An Introduction to the
Physics and Technology
of e+e- Linear Colliders

Lecture 1: Introduction and Overview

Nick Walker (DESY)

USPAS Santa Barbara 16th June, 2003

Course Content

Lecture:

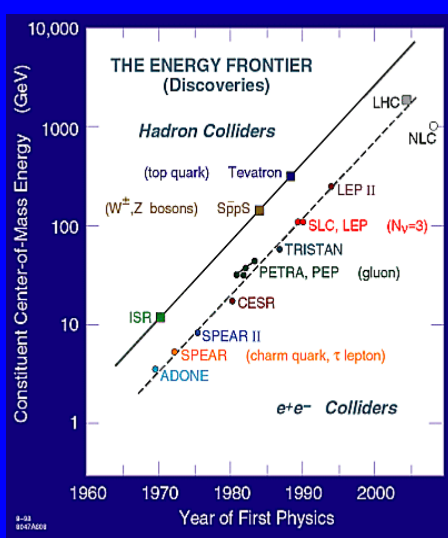
1. Introduction and overview
2. Linac part I
3. Linac part II
4. Damping Ring & Bunch Compressor I
5. Damping Ring & Bunch Compressor II
6. Final Focus Systems
7. Beam-Beam Effects
8. Stability Issues in Linear Colliders
9. the SLC experience and the Current LC Designs

This Lecture

- Why LC and not super-LEP?
- The Luminosity Problem
  - general scaling laws for linear colliders
- A introduction to the linear collider sub-systems:
  - main accelerator (linac)
  - sources
  - damping rings
  - bunch compression
  - final focus

during the lecture, we will introduce (revise) some important basic accelerator physics concepts that we will need in the remainder of the course.

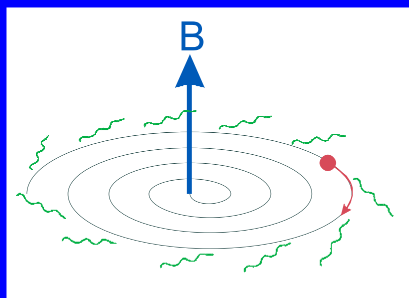
Energy Frontier e^+e^- Colliders



LEP at CERN, CH
E_{cm} = 180 GeV
P_{RF} = 30 MW

Why a Linear Collider?

Synchrotron Radiation from an electron in a magnetic field:



$$P_\gamma = \frac{e^2 c^2}{2\pi} C_\gamma E^2 B^2$$

Energy loss per turn of a machine with an average bending radius ρ:

$$\Delta E / rev = \frac{C_\gamma E^4}{\rho}$$

Energy loss must be replaced by RF system

Cost Scaling $$

- Linear Costs: (tunnel, magnets etc)

  $\$_{lin} \propto \rho$

- RF costs:

  $\$_{RF} \propto \Delta E \propto E^4/\rho$

- Optimum at

  $\$_{lin} = \$_{RF}$

Thus optimised cost ($\$_{lin}$+$\$_{RF}$) scales as E^2

The Bottom Line $$$

		LEP-II	Super-LEP	Hyper-LEP
E_{cm}	GeV	180	500	2000
L	km	27		
ΔE	GeV	1.5		
$\$_{tot}$	10^9 SF	2		

The Bottom Line $$$

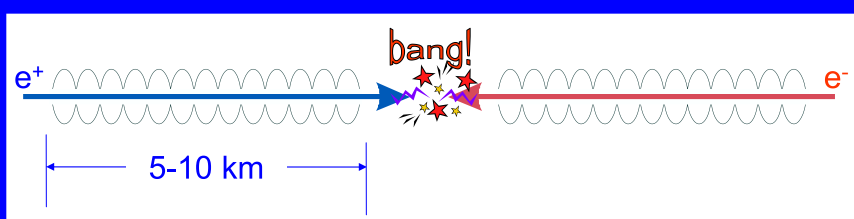
		LEP-II	Super-LEP	Hyper-LEP
E_{cm}	GeV	180	500	2000
L	km	27	200	
ΔE	GeV	1.5	12	
$\$_{tot}$	10^9 SF	2	15	

The Bottom Line $$$

		LEP-II	Super-LEP	Hyper-LEP
E_{cm}	GeV	180	500	2000
L	km	27	200	3200
ΔE	GeV	1.5	12	240
$\$_{tot}$	10^9 SF	2	15	240

solution: Linear Collider

No Bends, but *lots* of RF!



- long *linac* constructed of many RF accelerating structures
- typical gradients range from 25–100 MV/m

Note: for LC, $\$_{tot} \propto E$

A Little History

A Possible Apparatus for Electron-Clashing Experiments (*).

M. Tigner

Laboratory of Nuclear Studies. Cornell University - Ithaca, N.Y.

M. Tigner,
Nuovo Cimento **37** (1965) 1228

"While the storage ring concept for providing clashing-beam experiments ([1]) is very elegant in concept it seems worth-while at the present juncture to investigate other methods which, while less elegant and superficially more complex may prove more tractable."

A Little History (1988-2003)

- SLC (SLAC, 1988-98)
- NLCTA (SLAC, 1997-)
- TTF (DESY, 1994-)
- ATF (KEK, 1997-)
- FFTB (SLAC, 1992-1997)
- SBTF (DESY, 1994-1998)
- CLIC CTF1,2,3 (CERN, 1994-)

Over 14 Years of Linear Collider R&D

Past and Future

	SLC	LC	
E_{cm}	100	500–1000	GeV
P_{beam}	0.04	5–20	MW
σ^*_y	500 (≈50)	1–5	nm
$\delta E/E_{bs}$	0.03	3–10	%
L	0.0003	~3	10^{34} cm^{2}s^{-1}

generally quoted as 'proof of principle'

but we have a very long way to go!

LC Status in 1994

1994 E_{cm}=500 GeV

	TESLA	SBLC	JLC-S	JLC-C	JLC-X	NLC	VLEPP	CLIC
f [GHz]	1.3	3.0	2.8	5.7	11.4	11.4	14.0	30.0
$L\times10^{33}$ [cm^{-2}s^{-1}]	6	4	4	9	5	7	9	1-5
P_{beam} [MW]	16.5	7.3	1.3	4.3	3.2	4.2	2.4	~1-4
P_{AC} [MW]	164	139	118	209	114	103	57	100
$\gamma\varepsilon_y$ [$\times10^{-8}$m]	100	50	4.8	4.8	4.8	5	7.5	15
σ_y* [nm]	64	28	3	3	3	3.2	4	7.4

LC Status 2003

2003 E_{cm}=500 GeV

	TESLA	SBLC	JLC-S	JLC-C	JLC-X/NLC	VLEPP	CLIC
f [GHz]	1.3			5.7	11.4		30.0
$L\times10^{33}$ [$cm^{-2}s^{-1}$]	34			14	20		21
P_{beam} [MW]	11.3			5.8	6.9		4.9
P_{AC} [MW]	140			233	195		175
$\gamma\varepsilon_y$ [$\times10^{-8}$m]	3			4	4		1
σ_y* [nm]	5			4	3		1.2

The *Luminosity* Issue

Collider luminosity ($cm^{-2}\,s^{-1}$) is approximately given by

$$L = \frac{n_b N^2 f_{rep}}{A} H_D$$

where:

N_b = bunches / train
N = particles per bunch
f_{rep} = repetition frequency
A = beam cross-section at IP
H_D = beam-beam enhancement factor

For *Gaussian* beam distribution:

$$L = \frac{n_b N^2 f_{rep}}{4\pi\sigma_x\sigma_y} H_D$$

The *Luminosity* Issue: RF Power

Introduce the centre of mass energy, E_{cm}:

$$L = \frac{\left(E_{cm} n_b N f_{rep}\right) N}{4\pi\sigma_x\sigma_y E_{cm}} H_D$$

$$n_b N f_{rep} E_{cm} = P_{beams}$$
$$= \eta_{RF\to beam} P_{RF}$$

η_{RF} is RF to beam power efficiency.

Luminosity is proportional to the *RF power* for a given E_{cm}

$$L = \frac{\eta_{RF} P_{RF} N}{4\pi\sigma_x\sigma_y E_{cm}} H_D$$

The *Luminosity* Issue: RF Power

$$\boxed{L = \frac{\eta_{RF} P_{RF} N}{4\pi\sigma_x\sigma_y E_{cm}} H_D}$$

Some numbers:

$$\left.\begin{array}{ll} E_{cm} & = 500\ \text{GeV} \\ N & = 10^{10} \\ n_b & = 100 \\ f_{rep} & = 100\ \text{Hz} \end{array}\right\} P_{beams} = 8\ \text{MW}$$

Need to include efficiencies:

$$\left.\begin{array}{ll} \text{RF}\to\text{beam:} & \text{range 20-60\%} \\ \text{Wall plug}\to\text{RF:} & \text{range 28-40\%} \end{array}\right\} \textit{linac technology choice}$$

AC power > 100 MW just to accelerate beams and <u>achieve luminosity</u>

The *Luminosity* Issues: storage ring *vs* LC

LEP f_{rep} = 44 kHz

LC f_{rep} = few-100 Hz (power limited)

$$L = \frac{\eta_{RF} P_{RF} N}{4\pi \sigma_x \sigma_y E_{cm}} H_D$$

$\Rightarrow$ factor ~400 in L already lost!

Must push very hard on beam cross-section at collision:

LEP: $\sigma_x \sigma_y \approx 130 \times 6\ \mu m^2$

LC: $\sigma_x \sigma_y \approx (200\text{-}500) \times (3\text{-}5)\ nm^2$

factor of 10^6 gain!
Needed to obtain high luminosity of a few $10^{34}\ cm^{-2}s^{-1}$

The *Luminosity* Issue: intense beams at IP

$$L = \frac{1}{4\pi E_{cm}} \left(\eta_{RF} P_{RF}\right) \left(\frac{N}{\sigma_x \sigma_y} H_D\right)$$

choice of linac technology:
- efficiency
- available power

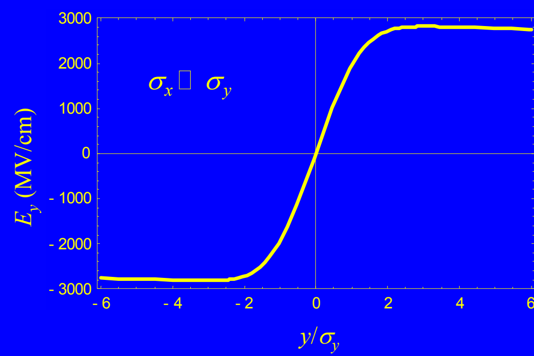
Beam-Beam effects:
- beamstrahlung
- disruption

Strong focusing
- optical aberrations
- stability issues and tolerances

The *Luminosity* Issue: Beam-Beam

see lecture 2 on beam-beam

- strong mutual focusing of beams (pinch) gives rise to luminosity enhancement H_D
- As $e^{\pm}$ pass through intense field of opposing beam, they radiate hard photons [beamstrahlung] and loose energy
- Interaction of *beamstrahlung* photons with intense field causes copious e^+e^- pair production [background]



The *Luminosity* Issue: Beam-Beam

see lecture 2 on beam-beam

beam-beam characterised by *Disruption Parameter:*

$$D_{x,y} = \frac{2 r_e N \sigma_z}{\gamma \sigma_{x,y}\left(\sigma_x + \sigma_y\right)} \approx \frac{\sigma_z}{f_{beam}}$$

σ_z = bunch length,
f_{beam} = *focal length* of beam-lens

for storage rings, $f_{beam} \gg \sigma_z$ and $D_{x,y} \ll 1$

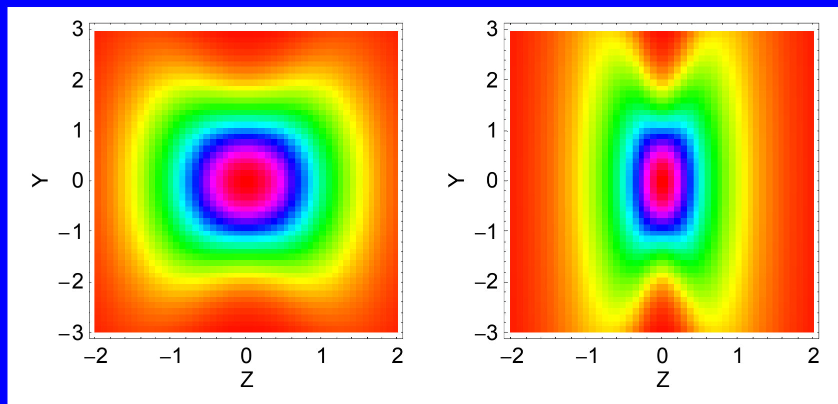
In a LC, $D_y \approx 10-20$ hence $f_{beam} < \sigma_z$

Enhancement factor (typically $H_D \sim 2$):

$$H_{Dx,y} = 1 + D_{x,y}^{1/4}\left(\frac{D_{x,y}^3}{1+D_{x,y}^3}\right)\left[\ln\left(\sqrt{D_{x,y}}+1\right) + 2\ln\underbrace{\left(\frac{0.8\beta_{x,y}}{\sigma_z}\right)}_{\text{'hour glass' effect}}\right]$$

The *Luminosity* Issue: Hour-Glass

see lecture 2 on beam-beam



β = "depth of focus"

reasonable lower limit for β is bunch length σ_z

The *Luminosity* Issue: Beamstrahlung

see lecture 2 on beam-beam

RMS relative energy loss $\quad \delta_{BS} \approx 0.86 \dfrac{e r_e^3}{2 m_0 c^2} \left(\dfrac{E_{cm}}{\sigma_z} \right) \dfrac{N^2}{(\sigma_x + \sigma_y)^2}$

we would like to make $\sigma_x \sigma_y$ <u>small</u> to maximise luminosity

BUT keep $(\sigma_x + \sigma_y)$ <u>large</u> to reduce δ_{SB}.

Trick: use "flat beams" with $\sigma_x \gg \sigma_y$ $\quad \delta_{BS} \propto \left(\dfrac{E_{cm}}{\sigma_z} \right) \dfrac{N^2}{\sigma_x^2}$

Now we set σ_x to fix δ_{SB}, and make σ_y as small as possible to achieve high luminosity.

For most LC designs, $\delta_{SB} \sim$ 3-10%

The *Luminosity* Issue: Beamstrahlung

Returning to our L scaling law, and ignoring H_D

$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \left(\frac{N}{\sigma_x} \right) \frac{1}{\sigma_y}$$

From flat-beam beamstrahlung $$\frac{N}{\sigma_x} \propto \sqrt{\frac{\sigma_z \delta_{BS}}{E_{cm}}}$$

hence $$\boxed{L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}^{3/2}} \frac{\sqrt{\delta_{BS} \sigma_z}}{\sigma_y}}$$

The *Luminosity* Issue: story so far

$$\boxed{L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}^{3/2}} \frac{\sqrt{\delta_{BS} \sigma_z}}{\sigma_y}}$$

For high Luminosity we need:

- high RF-beam conversion efficiency η_{RF}
- high RF power P_{RF}
- small vertical beam size σ_y
- large bunch length σ_z (will come back to this one)
- could also allow higher beamstrahlung δ_{BS} if willing to live with the consequences

Next question: *how to make a small* σ_y

The *Luminosity* Issue: A final scaling law?

$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}^{3/2}} \frac{\sqrt{\delta_{BS}\sigma_z}}{\sigma_y} \qquad \sigma_y = \sqrt{\frac{\beta_y \varepsilon_{n,y}}{\gamma}}$$

where $\varepsilon_{n,y}$ is the normalised vertical emittance, and β_y is the vertical β-function at the IP. Substituting:

$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}^{3/2}} \sqrt{\frac{\delta_{BS}\gamma}{\varepsilon_{n,y}}} \sqrt{\frac{\sigma_z}{\beta_y}} \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} \underbrace{\sqrt{\frac{\sigma_z}{\beta_y}}}_{\text{hour glass constraint}}$$

β_y is the same 'depth of focus' β for hour-glass effect. Hence $\beta_y \geq \sigma_z$

The *Luminosity* Issue: A final scaling law?

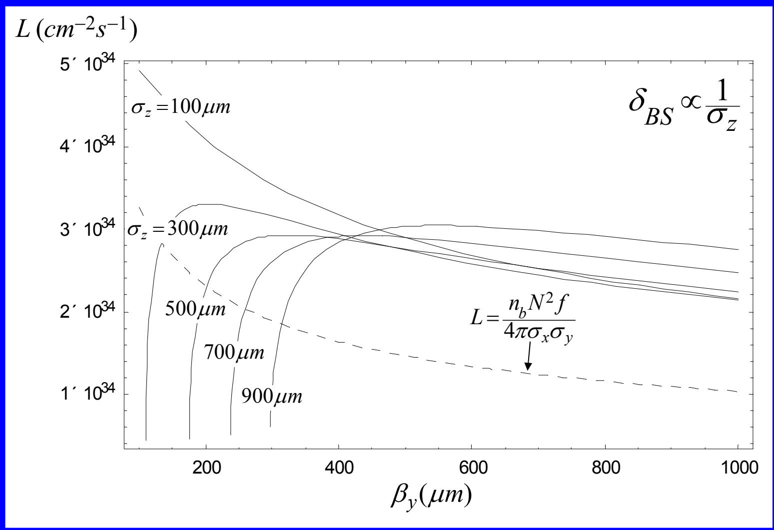
$$\boxed{L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} H_D} \qquad \beta_y \approx \sigma_z$$

- high RF-beam conversion efficiency η_{RF}
- high RF power P_{RF}
- small normalised vertical emittance $\varepsilon_{n,y}$
- strong focusing at IP (small β_y and hence small σ_z)
- could also allow higher beamstrahlung δ_{BS} if willing to live with the consequences

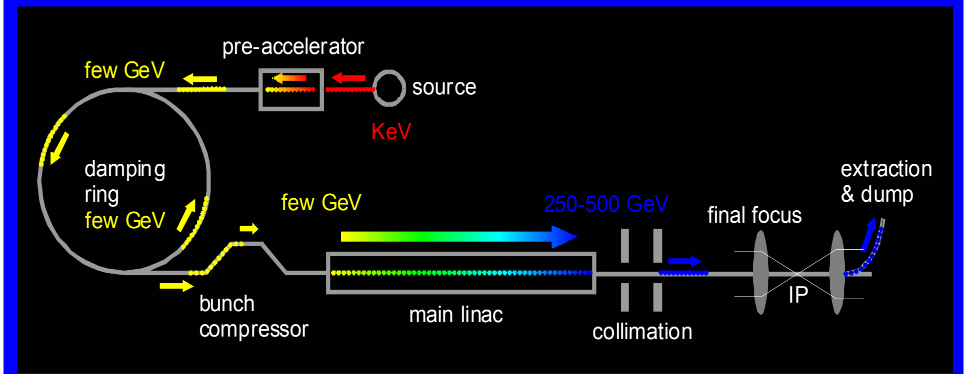
Above result is for the low beamstrahlung regime where δ_{BS} ~ few %

Slightly different result for high beamstrahlung regime

Luminosity as a function of β_y



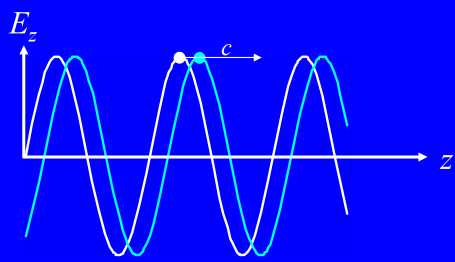
The 'Generic' Linear Collider



Each sub-system pushes the state-of-the-art in accelerator design

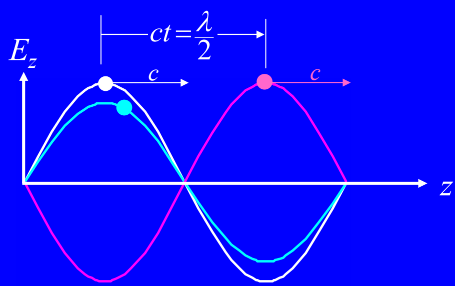
The Linear Accelerator (LINAC)

see lectures 3-4 on linac



<u>travelling wave structure</u>:
need *phase velocity = c*
(*disk-loaded structure*)

bunch sees constant field:
$E_z = E_0 \cos(\phi)$



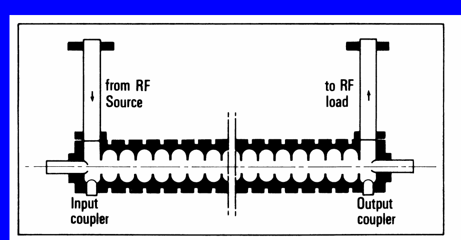
<u>standing wave cavity</u>:

bunch sees field:
$E_z = E_0 \sin(\omega t + \phi)\sin(kz)$
$= E_0 \sin(kz + \phi)\sin(kz)$

The Linear Accelerator (LINAC)

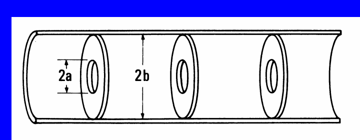
see lectures 3-4 on linac

Travelling wave structure



Circular waveguide mode TM_{01} has v_p>c

No good for acceleration!



Need to slow wave down using irises.

The Linear Accelerator (LINAC)

see lectures 3-4 on linac

- Gradient given by *shunt impedance*:
  - P_{RF} RF power /unit length
  - R_S shunt impedance /unit length

$$E_z = \sqrt{P_{RF} R_s}$$

- The cavity Q defines the *fill time:*
  - v_g = *group velocity,* l_s = *structure length*

$$t_{fill} = \begin{cases} 2Q/\omega & \text{SW} \\ 2\tau Q/\omega = l_s / v_g & \text{TW} \end{cases}$$

- For TW, τ is the structure attenuation constant:

$$P_{RF,out} = P_{RF,in} e^{-2\tau}$$

- RF power lost along structure (TW):

$$\frac{dP_{RF}}{dz} = -\underbrace{\frac{E_z^2}{R_s}}_{\text{power lost to structure}} - \underbrace{i_b E_z}_{\text{beam loading}}$$

η_{RF}

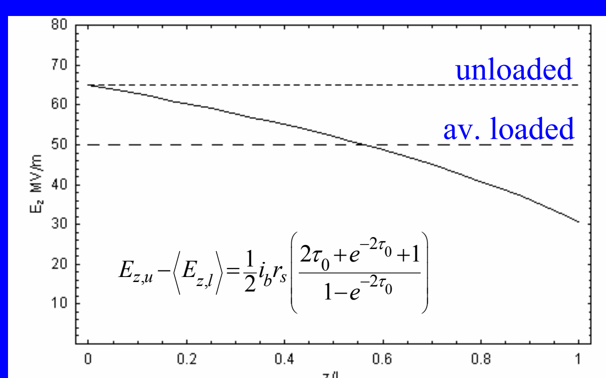
would like R_S to be as high as possible

$R_s \propto \sqrt{\omega}$

The Linear Accelerator (LINAC)

see lectures 3-4 on linac

- Steady state gradient drops over length of structure due to beam loading

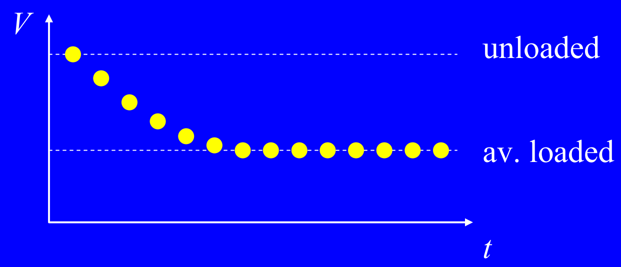


assumes constant (stead state) current

The Linear Accelerator (LINAC)

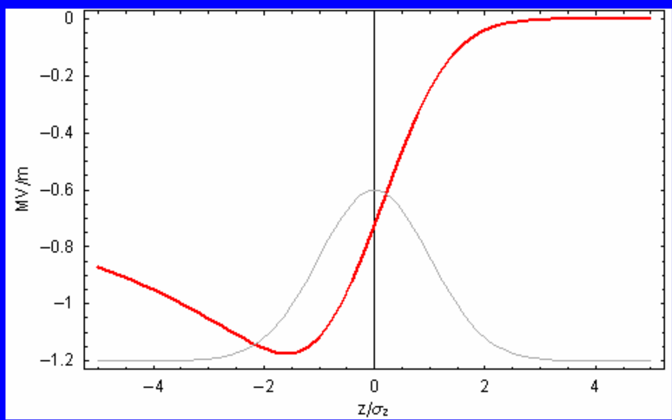
see lectures 3-4 on linac

- Transient beam loading
  - current not constant but pulses! ($t_{pulse} = n_b\, t_b$)
  - for all LC designs, long bunch trains achieve steady state quickly, and previous results very good approximation.
  - However, transient over first bunches needs to be compensated.



The Linear Accelerator (LINAC)

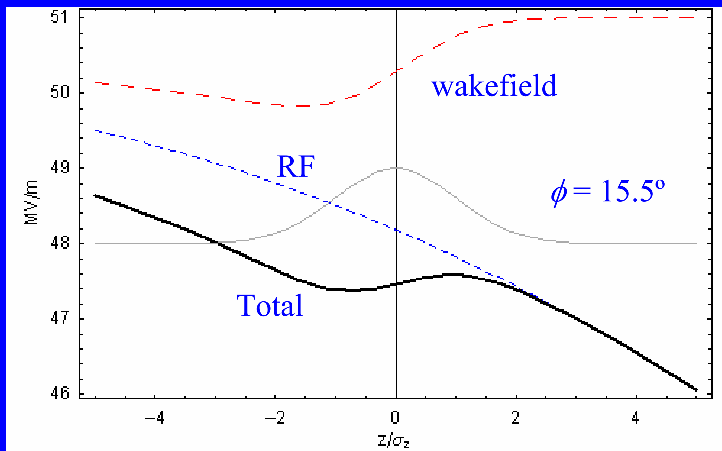
Single bunch beam loading: the Longitudinal wakefield



NLC X-band structure: $\langle \Delta E_z \rangle_{bunch} \approx 700\,\mathrm{kV/m}$

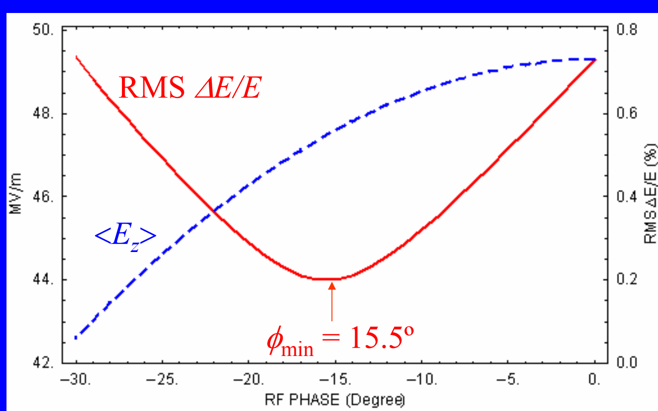
The Linear Accelerator (LINAC)

Single bunch beam loading Compensation using RF phase

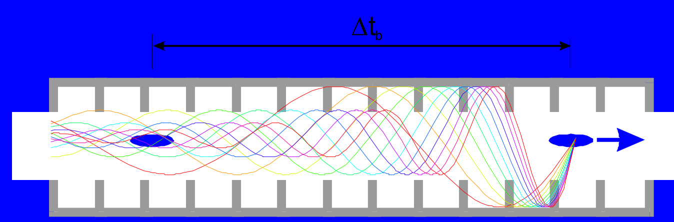


The Linear Accelerator (LINAC)

Single bunch beam loading: compensation



Transverse Wakes: The Emittance Killer!



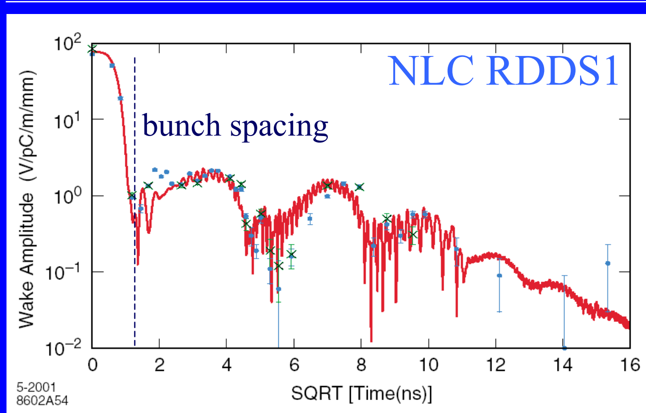
$$V(\omega,t) = I(\omega,t)Z(\omega,t)$$

Bunch current also generates transverse deflecting modes when bunches are not on cavity axis

Fields build up resonantly: latter bunches are kicked transversely

$\Rightarrow$ multi- and single-bunch beam breakup (MBBU, SBBU)

Damped & Detuned Structures



$$\Delta t \approx \frac{2Q_{\mathrm{HOM}}}{\Delta\omega}$$

Slight random detuning between cells causes HOMs to decohere.

Will recohere later: needs to be damped (HOM dampers)

Single bunch wakefields

Effect of coherent betatron oscillation

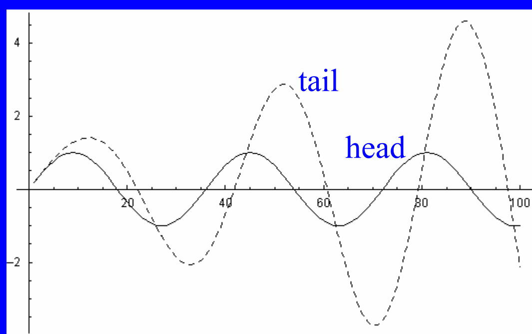
- head *resonantly* drives the tail

head eom:

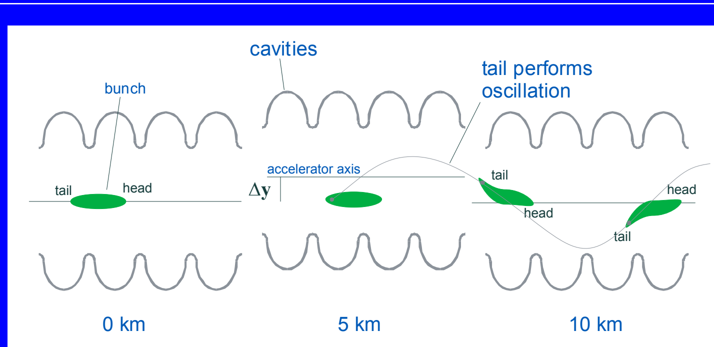
$$\frac{d^2 y_h}{ds} + k^2 y_y = 0$$

tail eom:

$$\frac{d^2 y_t}{ds} + k^2 y_t = k_{wf} y_h$$



Wakefields (alignment tolerances)



$$\delta Y_{\mathrm{RMS}} \propto \frac{1}{NW_{\perp}}\sqrt{\frac{E_z}{\beta}} \propto \frac{f^{-3}}{N}\sqrt{\frac{E_z}{\beta}}$$

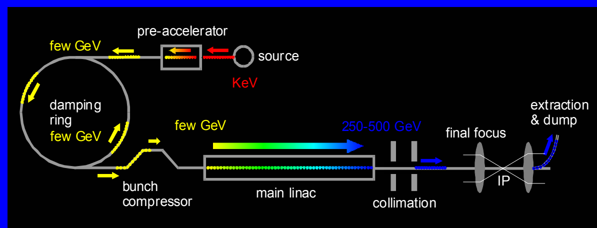
higher frequency = stronger wakefields

-higher gradients

-stronger focusing (smaller β)

-smaller bunch charge

The LINAC is only one part



Need to understand how to:

- Produce the electron charge?
- Produce the positron charge?
- Make small emittance beams?
- Focus the beams down to ~nm at the IP?

e^+e^- Sources

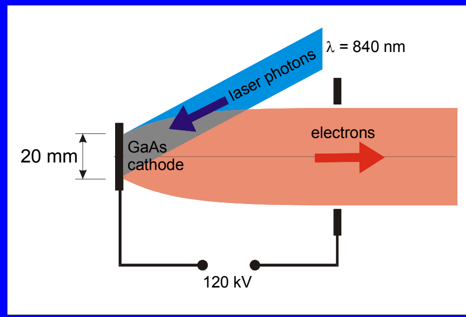
Requirements:

- produce long bunch trains of high charge bunches — 100-1000s @ 5-100 Hz, few nC
- with small emittances — $\varepsilon_{nx,y} \sim 10^{-6}, 10^{-8}$ m
- and *spin* polarisation (needed for physics) — mandatory for e^-, nice for e^+

Remember L scaling: $L \propto \frac{n_b N^2}{\sqrt{\varepsilon_n}}$

e^- Source

- laser-driven photo injector
- circ. polarised photons on GaAs cathode
  → long. polarised e^-
- laser pulse modulated to give required time structure
- very high vacuum requirements for GaAs ($<10^{-11}$ mbar)
- beam quality is dominated by *space charge* (note $v \sim 0.2c$)

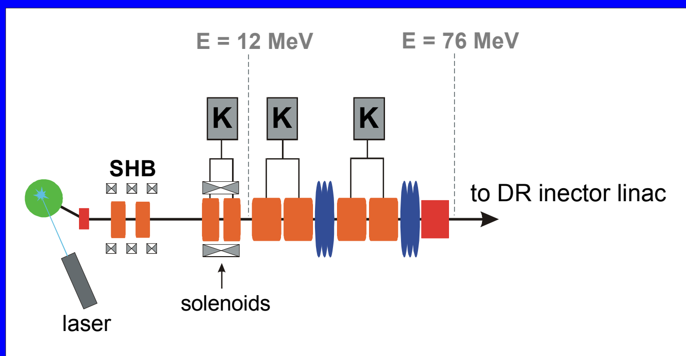


$$\varepsilon_n \approx 10^{-5} m$$

factor 10 in x plane

factor ~500 in y plane

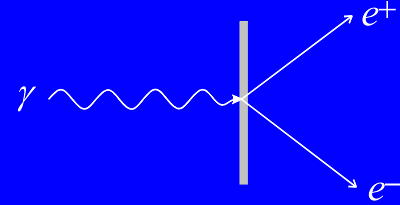
e^- Source: pre-acceleration



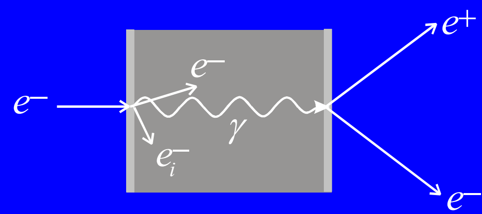
SHB = sub-harmonic buncher. Typical bunch length from gun is ~ns (too long for electron linac with $f \sim$ 1-3 GHz, need tens of ps)

e^{+} Source

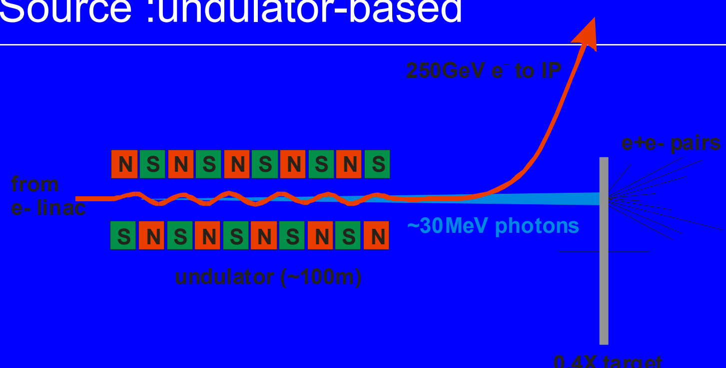
Photon conversion to $e^{\pm}$ pairs in target material



Standard method is e^{-} beam on ‘thick’ target (em-shower)



e^{+} Source :undulator-based



- SR radiation from undulator generates photons — ~ 30 MeV
- no need for ‘thick’ target to generate shower — $0.4X_0$
- thin target reduces multiple-Coulomb scattering: hence better emittance (but still much bigger than needed) — 10^{-2} m
- less power deposited in target (no need for mult. systems) — 5 kW
- Achilles heel: needs initial electron energy > 150 GeV!

Damping Rings

see lecture 5

- (storage) ring in which the bunch train is stored for T_{store} ~20-200 ms
- emittances are reduced via the interplay of synchrotron radiation and RF acceleration

initial emittance
(~0.01m for e$^+$)

$$\varepsilon_f = \varepsilon_{eq} + (\varepsilon_i - \varepsilon_{eq})e^{-2T/\tau_D}$$

final emittance

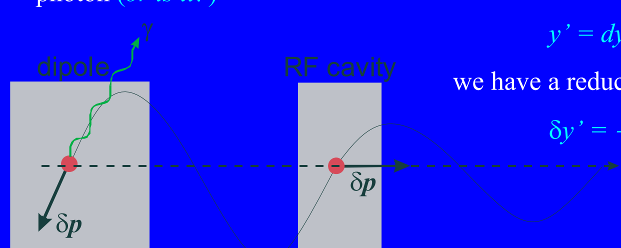
equilibrium emittance

damping time

Damping Rings: transverse damping

see lecture 5

y' not changed by photon (*or is it?*)



δ*p* replaced by RF such that $\Delta p_z = \delta p$.

since (adiabatic damping again)

$$y' = dy/ds = p_y/p_z,$$

we have a reduction in amplitude:

$$\delta y' = -\delta p\, y'$$

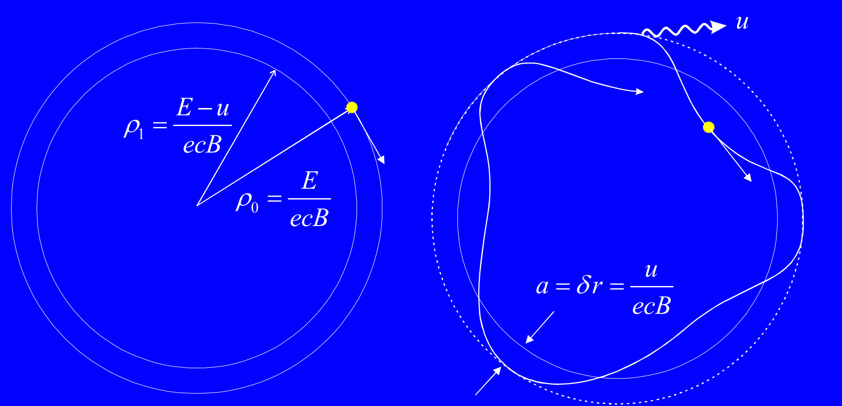
Must take average over all β-phases:

$$\tau_D \approx \frac{2E}{\langle P_\gamma \rangle} \quad \text{where} \quad \langle P_\gamma \rangle = \frac{c\,C_\gamma}{2\pi}\frac{E^4}{\rho^2} \quad \text{and hence} \quad \boxed{\tau_D \propto \frac{\rho^2}{E^3}}$$

LEP: E ~ 90 GeV, P_γ ~ 15000 GeV/s, τ_D ~ 12 ms

Damping Rings: Anti-Damping

see lecture 5



particle now performs β-oscillation about new closed orbit $\rho_1 \Rightarrow$ increase in emittance

$$\frac{d\varepsilon_x}{dt} = Q$$

Equilibrium achieved when

$$\frac{d\varepsilon_x}{dt} = 0 = Q - \frac{2}{\tau_d}\varepsilon_x$$

Damping Rings: transverse damping

see lecture 5

$\tau_D \propto \frac{\rho^2}{E^3}$ suggests high-energy and small ring. But

required RF power: $P_{RF} \propto \frac{E^4}{\rho^2} \times n_b N$

equilibrium emittance: $\varepsilon_{n,x} \propto \frac{E^2}{\rho}$

an example:

- Take $E \approx 2$ GeV
- $B_{bend} = 0.13$ T $\Rightarrow \rho \approx 50$ m
- $<P_\gamma> = 27$ GeV/s [28 kV/turn]
- hence $\tau_D \approx$ 148 ms - Few ms required!!!
  Increase $<P_\gamma>$ by ×30 using *wiggler magnets*

Remember: $8\times\tau_D$ needed to reduce e^+ vertical emittance.

Store time set by f_{rep}:

$$t_s \approx n_{train} / f_{rep}$$

radius:

$$\rho = \frac{n_{train} n_b \Delta t_b c}{2\pi}$$

Damping Rings: limits on vertical emittance

see lecture 5

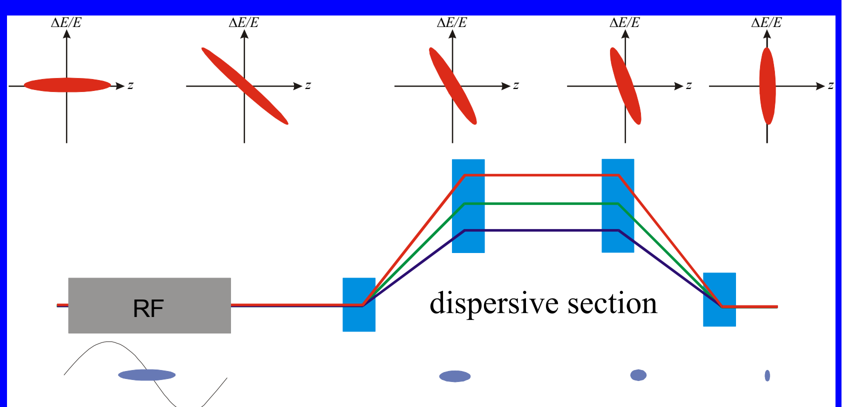
- Horizontal emittance defined by lattice
- theoretical vertical emittance limited by
  - space charge
  - intra-beam scattering (IBS)
  - photon opening angle
- In practice, ε_y limited by magnet alignment errors [cross plane coupling]
- typical vertical alignment tolerance: $\Delta y \approx 30\ \mu\text{m}$
  $\Rightarrow$ requires beam-based alignment techniques!

Bunch Compression

see lecture 6

- bunch length from ring ~ few mm
- required at IP 100-300 µm

long. phase space



The linear bunch compressor

see lecture 6

initial (uncorrelated) momentum spread:	δ_u
initial bunch length	$\sigma_{z,0}$
compression ration	$F_c=\sigma_{z,0}/\sigma_z$
beam energy	E
RF induced (correlated) momentum spread:	δ_c
RF voltage	V_{RF}
RF wavelength	$\lambda_{RF}=2\pi/k_{RF}$
longitudinal dispersion:	R_{56}

conservation of longitudinal emittance

$$F_c=\frac{\sqrt{\delta_c^2+\delta_u^2}}{\delta_u} \Leftrightarrow \delta_c=\delta_u\sqrt{F_c^2-1}$$

RF cavity

$$\delta_c \approx \frac{k_{RF}V_{RF}\sigma_{z,0}}{E} \Leftrightarrow V_{RF}=\frac{E\delta_c}{k_{RF}\sigma_{z,0}}=\frac{E}{k_{RF}}\left(\frac{\delta_u}{\sigma_{z,0}}\right)\sqrt{F_c^2-1}$$

The linear bunch compressor

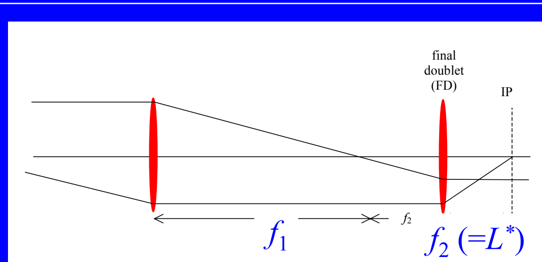
see lecture 6

chicane (dispersive section)

$$\Delta z \approx R_{56}\delta \qquad R_{56}=-\frac{\langle \delta z\rangle}{\delta^2}=-\frac{\delta_c\sigma_{z,0}}{F^2\delta_u^2}=\frac{k_{RF}V_{RF}}{E}\left(\frac{\sigma_{z,0}}{\delta_u}\right)^2\frac{1}{F^2}$$

$$\left.\begin{aligned}\sigma_{z,0}&=2\,\text{mm}\\ \delta_u&=0.1\%\\ \sigma_z&=100\,\mu\text{m}\Rightarrow F_c=20\\ f_{RF}&=3\,\text{GHz}\Rightarrow k_{RF}=62.8\,\text{m}^{-1}\\ E&=2\,\text{GeV}\end{aligned}\right\} \quad \begin{aligned}\delta&=2\%\\ V_{RF}&=318\,\text{MV}\\ R_{56}&=0.1\,\text{m}\end{aligned}$$

Final Focusing



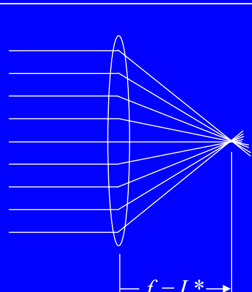
Use telescope optics to demagnify beam by factor $m = f_1/f_2 = f_1/L^*$

Need typically $m = 300$

putting $L^* = 2\text{m} \Rightarrow f_1 = 600\text{m}$

Final Focusing

see lecture 7



$L^* \approx 2 - 4\ \text{m}$

$\sigma_y = \sqrt{\varepsilon_{n,y}\beta_y / \gamma}$

$\sigma_y \approx 2 - 5\,\text{nm} \Rightarrow \beta_y \approx 100 - 300\ \mu\text{m}$

remember $\beta_y \sim \sigma_z$

at final lens $\beta_y \sim 100$ km

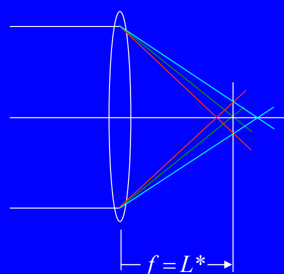
short f requires very strong fields (gradient): $dB/dr \sim 250$ T/m
pole tip field $B(r = 1\text{cm}) \sim 2.5$ T

normalised quadrupole strength: $K_1 = \frac{1}{B\rho}\frac{B_o}{r_0}$

where $B\rho$ = *magnetic rigidity* = $P/e \sim 3.3356\ P$ [GeV/c]

Final Focusing: chromaticity

see lecture 7



for a *thin-lens* of length l: $\quad \frac{1}{f} \approx K_1 l$

$$\Delta y'_{quad} \approx -K_1 l\, y_{quad} \frac{\delta}{1+\delta} \approx -K_1 l\, y_{quad}\, \delta$$

$$\Delta y_{IP} \approx f \Delta y'_{quad} = y_{quad} \delta$$

$$\left\langle \Delta y_{IP}^2 \right\rangle = \left\langle y_{quad}^2 \right\rangle \left\langle \delta^2 \right\rangle = \beta_{quad} \varepsilon_y \delta_{\mathrm{rms}}^2$$

for $\delta_{\mathrm{rms}} \sim 0.3\%$ $\quad \sqrt{\left\langle \Delta y_{IP}^2 \right\rangle} \approx 20-40\ \mathrm{nm}$

more general: $\quad \left\langle \Delta y_{IP}^2 \right\rangle = \xi_y \delta_{\mathrm{rms}}^2$

ξ is chromaticity $\quad \xi_y = \int K_1(s)\beta(s) ds$

chromaticity must be corrected using sextupole magnets

Final Focusing: chromatic correction

see lecture 7

magnetic multipole expansion:

$$B_y(x) = B\rho \left(\underbrace{\frac{1}{\rho}}_{\text{dipole}} + \underbrace{K_1 x}_{\text{quadrupole}} + \underbrace{\frac{1}{2} K_2 x^2}_{\text{sextupole}} + \underbrace{\frac{1}{3!} K_3 x^3}_{\text{octupole}} \ldots \right)$$

2nd-order kick: $\quad \Delta y' = \begin{cases} -k_1 y \delta & \text{quadrupole} \\ -k_2 x y & \text{sextupole} \end{cases} \qquad k_n \equiv \int_0^l K_n ds$

introduce horizontal dispersion D_x

$$x \to x + D_x \delta$$

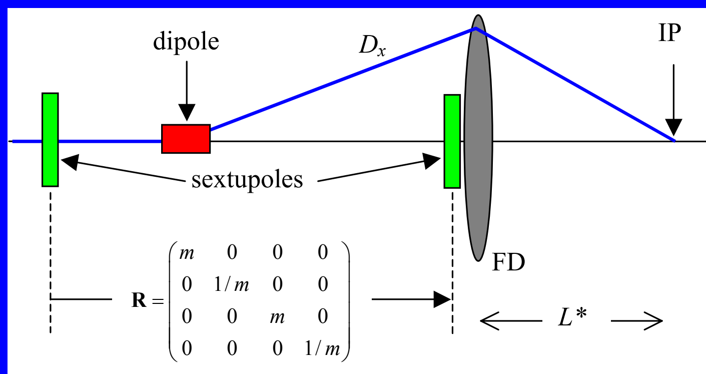
$$\Delta y' = \underbrace{-k_2 x y}_{geometric} - \underbrace{k_2 D_x y \delta}_{chromaticity}$$

chromatic correction when $\quad k_2 = -\frac{D_x}{k_1}$

need also to cancel geometric (xy) term! (second sextupole)

Final Focusing: chromatic correction

see lecture 7



Final Focusing: Fundamental limits

see lecture 7

Already mentioned that $\beta_y \geq \sigma_z$

At high-energies, additional limits set by so-called *Oide Effect*: synchrotron radiation in the final focusing quadrupoles leads to a beamsize growth at the IP

$$\left.\begin{aligned} &\text{minimum beam size:} && \sigma \approx 1.83\left(r_e \lambda\!\!\!^{-}_e F\right)^{1/7} \varepsilon_n^{5/7} \\ &\text{occurs when} && \beta \approx 2.39\left(r_e \lambda\!\!\!^{-}_e F\right)^{2/7} \varepsilon_n^{3/7} \end{aligned}\right\} \textit{independent of } E!$$

F is a function of the focusing optics: typically $F \sim 7$ (minimum value ~0.1)

Stability

see lecture 8

- Tiny (emittance) beams
- Tight component tolerances
  - Field quality
  - Alignment
- Vibration and Ground Motion issues
- Active stabilisation
- Feedback systems

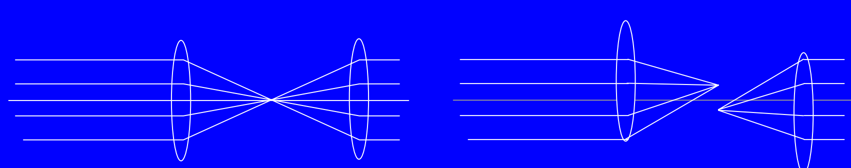
Linear Collider will be “Fly By Wire”

Stability: some numbers

see lecture 8

- Cavity alignment (RMS): ~ µm
- Linac magnets: 100 nm
- FFS magnets: 10-100 nm
- Final “lens”: ~ nm !!!

parallel-to-point focusing:

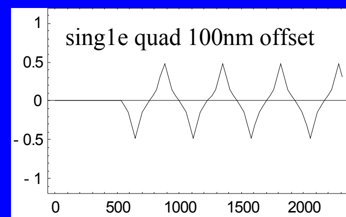


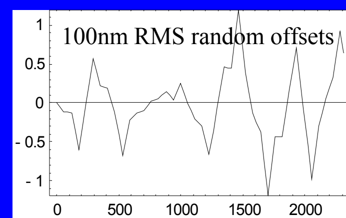
LINAC quadrupole stability

see lecture 8

$$y^* = \sum_{i=1}^{N_Q} k_{Q,i}\Delta Y_i g_i = k_Q \sum_{i=1}^{N_Q} \Delta Y_i g_i$$

$$g_i = \sqrt{\frac{\gamma_i}{\gamma^*}}\sqrt{\beta_i \beta^*}\,\sin(\Delta\phi_i)$$





for uncorrelated offsets

$$\left\langle y^{*2}\right\rangle = \frac{\beta^*\left\langle \Delta Y^2\right\rangle}{\gamma^*}\sum_{i=1}^{N_Q}\gamma_i k_{Q,i}^2 \beta_i \sin^2(\Delta\phi_{ij})$$

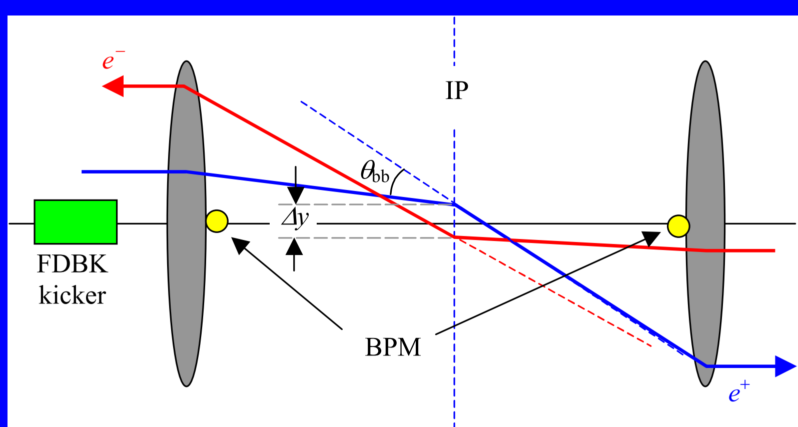
Dividing by $\sigma_y^{*2} = \beta^* \varepsilon_{y,n}/\gamma^*$
and taking average values:

$$\frac{\left\langle y_j^2\right\rangle}{\sigma_y^{*2}} \approx \frac{N_Q k_Q^2 \bar{\beta}\bar{\gamma}}{2\varepsilon_{y,n}}\sigma_{\Delta Y}^2 \le 0.3^2$$

take $N_Q = 400$, $\varepsilon_y \sim 6\times10^{-14}$ m, $\beta \sim 100$ m, $k_1 \sim 0.03$ m^{-1} $\Rightarrow$ ~25 nm

Beam-Beam orbit feedback

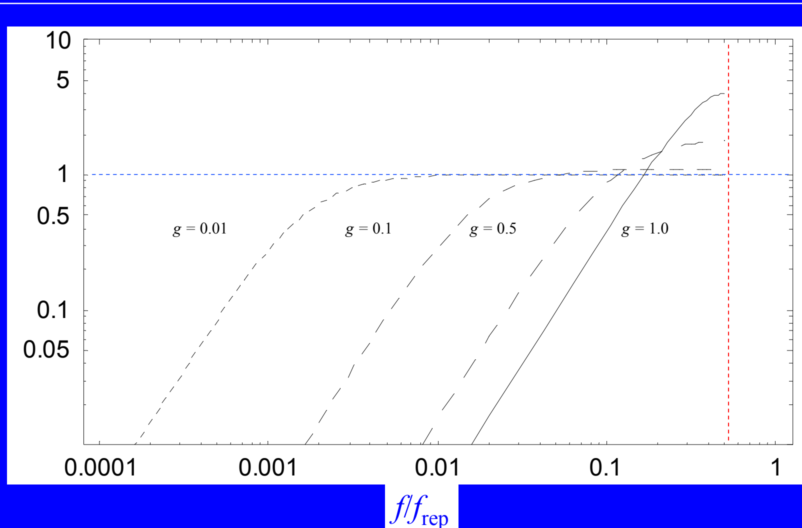
see lecture 8



use strong beam-beam kick to keep beams colliding

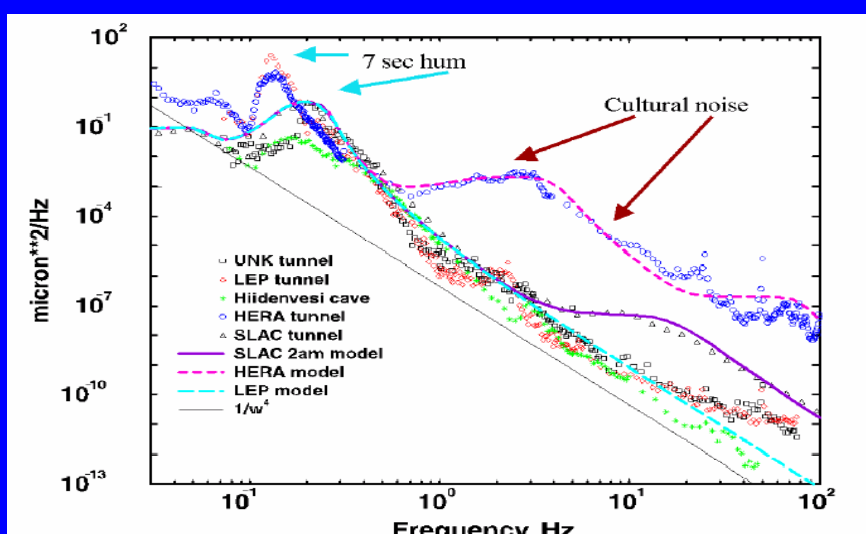
Generally, orbit control (feedback) will be used extensively in LC

Beam based feedback: bandwidth



Good rule of thumb: attenuate noise with $f < f_{rep}/20$

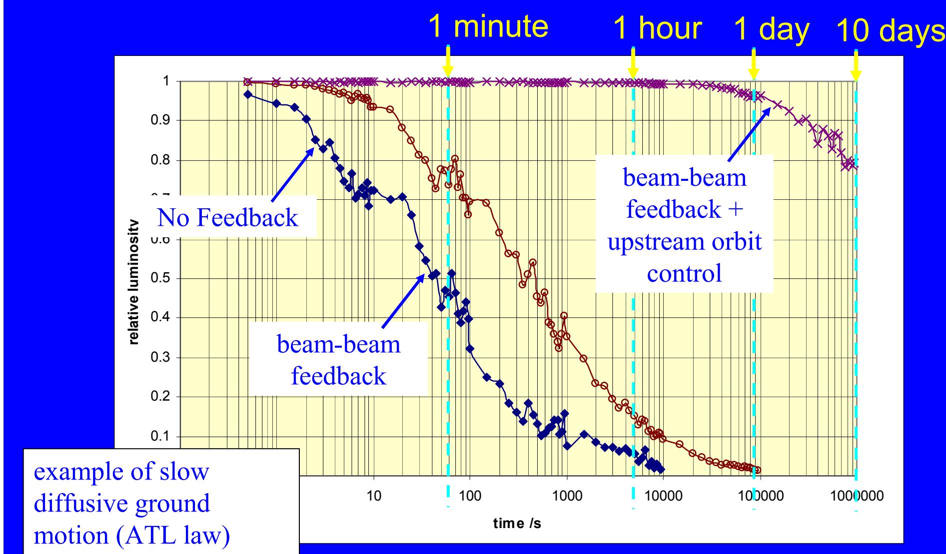
Ground motion spectra



Long Term Stability

see lecture 8

understanding of ground motion and vibration spectrum important



Here Endeth the First Lecture