An Introduction to the Physics and Technology of e+e- Linear Colliders

Lecture 3a: Main Linac, Continued

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Introducing the Beam

Let's consider a beam which is a steady stream of bunches, RMS length of 1 bunch = σ_z . The bunches are spaced at a harmonic of the microwave frequency of the structures, and are timed to arrive at the RF crest (for now).

Consider an infinite train of bunches forming an average current I_{beam}...

The beam is accelerated and thus removes energy from the RF structure. In the steady state, we can conserve energy:

$$\frac{dP}{dz} = -p_w(z) - I_{beam} E(z)$$

Introducing the Beam (2)

 $\omega P(z)$

 $v_{gr}Q_{w}$

ince:
$$\frac{G_0^2}{2}$$

(z)

taking a z-derivative of both sides yields:

$$2G_0(z)\frac{dG_0(z)}{dz} = \frac{r_i\omega}{v_{gr}Q_w}\frac{dP(z)}{dz}$$
 And we can replace dP/dz:

$$2G_{0}(z)\frac{dG_{0}(z)}{dz} = \frac{r_{l}\omega}{v_{gr}Q_{w}} \left[-p_{w}(z) - I_{beam}G_{0}(z)\right]$$
$$= -\frac{r_{l}\omega}{v_{gr}Q_{w}} \left[I_{beam}G_{0}(z) + \frac{G_{0}^{2}(z)}{r_{l}}\right]$$

A differential equation for the accelerating gradient, including the unloaded term (last term) and the beam current term...

Steady-State Beam Loading

For a Constant Impedance Structure:

$$V = \sqrt{r_{j}LP_{0}} \sqrt{2\tau} \frac{1 - e^{-\tau}}{\tau} - r_{j}LI_{hours} \left[1 + \frac{1}{\tau} \left(1 - e^{-\tau} \right) \right]$$

For the Constant Gradient Structure:

$$V = \sqrt{r_{l}LP_{0}}\sqrt{1 - e^{-2\tau}} - \frac{1}{2}r_{l}LI_{beam}\left(1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}}\right)$$

Note that if $P_0 \rightarrow 0$ the first term (voltage from applied RF power) is zero but second term (decelerating voltage from beam) is unchanged.

:: Beam passing thru the structure excites the structure in a decelerating phase at its resonance!

Acceleration Efficiency

How efficient is our accelerator? Consider the power extracted by the beam $(I_{beam}V)$ compared to the power supplied (P_0) . For a CG structure:

$$\eta = \frac{VI_{beam}}{P_0} = I_{beam} \sqrt{\frac{r_l L}{P_0}} \sqrt{1 - e^{-2\tau}} - \frac{1}{2} I_{beam}^2 r_l \frac{L}{P_0} \left(1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}}\right)$$

The current which maximizes the efficiency can be determined:

$$I_{opt} = \sqrt{\frac{P_0}{r_l L}} \frac{\left(1 - e^{-2\tau}\right)^{3/2}}{1 - (1 + 2\tau)e^{-2\tau}}$$

At this current, no power reaches the output coupler -- it all goes either into the structure walls or the beam. Also, the voltage is reduced by a factor of 2 from the unloaded case.

Efficiency (2)

For a given τ , there is a maximum efficiency: $\eta_{\max}(\tau) = \frac{1}{2} \frac{(1 - e^{-2\tau})^2}{1 - (1 + 2\tau)e^{-2\tau}}$

For small τ , $\eta_{max} \sim 1$, while for large $\tau \eta_{max} \rightarrow 0.5$.

Since we need to fill the structure with power before the beam shows up, and this takes a time t_f , there is another efficiency factor which is the ratio of the beam time to the sum of beam and fill time. When the dependence on current is folded in, we find:

$$\eta = \eta_{\max}\left(\tau\right) \left[2\frac{I_{beam}}{I_{opt}} - \frac{I_{beam}^{2}}{I_{opt}^{2}}\right] \frac{t_{beam}}{t_{beam} + t_{fill}}$$

High Beam Loading

Since electric power costs money, why would anyone ever operate below maximum efficiency? For one thing, the accelerating voltage at 100% loading is only 50% as large as the unloaded voltage, and some people like to operate at higher energy (**trading current for voltage**).

The accelerator is also more sensitive. If there is some train-to-train variation in current, this translates directly into voltage variation:



At full loading, 1% current variation = 1% voltage variation at 20% loading, 1% current variation = 0.2% voltage variation.

Transient Beam Loading

At the instant before the beam arrives, the structure has a voltage given by the unloaded expression.

After 1 filling time, the voltage has "sagged" to the steady-state value.

In-between, the beam acceleration varies from full to loaded. This is generally unacceptable, and requires compensation.

.Delta-T compensation: Inject the beam before 1 fill time is complete.

.Delta-V compensation: Vary the power in the first t_f to achieve a steady-state voltage when the beam first arrives, then go to nominal power.

.Delta-f compensation: Use structures with different frequencies; the beam is off-crest for the off-frequency units and gets a different acceleration.

Single-Bunch Loading

A passing beam bunch excites *all longitudinal modes* in the structure. Since they have different frequencies and many have low Q's, by the time the 2nd bunch arrives only the fundamental mode remains (usually).

Within 1 bunch, the higher modes can still affect the beam energy -- the passage of the bunch head lowers the voltage seen by the bunch tail.

The decelerating field a distance z behind a charge q in a structure is given approximately by $qW_1(z)$, where:

 $W_L(z) = \frac{Zc}{\pi a^2} \exp\left(-\sqrt{\frac{z}{s_z}}\right)$ $s_z \approx 0.41 \frac{a^{1.8} g^{1.6}}{a^{2.4}} \qquad \text{and } g = d - h.$

Self-Loading

Since $W_{L}(z) > 0$ for z=0, a single electron can load itself!

We can use conservation of energy to show that the self-loading field is given by:

$$E_{self} = \frac{q}{2} W_L \left(z = 0 \right)$$

ie, half as large as we would naively expect. This is called the *Fundamental Theorem of Beam Loading*.

Compensation of Short-Range Loading

Consider a 2-particle model in which we have 2 charges of q/2 separated by a distance of $2\sigma_z$ (so the RMS length is still σ_z). The mean and RMS energy loss can be analytically estimated:

$$\left\langle \Delta V \right\rangle = \frac{Lq}{4} W_L(0) \left(1 + e^{-\Delta} \right)$$
$$\sigma_V = \frac{Lq}{4} W_L(0) e^{-2\Delta}$$
$$\Delta = \sqrt{2\sigma_z / s_z}$$

$$\phi \approx \frac{LqW_L(0)}{8\pi V} \frac{\lambda}{\sigma_{-}}$$

The average loss must be tolerated.

The RMS loss can be compensated, since the tail loses more than the head.

Need to put the beam "ahead of the RF crest," so the sinusoidal change in voltage cancels the loading.

Note that we give up some acceleration to do this!

Transverse Wakefields: the HEM₁₁ Mode

In regular waveguide, TM_{11} mode has $k_c = 3.832 / b$

 TE_{11} mode has $k_c = 1.841 / b$

Impossible to have identical dispersion relations for these modes.

Consider a DLWG, limited to r < a. By excluding r=b, we eliminate the BC that $E_{r,\theta} = 0$ @ r = b, but $E_{\theta} \rightarrow 0$ @ r = a.

This can be done by making E_{θ} from the two modes cancel \rightarrow a relationship in the amplitudes of the modes!

Thus a hybrid of TE_{11} and TM_{11} -- the HEM₁₁ mode -- can propagate in a DLWG.

HEM₁₁ Mode and Beam Break Up (BBU)

The HEM_{11} mode is a dipole mode -- beam on-axis in the DLWG does not induce it, only off-center beam.

Causes a dipole kick to the beam (beam passing thru DLWG above the axis gets an upward kick).

Consider a train of bunches with an initial offset w.r.t. the accelerator. Each bunch excites the HEM_{11} mode -- it's not resonant with the bunch spacing (hopefully!) so each bunch in the train gets a different kick, but later bunches get a larger kick than earlier ones (more bunches driving the mode).

The kicks add coherently down the linac (180° in betatron phase later, beam positions and offsets have both changed signs) -- initially straight train gets a curvature to it which grows along the linac.

Beam Break-Up instability (BBU).

How to fight BBU

- Wait for HEM₁₁ excitation to decay between bunches
 - usually takes too long
- Use a low fundamental frequency – HEM₁₁ mode deflection ~ ω^3
- Use a low charge
- Limit injection jitter
- Strengthen the focusing lattice – ie, more quadrupoles
- Damp the dipole modes
 - so they decay faster

How to fight BBU (2) -- detuning

Adjust the parameters of the cells s.t. they have the same fundamental mode frequency but different HEM_{11} frequencies.

The beam will excite the HEM_{11} modes in each cell

The different frequencies will cause the deflections in 1 structure to "beat" against one another.

In theory, if RMS spread in HEM₁₁ frequencies is σ_{ω} , net deflection will decay with exp(-t² $\sigma_{\omega}^{2}/2$) (ie, very fast).

In practice: limited number of cells/structure makes the decay slower, and modes can *recohere* after some time.

Short-Range Transverse Wakefields

Like beam loading, excitation of transverse modes by a single bunch can cause beam-dynamical effects within that bunch.

The deflecting field a distance z behind a particle with charge q and transverse offset $x = xqW_{\perp}(z)$, where:

$$W_{\perp}(z) = \frac{4Zcs_{\perp}}{\pi a^4} \left[1 - \left(1 + \sqrt{\frac{z}{s_{\perp}}} \right) \exp\left(-\sqrt{\frac{z}{s_{\perp}}} \right) \right]$$
$$s_{\perp} = 0.169 \frac{a^{1.79} g^{.38}}{a^{1.17}}$$

Note that $W_{\perp}(0) = 0$ (a single electron can't deflect itself),

and W' \perp (0) = 2Zc/ πa^4

Short-range Transverse Wakefields (2)

To estimate the effect of transverse wakes, consider again a bunch represented by 2 macroparticles moving through an accelerator with constant transverse focusing (coefficient k_{β}). For now neglect acceleration (beam energy constant) and assume that the bunch is short enough to approximate $W_{\perp}(2\sigma_z) \sim 2\sigma_z W_{\perp}^2(0)$.

The first particle undergoes a $\ddot{y}_1 + k_\beta^2 y_1 = 0$ betatron oscillation:

The second particle sees the same focusing, plus a driving term from the wakefield of the first particle:

$$\ddot{y}_2 + k_\beta^2 y_2 = y_1 \frac{W'_\perp \frac{q}{2} 2\sigma_z}{E_{beam}}$$

Short-Range Transverse Wakefields (3)

If both macroparticles have an initial offset y_0 then particle 1 undergoes a sinusoidal oscillation, $y_1 = y_0 \cos(k_B s)$. What happens to particle 2?

$$y_{2} = y_{0} \left[\cos\left(k_{\beta}s\right) + s\sin\left(k_{\beta}s\right) \frac{W'_{\perp} q\sigma_{z}}{2k_{\beta}E_{beam}} \right]$$

Qualitatively: an additional oscillation out-of-phase with the betatron term which grows monotonically with s.

How do we beat it? Higher beam energy, stronger focusing, lower charge, shorter bunches, or a damping technique recommended by Balakin, Novokhatski, and Smirnov (*BNS Damping*)

BNS Damping

Imagine that the two macroparticles have different betatron frequencies, represented by different focusing constants k_{B1} and k_{B2}

The second particle now acts like an undamped oscillator driven off its resonant frequency by the wakefield of the first. The difference in trajectory between the two macroparticles is given by:

$$y_{2} - y_{1} = y_{0} \left(1 - \frac{W'_{\perp} q \sigma_{z}}{E_{beam}} \frac{1}{k_{\beta 2}^{2} - k_{\beta 1}^{2}} \right) \left[\cos(k_{\beta 2} s) - \cos(k_{\beta 1} s) \right]$$

BNS Damping (2)

Two approaches to curing the short-range wakefield via the BNS mechanism:

1. Adjust the focusing of the macroparticles to achieve beating between their oscillations, ie,

 $\cos(k_{\beta 2}s) - \cos(k_{\beta 1}s) = 0$

Assuming we achieve this at the end of the linac (can't be true for all s), then

$$k_{\beta 2} - k_{\beta 1} = \frac{2\pi n}{L_{linac}}$$

Note that in this case the variation in focusing needed is independent of the wakefield strength.

BNS Damping (3)

2. The wakefield can be locally cancelled (ie, cancelled at all points down the linac) if:

$$\frac{W'_{\perp} q\sigma_z}{E_{beam}} \frac{1}{k_{\beta 2}^2 - k_{\beta 1}^2} =$$

This condition is often known as "autophasing."

It can be achieved by introducing an energy difference between the head and tail of the bunch. When the requirements of discrete focusing (ie, FODO lattices) are included, the autophasing RMS energy spread is given by:

$$\frac{\sigma_E}{E_{beam}} = \frac{1}{16} \frac{W'_{\perp} q \sigma_z}{E_{beam}} \frac{L_{cell}^2}{\sin^2(\pi \nu_{\beta})}$$

Off-Crest Acceleration

Both BNS damping and short-range wakefield compensation require accelerating the beam off the crest of the RF. What are the implications of this?

Recall that the steady-state loading is always at the beam phase, while the acceleration is not. For acceleration at a phase ϕ from the RF crest, the optimal current and acceleration efficiency become:

$$I_{\phi} = I_{opt} \cos \phi$$

$$\eta_{\phi}(\tau) = \eta_{max}(\tau) \cos^{2} \phi$$

$$\eta = \eta_{max}(\tau) \cos^{2} \phi \left[2 \frac{I_{beam}}{I_{opt} \cos \phi} - \frac{I_{beam}^{2}}{I_{opt}^{2} \cos^{2} \phi} \right] \frac{t_{beam}}{t_{beam} + t_{f}}$$

Off-Crest Acceleration (2)

The purpose of off-crest acceleration is to introduce a variation in the energy gain -- either for BNS damping or for the cancellation of short-range loading. Since

$$V_{beam} = V_{noload} \cos \phi + V_{mbload} + V_{sbload}$$
$$V'_{beam} = k_z V_{noload} \sin \phi + V'_{sbload}$$

The phase for off-crest running must be calculated using the desired voltage slope and the *unloaded* voltage, not the *loaded* voltage!

Field Emission and Dark Current

Electrons in the surface of an RF structure are held by fields on the order of eV/angstrom (10⁴) MeV/meter, while a very high-gradient accelerator can reach 10² MeV/m. So the RF field in an accelerator should never be able to extract electrons from the surface of the structure.

On the other hand, field emission is an empirically-observed fact. How can this be?

.Microscopic imperfections (bumps and scratches) can raise the field on the surface of the structure by a factor of 100.

.Quantum tunneling permits field emission at gradients which are too low for classical emission.

Field Emission and Dark Current (2)

Electrons emitted from the surface have very low energies and velocities. They may be captured by the RF if the field is high enough to get the particles relativistic (ie, synchronous with the RF) before the decelerating phase of RF overtakes them. Mathematically,

$$E_{capture} = 1.6 MeV \frac{\omega}{2\pi c}$$

RF Breakdown

"RF Breakdown" is a poorly-understood but omnipresent phenomenon in which the following behaviors are observed:

- Sudden increase in dark current and X-rays emitted from RF structure
- Simultaneously, RF input power to structure is partially or completely reflected back to the power source
- As a result of the loss of incoming RF power the accelerating field in the structure drops.
- Simultaneously with all this, the pressure in the structure rises suddenly.

No structure can operate acceptably while breaking down.

RF Breakdown (2)

It appears that RF breakdown begins with field-emission at a site in the structure. A large field-emitted current flow causes surface heating, leading to vaporization and plasma formation.

The plasma forms an arc which acts like a wire in the structure, absorbing huge amounts of energy and causing a local change in the structure's impedance.

RF Processing

A newly-fabricated RF structure will break down frequently at low gradient.

As the structure is operated the breakdown rate at a given gradient (or RF pulse length) decreases gradually, and the gradient and pulse length can be increased (thus increasing the breakdown rate again).

This cycle, called "RF processing" is repeated until at some point no further progress can be made -- no amount of running will reduce the breakdown rate at a given gradient and pulse length.

The reason appears to be that processing "polishes away" (vaporizes) small surface features; in the process, some molten metal splashes from the vaporization point to nearby ones, forming new features.

The size of the new feature depends on the input power. At some power level, a given feature's destruction creates a new feature of equal size.

RF Processing (2)

At this time there isn't even a good empirical means to estimate the peak gradient a structure can maintain. It does appear to increase as the frequency is raised (goes ~sqrt(frequency)) and as the pulse length is reduced (maybe goes like the 1/fourth root of pulselength).

Pulsed Heating

Thermal cycling of the surface of an accelerator structure can eventually cause cracking and roughening of the surface (leads to increased RF breakdowns).

The empirical limit for pulsed temp rise in copper seems to be in the tens of degrees regime. This should be adequate for any reasonable set of RF structure parameters.

Caveat: special features (input/output couplers, damping slots, etc) can have much higher pulsed temperature increases!

Superconducting Cavity Limits

Superconducting cavities have certain common behaviors at high accelerating fields:

.Everything is basically fine up to some gradient

As the gradient is increased above this level, the Q of the cavity begins to fall off (cause not well known, but field emission looks likely). Causes increased heat load and decreased shunt impedance (more power needed).

Above some gradient the cavity quenches (goes normal). This occurs when the surface magnetic field exceeds a critical level.

The gradient limit for a cavity, set by the critical field for Niobium, should be about 50 MeV/m. In practice it's somewhat lower (local field enhancements?).



The SLAC Linac Structure

The SLAC Structure (2)			
Parameter	Symbol	Unit	Value
Frequency	ω/2π	MHz	2856
Length	L	m	3.048
Cell Radius	b	cm	4.174.09
Iris Radius	а	cm	1.310.96
Cell Length	d	cm	3.50
Phase Advance per Cell	Ψ	-	2π/3
Disc Thickness	h	cm	0.584
Quality Factor	Q	-	13,000
Shunt Impedance per Meter	r _l	MΩ/m	5260
Filling Time	t _f	nsec	830
Group Velocity	V _{gr}	% c	2.00.65
Attenuation	τ	"nepers"	0.57
Typical Unloaded Gradient	G ₀	MV/m	21
Typical Input Power	P ₀	MW	35

SLAC Structure for a Linear Collider

Assume that we have parameters typical of a 500 GeV CM LC:

- Bunch Length: 200 μm
 - to match the IP betatron functions
- Bunch Charge: 1.6 nC
 to limit the severity of the beam-beam interaction
- Beam Power: 10 MW
 - to achieve 10³⁴ luminosity with other IP parameters that aren't completely crazy
- Beam Energy: 10 GeV -- 250 GeV
 - beam comes in with some energy
- Linac Lattice: 90°/cell, 4 structures/quad
 - typical SLAC structure parameters

Single Bunch Requirements:

- Loading compensation -- severe because of short bunch
 - at 21 MeV/m, need to run 26° ahead of the RF crest!
 - 10% reduction in gradient
- BNS Damping
 - need 50 MeV RMS head-tail energy spread
 - can be achieved by running at the crest for 450 meters, then switch to 26 ° ahead
 - At end of linac, yields 0.02% RMS energy spread

Bunch Train Requirements

- Average current = 42 microamperes
 - implies avg of 1 bunch per 38 microseconds
 - too long! Fill time = 830 nsec! Need bunch trains!
- At 21 MeV/m, full loading = 810 milliamps
 - Too much! Try 20% loading (160 mA)
 - Corresponds to 1 bunch per 10 nsec (29 RF cycles)
- Frequency of lowest dipole mode ~4.1 GHz
 - Assume we detune with 10% bandwidth, 3σ cutoff
 - What does HOM amplitude vs time look like?



Length of Bunch Train

- Want longest possible -- minimize fill-time efficiency effect
- Pulsed heating no problem -- 1 msec pulse OK
- RF power: @ 3 GHz, 160 MW x 3 µsec best achieved
 - Trade pulse width for peak power: 35 MW x 13.7 μsec
 - 0.8 μsec for filling, 12.9 μsec for beam
 - 3.8 km bunch train (need HERA-e as damping ring)
 - 20 trains per second for average current requirements.
 - Can "trade:" 120 trains per second, 650 m train (if DR size is critical or higher train rate desired)

LC-SLAC: Evaluation

Low gradient

- 17 MeV/meter with loading and off-crest running
- 28 km of linac for 500 GeV CM
- 17%--23% efficiency (depending on train length/rate)
- Poor upgradability
 - To double E_{CM}, quadruple structure power to 140 MW
 - Pulse length limited to 3 µsec
 - Current must double
 - halve bunch spacing? Bad for BBU
 - Double bunch charge? Bad for beam-beam

LC-SLAC: Evaluation (2)

- SLAC structure takes a lot of energy to achieve its gradient
 - Advantage: can accelerate high bunch charge
 - But IP limits bunch charge -- can't use this advantage!
- Consider Alternative structure configuration
 - Higher frequency
 - can achieve higher gradient, shorter linac
 - Low frequency, superconducting
 - higher accelerating efficiency, lower power bill

Lecture 3b: RF Power Sources

Klystrons

Klystrons have been the principal source of high-power (>1 MW) RF since the beginning of time, and no alternative technology appears poised to replace them.

What are klystrons?

A klystron is a narrow-band vacuum-tube amplifier at microwave frequencies (an electron-beam device).



How the Klystron Works

- DC Beam at high voltage (<500 kV, <500 A) is emitted from the gun
- A low-power signal at the design frequency excites the input cavity
- Particles are accelerated or decelerated in the input cavity, depending on phase/arrival time
- Velocity modulation becomes time modulation in the long drift tube (beam is bunched at drive frequency)
- Bunched beam excites output cavity at design frequency (beam loading)
- Spent beam is stopped in the collector.



Space-Charge Limited Beam Current

Consider a klystron gun which is infinite in extent in x/y, and has a cathode-anode separation d in the z direction.

In the separation, the charge density is $\rho(z)$ and the current density is $J = \rho(z)v(z)$ (constant with z).

The voltage as a function of z is given by: $\nabla^2 V = \frac{\partial^2 V}{dz^2} = \frac{\rho(z)}{\varepsilon_0}$

For non-relativistic beams, the kinetic energy of each electron = $eV(z) = m_e v^2/2$. We can perform some rearrangements to find:

$$\frac{\partial^2 V}{\partial z^2} = \frac{J}{\varepsilon_0} \sqrt{\frac{m_e}{2e}} V^{1/2}(z)$$

Space-Charge Limited Beam Current (2)

With appropriate boundary conditions (V=0 @ z=0; in the space-charge limit dV/dz=0 @ z=0) we can find the beam current at the point where the charge at the cathode is so high it cancels the electric field of the anode (space charge limit):

$$J = \frac{4\varepsilon_0}{9} \sqrt{\frac{2e}{m_e} \frac{V_{klys}^{3/2}}{d_{gun}^2}}$$

In general, in the space-charge limit $I = kV^{3/2}$. k is known as the klystron perveance. Units ampere/volt^{3/2} are implicit -- klystron makers usually just say "perveance of 10⁻⁶" or "microperveance of 1" and leave off the units.

Temperature-Limited Current

The klystron cathode uses thermionic emission to free electrons. Thus the klystron cathode can never exceed the thermionic current density:

$$J_{th} = K_{RD} T^2 e^{-\phi}$$

where T is the temperature, k is the Boltzmann constant (1.38x10⁻²³ J/K), K_{RD} = Richardson-Dushman constant (1.204x10⁶ A/m²/K²), ϕ is the work function of the cathode.

Klystron engineers like to design tubes which will run in saturation, so that small variations in voltage have relatively little impact on the power output.

Modulators

The modulator is the DC power supply which drives the klystron beam.

Typically it cannot reach the klystron drive voltage directly -- a transformer is needed to reach the desired voltage.

Pulse Compression

Room-temperature accelerator structrures require a short pulse of high RF power to reach their desired gradients.

Klystrons run efficiently when they produce a long pulse of relatively low power (minimize inefficiency from modulator rise/fall time etc).

Matching these different time structures is done by pulse compression.

Pulse compression in turn relies on the magic of the 3-db directional coupler to succeed.

3-db Directional Coupler

The 3-db coupler is a passive device with 4 input/output ports passing thru a central nexus:



The key feature of the coupler is that the diagonal pathways are longer by 90° than the straight pathways. What does that do for us?





At some point in the klystron pulse, the phase of the klystron is reversed so that the stored energy interferes constructively...

SLED Pulse Compression

Assume that the amplitude at each SLED-head is a constant E_{SLED} . The RF is on for a time t_2 , and at $t_1 < t_2$ the phase flips. The output amplitude as a function of time is:

$$\begin{split} E_{out,SLED} &= E_{SLED} \frac{2\beta_{SLED}}{1+\beta_{SLED}} \left(1-e^{-t/t_c}\right) - 1, t \le t_1 \\ &= E_{SLED} \frac{2\beta_{SLED}}{1+\beta_{SLED}} \Big[\left(2-e^{-t_1/t_c}\right) e^{-(t-t_1)/t_c} - 1 \Big] + 1, t_1 < t \le t_2 \\ &= E_{SLED} \frac{2\beta_{SLED}}{1+\beta_{SLED}} \Big[\left(2-e^{-t_1/t_c}\right) e^{-(t_2-t_1)/t_c} - 1 \Big] e^{-(t-t_2)/t_c}, t > t_2 \end{split}$$

Note that the maximum amplification factor is 3 (for a maximum power gain of 9).

SLED Limitations

- Factor of 3 amplitude limit (factor of 9 power limit)
- For large amplitude gains, efficiency is low
 - Large value of t_1 required w.r.t. t_c
 - Long time when most RF power is reflected, not stored
- Output pulse not flat
 - Exponential character due to standing-wave SLED character



SLED-II (2)

Assume that the amplitude at each SLED-head is a constant E_{SLED2} . The RF is on for a time t, and the round-trip time in the SLED-II line is t_{SLED2} . We constrain $t = nt_{SLED2}$, n integer. The emitted amplitude is:

$$E_{e} = \frac{1 - s^{2}}{1 - s} \left(1 - s^{ntrip-1} \right) E_{SLED2}$$

ie, ntrip=1 during the 1st round trip, 2 during the 2nd, etc.

If we flip the klystron phase at $t=(n-1)t_{SLED2}$, the amplitude during the last round-trip time becomes

$$E_{out} = E_{SLED2} \left(1 + 2s - s^{n-1} - s^n \right)$$

SLED-II Limitations

- Same factor of 3 amplitude gain limit as SLED
- Same limited efficiency -- early round-trips include reflected/emitted power which is lost to the system
- SLED-II travelling-wave lines may be physically long.











Delay Line Distribution System (4)

- Intrinsic efficiency = 100%
- Flat output pulse produced
- Can only amplify peak power by factor of 2ⁿ
- Requires 2 klystrons for 2x amplification, 4 klystrons for 4x amplification, etc.
- Lots of waveguide required, though (at least for 2x pulse compression) less than BPC.