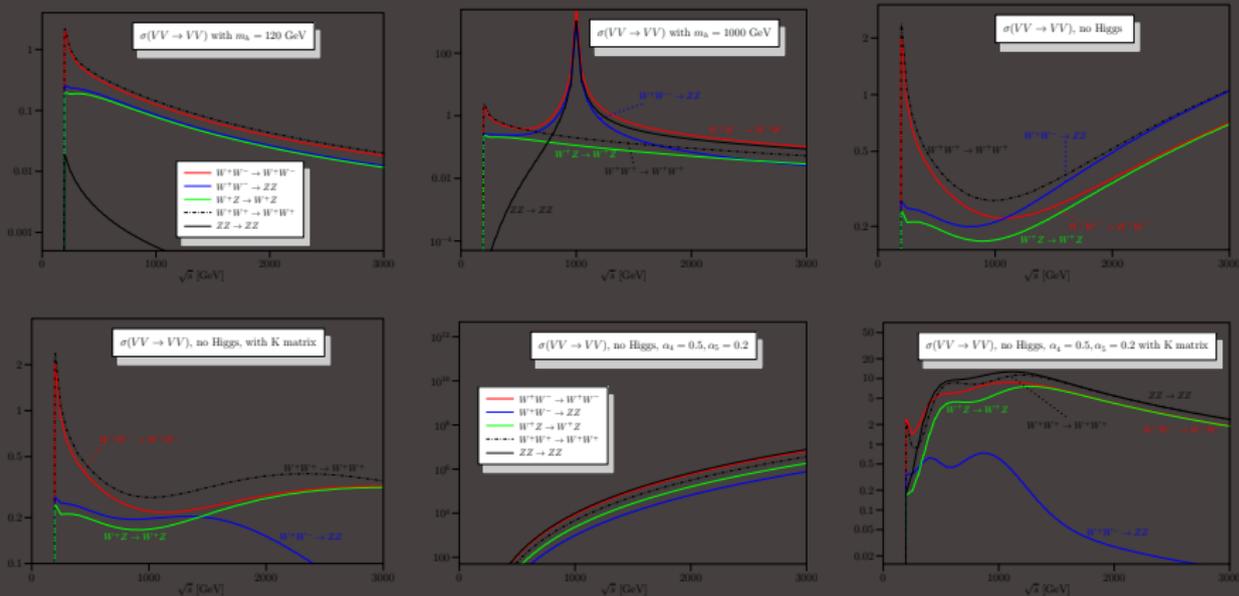

- Cross sections (in nb)

"Partonic" cross sections (I)

► Cross sections (in nb)

"Partonic" cross sections (II)
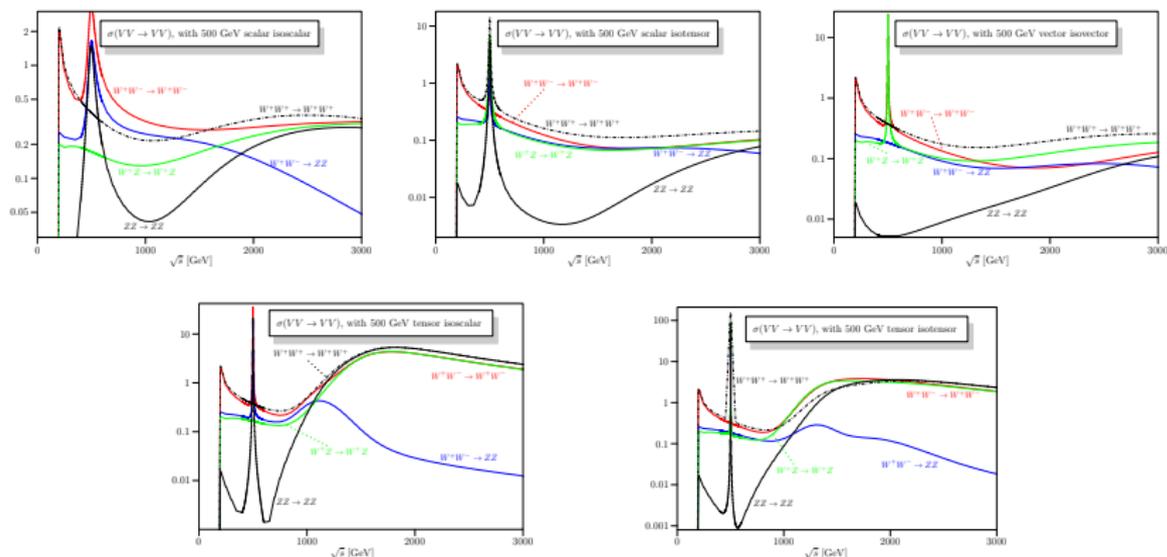
- $\sigma(\mathcal{VV} \to \mathcal{VV})$ in nb $M_R = 500$ GeV
- all amplitudes K-matrix unitarized
- Cut of $15°$ around the beam axis

"Partonic" cross sections (II)
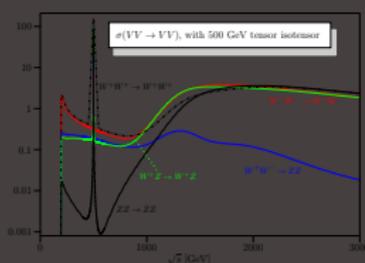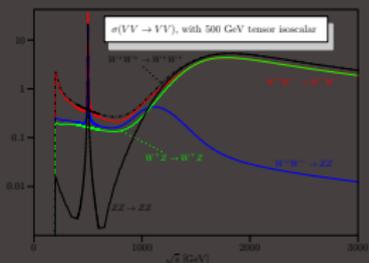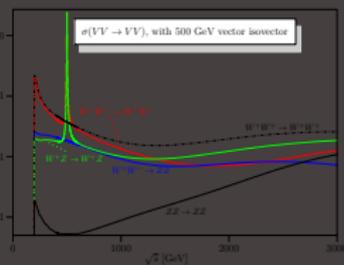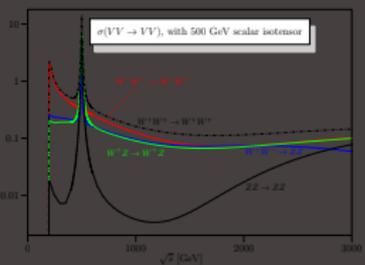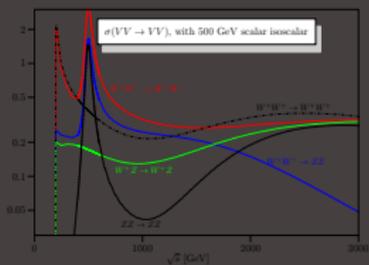
- $\sigma(\mathcal{VV} \to \mathcal{VV})$ in nb $M_R = 500$ GeV
- all amplitudes K-matrix unitarized
- Cut of $15°$ around the beam axis

The Effective W approximation

▶ $M_\mathcal{V}$, $\hat{t}_i$ small corrections, $\mathcal{V}$ nearly onshell:

$$\sigma(q_1 q_2 \rightarrow q_1' q_2' \mathcal{V}_1' \mathcal{V}_2') \approx \sum_{\lambda_1, \lambda_2} \int dx_1 \, dx_2 \, F_{q_1 \rightarrow q_1' \mathcal{V}_1}^{\lambda_1}(x_1) \, F_{q_2 \rightarrow q_2' \mathcal{V}_2}^{\lambda_2}(x_2) \, \sigma_{\mathcal{V}_1 \mathcal{V}_2 \rightarrow \mathcal{V}_1' \mathcal{V}_2'}^{\lambda_1 \lambda_2}(x_1 x_2 s)$$

▶ In addition to Weizsäcker-Williams: longitudinal polarisation

$$F_{q \rightarrow q' \mathcal{V}}^{+}(x) = \frac{(V - A)^2 + (V + A)^2 (1 - x)^2}{16\pi^2 \ x} \left[\ln \left(\frac{p_{\perp,\max}^2 + (1 - x) m_\mathcal{V}^2}{(1 - x) m_\mathcal{V}^2} \right) - \frac{p_{\perp,\max}^2}{p_{\perp,\max}^2 + (1 - x) m_\mathcal{V}^2} \right]$$

$$F_{q \rightarrow q' \mathcal{V}}^{-}(x) = \frac{(V + A)^2 + (V - A)^2 (1 - x)^2}{16\pi^2 \ x} \left[\ln \left(\frac{p_{\perp,\max}^2 + (1 - x) m_\mathcal{V}^2}{(1 - x) m_\mathcal{V}^2} \right) - \frac{p_{\perp,\max}^2}{p_{\perp,\max}^2 + (1 - x) m_\mathcal{V}^2} \right]$$

$$F_{q \rightarrow q' \mathcal{V}}^{0}(x) = \frac{V^2 + A^2}{8\pi^2} \ \frac{2(1 - x)}{x} \ \frac{p_{\perp,\max}^2}{p_{\perp,\max}^2 + (1 - x) m_\mathcal{V}^2}$$

▶ Dominant contribution from small $\mathcal{V}$ virtualities
▶ Transverse momentum cutoff $p_{\perp,\max} \leq (1 - x)\sqrt{s}/2$:
   ▶ longitudinal pol.:   finite for $p_{\perp,\max} \rightarrow \infty$
   ▶ Transversal pol.:   logarithmic singularity
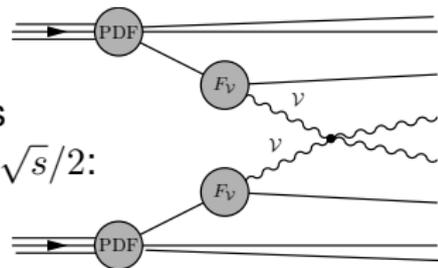
► EWA structure functions: W (left) and Z (right)
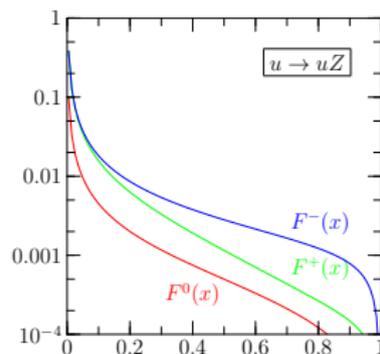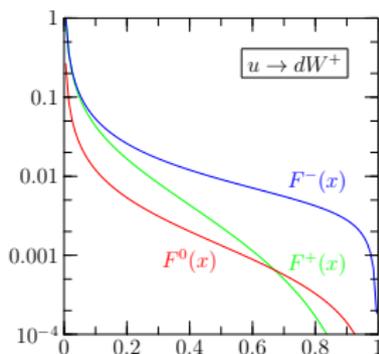
   – Emission from u, $\sqrt{s} = 2$ TeV   – preferred at high energy: transversal emission

► Problem: Irreducible background to weak-boson scattering
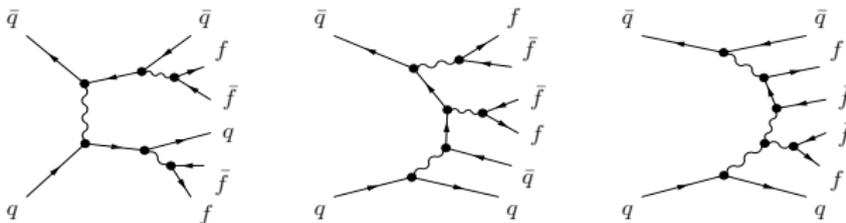
   – Double ISR/FSR   – t-channel like diagrams

► Coulomb-singularity (peak): cut on $p_{T,V} \gtrsim 30$ GeV
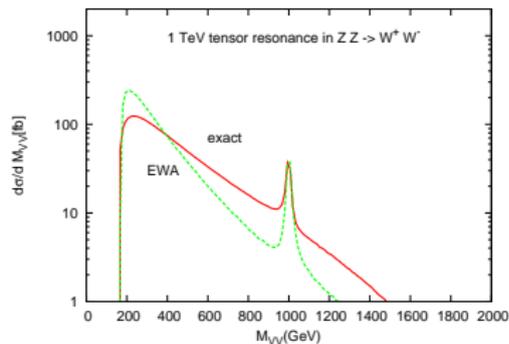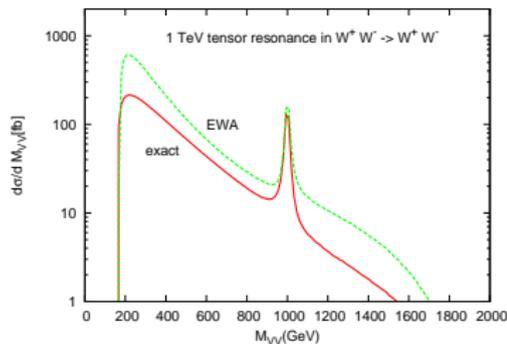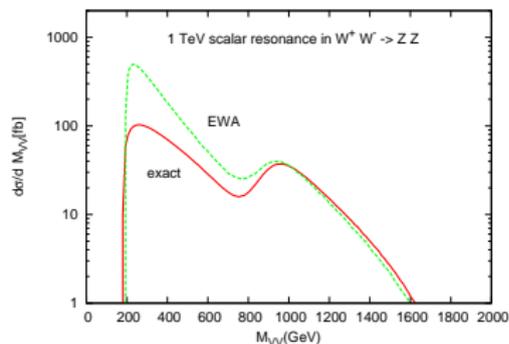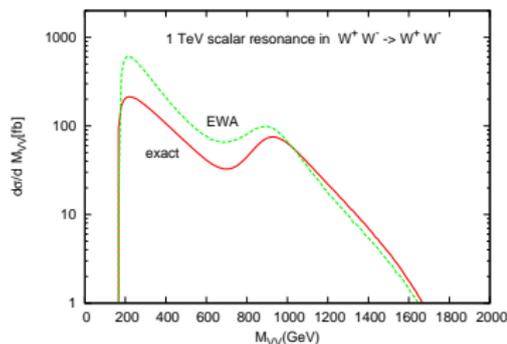
- ▶ Effective W approx. vs. WHIZARD full matrix elements
- ▶ Shapes/normalization of distributions heavily affected
- ▶ **EWA: Sideband subtraction completely screwed up!**

LHC Example: Vector Isovector

Alboteanu/Kilian/JR, 2008

- Example: 850 GeV vector resonance, coupling $g_\rho = 1$
- (Theory) Cuts:
  - $p_\perp(\ell\nu) > 30$ GeV
  - $|\delta R(\ell\nu)| < 1.5$
  - $\theta(u/d) > 0.5°$
- Integrated luminosity: 225 fb^{-1}
- Discriminator: angular correlations $\Delta\phi(\ell\ell)$
- Ongoing ATLAS study

 Kobel/JR/Schumacher

  - Cut analysis/NN
  - More kinematic observables
  - Geant4 FullSim (special points)
  - all resonances, parameter scans
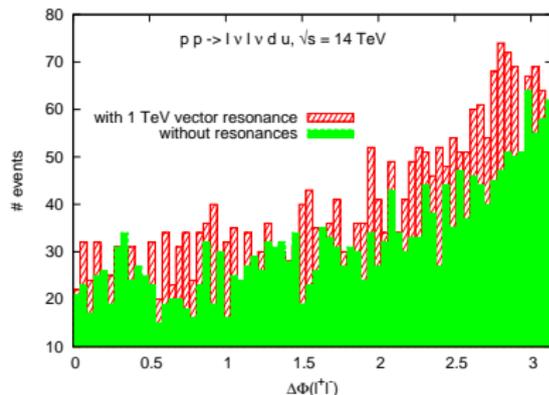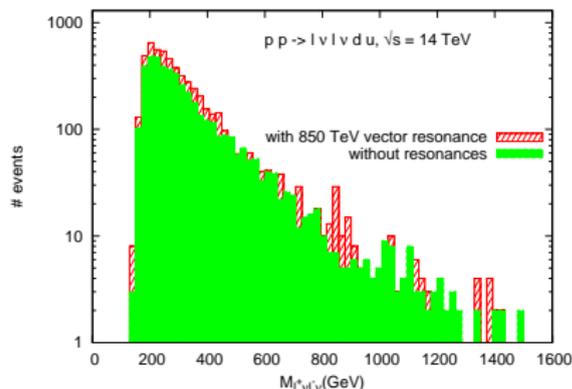
ILC Results: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4 + \alpha_6)$, $(\alpha_5 + \alpha_7)$, $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.

Simulation with WHIZARD   Kilian/Ohl/JR

1 TeV, $1\,\mathrm{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M_{WW}^2, M_{WZ}^2, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e_R^-, C) 80% e_R^-, 60% e_L^+

	WWZ			ZZZ	best
$16\pi^2 \times$	no pol.	e^- pol.	both pol.	no pol.	
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	−4.40	−3.34	−1.71	−3.53	−1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	−7.10	−6.40	−2.19	−3.53	−1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E_{\mathrm{mis}}^2 + p_{\perp,\mathrm{mis}}^2$

No angular correlations yet

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Polarization populates longitudinal modes, suppresses SM bkgd.
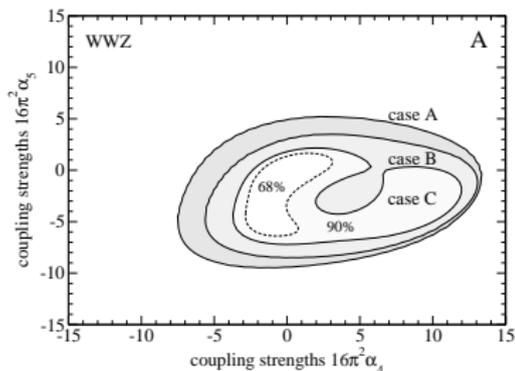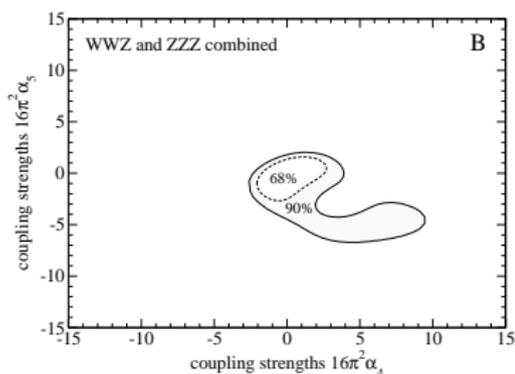
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Bkgd. $t\bar{t} \to 6$ jets

Veto against $E_{\text{mis}}^2 + p_{\perp,\text{mis}}^2$

No angular correlations yet

Vector Boson Scattering

$1\,\mathrm{TeV}$, $1\,\mathrm{ab}^{-1}$, full $6f$ final states, 80 % e_R^-, 60 % e_L^+ polarization, binned likelihood

Contributing channels: $WW \to WW, WW \to ZZ, WZ \to WZ, ZZ \to ZZ$

Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e \bar\nu_e q\bar q q\bar q$	$WW \to WW$	23.19
$e^+e^- \to \nu_e \bar\nu_e q\bar q q\bar q$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar\nu q\bar q q\bar q$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar q q\bar q$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar b X$	$e^+e^- \to t\bar t$	331.768
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar q$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar q$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar q$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

| coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55
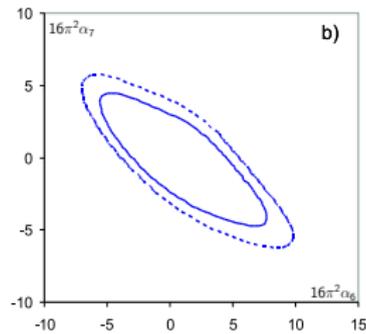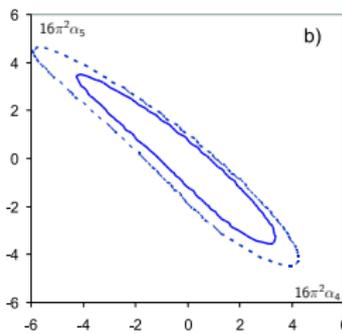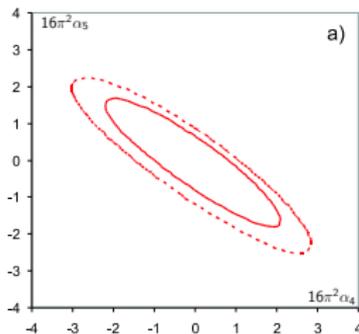
Interpretation as limits on resonances

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma / M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$
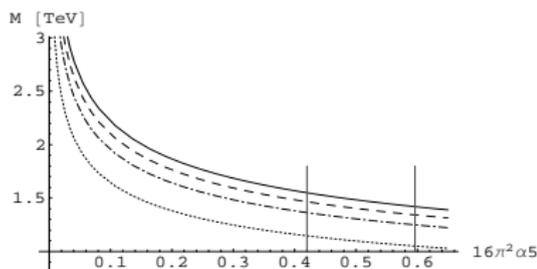

$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho\pm} = v \left(\frac{12\pi \alpha_4 f_{\rho\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2 (\alpha_4^\lambda)^2 /(2c_w^2)} \right)^{\frac{1}{4}}$$
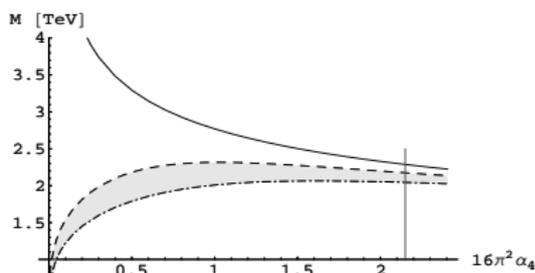
upper/lower limit from λ_Z, grey area: magnetic moments

Final result:

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.55	–	1.95
1	–	2.49	–
2	3.29	–	4.30

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Summary/Conclusions

▶ New Physics generically encoded in EW Chiral Lagrangian

▶ Triple/Quartic gauge couplings measured either

    – via triple boson production
    – via vector boson scattering

▶ interpreted as resonances coupled to EW bosons

▶ "Correct" description for first resonance (also [very] broad)

▶ Beyond that: assure unitarity (K matrix)

▶ Sensitivity rises with number of intermediate states:
    – LHC sensitivity limited in pure EW sector: $0.6 - 2\,\text{TeV}$
    – ILC   : $1.5 - 6\,\text{TeV}$

▶ Full analysis including all channels/backgrounds with WHIZARD

▶ Complete ATLAS study is under way

One Ring to Find them ... One Ring to Rule them Out

One Ring to Find them ... One Ring to Rule them Out