Linear Collider Damping Rings

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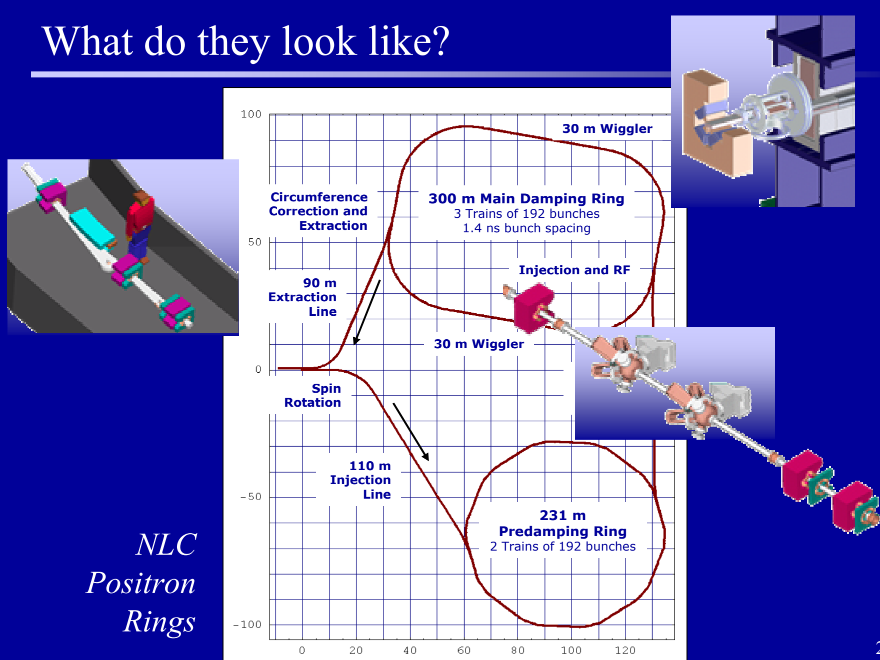
What do they look like?



NLC Positron Rings

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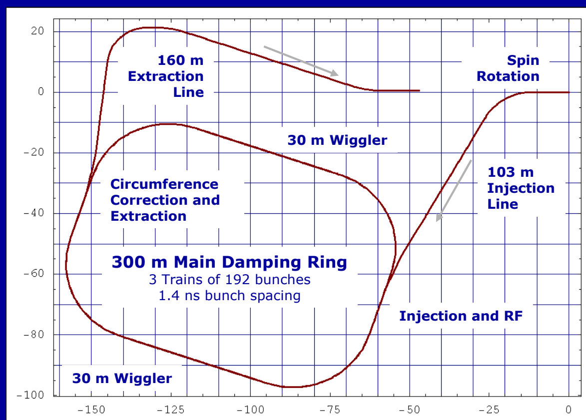
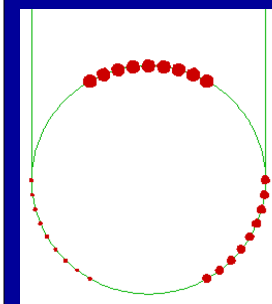
Operating Cycle in NLC/JLC MDRs

- Each bunch train is stored for three machine cycles
  - 25 ms or 25,000 turns in NLC
- Transverse damping time ≈ 4 ms
- Horizontal emittance ×1/50, vertical ×1/7500





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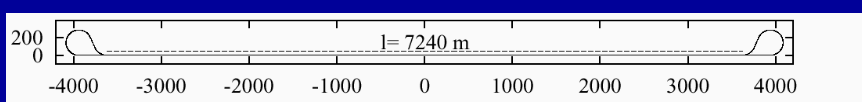

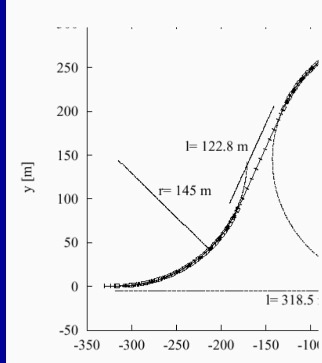

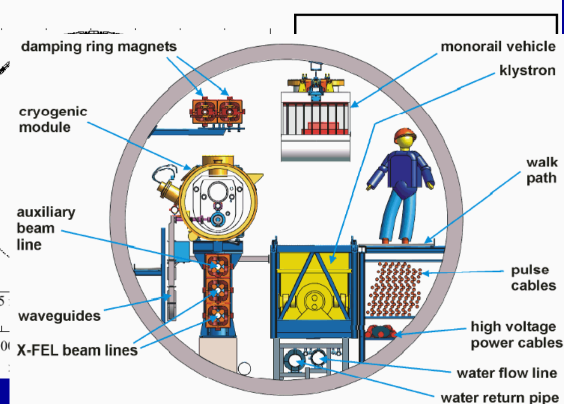

TESLA Damping Rings

Performance Specifications

	NLC MDR	TESLA e^+
Injected $\gamma\varepsilon$	150 µm rad	10 000 µm rad
Extracted Horizontal $\gamma\varepsilon$	3 µm rad	8 µm rad
Extracted Vertical $\gamma\varepsilon$	0.02 µm rad	0.02 µm rad
Injected Energy Spread	1% full width	1% full width
Extracted Energy Spread	0.1% rms	0.13%
Extracted Bunch Length	4 mm	6 mm
Bunch Spacing	1.4 ns	20 ns
Bunches per Train	192	2820
Repetition Rate	120 Hz	5 Hz

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Radiation Damping…

- Longitudinal phase space
  - Particles perform synchrotron oscillations in RF focusing potential
  - Higher energy particles radiate energy more quickly in bends
  - At the equilibrium energy, the revolution period is an integer times the RF period (the synchrotron principle…)
- Transverse phase space
  - Particles perform betatron oscillations around the closed orbit
  - Radiation is emitted in a narrow cone centered on the *instantaneous* direction of motion
  - Energy is restored by the RF cavities *longitudinally*
  - Combined effect of radiation and RF is a loss in transverse momentum
- Damping time in all planes is given by:

$$\mathcal{J}\tau = 2\frac{E_0}{U_0}T_0$$

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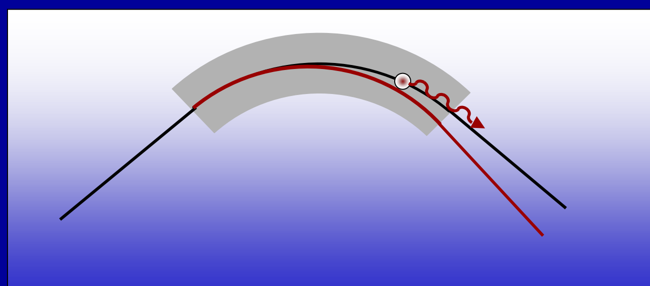
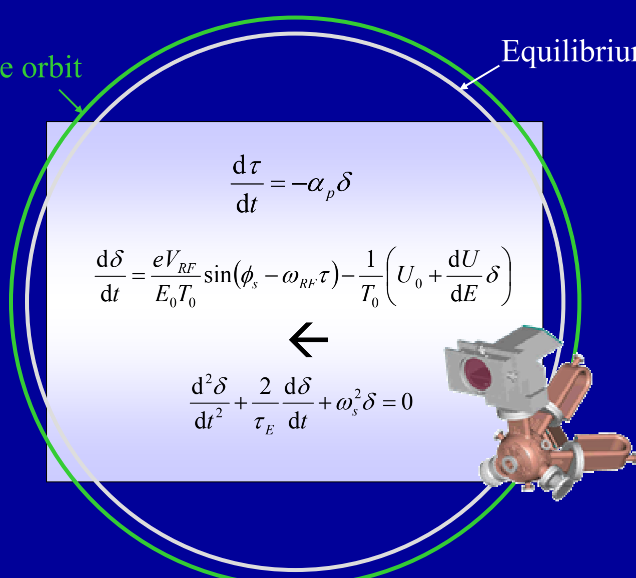
…and Quantum Excitation

- Radiation is emitted in discrete quanta
- Number and energy distribution etc. of photons obey statistical laws
- Radiation process can be modeled as a series of “kicks” that excite longitudinal and transverse oscillations



Synchrotron Oscillations



$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = -\alpha_p \delta$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{eV_{RF}}{E_0 T_0}\sin(\phi_s - \omega_{RF}\tau) - \frac{1}{T_0}\left(U_0 + \frac{\mathrm{d}U}{\mathrm{d}E}\delta\right)$$

$$\leftarrow$$

$$\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} + \frac{2}{\tau_E}\frac{\mathrm{d}\delta}{\mathrm{d}t} + \omega_s^2\delta = 0$$

Longitudinal Damping

$$\tau = \hat{\tau} e^{-t/\tau_E} \cos(\omega_s t - \theta_s)$$

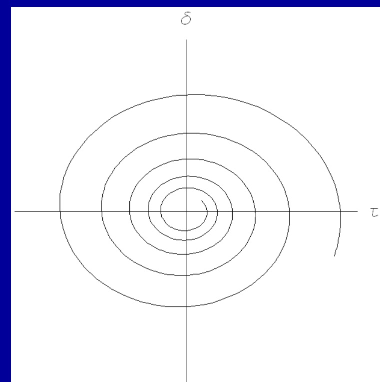
$$\delta = \frac{\omega_s}{\alpha_p} \hat{\tau} e^{-t/\tau_E} \sin(\omega_s t - \theta_s)$$

$$\omega_s^2 = -\frac{eV_{RF}}{E_0} \frac{\omega_{RF}}{T_0} \alpha_p \cos(\phi_s)$$

$$\mathcal{J}_E \tau_E = 2 \frac{E_0}{U_0} T_0$$

$$\mathcal{J}_E = 2 + \frac{I_4}{I_2}$$

$$I_2 = \oint \frac{1}{\rho^2} \mathrm{d}s \qquad I_4 = \oint \frac{\eta}{\rho} \left(\frac{1}{\rho^2} + 2k_1 \right) \mathrm{d}s$$



Problem 1
Show that:

$$\alpha_p = \frac{1}{C_0} I_1 = \frac{1}{C_0} \oint \frac{\eta}{\rho} \mathrm{d}s$$

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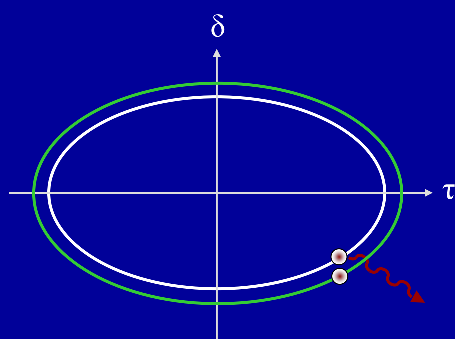
Quantum Excitation (Longitudinal)



$$\tau_1 = \frac{\alpha_p}{\omega_s} \hat{\delta}_1 \cos(\theta_1) = \frac{\alpha_p}{\omega_s} \hat{\delta} \cos(\theta)$$

$$\delta_1 = \hat{\delta}_1 \sin(\theta_1) = \hat{\delta} \sin(\theta) - \frac{u}{E_0}$$

$$\hat{\delta}_1^2 = \hat{\delta}^2 + \frac{u^2}{E_0^2} - 2 \frac{u}{E_0} \hat{\delta} \sin(\theta)$$

Including damping:

$$\frac{\mathrm{d}\hat{\delta}^2}{\mathrm{d}t} = \frac{1}{E_0^2 C_0} \oint N \langle u^2 \rangle \mathrm{d}s - 2 \frac{\hat{\delta}^2}{\tau_E}$$

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Equilibrium Longitudinal Emittance

- We have found that:

$$\frac{\mathrm{d}\hat{\delta}^2}{\mathrm{d}t} = \frac{1}{E_0^2 C_0}\oint N\langle u^2\rangle \mathrm{d}s - 2\frac{\hat{\delta}^2}{\tau_E}$$

- From synchrotron radiation theory:

$$\frac{1}{C_0}\oint N\langle u^2\rangle \mathrm{d}s = 4C_q \frac{\gamma^2 E_0^2}{\mathcal{J}_E \tau_E}\cdot\frac{I_3}{I_2} \qquad I_3 = \oint \frac{1}{|\rho|^3}\mathrm{d}s$$

Problem 2

Find an expression for the equilibrium energy spread, and show that:

$$\sigma_\delta = \sigma_{\delta 0}\mathrm{e}^{-t/\tau_\varepsilon} + \sigma_{\delta,equ}\left(1-\mathrm{e}^{-t/\tau_\varepsilon}\right)$$

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Betatron Oscillations: Action-Angle Variables

- It is often more convenient to describe betatron oscillations using action-angle variables:

$$2J = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

$$\tan(\phi) = -\alpha - \beta\frac{x'}{x}$$

- The old variables are related to the new ones by:

$$x = \sqrt{2\beta J}\cos(\phi)$$

$$x' = -\sqrt{\frac{2J}{\beta}}\left[\sin(\phi) + \alpha\cos(\phi)\right]$$

- The equations of motion take the simple form:

$$\frac{\mathrm{d}\phi}{\mathrm{d}s} = \frac{1}{\beta} \qquad \frac{\mathrm{d}J}{\mathrm{d}s} = 0$$

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Damping of Vertical Oscillations

- Radiation is emitted in a narrow cone (angle ~1/γ) around instantaneous direction of motion, so vertical co-ordinate and momentum are not changed by photon emission
- RF cavity changes longitudinal momentum, and hence the vertical direction of motion:

$$y_1 = y \qquad y_1' = \frac{p_y}{p + \delta p} \approx \frac{p_y}{p}\left(1 - \frac{\delta p}{p}\right) = y'\left(1 - \frac{\delta p}{p}\right)$$

- Averaging over all betatron phase angles gives (per turn):

$$\Delta J = -\frac{U_0}{E_0} J$$

Problem 3
Show this!

- Hence the equation of motion is:

$$\frac{\mathrm{d}J}{\mathrm{d}t} = -\frac{U_0}{E_0 T_0} J$$

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Damping of Horizontal Oscillations

- When a photon is emitted at a point where there is some dispersion, the co-ordinates with respect to the closed orbit change:

$$x_1 = x + \frac{u}{E_0}\eta \qquad x_1' = x' + \frac{u}{E_0}\eta'$$

- Taking the energy loss to first order and averaging around the ring, we find after some work:

$$\frac{\mathrm{d}J}{\mathrm{d}t} = -\left(1 - \frac{I_4}{I_2}\right)\frac{U_0}{E_0 T_0} J = -\mathcal{J}_x \frac{U_0}{E_0 T_0} J$$

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Quantum Excitation of Betatron Motion

- Let us now consider the second order effects. It is easy to show that the change in the action depends to second order on the photon energy as follows:

$$\Delta J = \frac{1}{2}\left(\frac{u}{E_0}\right)^2 \mathcal{H} \qquad \mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2$$

- Averaging over the photon spectrum and around the ring, and including the radiation damping gives:

$$\frac{\mathrm{d}J_x}{\mathrm{d}t} = \frac{1}{2E_0^2 C_0}\oint N\langle u^2\rangle \mathcal{H}\mathrm{d}s - \frac{2}{\tau_x}J_x$$

$$= C_q\gamma^2 \frac{2}{\mathcal{J}_x\tau_x}\frac{I_5}{I_2} - \frac{2}{\tau_x}J_x$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho|^3}\mathrm{d}s$$

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Summary of Dynamics with Radiation

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \varepsilon_{\mathrm{inj}}\mathrm{e}^{-2t/\tau} + \varepsilon_{\mathrm{equ}}\left(1-\mathrm{e}^{-2t/\tau}\right)$$

$$I_1 = \oint \frac{\eta}{\rho}\mathrm{d}s$$

$$I_2 = \oint \frac{1}{\rho^2}\mathrm{d}s$$

$$I_3 = \oint \frac{1}{|\rho|^3}\mathrm{d}s$$

$$I_4 = \oint \frac{\eta}{\rho}\left(\frac{1}{\rho^2} + 2k_1\right)\mathrm{d}s \qquad k_1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho|^3}\mathrm{d}s \qquad \mathcal{H} = \gamma\eta^2 + 2\alpha\eta\eta' + \beta\eta'^2$$

$$\mathcal{J}_x\tau_x = \mathcal{J}_y\tau_y = \mathcal{J}_E\tau_E = 2\frac{E_0}{U_0}T_0$$

$$\mathcal{J}_x = 1 - \frac{I_4}{I_2} \qquad \mathcal{J}_y = 1 \qquad \mathcal{J}_E = 2 + \frac{I_4}{I_2}$$

$$U_0 = \frac{C_\gamma}{2\pi}E_0^4 I_2$$

$$\sigma_\delta^2 = C_q\gamma^2\frac{I_3}{\mathcal{J}_E I_2} \qquad \sigma_\tau = \frac{\alpha_p}{\omega_s}\sigma_\delta$$

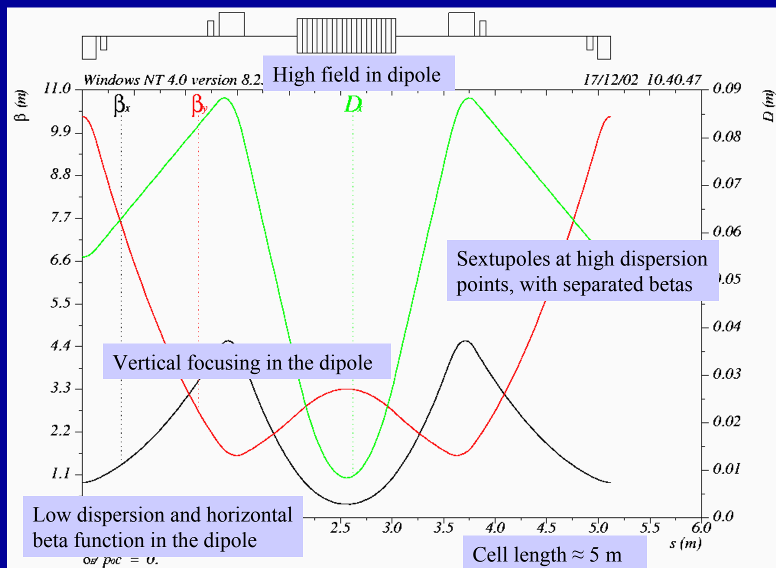
$$\alpha_p = \frac{I_1}{C_0}$$

$$\omega_s^2 = -\frac{eV_{RF}}{E_0}\frac{\omega_{RF}}{T_0}\alpha_p\cos(\phi_s)$$

$$\sin(\phi_s) = \frac{U_0}{eV_{RF}}$$

$$\varepsilon_0 = C_q\gamma^2\frac{I_5}{\mathcal{J}_x I_2}$$

The NLC TME Cell



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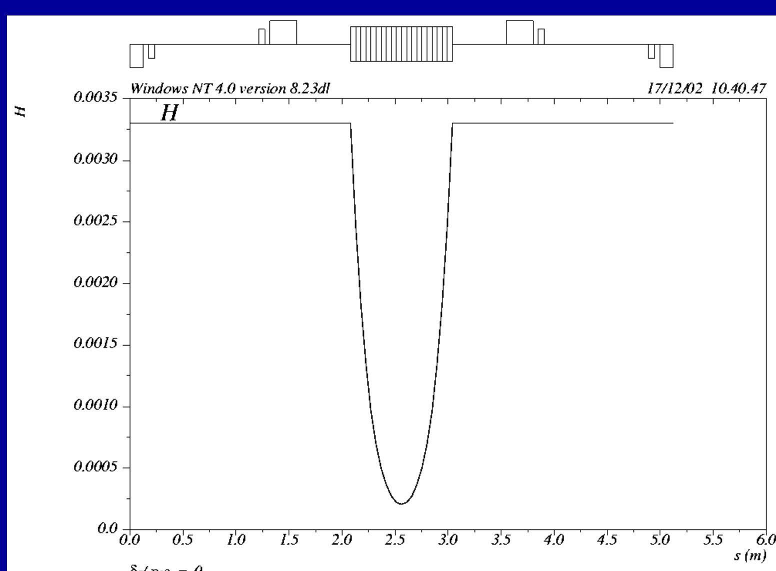
H Function in the NLC TME Cell



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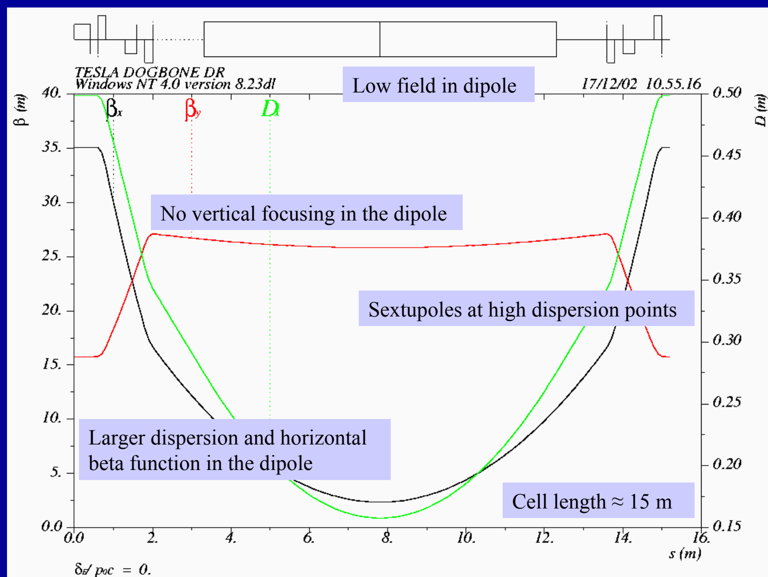
The TESLA TME Cell



NLC and TESLA TME Cells Compared

- NLC
  - Compact cell to keep circumference as short as possible
  - High dipole field for greater energy loss, reducing wiggler length
  - Short dipole requires very low values for dispersion and beta function
  - Gradient in dipole field to improve transverse dynamics
- TESLA
  - Circumference fixed by bunch train and kicker rise/fall time
  - Long dipole for larger momentum compaction, longer bunch
- Optimum lattice functions at center of dipole:

$$\beta_0 \approx \frac{L}{2\sqrt{15}} \qquad \eta_0 \approx \frac{L\theta}{24} \qquad \Rightarrow \qquad \varepsilon_{\min} \approx C_q \gamma^2 \frac{\theta^3}{J_x 12\sqrt{15}}$$

  - Obtained by minimizing I_5 for a ring without a wiggler
  - It is not usually possible to control the dispersion and beta function independently

Two Simple Scaling Relationships

Problem 4

Show that for an isomagnetic ring with the lattice functions tuned for minimum emittance:

$$N_{\text{cell}}B_0 = \frac{8\pi^2}{C_\gamma e m_e^2 c^6}\frac{L_{\text{cell}}}{\tau}\left(\frac{C_q}{12\sqrt{15}\gamma\varepsilon_0}\right)^{\frac{2}{3}}$$

$$2\pi\gamma B_0 = \frac{8\pi^2}{C_\gamma e m_e^2 c^6}\frac{L_{\text{cell}}}{\tau}\left(\frac{C_q}{12\sqrt{15}\gamma\varepsilon_0}\right)^{\frac{1}{3}}$$

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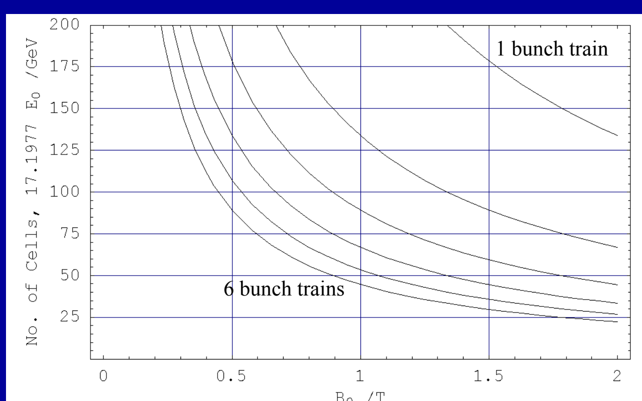
Scaling Relationships Applied to the NLC



$\gamma\varepsilon_0 = 3$ μm

$L_{\text{cell}} = 6$ m

$\tau = N_{\text{train}}$ 1.6 ms

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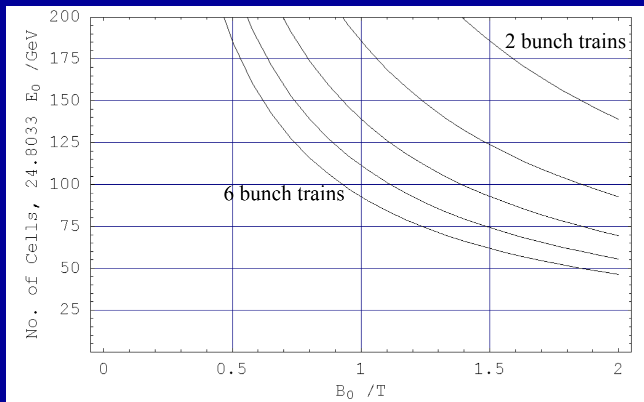
Scaling Relationships Applied to the NLC



$\gamma\varepsilon_0 = 1\ \mu\text{m}$
$L_{cell} = 6$ m
$\tau = N_{train}$ 1.6 ms

Damping Wiggler

- A wiggler reduces the damping time by increasing the energy loss per turn:

$$U_0 = \frac{C_\gamma}{2\pi} E_0^4 I_2 = \frac{e^2 c^2 C_\gamma}{2\pi} E_0^2 \oint B^2 \mathrm{d}s$$

- Wiggler must be located where nominal dispersion is zero, otherwise there can be a large increase in the natural emittance
- If horizontal beta function is reasonably small, wiggler can significantly *reduce* the natural emittance (through reduced damping time)
- Drawbacks include possible detrimental effect on beam dynamics

Types of Wiggler

- A wiggler is simply a periodic array of magnets, such that the field is approximately sinusoidal
- Different technologies are possible:
  - Electromagnetic
  - Permanent magnet
  - Hybrid (permanent magnets driving flux through steel poles)
- Choice of technology comes down to cost optimization for given requirements on field strength and quality
- Both TESLA and NLC damping rings have opted for hybrid technology

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Modeling the Dynamics in the Wiggler

- Magnet design is produced using a standard modeling code
- Field representation must be obtained in a form convenient for fast symplectic tracking

$$B_x = -\sum c_{mn} \frac{mk_x}{k_{y,mn}} \sin(mk_x x)\sinh(k_{y,mn} y)\cos(nk_z z)$$

$$\rightarrow \quad B_y = \sum c_{mn} \cos(mk_x x)\cosh(k_{y,mn} y)\cos(nk_z z) \quad \leftarrow$$

$$B_z = -\sum c_{mn} \frac{nk_z}{k_{y,mn}} \cos(mk_x x)\sinh(k_{y,mn} y)\sin(nk_z z)$$

$$k_{y,mn}^2 = m^2 k_x^2 + n^2 k_z^2$$

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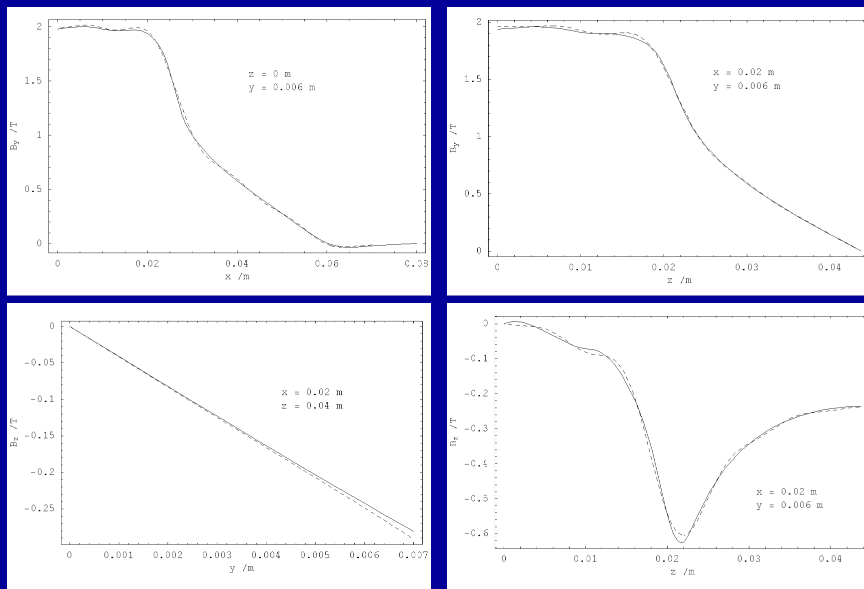
Fitting the Wiggler Field



Tracking Through the Field

- Using an appropriate field representation (that satisfies Maxwell's equations), one can construct a *symplectic integrator*:

$$\vec{x}_{\text{new}} = \vec{m}\left(\vec{x}_{\text{old}}\right)$$

- *M* is an explicit function of the phase-space co-ordinates, and satisfies the symplectic condition (so the dynamics obey Hamilton's Equations):

$$M \cdot S \cdot M^{\mathrm{T}} = S$$

$$M_{ij} = \frac{\partial m_i}{\partial x_j}$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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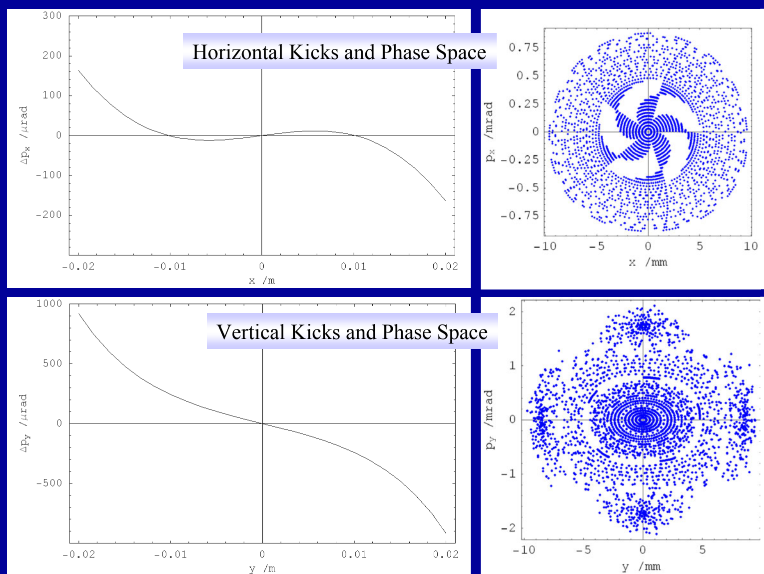
Dynamics in the NLC Wiggler



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Chromaticity

- Chromaticity is the tune variation with energy
- Quadrupole focusing strength gets smaller as particle energy increases
- It can easily be shown that:

$$\xi_x = \frac{\partial \nu_x}{\partial \delta} = -\frac{1}{4\pi}\oint \beta_x k_1 \mathrm{d}s$$

$$\xi_y = \frac{\partial \nu_y}{\partial \delta} = \frac{1}{4\pi}\oint \beta_y k_1 \mathrm{d}s$$

- Since beta functions peak at the focusing quadrupoles in the appropriate plane, the natural chromaticity is always negative
- Chromaticity is connected to beam instabilities
  - particles with large energy deviation cross resonance lines
  - some collective effects (e.g. head-tail instability) are sensitive to the chromaticity

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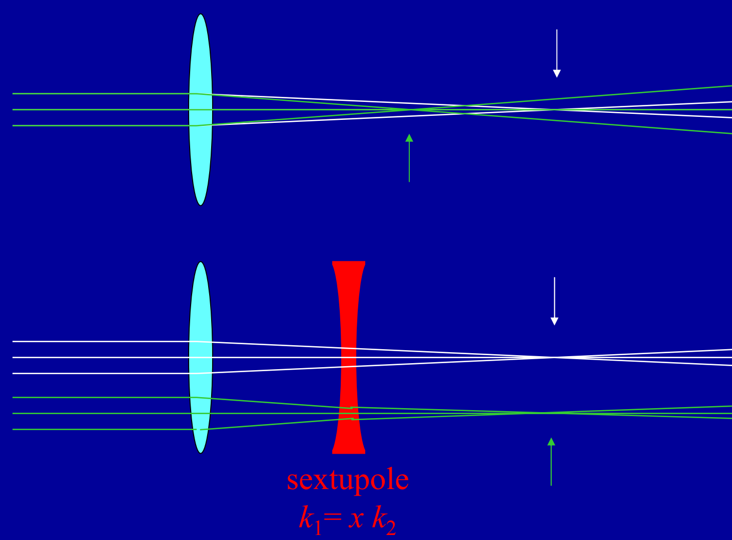
Correcting Chromaticity with Sextupoles



Dynamics with Sextupoles

- Sextupoles can be used to correct chromatic aberrations…

$$\xi_x = -\frac{1}{4\pi}\oint \beta_x k_1 - \eta_x \beta_x k_2 \, \mathrm{d}s$$

$$\xi_y = \frac{1}{4\pi}\oint \beta_y k_1 + \eta_x \beta_y k_2 \, \mathrm{d}s$$

- …but introduce geometric aberrations and coupling:

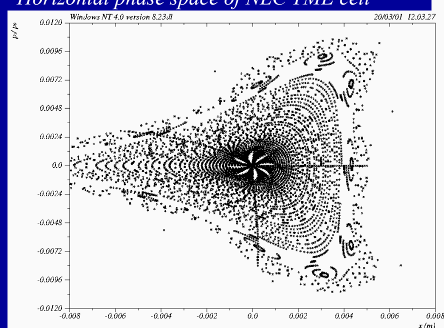
$$\Delta x' \approx \tfrac{1}{2} k_2 l \cdot x^2 \qquad \Delta y' \approx -k_2 l \Delta Y \cdot x$$

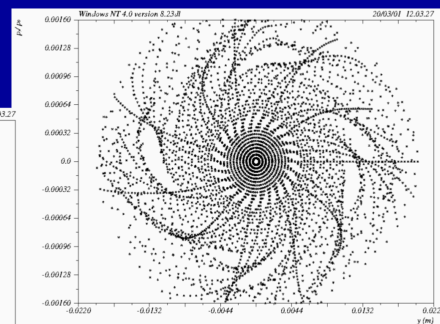
- It is important to keep the required strengths to a minimum by designing the linear lattice functions for effective sextupole location

Dynamic Aperture

- Geometric aberrations from sextupoles (and other sources) distort the transverse phase space, and limit the amplitude range of stable betatron oscillations

Horizontal phase space of NLC TME cell





Vertical phase space of NLC TME cell

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Transverse and Longitudinal Aperture

- Damping rings require a "large" dynamic aperture
  - Injected beam power ~ 50 kW average, and radiation load from any significant injection losses will destroy the ring
  - Nonlinear distortion of the phase space may lead to transient emittance growth from inability properly to match injected beam to the ring
  - For NLC Main Damping Rings, the target dynamic aperture is 15 times the injected rms beam size
- We also need a large momentum acceptance
  - Injected beam has a large energy spread
  - Particles may be lost from insufficient physical aperture in dispersive regions, or through poor off-momentum dynamics
  - Particles within a bunch can scatter off each other, leading to a significant change in energy deviation (Touschek Effect)
- It is important to perform tracking studies with full dynamic model and physical apertures

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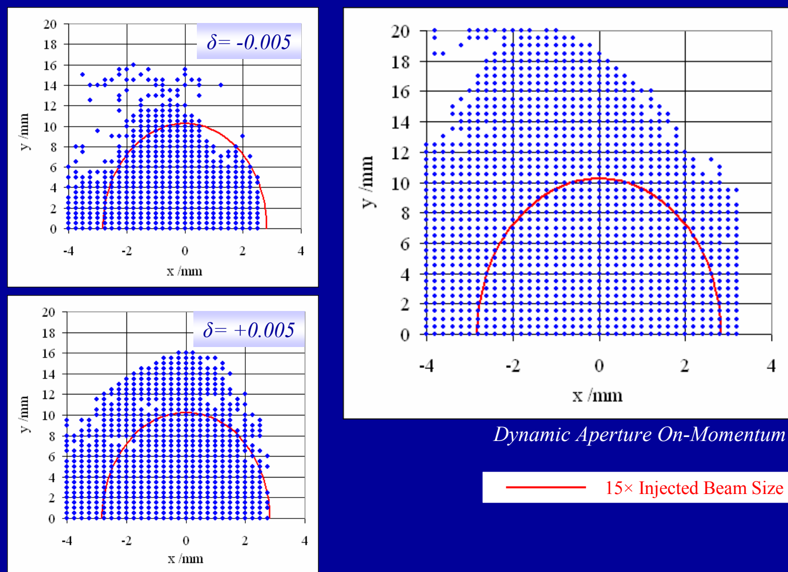
NLC Main Damping Ring Dynamic Aperture



Dynamic Aperture On-Momentum

15× Injected Beam Size

Longitudinal Acceptance

- The longitudinal acceptance has three major limitations:
  - Poor off-momentum dynamics
  - Physical aperture in dispersive regions
  - RF bucket height
- Off-momentum dynamics can be difficult to quantify
  - see previous slides
- Physical aperture can be a significant limitation
  - 1% momentum deviation in 1 m dispersion is a 1 cm orbit offset
- RF bucket height comes from non-linearity of the longitudinal focusing
  - Previous study of longitudinal dynamics assumed a linear slope of RF voltage around the synchronous phase
  - Valid for small oscillations with synchronous phase close to zero-crossing

RF Bucket Height

- The "proper" equations of longitudinal motion (without damping) are:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = -\alpha_p \delta$$
$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{eV_{RF}}{E_0 T_0}\left[\sin(\phi_s - \omega_{RF}\tau) - \sin(\phi_s)\right]$$

- These may be derived from the Hamiltonian:

$$H = -\tfrac{1}{2}\alpha_p \delta^2 - \frac{eV_{RF}}{E_0 T_0 \omega_{RF}}\left[\cos(\phi_s - \omega_{RF}\tau) - \sin(\phi_s)\omega_{RF}\tau\right]$$
$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{\partial H}{\partial \delta}$$
$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = -\frac{\partial H}{\partial \tau}$$

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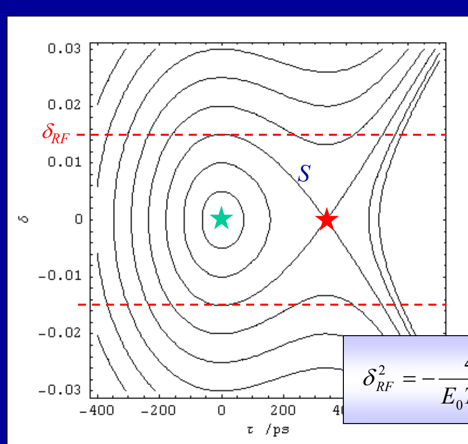
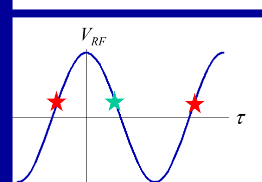
Longitudinal Phase Space

- The Hamiltonian is a constant of the motion, which allows us to draw a phase-space portrait



Stable fixed point
Unstable fixed point
S Separatrix



$$\delta_{RF}^2 = -\frac{4eV_{RF}}{E_0 T_0 \omega_{RF} \alpha_p}\left[\cos(\phi_s) + \left(\phi_s - \frac{\pi}{2}\right)\sin(\phi_s)\right]$$

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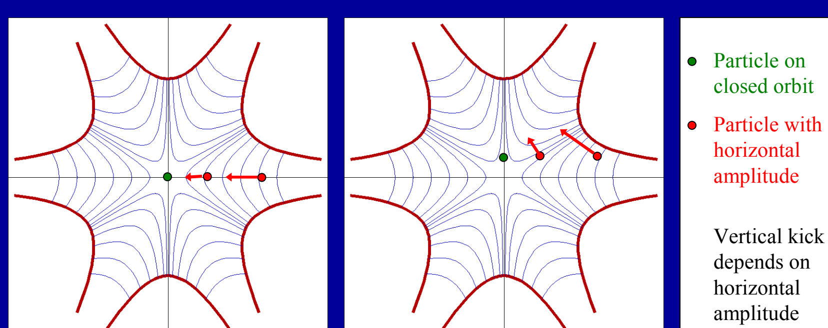
Alignment Issues

- The final luminosity of the collider is critically dependent on the vertical emittance extracted from the damping rings
- In a perfectly flat lattice, the lower limit on the vertical emittance comes from the opening angle of the radiation
  - Gives about 10% of the specified values for NLC and TESLA
- Magnet misalignments give the dominant contribution to the vertical emittance
  - Quadrupole vertical misalignments
    - Vertical dispersion
    - Vertical beam offset in sextupoles
  - Quadrupole rotations and sextupole vertical misalignments
    - Couple horizontal dispersion into the vertical plane
    - Couple horizontal betatron oscillations into the vertical plane

Betatron Coupling

- In a damping ring, the dominant sources of betatron coupling are skew quadrupole fields
  - Normal quadrupoles have some “roll” about the beam axis
  - Sextupoles have some vertical offset with respect to the closed orbit
- Particles with a horizontal offset get a vertical kick



Effects of Betatron Coupling

- In action-angle variables, the "averaged Hamiltonian" for a coupled storage ring can be written:

$$\frac{C_0}{2\pi} H = \nu_x J_x + \nu_y J_y + |\tilde{\kappa}_n| \sqrt{J_x J_y} \cos(\phi_x - \phi_y)$$

- The equations of motion are:

$$\frac{\mathrm{d}J_x}{\mathrm{d}s} = |\tilde{\kappa}| \sqrt{J_x J_y} \sin(\phi_x - \phi_y) \qquad \frac{\mathrm{d}\phi_x}{\mathrm{d}s} = \frac{2\pi}{C_0}\left[\nu_x + \frac{|\tilde{\kappa}|}{2}\sqrt{\frac{J_y}{J_x}} \cos(\phi_x - \phi_y)\right]$$

$$\frac{\mathrm{d}J_y}{\mathrm{d}s} = -|\tilde{\kappa}| \sqrt{J_x J_y} \sin(\phi_x - \phi_y) \qquad \frac{\mathrm{d}\phi_y}{\mathrm{d}s} = \frac{2\pi}{C_0}\left[\nu_y + \frac{|\tilde{\kappa}|}{2}\sqrt{\frac{J_x}{J_y}} \cos(\phi_x - \phi_y)\right]$$

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Solutions to the Coupled Hamiltonian

- The sum of the horizontal and vertical actions is conserved:

$$J_0 = J_x + J_y$$

$$\frac{\mathrm{d}J_0}{\mathrm{d}s} = 0$$

- There are fixed points at:

$$J_x = \frac{1}{2} J_0 \left(1 + \frac{\Delta}{\sqrt{\Delta^2 + |\tilde{\kappa}|^2}}\right)$$

$$J_y = \frac{1}{2} J_0 \left(1 - \frac{\Delta}{\sqrt{\Delta^2 + |\tilde{\kappa}|^2}}\right)$$

$$\Delta = |\nu_x - \nu_y|$$

- With radiation, the actions will damp to the fixed points

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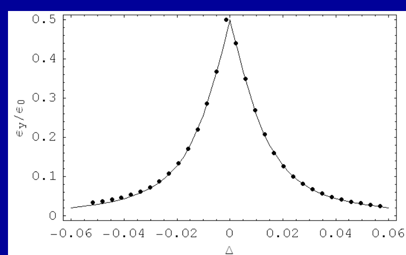
The Difference Coupling Resonance

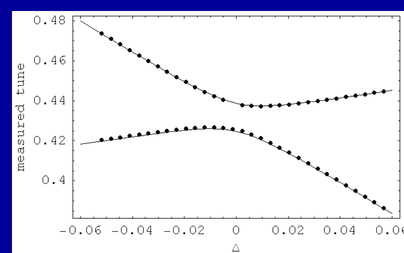
- The equilibrium emittance ratio is given by:

$$\frac{J_y}{J_x} = \frac{|\tilde{\kappa}|^2}{|\tilde{\kappa}|^2 + 4\Delta^2}$$

- The measured tunes are given by:

$$\frac{C_0}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}s} = \tfrac{1}{2}\Sigma \pm \tfrac{1}{2}\sqrt{\Delta^2 + |\tilde{\kappa}|^2} \qquad \Sigma = \nu_x + \nu_y$$





What is the Coupling Strength?

- We add up all the skew fields around the ring with an appropriate phase factor:

$$\tilde{\kappa}_n = \frac{1}{2\pi}\int_0^{C_0}\sqrt{\beta_x\beta_y}\,k_s e^{i\chi_n(s)}\mathrm{d}s \qquad \chi_n(s) = (\mu_x - \mu_y) - 2\pi(\nu_x - \nu_y - n)\frac{s}{C_0}$$

- k_s is the skew quadrupole k-value.
- For a rotated quadrupole or vertically misaligned sextupole, the equivalent skew fields are given by:

$$k_s = k_1 \sin(2\theta) \qquad k_s = k_2 \Delta y$$

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Vertical Dispersion

- In an electron storage ring, the vertical dispersion is typically dominated by betatron coupling
  - Emittance ratios of 1% are typical
- For very low values of the vertical emittance, vertical dispersion starts to make a significant contribution
- Vertical dispersion is generated by:
  - Vertical steering
    - vertically misaligned quadrupoles
  - Coupling of horizontal dispersion into the vertical plane
    - quadrupole rotations
    - vertical sextupole misalignments

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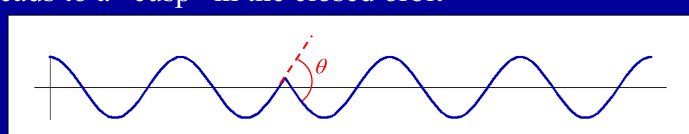
Vertical Steering: Closed Orbit Distortion

- A quadrupole misalignment can be represented by a kick that leads to a “cusp” in the closed orbit



- We can write a condition for the closed orbit in the presence of the kick:

$$\mathbf{M}\cdot\begin{pmatrix} y_0 \\ y_0' \end{pmatrix} = \begin{pmatrix} y_0 \\ y_0' - \theta \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} \cos(\mu_x) + \alpha_x \sin(\mu_x) & \beta_x \sin(\mu_x) \\ -\gamma_x \sin(\mu_x) & \cos(\mu_x) - \alpha_x \sin(\mu_x) \end{pmatrix}$$

- We can solve to find the distortion resulting from many kicks:

$$y(s_1) = \frac{\sqrt{\beta_y(s_1)}}{2\sin(\pi\nu_y)} \oint \frac{\sqrt{\beta_y(s)}}{\rho(s)} \cos\left(\pi\nu_y - \left|\phi_y(s_1) - \phi_y(s)\right|\right) \mathrm{d}s$$

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Vertical Steering: Vertical Dispersion

- The vertical dispersion obeys the same equation of motion as the vertical orbit, but with a modified driving term:

$$y'' - k_1 y = \frac{1}{(1+\delta)\rho} \quad \xrightarrow{\text{differentiate wrt } \delta} \quad \eta_y'' - k_1\eta_y = -k_1 y - \frac{1}{\rho}$$

- We can immediately write down the vertical dispersion arising from a set of steering errors:

$$\eta_y(s_1) = -\frac{\sqrt{\beta_y(s_1)}}{2\sin(\pi\nu_y)}\oint\sqrt{\beta_y(s)}\left(k_1 y + \frac{1}{\rho}\right)\cos\left(\pi\nu_y - \left|\phi_y(s_1) - \phi_y(s)\right|\right)\mathrm{d}s$$

- Including the effect of dispersion coupling:

$$\eta_y(s_1) = -\frac{\sqrt{\beta_y(s_1)}}{2\sin(\pi\nu_y)}\oint\sqrt{\beta_y(s)}\left[(k_1 - k_2\eta_x)y + k_s\eta_x + \frac{1}{\rho}\right]\cos\left(\pi\nu_y - \left|\phi_y(s_1) - \phi_y(s)\right|\right)\mathrm{d}s$$

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Effects of Uncorrelated Alignment Errors

- Closed orbit distortion from quadrupole misalignments:

$$\left\langle\frac{y^2}{\beta_y}\right\rangle = \frac{\left\langle Y_q^2\right\rangle}{8\sin^2(\pi\nu_y)}\sum\beta_y(k_1 l)^2$$

- Vertical dispersion from quadrupole rotation and sextupole misalignment:

$$\left\langle\frac{\eta_y^2}{\beta_y}\right\rangle = \frac{\left\langle\Theta_q^2\right\rangle}{2\sin^2(\pi\nu_y)}\sum\beta_y(k_1 l\eta_x)^2 \qquad \left\langle\frac{\eta_y^2}{\beta_y}\right\rangle = \frac{\left\langle Y_s^2\right\rangle}{8\sin^2(\pi\nu_y)}\sum\beta_y(k_2 l\eta_x)^2$$

- Vertical emittance generated by vertical dispersion:

$$\varepsilon_y \approx 2\frac{\mathcal{J}_E}{\mathcal{J}_y}\left\langle\frac{\eta_y^2}{\beta_y}\right\rangle\sigma_\delta^2$$

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Examples of Alignment Sensitivities

	APS	SLS	KEK-ATF	ALS	NLC MDR	TESLA DR
Energy [GeV]	7	2.4	1.3	1.9	1.98	5
Circumference [m]	1000	288	140	200	300	17,000
$\gamma\varepsilon_x$ [µm]	34	23	2.8	24	3	8
$\gamma\varepsilon_y$ [nm]	140	70	28	20	19	14
Sextupole vertical [µm]	74	71	87	30	53	11
Quadrupole roll [µrad]	240	374	1475	200	511	38
Quadrupole jitter [nm]	280	230	320	230	264	76

- Note:
  Sensitivity values give the random misalignments that will generate a specified vertical emittance. In practice, coupling correction schemes mean that significantly larger misalignments can be tolerated.

Collective Effects

- Issues of damping, acceptance, coupling are all *single particle* effects - they are independent of the beam current
- Particles in a storage ring interact with each other (directly or via some intermediary e.g. the vacuum chamber)
- A wide variety of collective effects limit the achievable beam quality, depending on the bunch charge or total current
- The consequences of collective effects are
  - Phase space distortion and/or emittance growth
  - Particle loss
- Damping rings have high bunch charges, moderate energies and small emittance
  - Vulnerable to a wide range of collective effects
- Too wide a subject to enter into here!