An Introduction to the Physics and Technology of e+e- Linear Colliders

Lecture 7: Beam-Beam Effects

Nick Walker (DESY)

USPAS, Santa Barbara, 16th-27th June, 2003

Introduction

- Beam-beam interaction in a linear collider is basically the same Coulomb interaction as in a storage ring collider. But:
  - Interaction occurs only once for each bunch (single pass); hence very large bunch deformations permissible.
  - Extremely high charge densities at IP lead to very intense fields; hence *quantum* behaviour becomes important
- Consequently, can divide LC beam-beam phenomena into two categories:
  - *classical*
  - *quantum*

Introduction (continued)

- Beam-Beam Effects
  - Electric field from a “flat” charge bunch
  - Equation of motion of an electron in flat bunch
  - The Disruption Parameter (D_y)
  - Crossing Angle and Kink Instability
  - Beamstrahlung
  - Pair production (background)

	E_{beam}	β_x	β_y	σ_x	σ_y	σ_z	N_e
	GeV	mm	mm	nm	nm	μm	$\times 10^{10}$
NLC	250	8	0.1	245	2.7	110	0.75
TESLA	250	14	0.4	550	5	300	2.0
CLIC	1500	8	0.15	43	1	30	0.42

Storage Ring Collider Comparison

Linear beam-beam tune shift $$\xi_{x,y} = \frac{r_e N_b}{2\pi\gamma} \frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Putting in some typical numbers (see previous table) gives:

$$\xi_x = 0.54$$
$$\xi_y = 1.44$$

Storage ring colliders try to keep $\xi_{x,y} < 0.05$

Electric Field from a Relativistic Flat Beam

- Highly relativistic beam $\mathbf{E}+\mathbf{v}\times\mathbf{B} \approx 2E$
- Flat beam $\sigma_x >> \sigma_y$ (*cf Beamstrahlung*)
- Assume
  - infinitely wide beam with constant density per unit length in x ($\equiv \rho(x)$)
  - Gaussian charge distribution in y:
  - for now, leave $\rho(z)$ unspecified

$$\rho(y) \approx \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right] \qquad \rho(x) \approx \frac{1}{\sqrt{2\pi}\sigma_x}$$

Electric Field from a Relativistic Flat Beam

Use Gauss' theorem: $\oint_s \mathbf{E}\cdot\mathbf{ds} = \frac{q}{\varepsilon_0}$

$$E_y(y,z)\Delta x\Delta z \approx \frac{qN\rho(x)\rho(z)\Delta x\Delta z}{\varepsilon_0}\int_{y'=0}^{y}\rho(y')dy'$$

$$E_y(y,z) = \frac{qN}{2\sqrt{2\pi}\varepsilon_0\sigma_x}\mathrm{Erf}\left(\frac{y}{\sqrt{2}\sigma_y}\right)\rho(z)$$

Assuming Gaussian distribution for z, the peak field is given by

$$\hat{E}_y = \frac{qN}{4\pi\varepsilon_0\sigma_x\sigma_z}$$

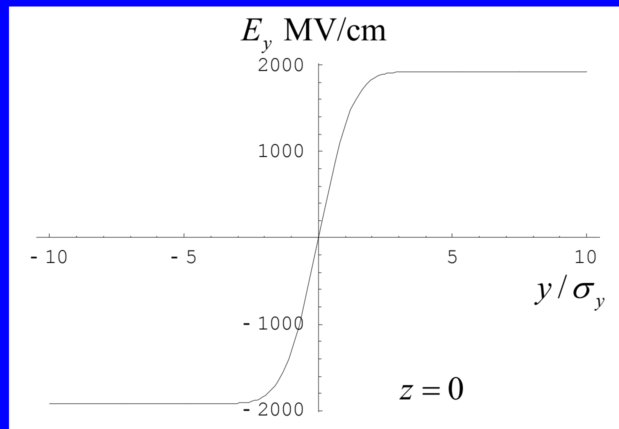
Electric Field from a Relativistic Flat Beam

$\sigma_x = 500\,\text{nm}$

$\sigma_y = 5\,\text{nm}$

$\sigma_z = 300\,\mu\text{m}$

$N = 10^{10}$



Note: $2\times E_y$ plotted

Assuming a Gaussian distribution for ρ(z)

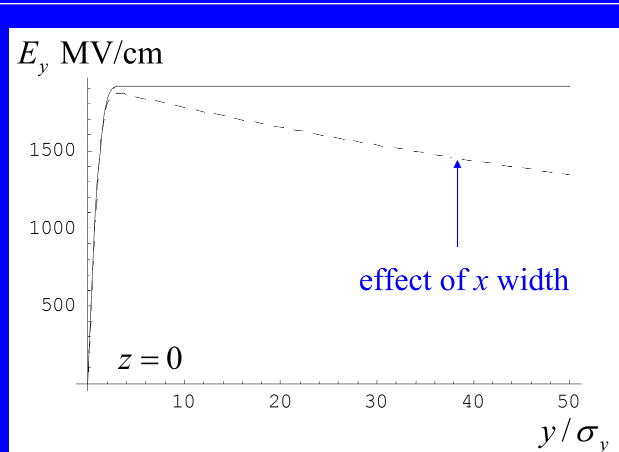
Electric Field from a Relativistic Flat Beam

$\sigma_x = 500\,\text{nm}$

$\sigma_y = 5\,\text{nm}$

$\sigma_z = 300\,\mu\text{m}$

$N = 10^{10}$



Assuming a Gaussian distribution for ρ(z)

Equation Of Motion

F = ma: $$\ddot{y}(t) = -\frac{2qE_y(y,t)}{\gamma m_0}$$

Changing variable to *z:* $$\ddot{y}(t) = c^2 y''(z)$$

$$y''(z) = -\frac{2qE_y(y,z)}{\gamma m_0 c^2}$$

$$= -\frac{q^2 N \,\mathrm{Erf}\left(\frac{y(z)}{\sqrt{2}\sigma_y}\right)\rho_z(2z)}{\sqrt{2\pi}\gamma\varepsilon_0\sigma_x m_o c^2}$$

why $\rho_z(2z)$?

$$r_e = \frac{q^2}{4\pi\varepsilon_0 m_0 c^2}$$

$$\boxed{y''(z) = -\frac{2\sqrt{2\pi}Nr_e}{\gamma\sigma_x}\mathrm{Erf}\left(\frac{y(z)}{\sqrt{2}\sigma_y}\right)\rho_z(2z)}$$

Linear Approximation and the Disruption Parameter

Taking only the linear part of the electric field: $$y''(z) \approx -\underbrace{\frac{4Nr_e\rho(2z)}{\gamma\sigma_x\sigma_y}}_{k^2(z)} y(z)$$

Take 'weak' approximation: $y(z)$ does not change during interaction $y(z) = y_0$

$$\Delta y' \approx -\frac{4Nr_e}{\gamma\sigma_x\sigma_y} y_0 \int_{-\infty}^{\infty} \rho_z(2z)dz$$

$$= -\frac{2Nr_e}{\gamma\sigma_x\sigma_y} y_0 \equiv -\frac{1}{f} y_0$$

Thin-lens focal length: $\frac{1}{f} = \frac{2Nr_e}{\gamma\sigma_x\sigma_y}$

Define Vertical *Disruption Parameter*

$$D_y \equiv \frac{\sigma_z}{f} = \frac{2Nr_e\sigma_z}{\gamma\sigma_x\sigma_y}$$

exact: $D_y = \frac{2Nr_e\sigma_z}{\gamma(\sigma_x + \sigma_y)\sigma_y}$

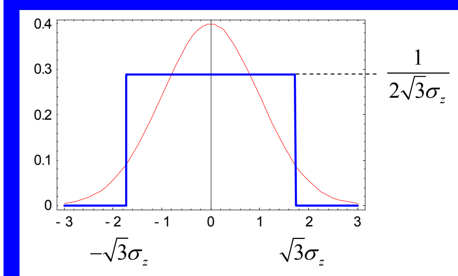
Number of Oscillations

Equation of motion re-visited:

$$y''(z) = -\frac{2D_y}{\sigma_z}\rho_z(2z)y(z)$$

Approximate $\rho(z)$ by rectangular distribution with same RMS as equivalent Gaussian distribution (σ_z)

half-length!



$$y''(z) = -\frac{D_y}{\sqrt{3}\sigma_z^2}y(z); \quad |z| \le \frac{\sqrt{3}}{2}\sigma_y$$

$$k^2 = \frac{D_y}{\sqrt{3}\sigma_z^2} \qquad \nu = \frac{\sqrt{3}k\sigma_z}{2\pi}$$

$$\Rightarrow \boxed{\nu = \frac{3^{1/4}\sqrt{D_y}}{2\pi} \approx 0.21\sqrt{D_y}}$$

Example of Numerical Solution

$N = 2\times10^{10}$

$\sigma_x = 500\,\mathrm{nm}$

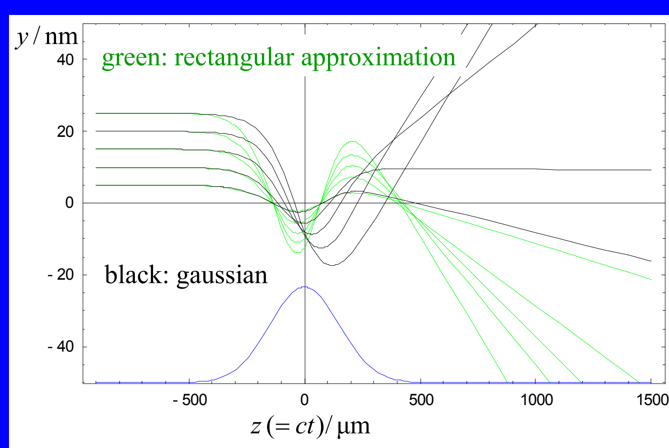
$\sigma_y = 5\,\mathrm{nm}$

$\sigma_z = 300\,\mu\mathrm{m}$

$E = 250\,\mathrm{GeV}$

$D_y = 27.7$

$\nu = 1.1$



Pinch Enhancement

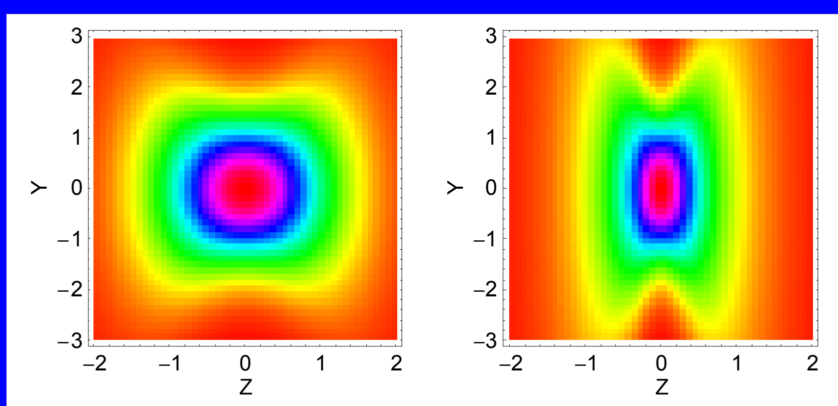
- Self-focusing (pinch) leads to higher luminosity for a head-on collision.

$$H_{Dx,y} = 1 + D_{x,y}^{1/4}\left(\frac{D_{x,y}^{3}}{1+D_{x,y}^{3}}\right)\left[\ln\left(\sqrt{D_{x,y}}+1\right) + 2\ln\underbrace{\left(\frac{0.8\beta_{x,y}}{\sigma_z}\right)}_{\text{‘hour glass’ effect}}\right]$$

Empirical fit to beam-beam simulation results

Only a function of disruption parameter $D_{x,y}$

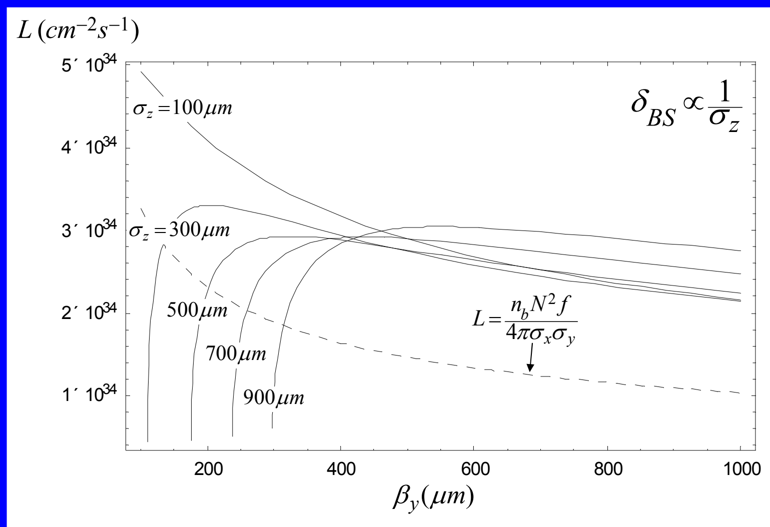
The *Luminosity* Issue: Hour-Glass



β = “depth of focus”

reasonable lower limit for β is bunch length σ_z

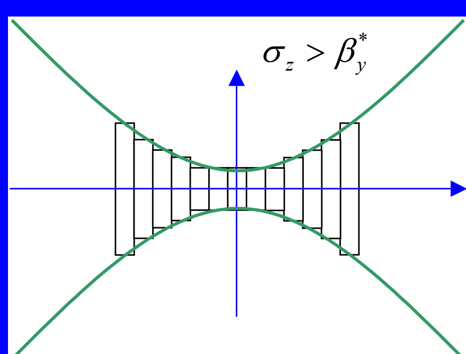
Luminosity as a function of β_y



Beating the hour glass effect

Travelling focus (Balakin)

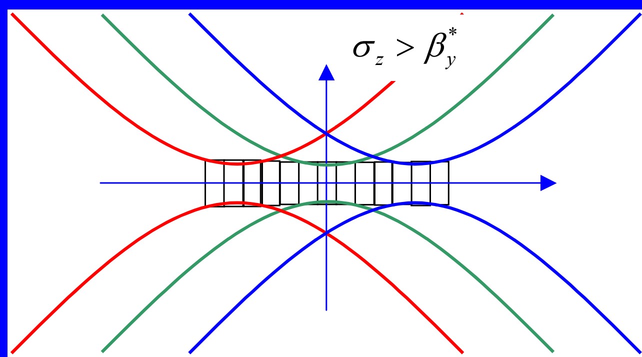
- Arrange for finite chromaticity at IP (how?)
- Create z-correlated energy spread along the bunch (how?)



Beating the hour glass effect

Travelling focus

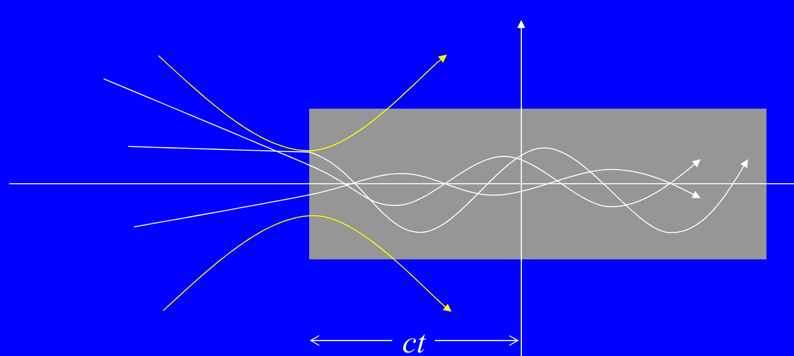
- Arrange for finite chromaticity at IP (how?)
- Create z-correlated energy spread along the bunch (how?)



Beating the hour glass effect

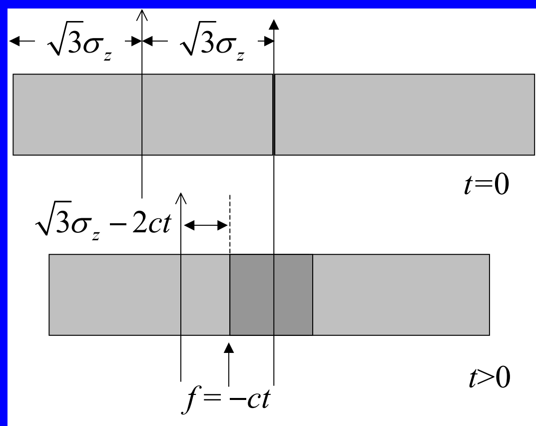
Travelling focus

- Arrange for finite chromaticity at IP (how?)
- Create z-correlated energy spread along the bunch (how?)



Beating the hour glass effect

foci 'travel' from $z = 0$ to $z = \pm\sqrt{3}\sigma_z$



chromaticity:

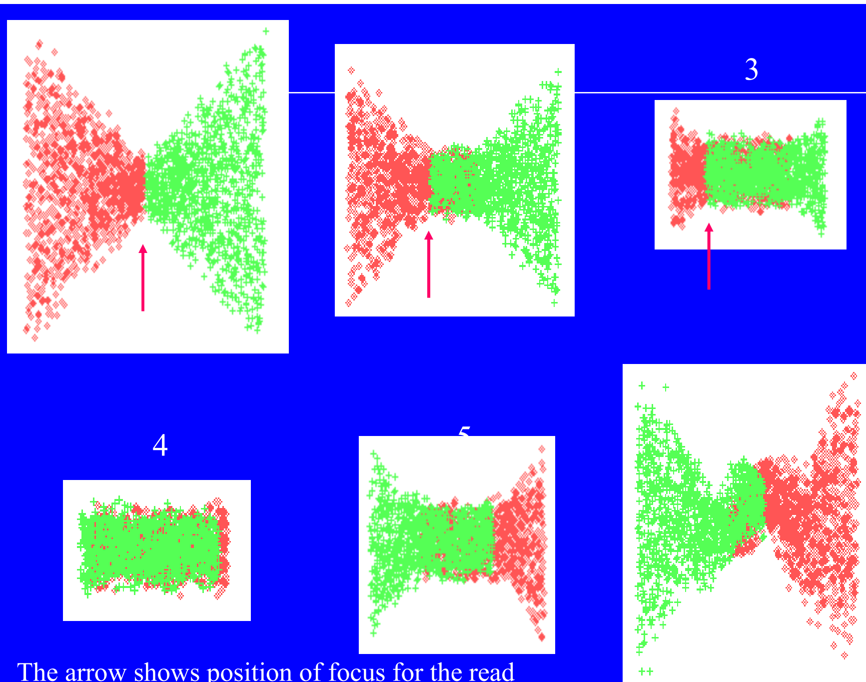
$$\Delta f_y = \xi_y \delta$$

travelling focus:

NB: z correlated!

$$\delta_{\mathrm{RMS}} \approx \frac{\sigma_z}{2\xi_y}$$

$$\langle z\delta \rangle \approx 2\xi_y \delta_{\mathrm{RMS}}^2 \approx \frac{\sigma_z^2}{2\xi_y}$$



The arrow shows position of focus for the read beam during travelling focus collision

Kink Instability

Simple model: 'sheet' beams with: $\rho_{x,z} = \frac{1}{2\sqrt{3}\sigma_{x,z}}$

Linear equation of motion becomes

$$y''(z) = \frac{\sqrt{2\pi}}{6}\frac{D_y}{\sigma_z^2} y(z); \quad |z| \le \frac{\sqrt{3}}{2}\sigma_z$$

Need to consider relative motion of *both* beams in t and z:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)^2 y_1(t,z) = -\omega_0^2 (y_1 - y_2)$$

$$\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial z}\right)^2 y_2(t,z) = +\omega_0^2 (y_1 - y_2)$$

$$\omega_0^2 = \frac{\sqrt{2\pi}}{6} c^2 \frac{D_y}{\sigma_z^2}$$

Classic coupled EoM.

Kink Instability

Assuming solutions of the form

$$y_{1(2)} = a_{1(2)} \exp\left[i\left(kz - \omega t\right)\right]$$

and substituting into EoM leads to the dispersion relation:

$$\omega^2 = c^2k^2 + \omega_0^2 \pm \sqrt{4\omega_0^2 c^2 k^2 + \omega_0^4}$$

Motion becomes unstable when $\omega^2 \le 0$, which occurs when

$$|k| \le \frac{\sqrt{2}\omega_0}{c}$$

Kink Instability

Exponential growth rate:

$$\left[\sqrt{4\omega_0^2 c^2 k^2 + \omega_0^4} - \omega_0^2 + c^2 k^2\right]^{1/2}$$

Maximum growth rate when $k = \pm\dfrac{\sqrt{3}}{2}\dfrac{\omega_0}{c}$; $\omega = i\dfrac{\omega_0}{2}$

Remember! $-\dfrac{\sqrt{3}}{2}\dfrac{\sigma_z}{c} \le t \le \dfrac{\sqrt{3}}{2}\dfrac{\sigma_z}{c}$ $\Rightarrow \Delta t = \sqrt{3}\sigma_z / c$

Thus 'amplification' factor of an initial offset is:

$$\exp\left(\frac{\omega_0 \Delta t}{2}\right) = \exp\left[\frac{1}{2}\left(\frac{\pi}{2}\right)^{1/4}\sqrt{D_y}\right] \approx e^{0.6\sqrt{D_y}}$$

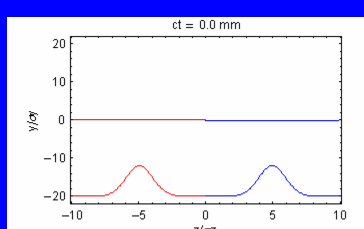
For our previous example with D_y ~28, factor ~ 3

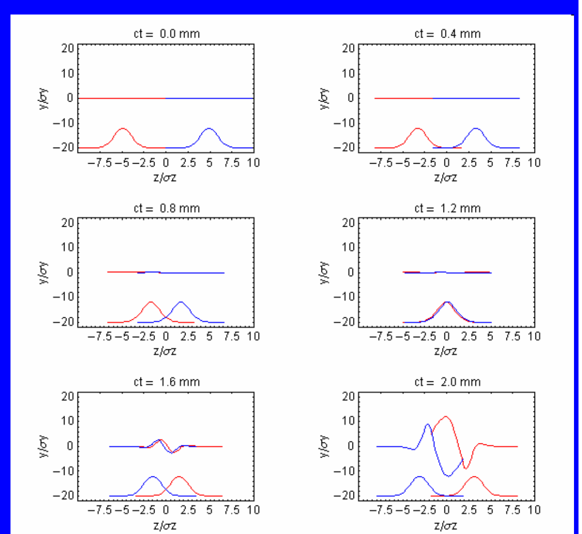
Kink Instability

$\Delta y = 0.1\sigma_y$

$D_y \approx 20$

$\sigma_z = 300\,\mu\text{m}$

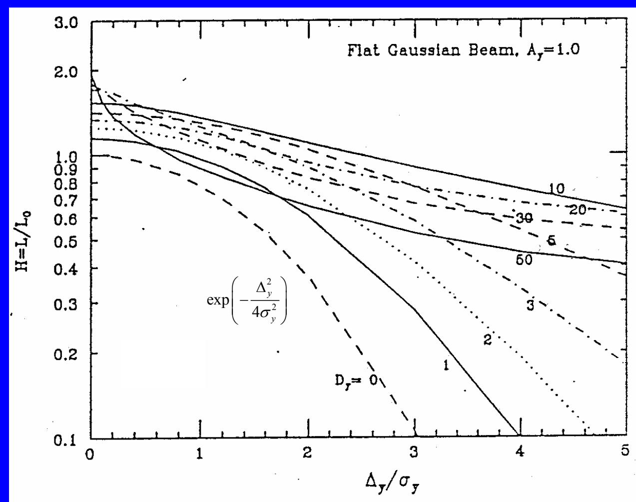




Pinch Enhancement

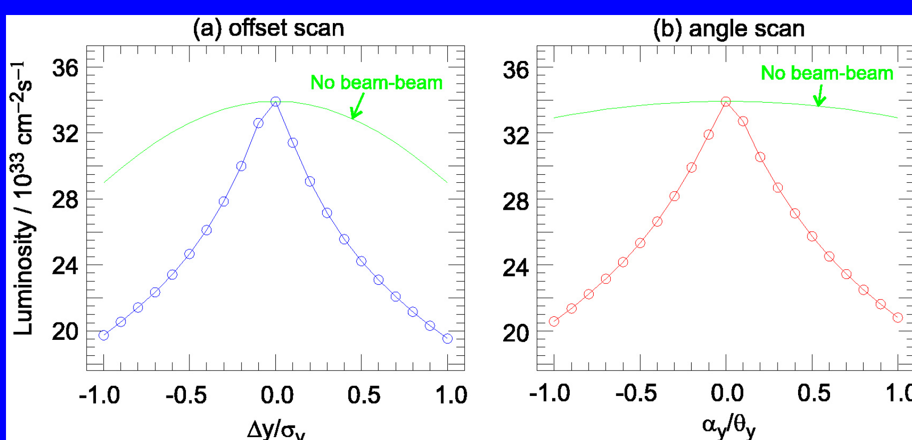
$L = L_{geom} H$ $\quad$ *H= enhancement factor*

results of simulations:



High D_y example: TESLA 500

$D_y \approx 24$



Disruption Angle

Remembering definition of D_y $\qquad D_y = \frac{\sigma_z}{f}$

The angles after collision are characterised by

$$\theta_0 \equiv \frac{D_x \sigma_x}{\sigma_z} = \frac{D_y \sigma_y}{\sigma_z} = \frac{2Nr_e}{\gamma(\sigma_x + \sigma_y)} \approx \frac{2Nr_e}{\gamma \sigma_x}$$

Numbers from our previous example give $\theta_0 \approx 467\ \mu\text{rad}$

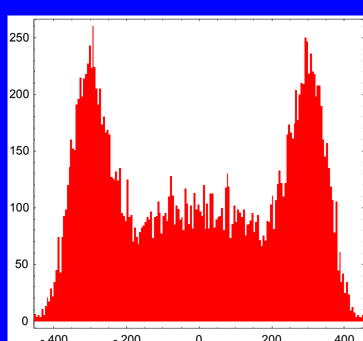
OK for horizontal plane where $D_x < 1$

For vertical plane (strong focusing $D_y > 1$), particles oscillate:

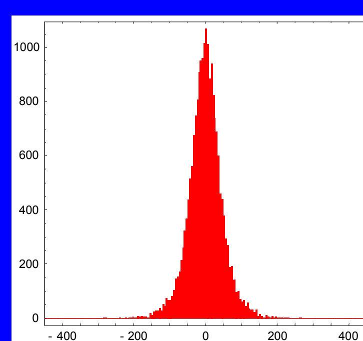
previous linear approximation:

$$y''(z) = -\frac{D_y}{\sqrt{3}\sigma_z^2} y(z); \quad |z| \le \frac{\sqrt{3}}{2}\sigma_y \qquad \theta \sim \frac{\sqrt{D_y}}{3^{1/4}\sigma_z}\sigma_y = \frac{\theta_0}{3^{1/4} D_y^{1/2}} \approx 67\ \mu\text{rad}$$

Disruption Angle: simulation results



horizontal angle (μrad)



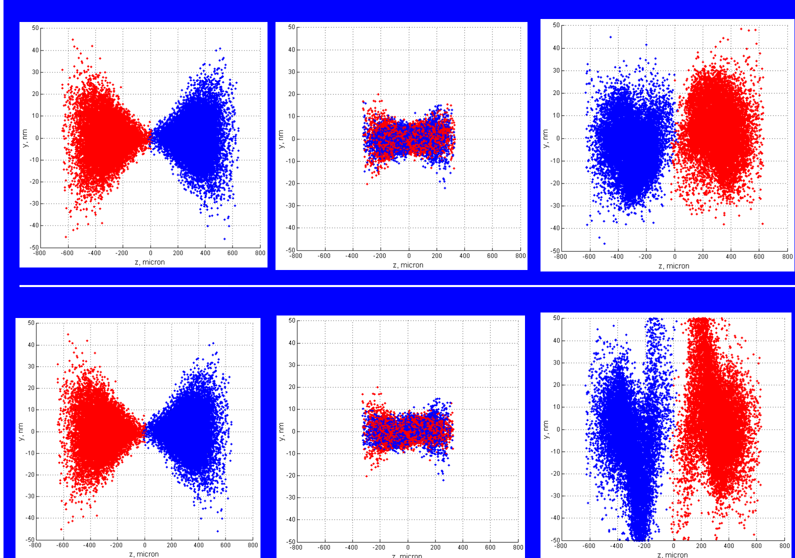
vertical angle (μrad)

Important in designing IR (spent-beam extraction)

Beam-Beam Animation Wonderland

Animations produced by A. Seryi using the **GUINEAPIG** beam-beam simulation code (written by D. Schulte, CERN).

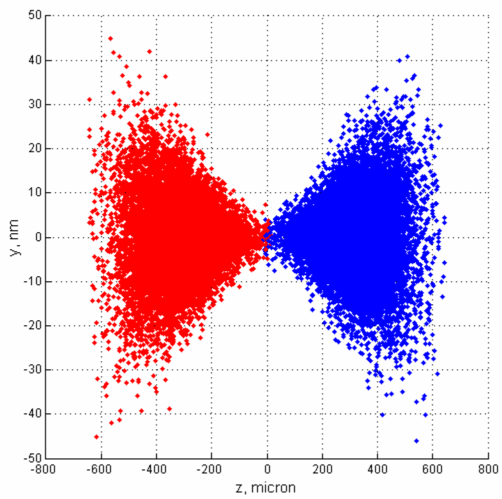
Examples of GUINEAPIG Simulations



NLC parameters
D_y~12

Nx2
D_y~24

Examples of GUINEAPIG Simulations

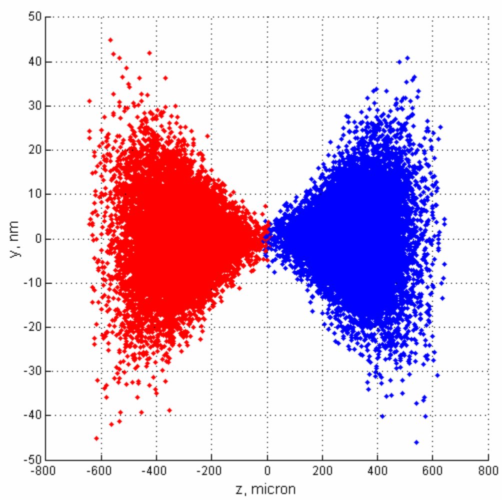


NLC parameters
D_y~12

Luminosity enhancement
H_D ~ 1.4

Not much of an instability

Examples of GUINEAPIG Simulations

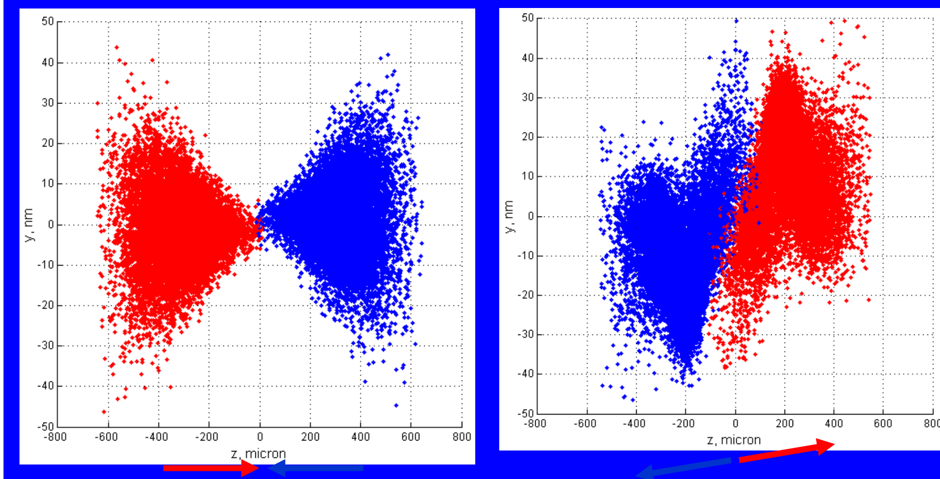


Nx2
D_y~24

Beam-beam instability is clearly pronounced

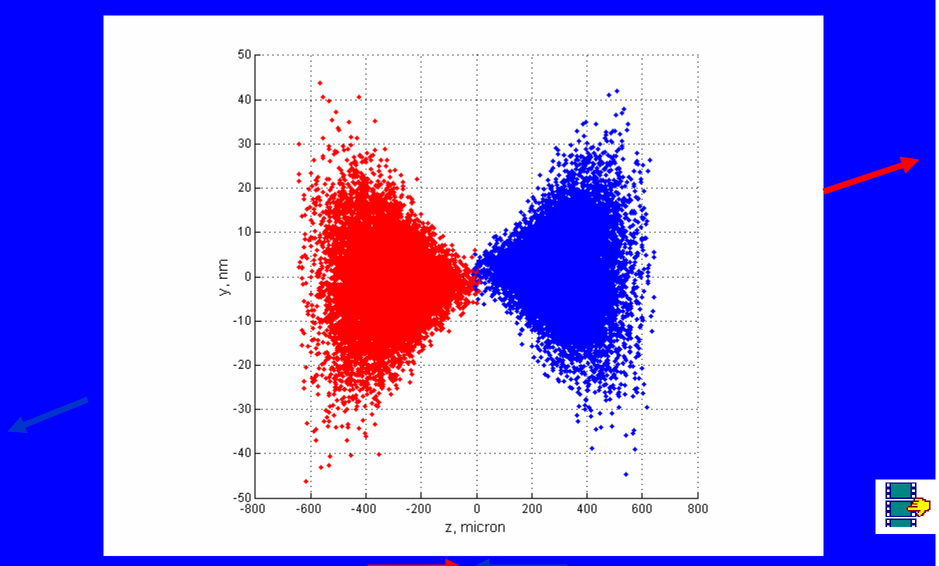
Luminosity enhancement is compromised by higher sensitivity to initial offsets

Beam-beam deflection

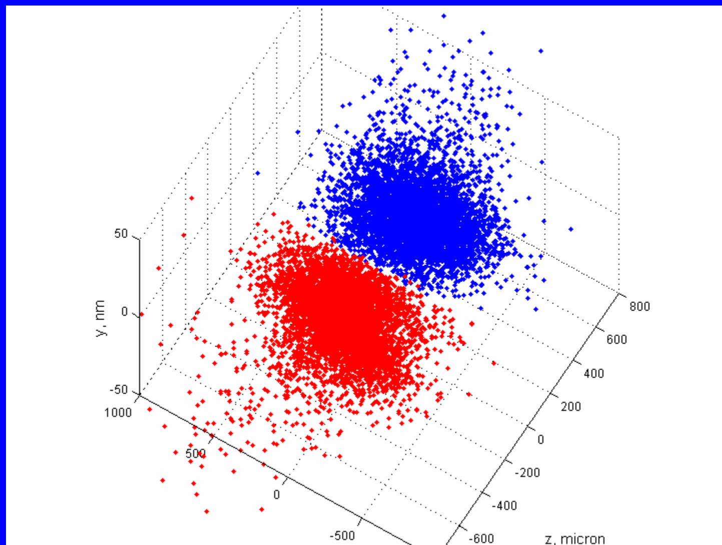


Sub nm offsets at IP cause large well detectable offsets (micron scale) of the beam a few meters downstream

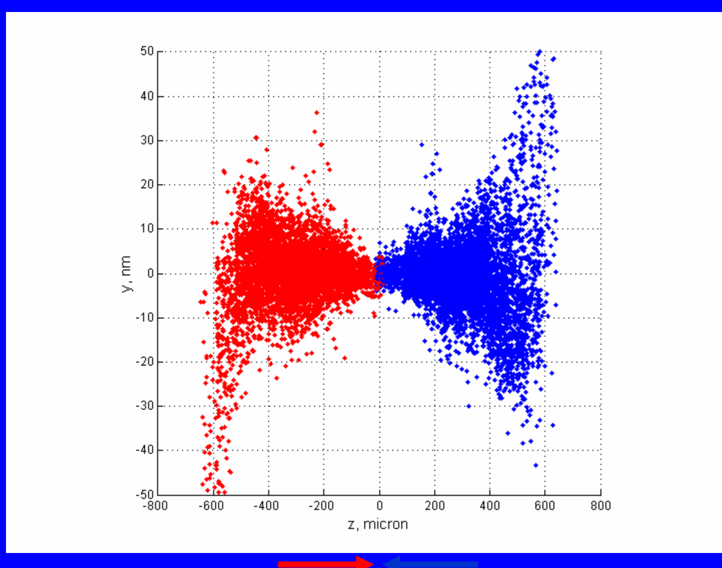
Beam-beam deflection allow to control collisions



Examples of GUINEAPIG Simulations



Examples of GUINEAPIG Simulations



Beam-Beam Kick

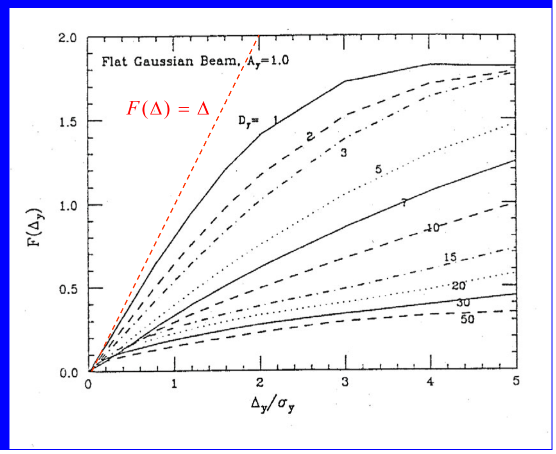
Beam-beam offset gives rise to an equal and opposite mean kick to the bunches – *important signal for feedback!*

For small disruption (D<<1) and offset (y/σ_y<<1)

$$\theta_{bb} = \frac{1}{2}\theta_0 \frac{y}{\sigma_y}$$

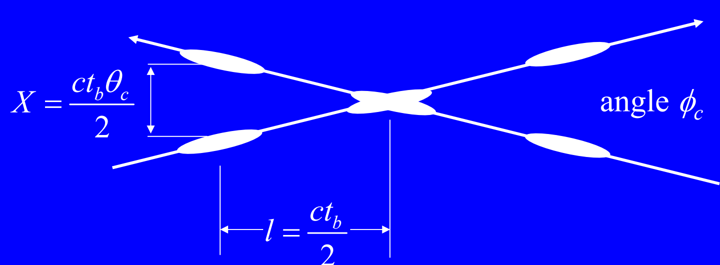
For large disruption or offset, we introduce the *form factor F:*

$$\theta_{bb} = \frac{1}{2}\theta_0 F\left(y/\sigma_y\right)$$



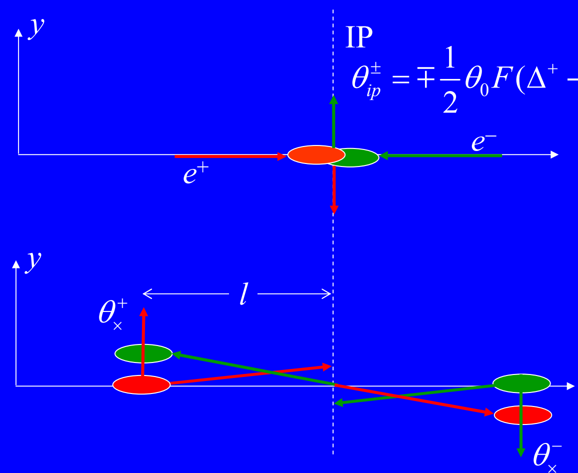
Long Range Kink Instability & Crossing Angle

To avoid parasitic bunch interactions in the IR, a horizontal crossing angle is introduced:



parasitic beam-beam kick: $\Delta r' = \dfrac{2Nr_e}{\gamma r} \quad r \gg \sigma_{x,y}$

Long Range Kink Instability & Crossing Angle



$$\theta_{ip}^{\pm} = \mp \frac{1}{2} \theta_0 F(\Delta^+ - \Delta^-)$$

small vertical offset ($\Delta^\pm = \delta y^\pm / \sigma_y$) gives rise to beam-beam kick

resulting vertical offset at parasitic crossing gives next incoming bunches additional vertical kick: IP offset increases

$$\theta_\times^{\mp} = \pm \frac{2Nr_e}{\gamma} \frac{l\theta_{ip}^{\pm}}{X^2} = \mp \frac{Nr_e}{\gamma} \frac{\theta_0 F(\Delta^+ - \Delta^-)}{l\phi_c^2}$$

$\Rightarrow$instability

Long Range Kink Instability & Crossing Angle

offset at IP of k-th bunch: $\Delta_k \equiv \Delta_k^+ - \Delta_k^-$

distance from IP to encounter with l-th bunch ($l < k$) $\quad l_{lk} = \dfrac{(k-l)t_b c}{2}$

contribution from encounter with l-th bunch:

$$\Delta_k^\pm = \Delta_{k,0}^\pm + l_{lk}\theta_{\times,lk}^\pm / \sigma_y$$
$$= \Delta_{k,0}^\pm \pm \frac{Nr_e}{\gamma} \frac{\theta_0}{\phi_c^2 \sigma_y} F(\Delta_l)$$
$$\Delta_k = \Delta_k^+ - \Delta_k^-$$
$$= \Delta_{k,0} + \frac{2Nr_e\theta_0}{\gamma\phi_c^2\sigma_y} F(\Delta_l)$$

NB. independent of l_{lk}

total offset: $\quad \Delta_k = \Delta_{k,0} + C\sum_{i=1}^{k-1} F(\Delta_i); \quad C = \dfrac{2Nr_e\theta_0}{\gamma\phi_c^2\sigma_y} = D_x D_y \left[\dfrac{\sigma_x/\sigma_z}{\phi_c}\right]^2$

Long Range Kink Instability & Crossing Angle

Assuming all bunches have same initial offset: $\Delta_{k,0} = \Delta_0$

For small disruption and small offsets, $F(\Delta_i) \approx \Delta_i$

$$\Delta_k = (1+C)^{k-1}\Delta_0$$

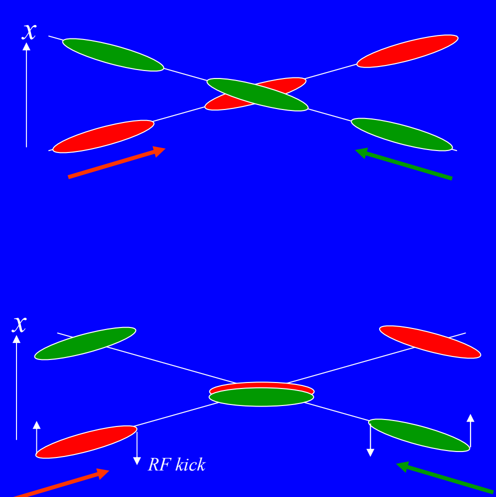
since $C = D_x D_y \left[\dfrac{\sigma_x/\sigma_z}{\phi_c}\right]^2 \ll 1 \qquad \dfrac{\Delta_k}{\Delta_0} \approx 1 + (k-1)C$

thus $(m_b - 1)C \le 1$ where m_b = number of interacting bunches

example:

$$\left.\begin{array}{ll} D_{x,y} & = 0.1,\ 10 \\ \sigma_x & = 220\ \text{nm} \\ \sigma_z & = 100\ \mu\text{m} \\ \phi_c & = 20\ \text{mrad} \end{array}\right\} \quad C = 0.012 \Rightarrow m_b < 80 \text{ for factor of 2 increase}$$

Crab Crossing



$$\sigma_{x,projected} \approx \sqrt{\sigma_x^2 + \phi_c^2\sigma_z^2}$$
$$\approx \phi_c\sigma_z$$
$$= 20\text{mr} \times 100\mu\text{m} \approx 2\mu\text{m}$$

factor 10 reduction in L!

use transverse (crab) RF cavity to ‘tilt’ the bunch at IP

$$V(z) = \hat{V}_{RF}\sin\left(\frac{2\pi z}{\lambda}\right) \approx \frac{2\pi\hat{V}_{RF}z}{\lambda}$$

$$\phi_c \approx 4\pi\sqrt{\beta_{cav}\beta_{ip}}\,\frac{\hat{V}_{RF}}{\lambda_{RF}}\sigma_z$$

Beamstrahlung

Magnetic field of bunch $B = E/c$

Peak field: $$B_{\max} = \frac{2E_{\max}}{c} = \frac{qN}{2\pi\varepsilon_0 c\sigma_x\sigma_z} = 1160\,\text{Tesla}$$

From classical theory, power radiated is given by

$$P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2}; \quad C_\gamma \approx 8.85\times10^{-5}\ \text{m.GeV}^{-3}$$

For E = 250 GeV and σ_z = 300μm:

$$\frac{1}{\rho} \approx \frac{cB}{E} \approx 1.4\,\text{m}^{-1}$$

$$\Delta E_{rad} \approx P_\gamma \Delta t \approx P_\gamma \frac{\sigma_z}{c} \approx 32\ \text{GeV} \qquad \Delta E/E \sim 12\%$$

note: $\sigma_z / \rho \approx 200\,\mu\text{rad} \gg 1/\gamma \approx 2\,\mu\text{rad}$

Beamstrahlung

Most important parameter is Υ

$$\Upsilon = \frac{2}{3}\frac{\hbar\omega_c}{E} = \frac{\lambda_e\gamma^2}{\rho} = \gamma\frac{2B}{B_s} = \frac{e}{m_0^2}\sqrt{\left|\left(F_{\mu\nu}p^\nu\right)^2\right|}$$

ω_c	critical photon frequency
λ_e	Compton wavelength
ρ	local bending radius
B	beam magnetic field
B_s	Schwinger's critical field (= 4.4 GTesla)
$F_{\mu\nu}$	em field tensor
p^ν	electron 4-momentum

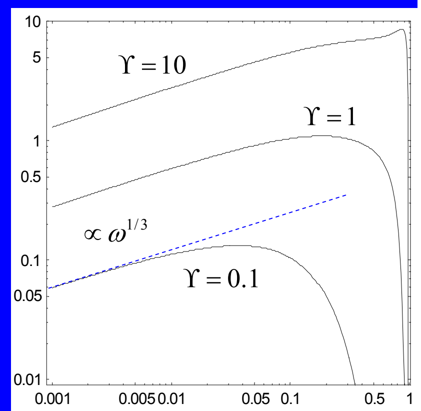
Beamstrahlung: photon spectrum

NOT classical synchrotron radiation spectrum!
Need to use Sokolov-Ternov formula:

$$F_{BS} = \underbrace{\int_{\xi}^{\infty} K_{5/3}(\xi')d\xi'}_{\text{classical}} + \underbrace{\frac{y^2}{1-y} K_{2/3}(\xi)}_{\text{quantum theoretical}}$$

$$y = \frac{\hbar\omega}{E}$$

$$\xi = \frac{2}{3\Upsilon}\left(\frac{y}{1-y}\right) = \left(\frac{\omega}{\omega_c}\right)\left(\frac{1}{1-y}\right)$$



$y = \hbar\omega / E$

Beamstrahlung Numbers

average and maximum Υ

$$\Upsilon_{avr} = \frac{5}{6}\frac{N r_e \lambda_e \gamma}{\sigma_z(\sigma_x + \sigma_y)}$$

$$\Upsilon_{max} \approx 2.4\Upsilon_{avr}$$

photons per electron:

$$n_\gamma \approx 2.54\left[\frac{\alpha\sigma_z}{\lambda_e\gamma}\right]\frac{\Upsilon_{avr}}{\sqrt{1+\Upsilon_{avr}^{2/3}}}$$

average energy loss:

$$\delta_{BS} = \left\langle -\frac{\Delta E}{E}\right\rangle \approx 1.24\left[\frac{\alpha\sigma_z}{\lambda_e\gamma}\right]\frac{\Upsilon_{avr}^2}{\left[1+(1.5\Upsilon_{avr})^{2/3}\right]^2}$$

δ_{BS} and n_γ (and hence Υ) talk directly to *luminosity spectrum* and *backgrounds*

Beamstrahlung energy loss

In lecture 1, we used the following equation to derive our luminosity scaling law

$$\delta_{BS} \approx 0.86 \frac{e r_e^3}{2 m_0 c^2} \left(\frac{E_{cm}}{\sigma_z} \right) \frac{N^2}{(\sigma_x + \sigma_y)^2}$$

Now we have this:

$$\delta_{BS} \approx 1.24 \left[\frac{\alpha \sigma_z}{\lambda_e \gamma} \right] \frac{\Upsilon_{avr}^2}{\left[1 + (1.5 \Upsilon_{avr})^{2/3} \right]^2}$$

with $$\Upsilon_{avr} = \frac{5}{6} \frac{N r_e \lambda_e \gamma}{\sigma_z (\sigma_x + \sigma_y)}$$

under what regime is our original expression valid?

Luminosity scaling revisited

$$\beta_y^* \approx \sigma_z$$

low beamstrahlung regime $\Upsilon \ll 1$: $$L \propto P_{\text{beam}} \frac{\sqrt{\delta_{\text{BS}}}}{\sqrt{\varepsilon_{y,n}}}$$

high beamstrahlung regime $\Upsilon \gg 1$: $$L \propto P_{\text{beam}} \frac{\delta_{\text{BS}}^{3/2}}{\sigma_z \sqrt{\varepsilon_{y,n}}}$$

homework: derive high BS scaling law

Beamstrahlung Numbers: example

$E = 500\,\text{GeV}$

$\sigma_x = 220\,\text{nm}$

$\sigma_y = 2\,\text{nm}$

$\sigma_x = 110\,\mu\text{m}$

$N = 0.75\times 10^{10}$

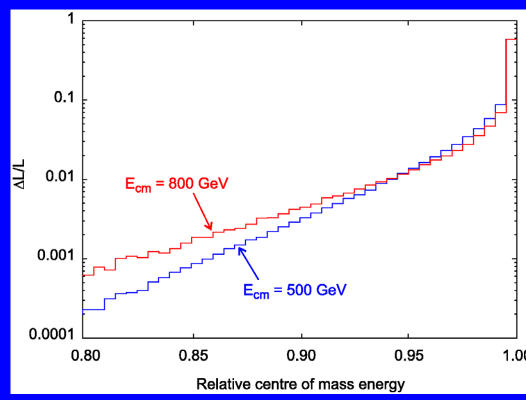
$\Upsilon_{avr} \approx 0.28 \quad \left(E_{\gamma,c} \approx 210\,\text{GeV}\right)$

$\Upsilon_{max} \approx 0.66$

$n_\gamma \approx 1.3$

$\delta_{BS} \approx 7.5\%$

TESLA



Why Beamstrahlung is bad

- Large number of high-energy photons interact with electron (positron) beam and generate *e^+e^- pairs*
  - Low energies (Υ<0.6), pairs made by *incoherent* process (photons interact directly with individual beam particles)
  - High energies (0.6<Υ<100), *coherent* pairs are generated by interaction of photons with macroscopic field of bunch.
  - Very high energies (Y>100), *coherent direct trident production* $e^\pm \to e^\pm e^+ e^-$

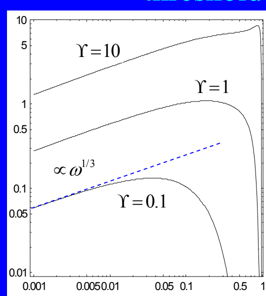
TESLA	0.5TeV	$\Upsilon \sim 0.06$
NLC	1TeV	$\Upsilon \sim 0.28$
CLIC	3TeV	$\Upsilon \sim 9$

- Beamstrahlung degrades *Luminosity Spectrum*

Pair Production

- Incoherent e^+e^- pairs $\Upsilon<0.6$
  - Breit-Wheeler: $\gamma\gamma \to e^+e^-$
  - Bethe-Heitler: $e^\pm\gamma \to e^\pm e^+e^-$
  - Landau-Lifshitz: $e^+e^- \to e^+e^-e^+e^-$
- Coherent e^+e^- pairs $0.6<\Upsilon<100$
  - threshold defined by

$$\chi \equiv \frac{\hbar\omega}{m_0c^2}\frac{2B}{B_s} = \frac{\hbar\omega}{E}\Upsilon \geq 1$$



$y=\hbar\omega/E$

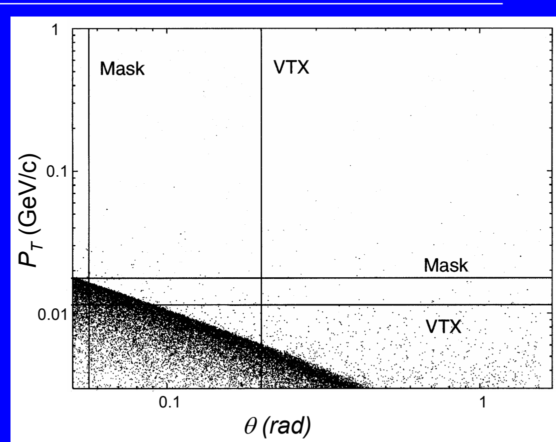
for $\Upsilon>1$ $\quad \hbar\omega/E \sim O(1) \quad \Rightarrow \chi \geq 1$

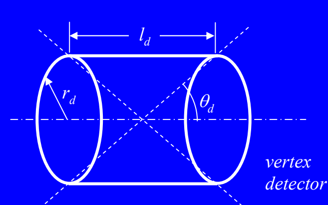
for intermediate colliders (E_{cm}<1TeV), incoherent pairs dominate

Pair Production

e^+e^- pairs are a potential major source of background

Most important: angle with beam axis (θ) and transverse momentum P_T.





pairs curl-up (spiral) in solenoid field of detector

$$r = \frac{P_T}{cB_z} < r_d$$

Summary

- Single pass collider allows us to use very strong beam-beam to increase luminosity
- beam-beam is characterised by following important parameters:
  - $D_y = \sigma_z/f$ — defines pinch effect (*HD*), kink instability, dynamics
  - Υ — QM effects, backgrounds,
  - δ_{BS} [$=f(\Upsilon_{av})$] — energy loss, lumi spectrum
- strong-strong regime requires simulation (e.g. GUINEAPIG). Analytical treatments limited.