Outline

• Damping Rings produce “long” bunches
  – quantum excitation in a storage ring produces longitudinal emittance that is relatively large compared to some modern particle sources
  – long bunches tend to reduce the impact of collective effects
    • large momentum compaction rapidly decoheres modes
    • the longer the bunch, the lower the charge density
  – bunch lengths in damping rings are ~ 5 mm

• Main Linacs and Interaction Point require “short” bunches
  – of the order 100 µm in NLC, 300 µm in TESLA

• Main issues are:
  – How can we achieve bunch compression?
  – How can we compensate for the effects of nonlinear dynamics?
  – What are the effects of (incoherent and coherent) synchrotron radiation?
Schematic Layout (NLC)

- Essential components of a bunch compression system include:
  - RF power
  - “Phase Slip”: variation of path length with energy

NLC Bunch Compressor (First and Second Stages)

Basic Principles

- A “rotation” of longitudinal phase space…
We would like to know:
- how much RF power
- how much wiggler (or chicane, or arc) are needed to achieve a given compression

We consider the changes in the longitudinal phase space variables of a chosen particle in each part of the compressor.

The RF section changes only the energy deviation:

\[
\begin{align*}
\tilde{z}_i &= z_0 \\
\delta_i &= \delta_0 + \frac{eV_{RF}}{E_0} \cos\left(\frac{\pi}{2} - k_{RF}z_0\right)
\end{align*}
\]

In a linear approximation, we can write:

\[
\begin{pmatrix}
\tilde{z}_i \\
\delta_i
\end{pmatrix} \approx 
\begin{pmatrix}
1 & 0 \\
R_{65} & 1
\end{pmatrix}
\begin{pmatrix}
z_0 \\
\delta_0
\end{pmatrix} \\
R_{65} = \frac{eV_{RF}}{E_0} \sin(\phi_{RF}) k_{RF}
\]

The wiggler (or arc) changes only the longitudinal co-ordinate:

\[
\begin{align*}
z_2 &= z_1 + R_{66} \delta_1 + T_{66} \delta_1^2 + U_{66} \delta_1^3 + \cdots \\
\delta_2 &= \delta_1
\end{align*}
\]

Again in a linear approximation:

\[
\begin{pmatrix}
z_2 \\
\delta_2
\end{pmatrix} \approx 
\begin{pmatrix}
1 & R_{66} \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
z_1 \\
\delta_1
\end{pmatrix}
\]

The full transformation can be written:

\[
\begin{pmatrix}
z_2 \\
\delta_2
\end{pmatrix} \approx \mathbf{M} \begin{pmatrix}
z_0 \\
\delta_0
\end{pmatrix} \\
\mathbf{M} = \begin{pmatrix}
1 + R_{66}R_{56} & R_{56} \\
R_{66} & 1
\end{pmatrix}
\]
Optimum Compression

• Since the transformation is symplectic (in the case of no acceleration from the RF) the longitudinal emittance is conserved

\[ \varepsilon = \sqrt{\sigma_x^2 \sigma_z^2 - \sigma_{xz}^2} \]

• For a given value of \( R_{65} \), the best compression that can be achieved is:

\[ \left( \frac{\sigma_z}{\sigma_z} \right)_{\min} = \frac{1}{1 + a^2} \quad a = \frac{\sigma_x}{\sigma_z} R_{65} \]

• This optimum compression is obtained with:

\[ R_{56} = -\frac{a^2}{1 + a^2} \frac{1}{R_{65}} \]

Limitations on Compression

• For final bunch length \(< < \) initial bunch length, we can make the approximations:

\[ \left( \frac{\sigma_z}{\sigma_z} \right)_{\min} \approx \frac{1}{a} \quad R_{56} \approx -\frac{1}{R_{65}} \]

\[ a = \frac{\sigma_x}{\sigma_z} R_{65} \gg 1 \quad R_{65} = \frac{eV_{RF}}{E_0} \frac{\omega_{RF}}{c} \]

• Clearly, we can make the final bunch length shorter simply by
  – increasing the RF voltage, and/or
  – increasing the RF frequency
  and adjusting \( R_{56} \) appropriately.

• In practice, the compression that can be achieved is limited by:
  – available RF power
  – increase in energy spread of the bunch (emittance is conserved)
  – nonlinear dynamics, CSR etc.
Nonlinear Effects

• So far, we have made linear approximations for
  – the energy change variation with position in bunch (in the RF section)
  – the path length variation with energy (in the wiggler or arc), also known as nonlinear phase slip

• The nonlinear phase slip is dependent on the linear slip
  – for an arc, $T_{566} \approx 1.9R_{56}$
  – for a chicane or wiggler, $T_{566} \approx -1.5R_{56}$

Bunch compression in TESLA. The pictures show the initial (left) and final (right) longitudinal phase space, excluding (red) and including (black) the nonlinear phase slip terms.

Nonlinear Effects

• The nonlinear phase slip introduces a strong correlation between $z$ and $\delta^2$
• Since the phase space is rotated by $\sim \pi/2$, we can compensate this with a correlation between $\delta$ and $z^2$ at the start of the compressor
• Note that the energy map (for a general RF phase) looks like:

$$\delta \approx \delta_0 \left( 1 - \frac{eV_{\text{RF}}}{E_0} \cos(\phi_{\text{RF}}) \right) + \frac{eV_{\text{RF}}}{E_0} \left[ \cos(\phi_{\text{RF}} - k_{\text{RF}} z_0) \cos(\phi_{\text{RF}}) \right]$$

• Choosing an appropriate value for the RF phase introduces the required correlation between $\delta$ and $z^2$ to compensate the nonlinear phase slip
Compensation of Nonlinear Phase Slip

- An expression for the RF phase required to compensate the nonlinear phase slip can be found as follows:
  - calculate the complete map for the bunch compressor up to second order in the phase space variables
  - select the coefficient of $\delta^2$ in the expression for $z$, and set this to zero
- We find that the required RF phase is given by:
  \[
  \cos(\phi_{\text{RF}}) = \frac{\sqrt{1 + 8(1 + 2r)\theta^2} - 1}{2(1 + 2r)\theta} = 20\theta
  \]
  \[
  \theta = \frac{eV_{\text{RF}}}{E_0}, \quad r = \frac{T_{\text{stab}}}{R_{\text{st}}}
  \]
- The optimum (linear) phase slip is now given by:
  \[
  R_{\text{st}} = \frac{\alpha^2}{1 + \alpha^2} \frac{1}{R_{\text{st}} R_{\text{ss}}}
  \]

Compensation of Nonlinear Phase Slip - TESLA

- Entrance of Bunch Compressor
- After RF
- After RF and chicane
Two-Stage Compression

- The NLC uses a two-stage bunch compressor:
  - Stage 1 at low energy (1.98 GeV), bunch length reduced from ~ 5 mm to 500 µm
  - Stage 2 at higher energy (8 GeV), bunch length reduced to ~ 110 µm
- Advantages:
  - Acceleration provides adiabatic damping of energy spread, so the maximum energy spread anywhere in the system is less than 2%
  - High frequency RF can be used in Stage 2, where the bunch length is already short
- Disadvantage:
  - More complex, longer system

Two-Stage Compression in NLC

- Phase errors at the entrance to the main linac are worse than energy errors
  - Energy error becomes adiabatically damped in the linac
  - Phase error at the entrance leads to large energy error at the exit
- First stage rotates longitudinal phase space ~ $\pi/2$
  - Energy of beam extracted from Damping Rings is very stable
  - Phase errors from beam loading in the damping ring become energy errors at the exit of the first stage of bunch compression
- Second stage rotates phase space by $2\pi$
  - Energy errors from imperfect beam loading compensation in the prelinac stay as energy errors
Two-Stage Compression in NLC

- How do we achieve compression with a rotation through $2\pi$?
- NLC Stage 2 compressor uses a sequence of systems:
  - RF
  - arc
  - RF
  - chicane

Longitudinal Phase Space Telescope

- The linear map for the NLC Stage 2 compressor is as follows:

\[
M = \begin{pmatrix} 1 + R_{66}^{(2)} R_{56}^{(2)} & R_{65}^{(1)} R_{56}^{(2)} + R_{55}^{(2)} + R_{56}^{(1)} + R_{56}^{(2)} & R_{65}^{(2)} R_{56}^{(1)} R_{56}^{(2)} + R_{56}^{(1)} + R_{56}^{(2)} \\
R_{55}^{(1)} + R_{55}^{(2)} + R_{56}^{(1)} R_{56}^{(2)} R_{56}^{(1)} & \ldots & \end{pmatrix}
\]

- With appropriate choices for the parameters:

\[
1 + R_{66}^{(2)} R_{56}^{(2)} = \pm \frac{1}{m} \quad R_{56}^{(1)} = \mp m R_{56}^{(2)}
\]

this can be written:

\[
M = \begin{pmatrix} \pm \frac{1}{m} & \frac{1}{m} & 0 \\
R_{55}^{(1)} + R_{55}^{(2)} + R_{56}^{(1)} R_{56}^{(2)} R_{56}^{(1)} & \pm m & \end{pmatrix}
\]
### Effects of Synchrotron Radiation

- Synchrotron radiation is emitted in the arcs or wiggler/chicane used to provide the phase slip in a bunch compressor.
- Effects are:
  - Transverse emittance growth
  - Increase in energy spread
- For very short bunches at low energy, coherent synchrotron radiation (CSR) may be more of a problem than incoherent synchrotron radiation.
- Weaker bending fields produce less radiation, and therefore have less severe effects.
- CSR may also be limited by “shielding” the radiation using a narrow aperture beam pipe.
Incoherent Synchrotron Radiation

- Transverse and longitudinal emittance growth is analogous to quantum excitation in storage rings
- Transverse emittance growth is given by:
  \[ \Delta \gamma = \frac{i}{2} C_\gamma r_s \gamma^3 I_s \]
  \[ I_s = \int \frac{H}{|A|} \, ds \]
- The energy loss from incoherent synchrotron radiation is:
  \[ U_0 = \frac{C}{2\pi} E_0^2 I_2 \]
  \[ I_2 = \int \frac{1}{\rho^2} \, ds \]
- The increase in energy spread is given by:
  \[ \Delta \sigma_\gamma^2 = \frac{i}{2} C_\gamma r_s \gamma^3 I_s \]
  \[ I_s = \int \frac{1}{|A|} \, ds \]

Coherent Synchrotron Radiation

- A bunch of particles emits radiation over a wide spectrum
- For regions of the spectrum where the radiation wavelength is much less than the bunch length, the emission is incoherent
  - for a bunch of \( N \) particles, radiation power \( \propto N \)
- Where the radiation wavelength is of the order of or longer than the bunch length, the bunch emits as a single particle
  - radiation power \( \propto N^2 \)
- Since \( N \) is of the order \( 10^{10} \), the coherence of the radiation represents a significant enhancement
- The radiation acts back on the beam, leading to a correlated energy spread within the bunch
Coherent Synchrotron Radiation