Q1

Assuming a simple isomagnetic ring with a circumference of C_0 = 300 ($r \approx 47.7\, m$) and an energy of 2 GeV, estimate the damping time.

Calculate power/electron: $P_g = \frac{cC_g}{2p}\frac{E^4}{r^2}$ = 29.7 GeV/s. The damping time is then given by $t = 2E / P_g = 134.8$ ms.

If we now split this ring into two halves, and insert a total of 50 m of wiggler (2´25 m), what is the peek magnetic field required to achieve a damping time of 5 ms (you may assume the wiggler field has a perfect sinusoidal variation long its length).

For $t = 5$ ms, we require a radiated power of $2E / t = 800$ GeV/s. The total 'circumference' of the ring is now 300+50=350 m. Thus we require an energy loss per turn of $800 \times 350 / c = 933.3$ kV. The energy loss in the arcs per turn is $29.7 \times 300 / c = 29.7$ kV: hence we require 933.3-29.7=903.6 kV from the wigglers. Energy loss from a length of wiggler is

$$\Delta E_w(\text{GeV}) = 1.27 \times 10^{-6} B^2(\text{T}) E^2(\text{GeV}) L(\text{m})\,.$$

Putting in the numbers, we arrive at B^2 = 3.56 T. Remembering that this is the average value of B^2 over the wiggler, the peak field is $\hat{B} = \sqrt{2B^2} \approx 2.7\,\text{T}$

For the original ring, let is define the fill factor f as the fraction of the circumference that is actually dipole magnet ($f = r_d / r = l_d / C_0$, where r_d and l_d are the dipole bending radius and the total dipole length respectively). Show that the damping time scales as $t \propto f$.

The radiated power can be written as $P_g = c\Delta E_{turn} / C_0$. Now $\Delta E_{turn} \propto L_d / r_d^2$ where L_d is the total length of dipole in the ring ($L_d = C_0 f$). Since both L_d and r_d scale as *f*, then $\Delta E_{turn} \propto 1/ f \propto P_g$: hence $t_D \propto P_g^{-1} \propto f$

For an arc fill factor of 0.1, re-calculate the peak wiggler field. What is the field for the arc dipole magnets.

In our original calculation, we must increase the energy lost in the arcs by a factor of 10 due to the fill factor. Hence the total energy lost per turn in the wigglers is now 933.3-10×29.7=636.3 kV. It follows that the peek wiggler field is 2.26 T. The arc dipole bending radius is 0.1×47.7 = 4.77 m. Using $Br \approx 3.34E(\text{GeV})$, we arrive at $B \approx 1.4$ T.

Q2

The NLC main linac parameters are:

RF frequency	*11.4 GHz*
structure length	*0.9 m*
fill time	*120 ns*
Q	*9000*
r/Q	*9 kW/m*
unloaded gradient	*65 MV/m*
particles per bunch	$0.75´10^{10}$
bunches per pulse	*192*
bunch spacing	*1.4 ns*

Assuming constant gradient structures, calculate

(a) *The attenuation constant* t

$$t_{\text{fill}} = \frac{2Qt}{w}$$

$$t = \frac{w t_{\text{fill}}}{2Q} \approx 0.48$$

(b) *The RF peak power required to maintain the unloaded gradient*

$$r_l = \left(\frac{r}{Q}\right) Q \approx 81\,\mathrm{M\Omega/m}$$

$$V_u = \sqrt{r_l L P_0 \left(1 - e^{-2t}\right)}$$

$$P_0 = \frac{G_u^2 L}{r_l \left(1 - e^{-2t}\right)} = 76\,\mathrm{MW}$$

(c) *The loaded gradient (assuming a steady state beam current)*

$$G_l = G_u - \frac{1}{2} r_l i_{beam} \left(1 - \frac{2t\, e^{-2t}}{1 - e^{-2t}}\right)$$

$$i_{beam} = q n_b / \Delta t_b = 0.86\,\mathrm{A}$$

$$G_l \approx 50.9\mathrm{MV/m}$$

(d) *The optimal current (100% beam loading)*

$$i_{\text{opt}} = \sqrt{\frac{P_0}{r_l L}} \left(\frac{(1 - e^{-2t})^{3/2}}{1 - (1 + 2t)e^{-2t}} \right)$$

$$\approx 1.98\,\mathrm{A}$$

(e) The effective RF-to-beam power transfer efficiency, including the fill time.

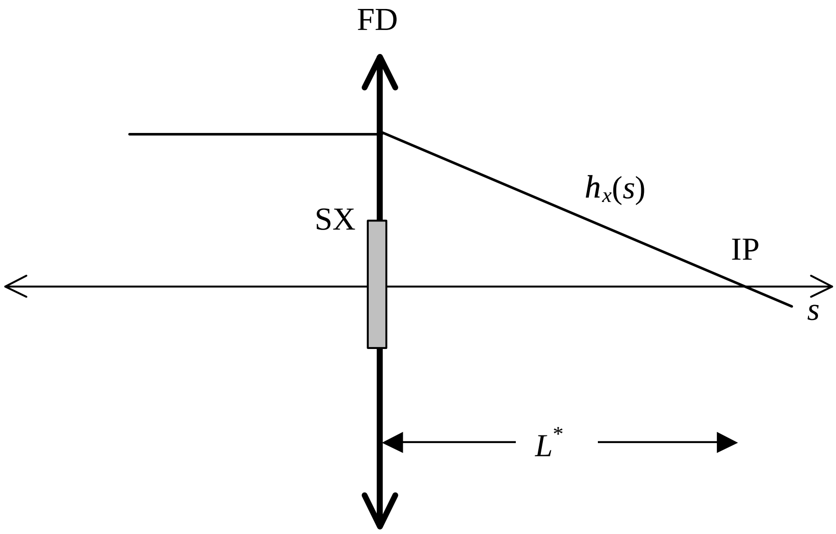
$$\eta = \left(\frac{V i_{\text{beam}}}{P_0}\right)\left(\frac{t_{\text{beam}}}{t_{\text{beam}} + t_{\text{fill}}}\right) = 36\%$$

Why do we run the linac at a smaller current than the optimal?

At the optimum current (100% beam loading) there is a one-to-one correspondence between current fluctuation and voltage (1% in current = 1% in voltage). This makes the linacs sensitive to charge fluctuations. Typical loading values of <50% make the voltage less sensitive to current variations.

Q4

Consider the following simple final focus (final doublet) system:



where FD and SX are a thin-lens quadrupole and sextupole respectively. If the angular dispersion at the IP is η'^*, show that the system is chromatically corrected when the sextupole strength is

$$S = -\frac{1}{L^{*2}\eta'^*}$$

answer:

$$\Delta y'_Q = -\frac{y\delta}{L^*}$$

$$\Delta y'_{SX} = S(x + \eta\delta)y$$

We want to cancel the $y'\delta$ term. Hence

$$Sh - \frac{1}{L^*} = 0$$

$$S = \frac{1}{L^* h}$$

$$= \frac{1}{L^{*2} h'^*}$$

Assuming that SX is now adjusted to compensate the second-order dispersion (d^2) term in the horizontal plane, show that relative chromatic aberration in the vertical plane is

$$\frac{\Delta y^*_{\mathrm{RMS}}}{s^*_y} \approx \frac{L^* d_{\mathrm{RMS}}}{b^*_y}$$

In the horizontal plane:

$$\Delta x'_Q = -\frac{(x + hd)d}{L^*}$$

$$\Delta x'_{SX} = -\frac{1}{2} S (x + hd)^2$$

To cancel the d^2 we need

$$+\frac{h}{L^*} - \frac{Sh^2}{2} = 0$$

$$S = \frac{2}{hL^*}$$

which is twice the value needed to compensate the chromaticity. Hence we will have the same magnitude chromaticity. Thus we have the same uncorrected FD chromaticity but with the opposite sign:

$$\Delta y' = +\frac{yd}{L^*}$$

$$= -y'd$$

$$\Delta y^* = -L^* y'd$$

$$\left\langle \Delta y^{*2} \right\rangle = \Delta y^{*2}_{\mathrm{RMS}} = L^{*2} q^{*2}_y d^2_{\mathrm{RMS}}$$

$$\frac{\Delta y^{*2}_{\mathrm{RMS}}}{s^{*2}_y} = \frac{L^{*2} q^{*2}_y d^2_{\mathrm{RMS}}}{b^*_y e_y} = \frac{L^{*2} d^2_{\mathrm{RMS}}}{b^{*2}_y}$$

$$\frac{\Delta y^*_{\mathrm{RMS}}}{s^*_y} = \frac{L^* d_{\mathrm{RMS}}}{b^*_y}$$

How are the pure geometric terms generated by SX compensated?

By placing a second sextupole upstream of the first at a location where the transport matrix between the two is

$$R = \begin{pmatrix} m & & & \\ & 1/m & & \\ & & m & \\ & & & 1/m \end{pmatrix}$$

A damping ring with an energy of 3 GeV produces a longitudinal emittance of 6mm ´ 0.1%. Calculate the required RF voltages and R_{56} values for a two stage compressor system (two 90° phase space rotations using wigglers) to compress the bunch to 100 mm with an energy spread of 2%. Take 1 GHz and 8 GHz RF for the first and second stage respectively. Assume that the beam is ideally accelerated between the stages. Assume compression ratios of 10 and 6 for the two stages respectively.

First stage:

$$V_{RF} = \frac{E}{k_{RF}}\left(\frac{d_u}{s_{z,0}}\right)\sqrt{F^2 - 1}$$
$$F = 10$$
$$V_{RF} \approx 238\,\mathrm{MV}$$

$$R_{56} = \frac{k_{RF}V_{RF}}{E}\left(\frac{s_{z,0}}{d_u}\right)^2 \frac{1}{F^2}$$
$$R_{56} \approx 0.6\,\mathrm{m}$$

Second stage:

Energy of the second stage is obtained from

$$d_f = F_T \frac{E_0}{E_0 + \Delta E} d_i$$

Hence

$$F_T = 60$$
$$E_0 = 3\text{ GeV}$$
$$d_i = 0.1\%$$
$$d_f = 2\%$$
$$\Delta E = 6\text{ GeV}$$

Thus the energy of the second compressor is 9 GeV. Our initial values for the second compressor stage are therefore:

$$\begin{aligned} E &= 9\text{GeV} \\ s_z &= 0.6\text{mm} \\ d_u &= 2\%/6 = 0.33\% \\ F &= 6 \\ f_{\text{RF}} &= 8\text{GHz} \end{aligned}$$

$$\begin{aligned} V_{RF} &\approx 1.8\text{GV} \\ R_{56} &\approx 3\text{cm} \end{aligned}$$

Q6

As a tolerance, we normally take an allowed beam offset at the IP of $s_y^*/3$ *(this corresponds to a 2% loss of geometric luminosity). Show that the vibration tolerance on a single quadrupole is given by*

$$\Delta y_{quad} \le \frac{1}{3}\left(\frac{ge_y}{g_Q b_Q}\right)^{\frac{1}{2}} \frac{1}{\sin(f_Q)K}$$

where ge_y *is the normalised vertical emittance,* b_Q *is the beta function at the quadrupole, K the strength of the quadrupole (=* f^{-1}*),* f_Q *is the quadrupole phase relative to the IP, and* $g_Q = E_Q/m_0c^2$ *is the relativistic factor at the quad.*

The kick from the quadrupole is $\Delta y' = K\Delta y_{quad}$. From basic optics we know that

$$\Delta y^* = R_{34}\Delta y' = K\Delta y_{quad}\sqrt{\frac{g_Q}{g_{IP}}}\sqrt{b_Q b_y^*}\sin(f_Q)$$

Diving both sides by the nominal IP beam size $s_y^* = \sqrt{b_y^*(ge_y)/g_{IP}}$, we obtain

$$\frac{\Delta y^*}{s_y^*} = K\Delta y_{quad}\sqrt{g_Q}\sqrt{\frac{b_Q}{ge_y}}\sin(f_Q) \le \frac{1}{3}$$

Rearranging yields the required inequality.

For the special case of the thin-lens FD of question 4, show that the tolerances reduces to

$$\Delta y_{FD} \le \frac{s_y^*}{3}$$

The beta function at the FD is approximately given by

$$b_{y,FD} = b_y^* + L^{*2} / b_y^* \approx L^{*2} / b_y^*$$

Setting $f_{FD} = p / 2$ and remembering that $K_{FD} = 1/L^*$, we arrive at

$$\Delta y_{quad} \leq \frac{1}{3}\left(\frac{ge_y b_y^*}{g_{IP} L^{*2}}\right)^{\frac{1}{2}} L^* = \frac{1}{3}\left(\frac{ge_y b_y^*}{g_{IP}}\right)^{\frac{1}{2}} = \frac{s_y^*}{3}$$