test

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Assume a non invertible matrix $\mathbf{M}$ defined as

$$\mathbf{M} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \tag{1}$$

and a truth distribution $\mathbf{T}$ defined as

$$\mathbf{T} = \begin{pmatrix} a \\ b \end{pmatrix}. \tag{2}$$

In building this matrix, the events that migrate to the neighbor bin are those "not matched". The folding equation $\mathbf{R} = \mathbf{MT}$ leads to a reco spectrum

$$\mathbf{R} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{2} \begin{pmatrix} a+b \\ a+b \end{pmatrix}. \tag{3}$$

Given the singularity of $\mathbf{M}$, we can't use the unfolding equation to retrieve $\mathbf{T}$ from $\mathbf{R}$.

The reco-level matching correction used in the ljets paper can be defined as a diagonal matrix $\mathbf{C}$ whose elements are built from the ratio of the "matched" reco events and the reco events:

$$\mathbf{C} = \begin{pmatrix} \frac{a}{a+b} & 0 \\ 0 & \frac{b}{a+b} \end{pmatrix}. \tag{4}$$

We also have to defined a "matching efficiency", that accounts for the events generated in bin i which are not matched. Given the simplicity of the problem, the definition is trivially

$$\epsilon = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \tag{5}$$

since we said that half of the events end up not matched.

With these conditions, the new migration matrix $\mathbf{M}^{'}$ becomes

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}, \tag{6}$$

which, magically, becomes invertible.

The job of the **C** correction is to rotate and rescale the data vector in a base where the matrix becomes not-singular, while ϵ is needed to rotate and scale back to the original space.

At this point, it is possible to retrieve **T** from **R** using the extended formula form the ljets analysis

$$\mathbf{T} = \epsilon^{-1}\left(M^{'}\right)^{-1}\mathbf{CR} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{a}{a+b} & 0 \\ 0 & \frac{b}{a+b} \end{pmatrix}\frac{1}{2}\begin{pmatrix} a+b \\ a+b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (7)$$

In this formula, the corrections ϵ, $\mathbf{M}^{'}$ and **C** are all evaluated via MC and all the systematic uncertainties are taken into account for each of them when evaluating the final uncertainties.

One could say that we could, a priory, remove the off-diagonal values in **M** and add by hand the **C** and ϵ corrections. But that is not what is done in the lepton+jets, since the changes from **M** to $\mathbf{M}^{'}$ are not arbitrarily applied to the matrix, but are the result of the modelling of the final state and the corresponding detector response.

This procedure on one hand increases the extrapolation (since we are further reducing the space in which we consider our events) but on the other hand seems to stabilise the unfolding procedure.