LATTICE MATCHING WITH A QUADRUPOLE MISSING

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Abstract

The lattice for the present design of the TESLA Linear Collider with integrated X-Ray Laser Facility is basically a FODO structure with constant beta-function. There are more than 800 individually powered superconducting quadrupoles to focus the beam in the two main linear accelerators (each with a length of 15 km). For the availability of the beams it is important that a power supply failure or even a magnet failure does not cause a significant down time of the linac. It is demonstrated that the beta-function can be matched with a quadrupole "missing". One part of the main linac (up-to 50 GeV) will be used to accelerate the High Energy Physics beam and the Free Electron Laser drive beam with different accelerating gradients. Therefore the betatron phase advance will be 60° or 90° depending on the beam energy. It is shown that even in that case it is possible to match the lattice simultaneously for both beams with a quadrupole missing.

1 INTRODUCTION

The conceptual design of the TESLA linear collider with integrated x-ray laser facility [1] requires that one of the two 15 km long super-conducting linear accelerator has to provide beams for high energy physics (HEP) experiments and beams for the x-ray free electron laser (FEL) facility. Fig. 1 shows the e$^-$ linear accelerator which is used to ac-

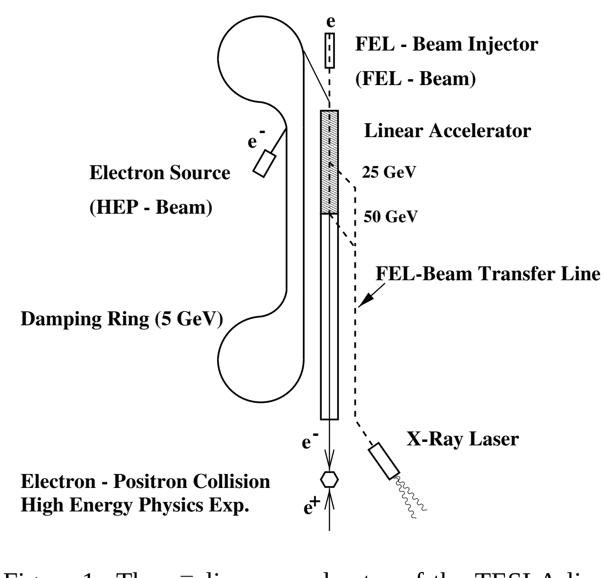

Figure 1: The e$^-$ linear accelerator of the TESLA linear collider with integrated x-ray laser facility.

celerate the HEP-beam to 250 GeV and the FEL-beam to energies up-to 50 GeV. There are 25 GeV and 50 GeV extraction points in the linac. The HEP beam is injected from the dog-bone damping ring into the main linear accelerator while the FEL beam is generated with an rf-gun and accelerated to 5 GeV in an injector linac before it is also injected into the main linear accelerator.

The first part of the e$^-$ linear accelerator is operated at a duty cycle of 10 Hz providing HEP (2882 bunches) and FEL (11315 bunches) pulses in a alternating way. The pulse structure is illustrated in Fig. 2. The FEL-pulses are accelerated at the same gradient of 17 MV/m but are ejected at 25 GeV or 50 GeV beam energy to be used for different undulators.

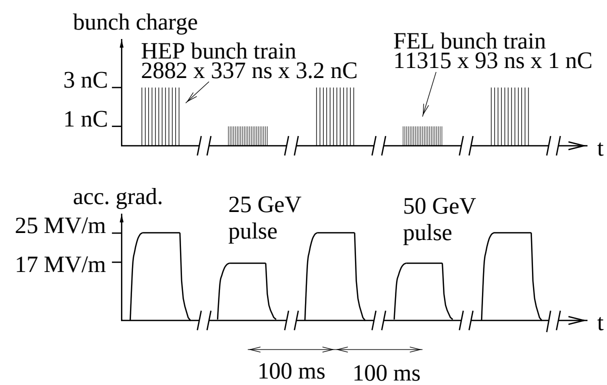

Figure 2: HEP and FEL beam pulse structure

The basic optical building block is a FODO cell with a length of 65 m for the first half of the linac (up-to 125 GeV) and a length of 97 m for the second half. Two cryo-modules, housing 12 (7-cell) cavities each, will fit between two quadrupoles of the 65 m long FODO cell, while three cryo-modules will fit between the quadrupoles of the 97 m long cell. The quadrupole strength $k = g\,e/p$ (g the gradient of the magnet, p the momentum of the beam) is different for the FEL and HEP beams due to the different accelerating gradients: $k_{FEL} = k_{HEP} \cdot 25/17$. A betatron phase advance of 60° is chosen for the HEP beam optics. This corresponds to a phase advance of about 90° for the FEL beam since the focusing is stronger than that of the HEP beam due to the lower momentum.

For the availability of the beams it is important that a power supply failure or even a magnet failure does not cause a significant down time of the linac. In the next section it will be shown how the lattice can be matched with a quadrupole "missing" in the case of the 60°-FODO lattice. Finally it is demonstrated that it is possible to match the lattice *simultaneously* for both beams.

2 60°-FODO LATTICE WITH A QUADRUPOLE MISSING

The beta-functions of a standard (i.e. length $L_{cell} = 65\,\mathrm{m}$) FODO cell with 60° phase advance is shown in Fig. 3. The quadrupole strength is $k = 0.0516/\mathrm{m}^2$ for a quadrupole length of 0.6 m. The focusing effect of the cavities can be neglected for a high energy beam. The minimal and maximal beta-functions are $\check{\beta} = 37.58\,\mathrm{m}$ and $\hat{\beta} = 112.41\,\mathrm{m}$ for the HEP beam. The focussing effect of the quadrupole is a

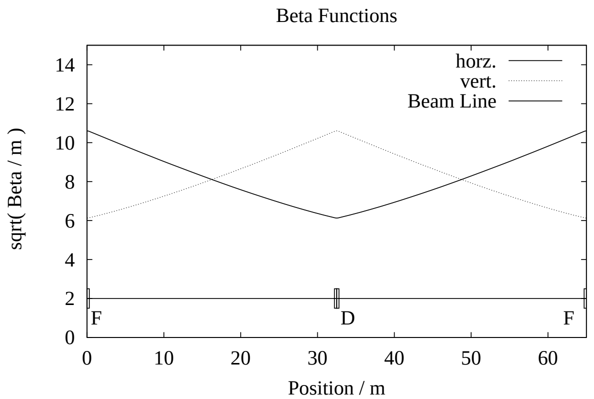

Figure 3: Standard FODO cell of the linac (60° phase advance). The square root of the horizontal (solid line) and vertical (dashed line) beta-function are shown versus the longitudinal position for the HEP beam.

factor 1.47 stronger for the FEL-beam resulting in beta-functions $\check{\beta} = 17.32\,\mathrm{m}$ and $\hat{\beta} = 112.12\,\mathrm{m}$ which are shown in Fig. 4.

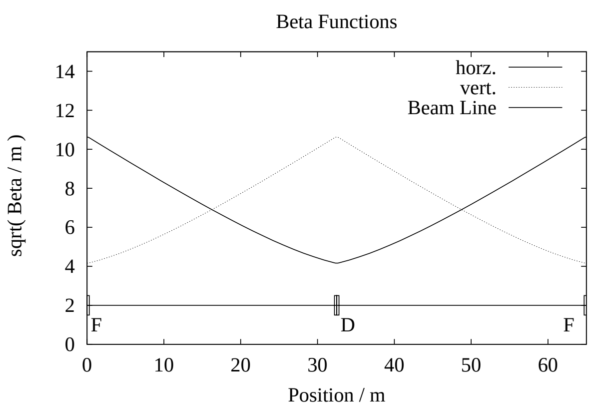

Figure 4: The square root of the horizontal (solid line) and vertical (dashed line) beta-function of the FEL beam.

Now it is assumed that one power supply failed causing a significant betatron-mismatch (see Fig. 5). Using the computer code COMFORT [2] it is possible to rematch the beta-function such that two FODO cells down-stream from the missing quadrupole the original periodic solution is recovered. Actually there exist several solutions which rematch the lattice. The solution presented in Fig. 6 fulfills the additional constraints that the strength of all quadrupoles does not exceed the design values of the standard 60° lattice and that the polarity is unchanged, i.e. an F(D)-quad remains an F(D)-quad. The relative quadrupole strengths of the six matching quadrupoles (k_n/k, $n = 1, \ldots, 6$) are summarized in table 1.

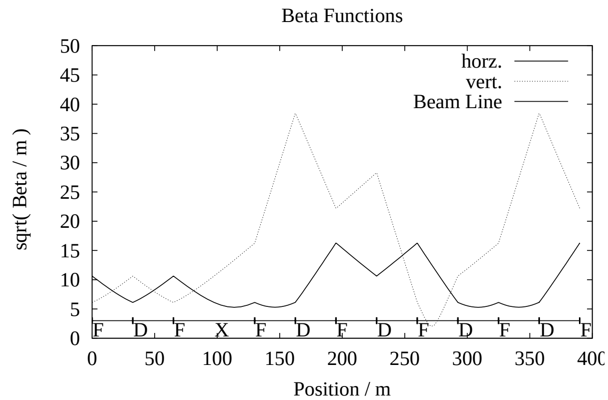

Figure 5: Lattice with a missing quad. The square root of the horizontal (solid line) and vertical (dashed line) beta-function of the HEP beam are shown.

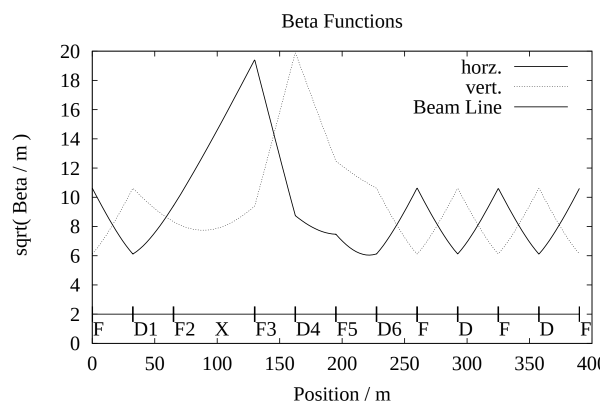

Figure 6: Re-matched beta-functions with a missing quad. The square root of the horizontal (solid line) and vertical (dashed line) beta-function of the HEP beam are shown.

Quad	k_n/k
D 1	0.7694
F 2	0.0
F 3	0.8283

Quad	k_n/k
D 4	0.9118
F 5	0.4056
D 6	0.3616

Table 1: Relative quadrupole strength of the 6 matching quadrupoles. All quadrupole strengths are normalized to the quadrupole of a regular FODO-cell.

While the solution presented in Fig. 6 is perfect for the HEP beam, the FEL beam is still mismatched. It is of course possible to find another solution for the FEL beam but that solution would not fulfill the matching conditions for the HEP beam. In the next section a solution is shown which matched the lattice for the HEP and FEL beam simultaneously.

3 SIMULTANEOUS LATTICE MATCHING FOR THE HEP AND FEL BEAM

Simultaneous matching of the lattice for the HEP and FEL beams requires a solution to the following four equations:

$$T(k_1,\ldots,k_n)\begin{pmatrix}\beta_{HEP,j}\\ \alpha_{HEP,j}\\ \gamma_{HEP,j}\end{pmatrix} = \begin{pmatrix}\beta_{HEP,j}\\ \alpha_{HEP,j}\\ \gamma_{HEP,j}\end{pmatrix}$$

$$j = x, y$$

$$T(\lambda\, k_1,\ldots,\lambda k_n)\begin{pmatrix}\beta_{FEL,j}\\ \alpha_{FEL,j}\\ \gamma_{FEL,j}\end{pmatrix} = \begin{pmatrix}\beta_{FEL,j}\\ \alpha_{FEL,j}\\ \gamma_{FEL,j}\end{pmatrix},$$

where T is the 3×3 lattice transfer matrix depending on the quadrupole strengths k_i ($i = 1 \ldots n$), where $\beta_{HEP,j}$, $\alpha_{HEP,j}$, $\gamma_{HEP,j}$, $\beta_{FEL,j}$, $\alpha_{FEL,j}$, $\gamma_{FEL,j}$ ($j = x, y$) are the horizontal and vertical twiss parameter of the HEP and FEL beam corresponding to the periodic solutions of a standard cell (see Fig. 3 and Fig. 4), and $\lambda = 1.47$ is the ratio of the beam momenta of the HEP and FEL beam.

Commonly used beam optics computer codes like COMFORT cannot match the beta-functions for two beams of different momenta simultaneously. Using the code Mathcad [3] it is possible to solve the above equations numerically using $n = 14$ quadrupoles. The algorithm is based on a quasi-Newton method [3]. The result of the computations is shown in Fig. 7 for the HEP beam and in Fig. 8 for the FEL beam. Please note that the magnetic gradient of all quads is the same in both cases only the beam momenta differ. The relative strengths of the 14 matching quadrupoles are given in table 2. One quadrupole is 22 % stronger than a quadrupole of a regular FODO cell, which is not critical since the simultaneous matching solution is only required for beam energies up-to 50 GeV.

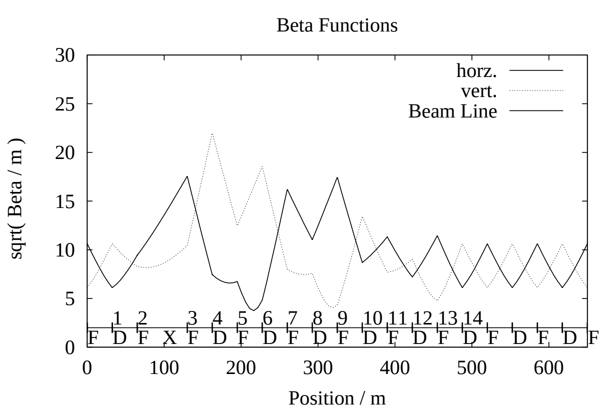

Figure 7: Simultaneous beam matching with a missing quad. The square root of the horizontal (solid line) and vertical (dashed line) beta-function of the HEP beam is shown.

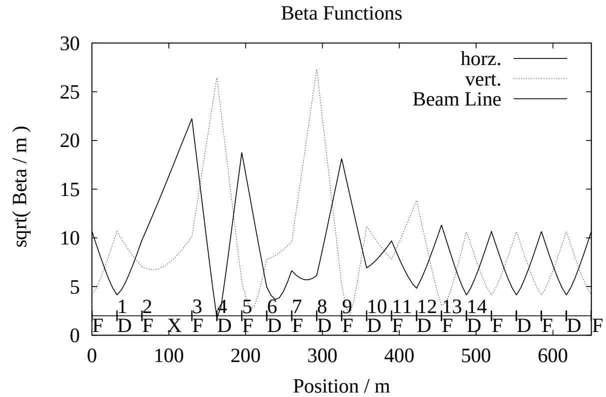

Figure 8: Simultaneous beam matching with a missing quad. The square root of the horizontal (solid line) and vertical (dashed line) beta-function of the FEL beam is shown.

Quad	k_n/k	Quad	k_n/k
D 1	0.1075	**D 8**	0.9962
F 2	0.7727	**F 9**	0.8883
F 3	0.8390	**D 10**	1.1831
D 4	0.9646	**F 11**	0.6901
F 5	1.2198	**D 12**	0.8955
D 6	0.9129	**F 13**	0.9455
F 7	1.0559	**D 14**	1.1530

Table 2: Relative quadrupole strengths of the 14 matching quadrupoles. All quadrupole strengths are normalized to the quadrupole of a regular FODO-cell.

4 CONCLUSION

It has been demonstrated that there exist matched beam optics with a missing quadrupole for the HEP beam and even simultaneously for the HEP and FEL beam. Therefore a power supply failure will not cause a significant down-time of the linear accelerator since the operation can continue with a re-matched lattice.

Acknowledgment

I would like to thank R. Brinkmann for discussions and suggestion for matching of the 60°-lattice. Thanks go also to S.G. Wipf for carefully reading the manuscript.

5 REFERENCES

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