A Short Introduction to Bunch Compressors for Linear Colliders

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1 Introduction

The length of a bunch extracted from a damping ring is generally a few millimeters, while the main linacs and final focus systems require bunches in the range approximately 100 µm (NLC/JLC) to 300 µm (TESLA). Collective effects in storage rings prevent the damping rings operating in a regime where the required bunch lengths could be achieved directly, so an additional system – a bunch compressor – is required between the damping ring and the main linac.

The damping rings are expected to provide highly stable output, in terms of transverse bunch dimensions, bunch length, energy spread etc. However, there are effects that lead to undesirable features in the extracted beam. For example, beam loading in the RF cavities will result in phase variation along the bunch train (this is inherent in the NLC design – whether this is an issue for TESLA depends on whether or not they implement ion clearing gaps, and how these gaps are structured). The bunch compressors should be able to compensate for such effects, and not introduce any additional instability, thus producing a properly matched, highly stable beam with the desired parameters for the main linacs.

The *absolute* energy spread of a bunch is preserved as a bunch is accelerated in a linac. Thus, as the energy increases, the *fractional* energy spread decreases in proportion; this is just the adiabatic damping of the energy spread. However, the bunch length is preserved during acceleration. Furthermore, Liouville's theorem forbids any change in the longitudinal emittance in the absence of any non-conservative processes. In other words, damping of longitudinal phase space requires either a damping ring (which we have already used) or a linac (which has no effect on the bunch length). How then do we achieve a compression of the bunch length? The answer is that we *rotate* the longitudinal phase space, preserving the longitudinal emittance, but reducing the bunch length at the expense of the energy spread. We show how this is achieved and consider some of the technical details in the following sections.

2 Linear Theory

2.1 Phase Space Rotation Through π/2

For the moment, we consider only the longitudinal phase space. Let us consider the effect of passing a bunch through an RF cavity on the zero crossing of the voltage (i.e. without acceleration), followed by transporting the bunch around an arc. The RF cavity has no effect on the longitudinal position of a particle with respect to the bunch center, but changes the energy deviation by an amount depending on its position:

$$z_1 = z_0$$
$$\delta_1 = \delta_0 + \frac{eV_{RF}}{E_0}\cos\left(\frac{\pi}{2} - k_{RF} z_0\right)$$

where $k_{RF} = 2\pi f_{RF}/c$. Note that a *positive* z means that the particle is *ahead* of the reference particle that has $z = 0$, and that the RF voltage is cosine-like. More generally, we can adjust the RF phase so that the reference particle crosses at some phase ϕ_{RF} that may or may not be the zero crossing. In this case, we need to account for the change in reference energy of the beam (from E_0 to E_1). We can write for the initial and final energies of a given particle:

$$E_i = E_0(1+\delta_0)$$
$$E_f = E_1(1+\delta_1) = E_i + eV_{RF}\cos(\phi_{RF} - k_{RF} z_0)$$
$$E_1 = E_0 + eV_{RF}\cos(\phi_{RF})$$

After a little algebra, we find:

$$\delta_1 = \frac{E_0(1+\delta_0) + eV_{RF}\cos(\phi_{RF} - k_{RF} z_0)}{E_0 + eV_{RF}\cos(\phi_{RF})} - 1$$

To first order in $eV_{RF}/E_0 << 1$, the full map may be written:

$$z_1 = z_0$$
$$\delta_1 = \delta_0\left(1 - \frac{eV_{RF}}{E_0}\cos(\phi_{RF})\right) + \frac{eV_{RF}}{E_0}[\cos(\phi_{RF} - k_{RF} z_0) - \cos(\phi_{RF})]$$

Taking a linear approximation for the RF, we can write the map in matrix form:

$$\begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix} \approx \begin{pmatrix} 1 & 0 \\ R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}$$

where

$$R_{65} = \frac{eV_{RF}}{E_0}\sin(\phi_{RF})k_{RF}$$

$$R_{66} = 1 - \frac{eV_{RF}}{E_0}\cos(\phi_{RF})$$

The arc has the opposite effect to the RF: it leaves the energy deviation of each particle unchanged (to the extent we can ignore synchrotron radiation), but the path length depends on the energy:

$$z_2 = z_1 + R_{56}\delta_1 + T_{566}\delta_1^2 + U_{5666}\delta_1^3 \dots$$
$$\delta_2 = \delta_1$$

Again making a linear approximation ($T_{566}\delta_1 << R_{56}$), we can write:

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \begin{pmatrix} 1 & R_{56} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix}$$

The total transformation is given by:

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} \approx \mathbf{M} \cdot \begin{pmatrix} z_0 \\ \delta_0 \end{pmatrix}$$
$$\mathbf{M} = \begin{pmatrix} 1 + R_{65}R_{56} & R_{56}R_{66} \\ R_{65} & R_{66} \end{pmatrix}$$

Note that for $\phi_{RF} = \pm\frac{\pi}{2}$ (i.e. no acceleration), R_{66} = 1, and the transformation matrix is symplectic, which means we can introduce the longitudinal emittance as a conserved quantity:

$$\varepsilon = \sqrt{\sigma_z^2\sigma_\delta^2 - \sigma_{z\delta}^2} \tag{1}$$

and longitudinal Twiss parameters:

$$\sigma_z^2 = \langle z^2 \rangle = \beta\varepsilon$$
$$\sigma_\delta^2 = \langle \delta^2 \rangle = \gamma\varepsilon$$
$$\sigma_{z\delta} = \langle z\delta \rangle = \alpha\varepsilon$$

where $\beta\gamma - \alpha^2 = 1$, and we expect $\alpha_0 = 0$ for the beam from the damping ring (we use the subscript zero to denote the initial value).

In the more general case where we run the beam through the RF away from the zero-crossing, $R_{66} \neq 1$, and there is some damping (or antidamping) of the longitudinal phase space, associated with the acceleration (or deceleration). The RF phase may be chosen to be other than the zero crossing to compensate the effect of the nonlinear phase slip – this is discussed in more detail below. For the present, we consider only the symplectic case, with R_{66} = 1, and no acceleration.

The effect of the symplectic transformation on the longitudinal Twiss parameters may be written:

$$\mathbf{A}_1 = \mathbf{M} \cdot \mathbf{A}_0 \cdot \mathbf{M}^{\mathrm{T}} \tag{2}$$

where

$$\mathbf{A} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

We find that the final value of β is given by:

$$\beta_1 = \beta_0 \left(1 + R_{56} R_{65}\right)^2 - 2\alpha_0 R_{56} \left(1 + R_{56} R_{65}\right) + \gamma_0 R_{56}^2$$

Note that if we take $R_{56} \to 0$ while at the same time $R_{56} R_{65} \to -1$, then $\beta_1 \to 0$. In other words, if we are willing to provide an infinite RF voltage and tolerate an infinite incoherent energy spread, there is no limit to how short we can make the bunch. In practice, the RF voltage is limited and there is also some limit to the incoherent energy spread that can be allowed, so we have some given value for R_{65}. In this case, the minimum value for β_1 is given by:

$$\breve{\beta}_1 = \frac{1}{\gamma_0 - 2R_{65}\alpha_0 + R_{65}^2 \beta_0} \tag{3}$$

which is achieved with

$$R_{56} = \breve{\beta}_1 \left(\alpha_0 - R_{65} \beta_0\right) \tag{4}$$

Note that the phase of the RF is determined by the method of generating the R_{56}. Using the convention that particles at the head of the bunch have $z > 0$ (as in the codes MAD and Merlin), an arc has $R_{56} < 0$, in which case we need $R_{65} > 0$, and thus $\phi_{RF} = \pi/2$. On the other hand, a chicane has $R_{56} > 0$, thus $R_{65} < 0$ and $\phi_{RF} = -\pi/2$. If the bunch compressor uses an arc to generate the R_{56}, particles at the *head* of the bunch are *increased* in energy, so in the arc they take a longer path (the bending radius is larger), and thus move towards the back of the bunch. Conversely, particles at the back of the bunch are reduced in energy, take a shorter path through the arc, and move towards the head of the bunch. The opposite arguments apply if a chicane is used instead of an arc. It is also worth remembering that some codes (Elegant, Parmela) use the convention that $z < 0$ for particles at the head of the bunch, so arcs have $R_{56} > 0$ and chicanes have $R_{56} < 0$. Confused? The rule is: before doing a calculation, declare up front what convention you are using, and stick to it for that calculation. This might not solve your confusion, but it gives other people a fighting chance of understanding your work.

In the special case $\alpha_0 = 0$, the minimum compression ratio, defined as the ratio of the bunch length after compression to the bunch length before compression, is given by:

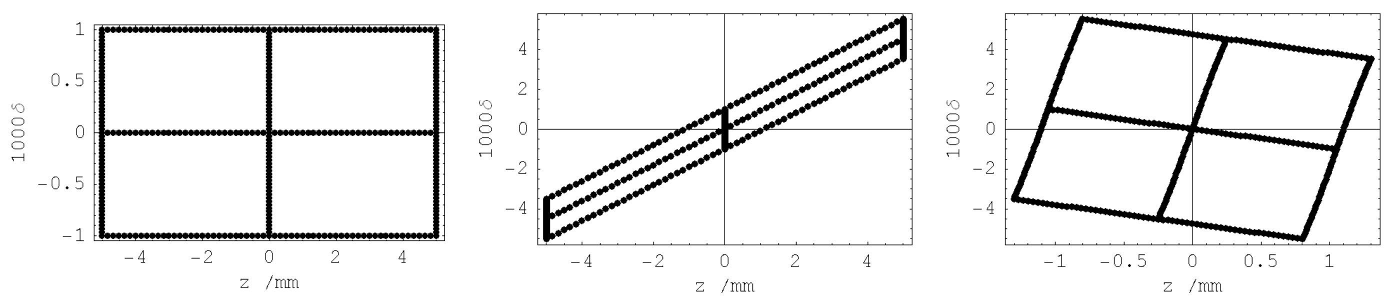
$$\left(\frac{\sigma_{zf}}{\sigma_{zi}}\right)_{\min} = \frac{1}{\sqrt{1 + a^2}} \qquad a = \frac{\sigma_{zi}}{\sigma_{\delta i}} R_{65} \tag{5}$$

and this is achieved with a phase slip given by:

$$R_{56} = -\frac{a^2}{1+a^2} \cdot \frac{1}{R_{65}} \tag{6}$$

Note that this expression for the required phase slip is modified if the RF accelerates or decelerates the beam (i.e. if $R_{66} \neq 1$).

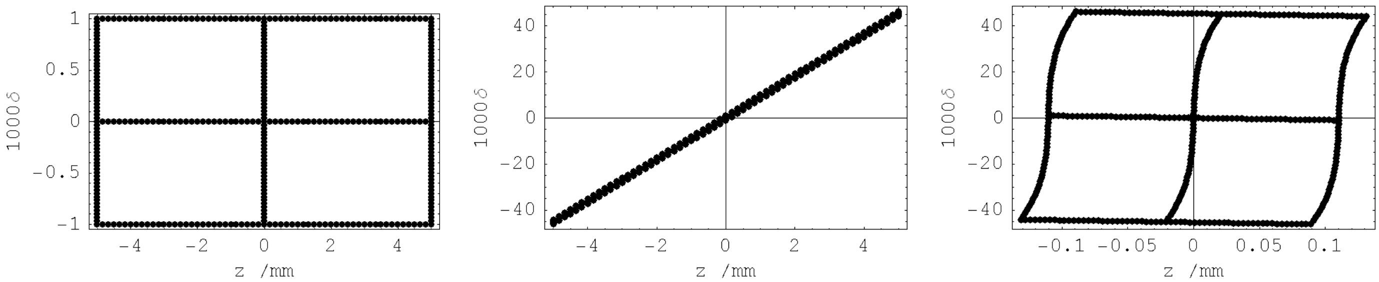
Examples of the transformation of longitudinal phase space in a bunch compressor are shown in Figure 1 and Figure 2. We have set up a bunch of particles appearing as a "window frame" in longitudinal phase space to illustrate more clearly the effect of the RF and the arc. In reality, a gaussian bunch distribution is expected. The parameters for the beam extracted from the damping ring are appropriate for the NLC. The specification for the beam injected into the NLC main linac is a bunch length of 110 µm. In each example, the value of R_{56} is chosen so as to optimize the bunch compression, and T_{566} is assumed to be zero. In the example shown in Figure 2, this leads to a phase space rotation of close to $\pi/2$, but note the distortion of the phase space that comes from the RF curvature (we have used the nonlinear RF map rather than the linear approximation). Note also that in this case (no initial or final $\delta - z$ correlation or acceleration) the product of the bunch length and energy spread is conserved. More generally (including $\delta - z$ correlation but no acceleration), the longitudinal emittance defined by (1) is conserved. More generally still (including $\delta - z$ correlation and acceleration), the *normalized* longitudinal emittance obtained by multiplying the longitudinal emittance (1) by the energy, is conserved.



(a) Phase space at entrance (b) After 60 MV RF (c) After RF and arc, R_{56} = -1.05 m

Figure 1

Transformation of longitudinal phase space at different stages in a bunch compressor. Note the different scales for each plot. The RF voltage is 60 MV, the frequency is 1.428 GHz, and the phase is $\pi/2$. The beam energy is 1.98 GeV, and R_{56} = -1.05 m. The bunch length is reduced from 5 mm to 1.1 mm.



(a) Phase space at entrance (b) After 600 MV RF (c) After RF and arc, $R_{56} = $-0.110 m

Figure 2

Transformation of longitudinal phase space as shown in Figure 1, but with different parameters for the compressor. The RF voltage is 600 MV, the frequency is 1.428 GHz, and the phase is $\pi/2$. The beam energy is 1.98 GeV, and $R_{56} = $-0.110 m. The bunch length is reduced from 5 mm to 0.11 mm.

To achieve the specified compression in a single stage bunch compressor for the NLC, we have needed an RF voltage of 600 MV, and the energy spread increases to about 4.5%. This is a large energy spread to consider transporting efficiently through any beam line (i.e. without significant particle loss or degradation of beam quality). In particular, the larger the energy spread, then the greater the sensitivity to the nonlinear phase slip terms T_{566}, U_{5666} etc.

As an alternative to the single stage compression used above, a two-stage compressor can be used. The compressors are separated by a linac that accelerates the beam, thus reducing the energy spread at an intermediate stage through adiabatic damping. This has the advantage that the energy spread never gets beyond an acceptable level, and is the option that has currently been adopted by the NLC. The second compressor is actually a telescope in longitudinal phase space, using the principles that we now describe.

2.2 Phase Space Rotation Through nπ

Let us consider a beamline comprising an RF section followed by some bending, then a second RF section followed by more bending. In each case, the bending can be provided either by an arc or a chicane. It is straightforward to show that the total transfer matrix for such a system is given by:

$$\mathbf{M} = \begin{pmatrix} 1 + R_{65}^{(2)} R_{56}^{(2)} + R_{65}^{(1)} \left(R_{65}^{(2)} R_{56}^{(1)} R_{56}^{(2)} + R_{56}^{(1)} + R_{56}^{(2)} \right) & R_{65}^{(2)} R_{56}^{(1)} R_{56}^{(2)} + R_{56}^{(1)} + R_{56}^{(2)} \\ R_{65}^{(1)} + R_{65}^{(2)} + R_{65}^{(1)} R_{65}^{(2)} R_{56}^{(1)} & \cdots \end{pmatrix}$$

M_{22} is readily found by the matrix multiplication or the symplectic condition; we shall not need an explicit expression for our purposes. We like to have the final phase independent of the initial energy so that an initial energy error does not convert to a phase error on entry to the main linac. To achieve this we set M_{12} to zero. It would also be nice for the final energy to be independent of the initial phase, which would require M_{21} also to be zero (but since the initial phase errors are expected to be small, and energy errors are adiabatically damped in the main linac, this is not essential). Finally, for bunch compression by a factor m, M_{11} must be equal to $\pm 1/m$. Putting the requirements together, we have:

$$1 + R_{65}^{(2)} R_{56}^{(2)} = \pm 1/m \qquad R_{56}^{(1)} = \mp m R_{56}^{(2)}$$

and the transfer matrix becomes:

$$\mathbf{M} = \begin{pmatrix} \pm 1/m & 0 \\ R_{65}^{(1)} + R_{65}^{(2)} + R_{65}^{(1)} R_{65}^{(2)} R_{56}^{(1)} & \pm m \end{pmatrix}$$

This gives a rotation in phase space of either π (lower signs) or 2π (upper signs). If this is not immediately obvious, it is only necessary to decompose **M** using the normalizing transformation in terms of the longitudinal Twiss parameters:

$$\mathbf{M} = \begin{pmatrix} \dfrac{1}{\sqrt{\beta_f}} & 0 \\ \dfrac{\alpha_f}{\sqrt{\beta_f}} & \sqrt{\beta_f} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ \sin(\varphi) & \cos(\varphi) \end{pmatrix} \cdot \begin{pmatrix} \dfrac{1}{\sqrt{\beta_i}} & 0 \\ \dfrac{\alpha_i}{\sqrt{\beta_i}} & \sqrt{\beta_i} \end{pmatrix}$$

If we assume that there is no initial correlation between phase and energy, then $\alpha_i = 0$, $\varphi = \pi$ or $\varphi = 2\pi$, and $\sqrt{\beta_f / \beta_i} = 1/m$. In general, the beam at the end of the compressor has some correlation between phase and energy, which can be found from the value of α_f, but it is easy to choose an appropriate value for $R_{65}^{(1)}$ to eliminate this correlation.

By analogy with light optics, a system performing a phase space rotation through $n\pi$ (with n an integer) is referred to as a telescope. The advantage of this system compared with the $\pi/2$ rotation is that initial energy errors become final energy errors, as opposed to final phase errors. As we mentioned above, the NLC uses a telescope in longitudinal phase space as the second (high energy) bunch compressor. A schematic is shown in Figure 3. The $R_{65}^{(1)}$ is provided by accelerating the beam off-crest in the pre-linac, and the $R_{56}^{(1)}$ by the 180° arc that is suggested by the overall layout of the machine. A further RF section (200 m S-band) and a chicane provide the remaining transformations. Since $R_{56}^{(1)}$ is provided by and arc and $R_{56}^{(2)}$ by a chicane, these quantities have opposite signs, and the overall transformation is a rotation through 2π rather than π.

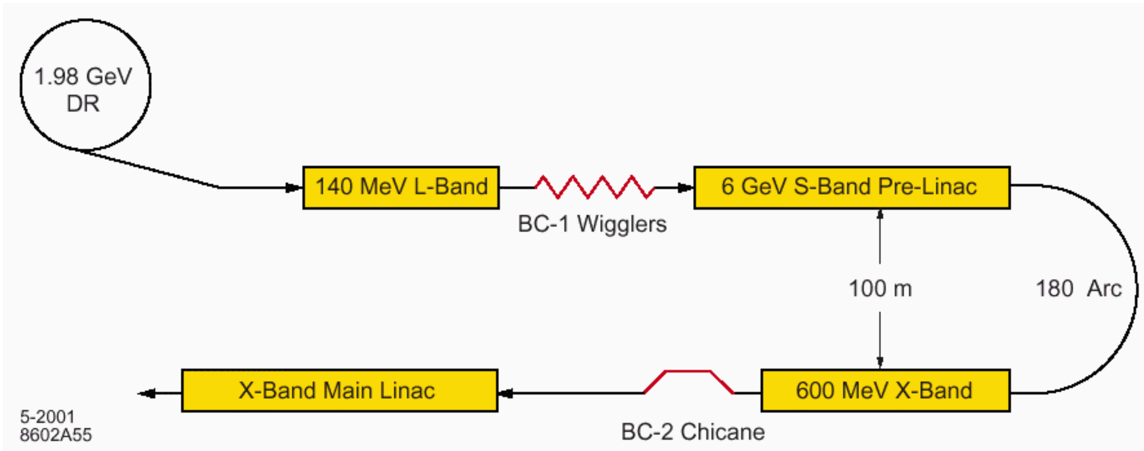


Figure 3

Schematic of the NLC bunch compressor system (from the NLC 2001 Report, SLAC-R-571)

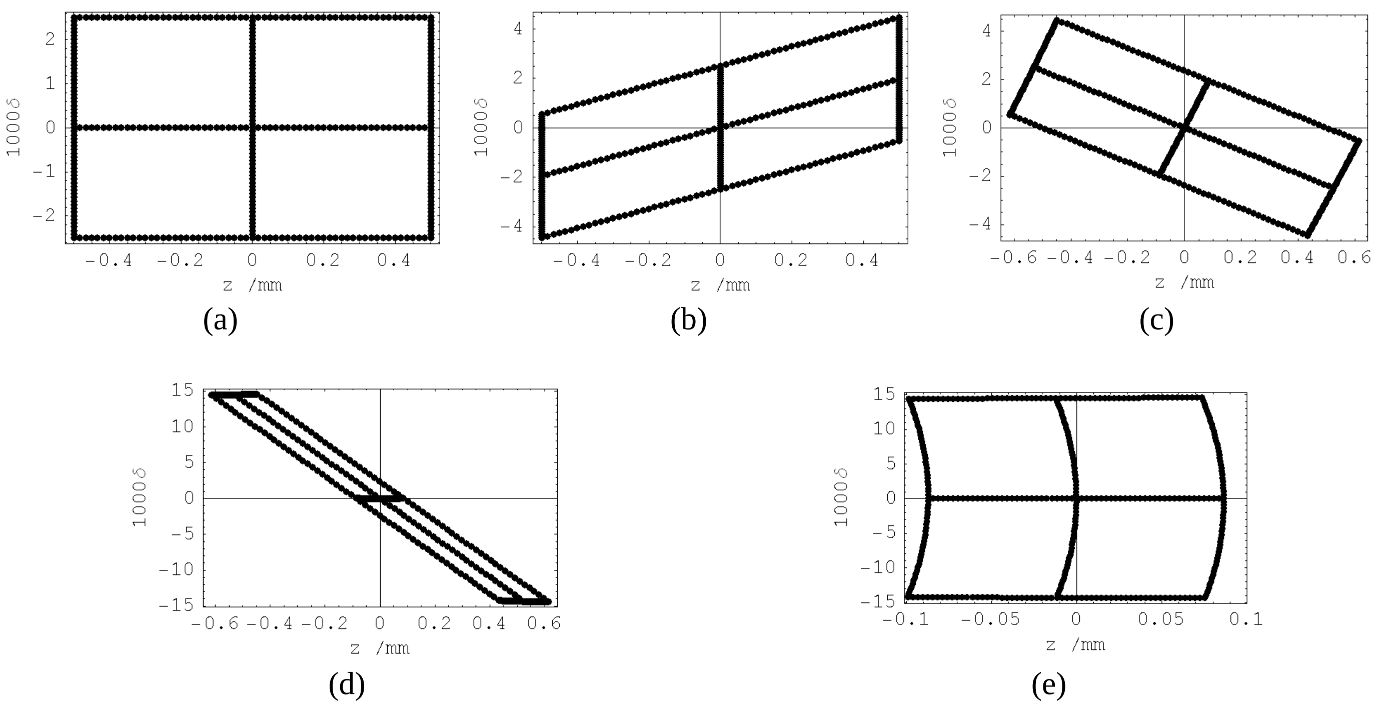


Figure 4

Transformations of longitudinal phase space in the NLC high-energy bunch compressor. (a) Initial phase space, bunch length 500 μm and energy spread 0.25%. (b) After initial RF, $R_{65}^{(1)} \approx 3.9\ \mathrm{m}^{-1}$. (c) After 180º arc, $R_{56}^{(1)} \approx -0.21$ m. (d) After second RF section, $R_{65}^{(2)} \approx -22.6\ \mathrm{m}^{-1}$. (e) After final chicane, $R_{56}^{(1)} \approx -0.037$ m. We include the effects of RF curvature and second order phase slip, but no compensation for the nonlinear phase slip has been applied.

Transformations of the longitudinal phase space through the NLC high-energy bunch compressor are shown in Figure 4. Further details of the NLC bunch compressors are provided in the NLC ZDR.

3 Compensation of Nonlinear Phase Slip

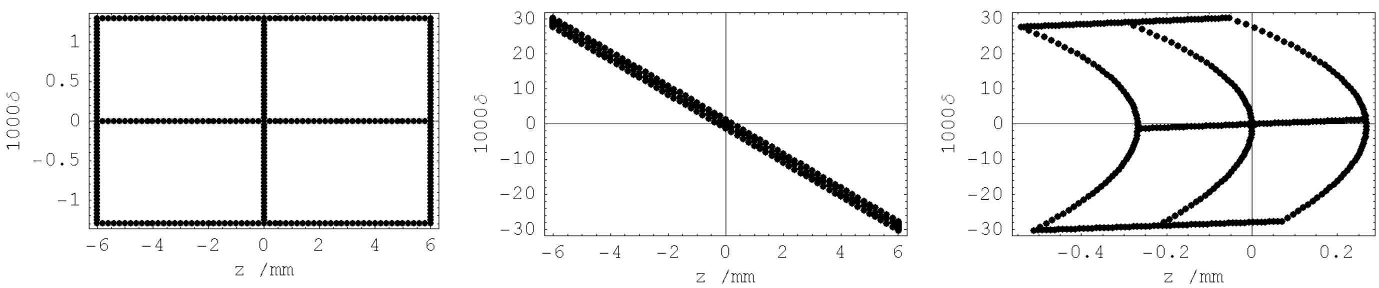
Although we included the effects of the RF curvature in the previous section, we assumed that the phase slip was linear, i.e. we set $T_{566} = 0$. In reality, higher order phase slip terms are intrinsic to dipoles, and can be estimated using some simple approximations. For an arc, we find:

$$T_{566} \approx 1.9 R_{56}$$

and for a chicane (see Problem 2):

$$T_{566} \approx -\frac{3}{2} R_{56}$$

Let us consider the case of the TESLA bunch compressor, which uses a chicane to generate the R_{56}. Including the nonlinear phase slip, we observe the transformations of the longitudinal phase space shown in Figure 5.



(a) Phase space at entrance (b) After RF (c) After RF and chicane

Figure 5

Transformation of longitudinal phase space in the TESLA bunch compressor, without compensation of the nonlinear phase slip. The beam energy is 5.0 GeV. The RF frequency is 1.3 GHz, and the phase is $-\pi/2$.

Clearly, the nonlinear phase slip introduces a strong correlation between z and δ^2. This spoils the phase space distribution of the beam, and is an undesirable effect that we wish to eliminate somehow. Since the phase space is rotated by nearly $\pi/2$, we can consider compensating the distortion by introducing a canceling correlation between δ and z^2 at the start of the compression. In fact, we can do this in the RF section by adjusting the phase of the RF seen by the bunch. In effect, we use a nonlinear term in the RF map to cancel a nonlinear term in the chicane map. The appropriate phase angle is given by:

$$\cos(\phi_{RF}) = \frac{\sqrt{1+8(1+2r)r\theta^2} - 1}{2(1+2r)\theta} \approx 2\theta r \qquad (7)$$

where

$$\theta = \frac{eV_{RF}}{E_0} \qquad r = \frac{T_{566}}{R_{56}}$$

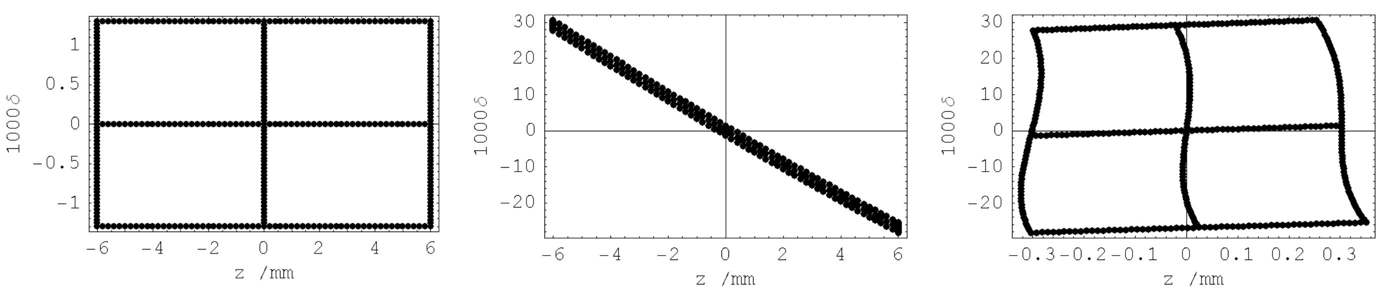
and the final approximation in (7) is valid for $8(1+2r)r\theta^2 << 1$. Note that r is fixed by the system used to provide the phase slip, and has the approximate value 1.9 for an arc, or –1.5 for a wiggler or chicane.

Since the RF phase is now moved off the zero crossing, the RF now accelerates (or decelerates) the beam. The value of R_{65} is fixed by the required compression ratio (5) (in the symplectic approximation), and one first calculates the required RF voltage with the assumption that the phase is set to the zero crossing. The phase adjustment is then calculated from (7). Depending on the accuracy required (and the size of the phase adjustment) it may be necessary to iterate the calculation, since the change of phase changes the compression ratio.

Note that the value of R_{56} required for optimum compression is now given by:

$$R_{56} = -\frac{a^2}{1+a^2} \cdot \frac{1}{R_{66}R_{65}}$$

For the TESLA bunch compressor with optimum values for the RF phase and voltage, we see the transformations of phase space shown in Figure 6. The effect of the curvature introduced by the RF to compensate the nonlinear phase slip is not obvious until after the bunch has passed through the chicane.



(a) Phase space at entrance (b) After RF with phase adjustment (c) After RF and chicane

Figure 6

Transformation of longitudinal phase space in the TESLA bunch compressor, including compensation of the nonlinear phase slip by adjustment of the RF phase.

Some basic parameters for the TESLA and NLC Low Energy bunch compressors are given in Table 1.

Table 1

Some basic parameters for the TESLA and NLC Low Energy bunch compressors.

	TESLA	NLC Low Energy
Beam Energy /GeV	5.0	1.98
Injected σ_δ	1.3×10^{-3}	1.0×10^{-3}
Injected σ_z /mm	6.0	5.0
Extracted σ_z /mm	0.3	0.5
RF frequency /GHz	1.3	1.428

4 Effects of Synchrotron Radiation

Generating the phase slip for bunch compression requires bending the beam in an arc, a chicane, or a wiggler. Whenever relativistic charged particles pass through a magnetic field, synchrotron radiation is produced. In a storage ring, the combined effect of the synchrotron radiation with the energy restoration by the RF cavities leads to radiation damping, with the equilibrium emittances determined by the damping rate and the quantum excitation. Although we do not have the same damping mechanism, the synchrotron radiation in the dipoles or wiggler of a bunch compressor does lead to quantum excitation, and a poorly designed bunch compressor risks growth of the transverse and longitudinal emittances that will significantly degrade the luminosity of the collider.

Synchrotron radiation can be incoherent or coherent. Incoherent synchrotron radiation, like that produced by third generation light sources, is the result of individual electrons randomly emitting photons. The power of the radiation is proportional to the square of the charge of an individual electron, and rises linearly with the number of electrons in the bunch. Coherent synchrotron radiation (CSR) is produced when a group of electrons collectively emit photons in phase. This can occur when the bunch length is shorter than the radiation wavelength, and the bunch effectively behaves as a single charged particle emitting photons. The radiation power rises as the square of the charge of the particle; in other words, the power rises as the square of the number of particles. Since there are typically of the order 10^{10} particles per bunch in a linear collider, CSR can potentially lead to very large amounts of radiated energy, with consequently significant increases in the transverse and longitudinal emittances. Since the bunch length is an important parameter, CSR is more likely to be a problem in the bunch compressors than the damping rings (but should not be forgotten about entirely in the damping rings).

4.1 Incoherent Synchrotron Radiation

4.1.1 Transverse Emittance Growth

Recall that when a particle emits a photon of energy *u*, the change in the betatron action is given by:

$$\Delta J = \frac{1}{2}\left(\frac{u}{E_0}\right)^2 \mathcal{H}^2$$

where the $\mathcal{H}$ -function is defined as usual in terms of the dispersion and transverse Twiss parameters:

$$\mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2$$

Using expressions for the second moment of the emitted photon energy, it can then be shown that the emittance growth from (incoherent) synchrotron radiation in any beamline is given by:

$$\Delta(\gamma\varepsilon) = \tfrac{2}{3} C_q r_e \gamma^6 I_5 \qquad (8)$$

where $r_e \approx 2.818\times10^{-15}$ m is the classical electron radius, and I_5 is the fifth synchrotron radiation integral. Note that for the normalized emittance, this goes as the sixth power of the energy, and is proportional to the fifth synchrotron radiation integral.

4.1.2 Longitudinal Effects

Assuming that the energy loss is small compared to the particle energy, the energy lost through synchrotron radiation by a particle in a beamline is given by:

$$U_0 = \frac{C_\gamma}{2\pi} E_0^4 I_2$$

where I_2 is the second synchrotron radiation integral. It is straightforward to show (using a procedure analogous to the calculation of the transverse emittance growth) that the increase in energy spread is given by:

$$\Delta\sigma_\delta^2 = \tfrac{4}{3} C_q r_e \gamma^5 I_3 \qquad (9)$$

where I_3 is the third synchrotron radiation integral. We have again assumed that the energy loss is small compared to the energy of the beam.

4.2 Coherent Synchrotron Radiation

Coherent synchrotron radiation can affect the beam in two ways. First, there is the energy loss itself, which can have a similar impact to the incoherent synchrotron radiation. Second, the radiation emitted by a group of particles at the tail of the bunch can follow a straight path between two points on the curved particle trajectory, and affect particles nearer the head of the bunch. A wake field, similar to the standard description of impedance effects, may be used to describe this effect with the difference that the tail affects the head of the bunch rather than the other way around. CSR effects may be minimized by ensuring a sufficiently large bending radius in the dipoles, and if necessary, by shielding the long wavelength radiation using a narrow vacuum chamber aperture.

Some observations of CSR in a bunch compressor at the CLIC Test Facility (CTF II) are described in:

- "Emittance Growth and Energy Loss due to Coherent Synchrotron Radiation in a Bunch Compressor", PhysRevSTAB Volume 3, 124402 (2000).

5 Problems for the Student

1. Starting from equation (2), derive equations (3) through (6).

2. Consider a chicane constructed from four symmetrically placed short dipoles. Show that the change in path length for a particle with normalized energy deviation δ with respect to the nominal energy is given approximately by:

$$\Delta s \approx \tfrac{1}{2} R_{56} \left(1 - \frac{1}{(1+\delta)^2} \right)$$

   Hence derive a general expression for the higher order phase slip coefficients, in terms of the linear phase slip R_{56}. Explain why your expressions are not valid if there are quadrupoles between the dipoles.

3. Using the linear model, calculate from the TESLA, NLC Stage I and NLC Stage II bunch compressors:
   a) the required RF voltage;
   b) the required phase slip R_{56}.

4. Calculate the nonlinear phase slip compensation for the TESLA, NLC Stage I and NLC Stage II bunch compressors.

5. Derive equations (8) and (9).

6. A magnetic chicane by a bunch compressor may be constructed from achromat cells bending the beam alternately left and right. Show that, for such a chicane constructed from short dipoles of bending angle θ, and consisting of N periods ($2N$ cells) of length λ:

$$R_{56} \approx \tfrac{1}{4} N \lambda \theta^2$$

7. Suggest a parameter set for the magnetic chicane for the TESLA bunch compressor. You should specify the dipole field and bending angle, and the dipole spacing. Considerations should include the maximum values for the lattice functions, and the growth in emittance and energy spread from (incoherent) synchrotron radiation.