# Electron/positron polarisation?

.

**Desmond Barber**

.

*Deutsches Elektronen–Synchrotron (DESY), Germany*

*University of Liverpool, UK*

*Cockcroft Institute, Daresbury, UK*

*18 February 2009*

## Plan

- Reminders: HERA
- Some theory and phenomenology
- eRHIC ring-ring: calculations
- SPEAR
- Beam-beam forces
- Detector fields

# REMINDERS

- Electrons (positrons) in storage rings can become spin POLARISED due to emission of synchrotron radiation: Sokolov–Ternov effect (1964).

- The polarisation is perpendicular to the machine plane in simple rings.

- The maximum value is then $P_{st} = 92.4\%$.

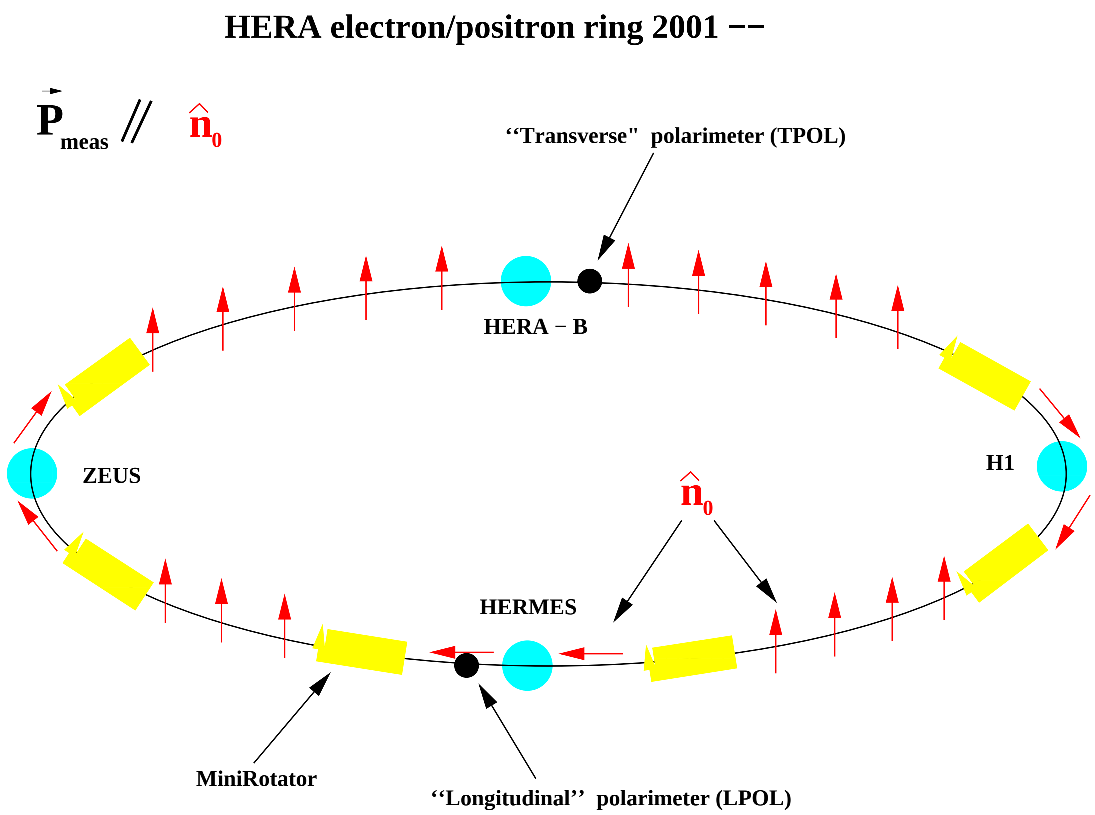
**BUT!**

- Sync. radn. also excites orbit motion. This leads to DEPOLARISATION!
- For longitudinal polarisation the polarisation vector must be rotated into the longitudinal direction before an IP and back to the vertical afterwards ===> spin rotators.
- Vertical bends must be neutralised – otherwise $\hat{n}_0$ is not vertical in the arcs $\Longrightarrow$ strong depolarisation
- Depolarisation can be strongly enhanced by misalignments, regions where the polarisation vector ($\hat{n}_0$) is horizontal between spin rotators etc, etc.....
- Beam-beam forces

# HERA

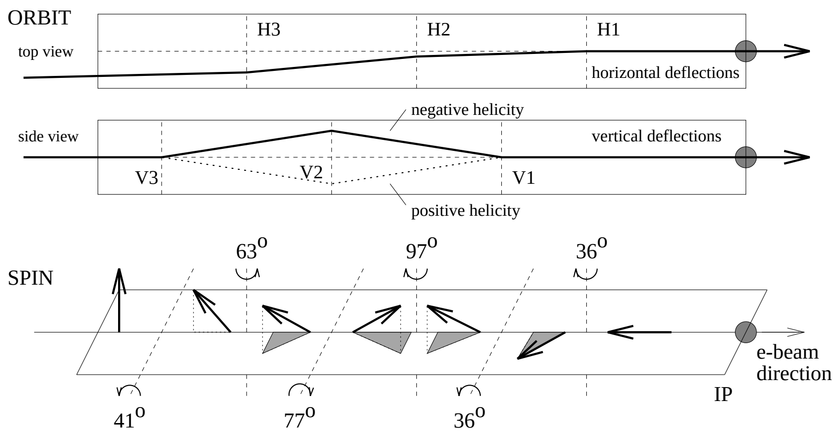
The first and only $e^{\pm}$ ring to supply longitudinal polarisation at high energy
— via the Sokolov-Ternov effect – also at 3 IP's simultaneously!
$\approx$ 30 GeV, $\tau_{st} \approx$ 30 mins. Depolarisation not too strong.
Perfectly balanced parameters

BUT influence of beam-beam forces!




Polarisation vertical in the arcs – to drive the Sokolov-Ternov effect

# HERA MiniRotator: Buon + Steffen

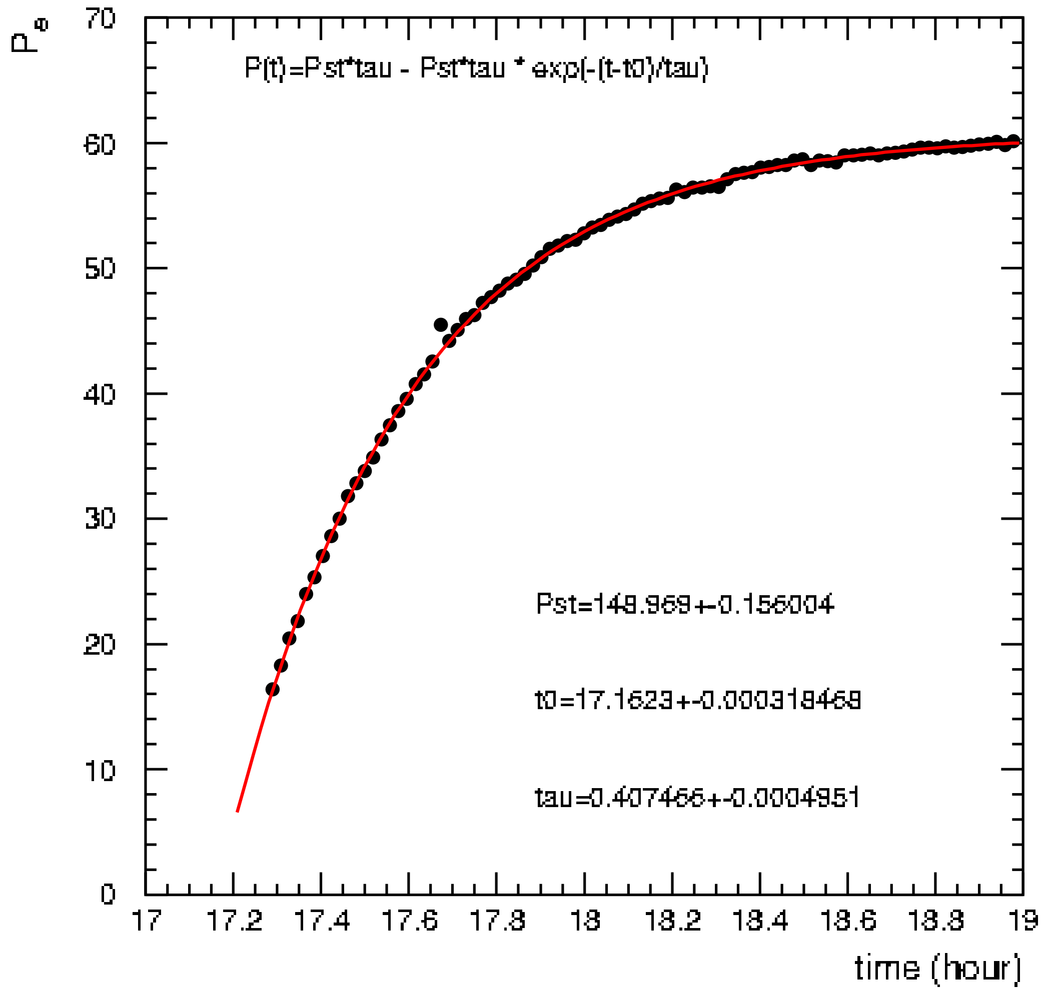



**56 m ("short") → no quads.**

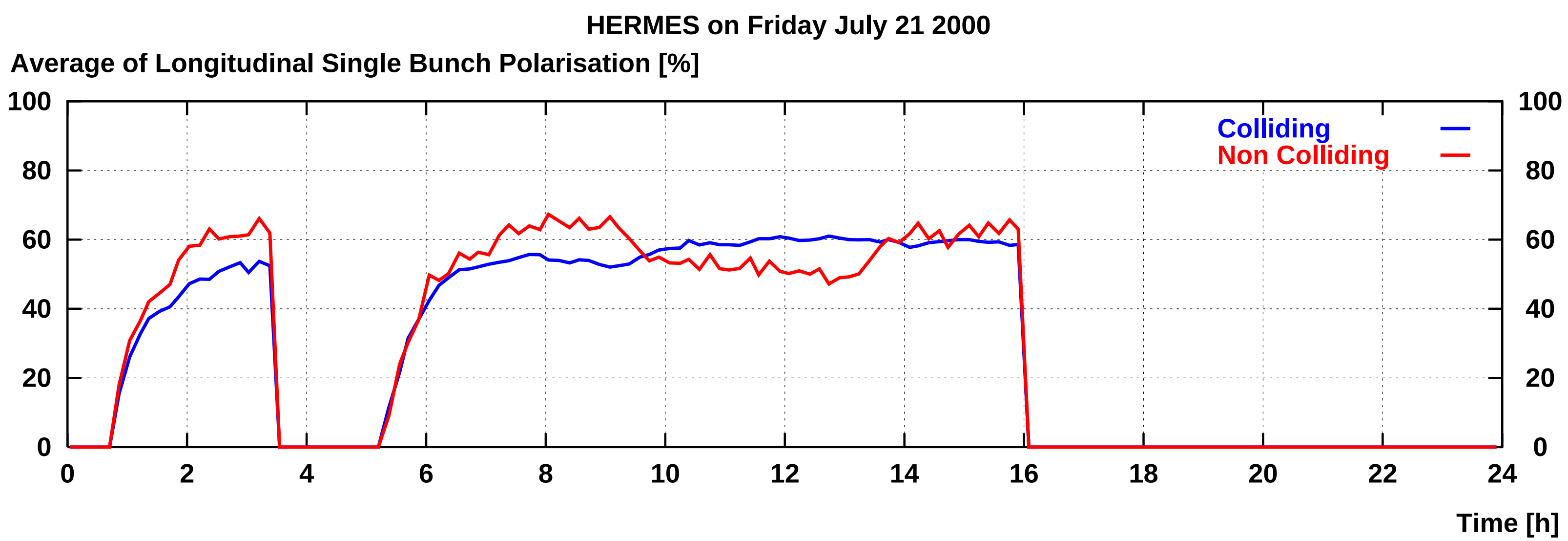
**27 – 39 GeV, both helicities, variable geometry**

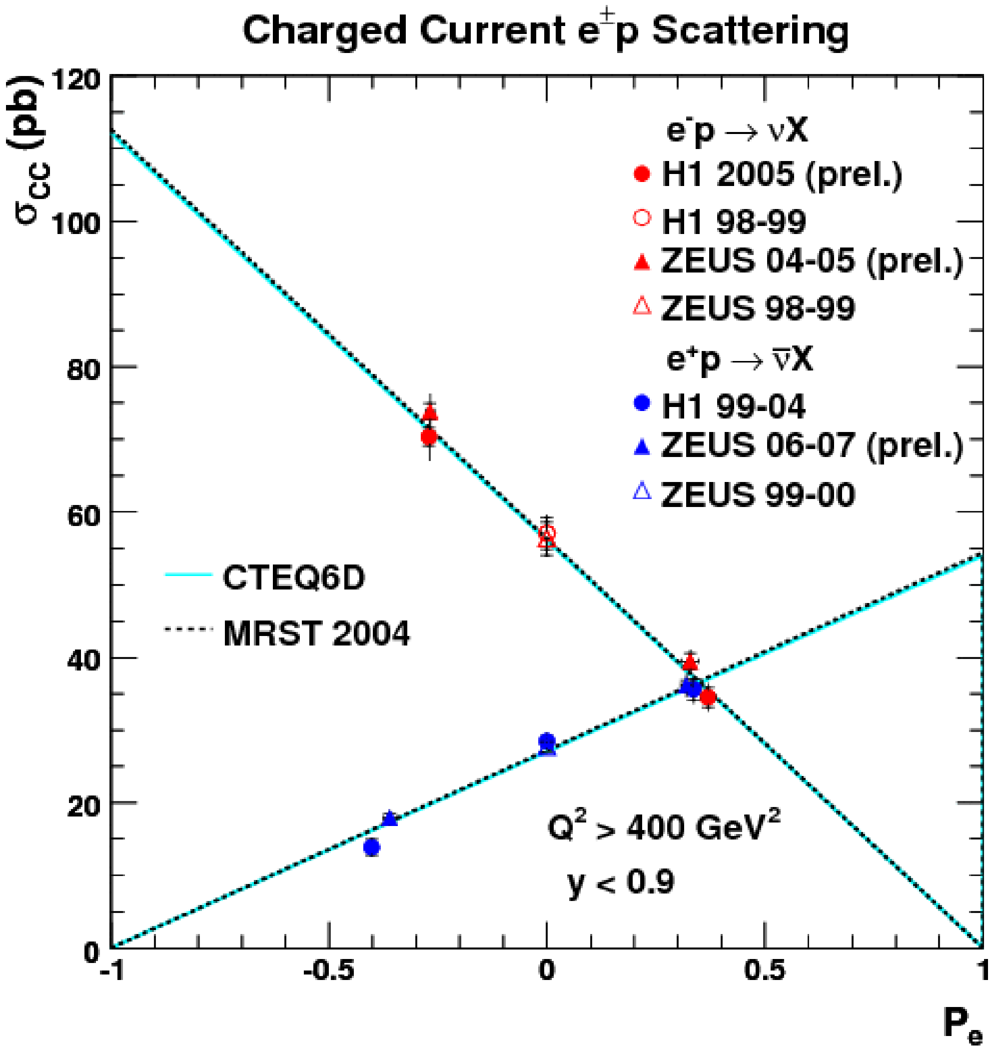
NO INTERNAL QUADRUPOLES!

June 2007, the Fabry-Perot-Compton polarimeter of the POL2000 Project: Calibrating polarimeters




3 pairs of rotators (so max. Sokolov-Ternov polarisation = 83 %), solenoids on, no beam-beam


HERMES on Friday July 21 2000
Average of Longitudinal Single Bunch Polarisation [%]
Colliding
Non Colliding
Time [h]
0
20
40
60
80
100
0
2
4
6
8
10
12
14
16
18
20
22
24



Charged Current $e^{\pm}p$ Scattering
$\sigma_{CC}$ (pb)
120
100
80
60
40
20
0
-1
-0.5
0
0.5
1
$P_e$
$e^-p \rightarrow \nu X$
H1 2005 (prel.)
H1 98-99
ZEUS 04-05 (prel.)
ZEUS 98-99
$e^+p \rightarrow \bar{\nu} X$
H1 99-04
ZEUS 06-07 (prel.)
ZEUS 99-00
CTEQ6D
MRST 2004
$Q^2 > 400$ GeV$^2$
$y < 0.9$

# THEORY and PHENOMENOLOGY

# Spin motion of moving particles – the T-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

Use $a = \frac{g-2}{2} \quad (= G) : G \neq g$ !!

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(a\gamma+1)\,\vec{B} - \frac{a\gamma^2\beta^2}{1+\gamma}(\hat{\beta}\cdot\vec{B})\hat{\beta} - \frac{\beta\gamma}{c}\left(a+\frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right] .$$

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(a\gamma+1)\,\vec{B}_{\perp} + \frac{g}{2}\vec{B}_{//} - \frac{\beta\gamma}{c}\left(a+\frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right] .$$

Several equivalent forms. Very easy to get it wrong!

Now calculate w.r.t. the design orbit which itself rotates (precesses) with $\vec{\Omega}_{\mathrm{CO}} = -\frac{e}{m\gamma}\vec{B}_{\mathrm{guide}}$.

$$\vec{\Omega} = -\frac{e}{m\gamma}\left[(a\gamma+1)\,\vec{B} - \vec{B}_{\mathrm{guide}} - \frac{a\gamma^2\beta^2}{1+\gamma}(\hat{\beta}\cdot\vec{B})\hat{\beta} - \frac{\beta\gamma}{c}\left(a+\frac{1}{1+\gamma}\right)(\hat{\beta}\times\vec{E})\right] .$$

In transverse fields: $\delta\phi_{\mathrm{spin}} = (a\gamma+1)\,\delta\phi_{\mathrm{traj}}$

$\Longrightarrow$ In a flat ring the number of spin precessions per turn around the ring on the design orbit is

$\nu_0 \equiv a\gamma = G\gamma$!

$\nu_0$ is called the **spin tune on the design orbit**, Thomas precession: $a + \frac{1}{\gamma} = \frac{g}{2} - 1 + \frac{1}{\gamma}$

**Periodic solution $\hat{n}_0$ on closed orbit.**

The real unit eigenvector of:

$$R_{3\times 3}(s+C,s)\hat{n}_0 = \hat{n}_0$$

$\hat{n}_0$ is 1–turn periodic: $\hat{n}_0(s+C) = \hat{n}_0(s)$

$\hat{n}_0$: direction of measured equilibrium radiative polarisation.

Closed orbit spin tune $\nu_0$: number of precessions per turn around $\hat{n}_0$ for a spin on the closed orbit. Extract from the eigenvalues of $R_{3\times 3}(s+C,s)$

$$\nu_0 = a\gamma + \Delta \qquad a\gamma = \frac{E(GeV)}{0.440652..(GeV)}$$

## Sokolov-Ternov effect

$$W_{\uparrow\downarrow} = \frac{5\sqrt{3}}{16}\frac{r_{\rm e}\gamma^5\hbar}{m_{\rm e}|\rho|^3}\left(1+\frac{8}{5\sqrt{3}}\right)$$

$$W_{\downarrow\uparrow} = \frac{5\sqrt{3}}{16}\frac{r_{\rm e}\gamma^5\hbar}{m_{\rm e}|\rho|^3}\left(1-\frac{8}{5\sqrt{3}}\right)$$

For positrons, interchange plus and minus signs here and elsewhere.

The equilibrium polarisation in a uniform magnetic field is independent of $\gamma$,

$$P_{\rm st} = \frac{W_{\uparrow\downarrow}-W_{\downarrow\uparrow}}{W_{\uparrow\downarrow}+W_{\downarrow\uparrow}} = \frac{8}{5\sqrt{3}} = 0.9238$$

Why not 100 percent ?

Recall: $\Delta E_{SG} = -\vec{\mu}.\vec{B}_R \Rightarrow \vec{\Omega}.\vec{S}$

So perhaps the spins just want to fall into the lowest S-G. energy state.

WRONG! – but a useful memory device for electrons/positrons.

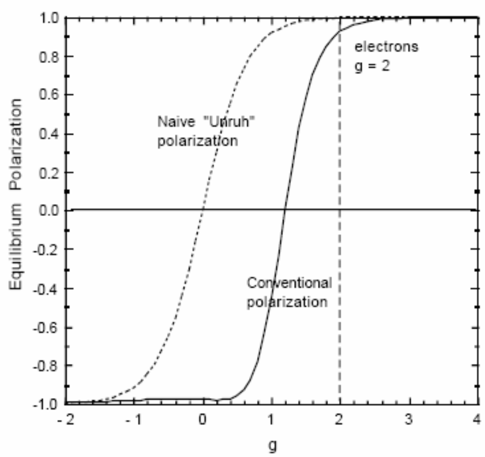



Figure 1. Equilibrium electron polarization in storage rings. Solid curve, conventional result[3], dashed curve, $P = \tanh(\pi g/2)$, the naive BL result[2]. For the range, $0 < g < 1.2$, the effective temperature $T_{eff}$ is negative ($gP < 0$).

Derbenev and Kondratenko 1973
Jackson 1976 and 1998.

Keep $\frac{g-2}{2}$ in the pure spin flip calculations. 92.38 % is not special.



Figure 1:

# Dirac eqn $\Longrightarrow$ Foldy-Wouthuysen trans $\Longrightarrow$ two component formalism

The Dirac equation contains unneeded information. So simplify.

$$h^{\rm dk} = h^{\rm dk}_{\rm orb} + \frac{\hbar}{2}\vec{\sigma}\cdot\vec{\Omega} \ ,$$

where:

$$h^{\rm dk}_{\rm orb} = c\sqrt{(\vec{p}-e\vec{A})^2 + m^2c^2} + e\Phi \ ,$$

The Heisenberg eqn of motion for $\vec{\sigma}$ is the T-BMT equation!!!!!

For radiation: replace classical fields by radiation fields:

$$h^{\rm dk}_{\rm tot} = h^{\rm dk} + e(\phi_{\rm rad} - \vec{\beta}\cdot\vec{A}_{\rm rad}) + \frac{\hbar}{2}\vec{\sigma}\cdot\vec{\Omega}_{\rm rad} \ ,$$

A photon emission causes both the spin **and orbital** quantum numbers to jump.

Thomas precession: $a + \frac{1}{\gamma} = \frac{g}{2} - 1 + \frac{1}{\gamma}$

So $g$ is effectively shifted – orbit to spin coupling.

Characteristic times of processes (Montague 1984)

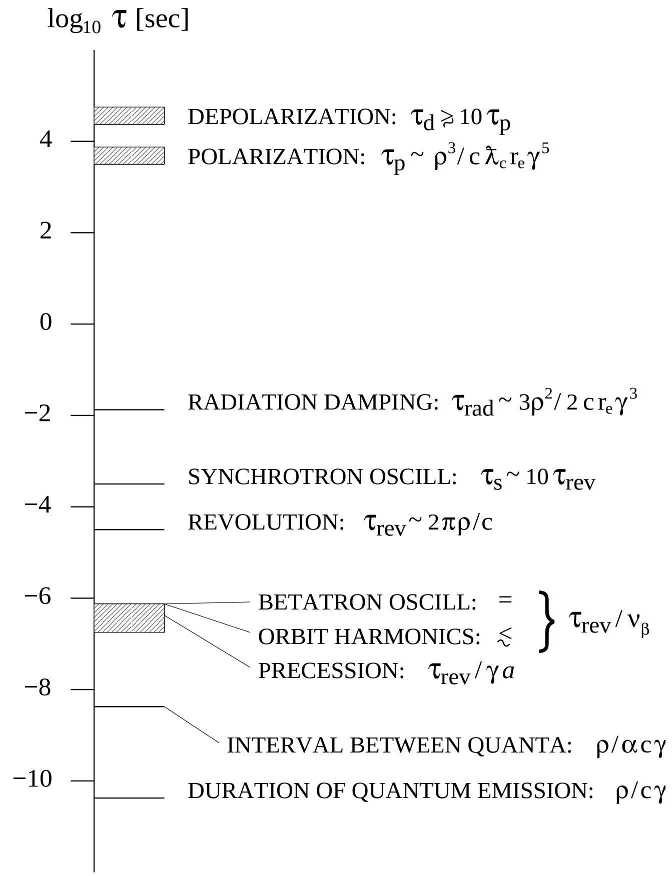



Figure 2: Characteristic time scales in a typical 25 GeV electron storage ring. Legend: $\rho$ = bending radius, $\lambda\!\!\!^{-}_c$ = Compton wavelength, $r_e$ = classical electron radius, $\nu_\beta$ = betatron tune, $\alpha$ = fine structure constant, $a$ = gyromagnetic anomaly. Although it is desirable that $\tau_d \geq 10\,\tau_p$, this is difficult to achieve in practice.

$\hat{n}_0$: direction of measured equilibrium radiative polarisation — **time scales**.

.

Without depolarisation

$$\vec{P}_{\rm bks} \;=\; -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint\{\frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}\}ds}{\oint ds\{\frac{1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2}{|\rho(s)|^3}\}} \quad\Longrightarrow -\frac{8}{5\sqrt{3}}\,\hat{n}_0 \;\;\text{for vertical}\;\; \hat{n}_0$$

The **value** of the polarisation is the same at all azimuths — **time scales**.

.

The BKS polarisation build-up rate is — **time scales**

$$\tau_{\rm bks}^{-1} \;=\; \frac{5\sqrt{3}}{8}\frac{r_{\rm e}\gamma^5\hbar}{m_{\rm e}}\frac{1}{C}\oint ds\,\frac{1-\frac{2}{9}\,(\hat{n}_0\cdot\hat{s})^2}{|\rho(s)|^3}$$

$$\tau_{\rm bks}^{-1}[{\rm s}^{-1}] \;\approx\; \frac{2\pi}{99}\frac{E[{\rm GeV}]^5}{C[{\rm m}]\rho[{\rm m}]^2}$$

## Orbit and spin motions

- **Protons**: largely deterministic, reversible and long memory— unless IBS etc.
- **Electrons/positrons**:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? ===>

  **Stochastic/damped** orbital motion due to synchrotron radiation:
  . irreversible and only short memory
  \+ **inhomogeneous fields**
  \+ **spin–orbit coupling via T–BMT**
  ===> spin diffusion i.e. depolarisation!!!

  Self polarisation: Balance of poln. and depoln.

## Orbital motion

Linearise the EOM and use symplectic $6 \times 6$ transport matrices.

$$u(s_2) = M(s_2, s_1)u(s_1)$$

with $u^T = (x,\ x',\ y,\ y',\ l,\ \delta)$ with $\delta = \frac{\Delta E}{E_0}$

$$u(s_N) = M(s_N, s_{N-1})M(s_{N-1}, s_{N-2})M(s_2, s_1)u(s_1)$$

Photon noise goes into $\delta$ and spreads into the rest.

With damping mechanisms, the phase space density in a bunch reaches equilibrium. Also need a non-symplectic $M_{\text{damp}}$.

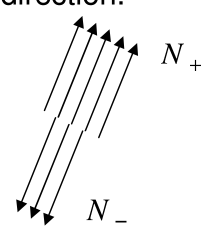
# The definition of polarisation

$$\vec{P} = \frac{1}{|\langle \vec{S} \rangle|} \frac{\sum_{i=1}^{1=N} \langle \vec{S}_i \rangle}{N}$$

Just an average of normalised expectation values – applicable both to pure and mixed states.

Obviously $|\vec{P}| \leq 1$

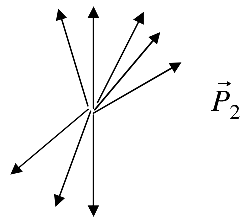
Example 1: a mixed fermion state consisting of $N_+$ $(N_-)$ spins pointing up (down) along some direction:




$$P_1 = \frac{N_+ - N_-}{N_+ + N_-}$$

along the common direction.

Example 2: a mixed state consisting of spins pointing is many directions.




For fermions: if $\vec{P}_1 = \vec{P}_2$ the two states are **completely indistinguishable** w.r.t. any observations.

The polarisation and the T-BMT equation are linear in the spins.
So the polarisation for spins in an infinitisimal volume of phase space around an orbit obeys the T-BMT equation.

For fermions: if $\vec{P}_1 = \vec{P}_2$ the two states are **completely indistinguishable** w.r.t. any observations.

The spin density matrix: $\rho_{1/2} = \frac{1}{2}\{I_{2\times 2} + \vec{P}.\vec{\sigma}\}$

**The SLIM formalism for estimating depolarisation at first order (Chao 1981).**

Skip the invariant spin field and the Derbenev-Kondratenko formula for today!
With the ISF, the complications from the motion in the quads etc are packed into classically calculable quantisation axes which depend on $u$ and $s$.

Heuristics instead!

$$\vec{S} = \sqrt{1-\alpha^2-\beta^2}\ \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s) \quad \approx \quad \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s)$$

$\hat{m}_0$ and $\hat{l}_0$ orthogonal to $\hat{n}_0$. All obey the T-BMT eqn.

$\alpha, \beta$: 2 small spin tilt angles — have subtracted out the big rotations!

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_2) \;=\; \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_1)$$

with $u^T = (x,\ x',\ y,\ y',\ l,\ \delta)$ and $\alpha, \beta$

## Spin–orbit covariance matrix

$$\begin{pmatrix}
\sigma_x^2 & \sigma_{xx'} & . & . & . & . & | & . & . \\
\sigma_{x'x} & \sigma_{x'}^2 & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & \sigma_\delta^2 & | & . & . \\
- & - & - & - & - & - & - & - & - \\
. & . & . & . & . & . & | & \sigma_\alpha^2 & \sigma_{\alpha\beta} \\
\sigma_{\beta x}. & . & . & . & . & . & | & \sigma_{\beta\alpha} & \sigma_\beta^2
\end{pmatrix}$$

$$\Delta P = 1 - \Delta < \sqrt{1-\alpha^2-\beta^2} > \approx -\frac{1}{2}\Delta(<\alpha^2+\beta^2>) = -\frac{1}{2}\Delta(\sigma_\alpha^2+\sigma_\beta^2) \Longrightarrow \frac{dP}{dt} \approx -\frac{1}{2}\frac{d}{dt}(\sigma_\alpha^2+\sigma_\beta^2)$$

Random walk in plane orthogonal to $\hat{n}_0$.

Calculate analytically (stochastic differential equations or Fokker-Planck methods) (Brownian motion). Everything needed is in 1-turn matrices.

Or brute force with a Monte-Carlo with multi-turn tracking – in fact the best in the end.

No damping mechanism for spin – but the S-T effect works to restore the polarisation along $\hat{n}_0$.

Just need the **slope** $\frac{dP}{dt}$.

## Spin–orbit resonances

Emerge automatically in analytical calculations.

$$\nu_0 \;=\; k + k_I\nu_I + k_{II}\nu_{II} + k_{III}\nu_{III}$$

$\nu_0$ : amplitude dependent spin tune ≈ closed orbit spin tune = precessions /turn on CO , $\nu_0$

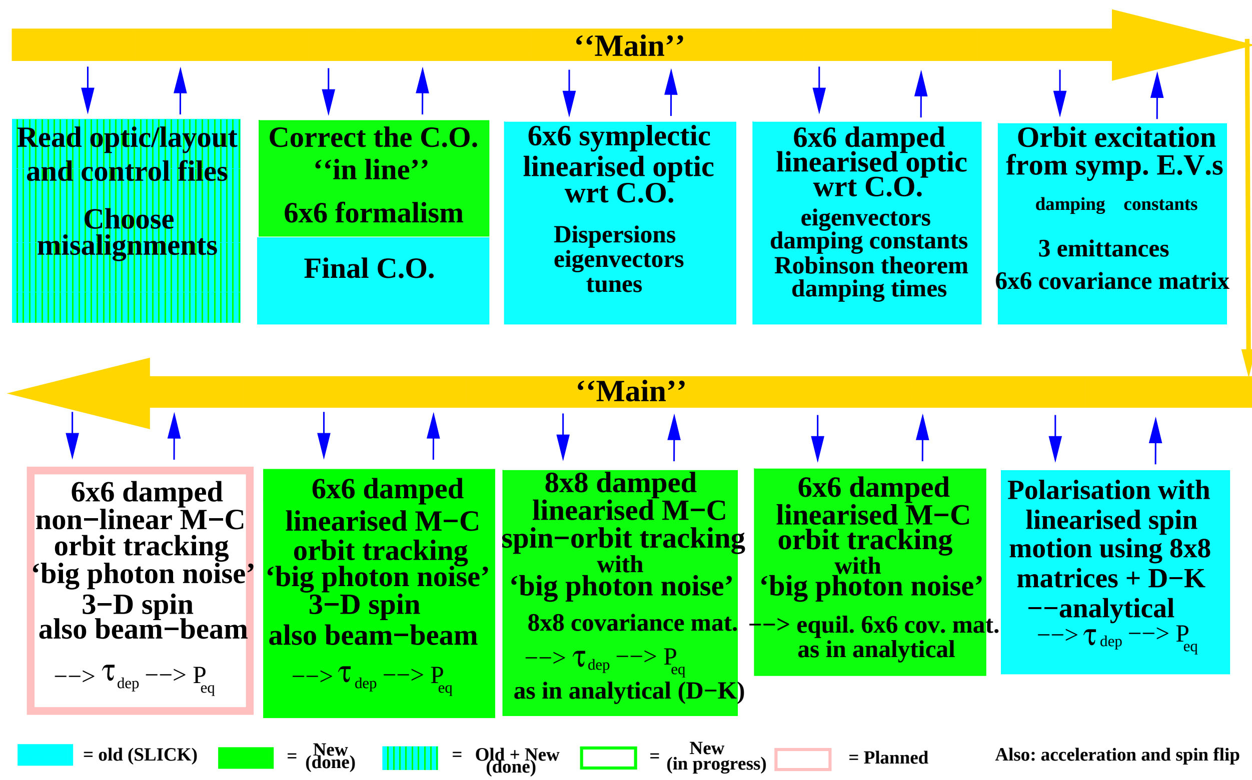
- Orbit "drives spins" ===> Resonant enhancement of spin diffusion.
- Resonance order: $|k_I| + |k_{II}| + |k_{III}|$
- First order: $|k_I| + |k_{II}| + |k_{III}| = 1$ e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

$$\nu_0 \;=\; k + k_i\nu_i + k_{III}\nu_{III}, \quad i = I, II \text{ or } III$$

Protons: narrow except at very high energy. Then only acceleration is a problem.

Electrons: Wide at fixed energy!

## The structure of SLICKTRACK



# Equilibrium polarisation is the result of the balance of poln. and depoln

$$P_{\infty} \approx P_{\rm bks} \frac{1}{1 \ + \ (\frac{\tau_{\rm dep}}{\tau_{\rm bks}})^{-1}} \qquad (P_{\rm st} \to P_{\rm bks})$$

In any case:

$$\tau_{\rm dep}^{-1} \ \propto \ \gamma^{2N} \ \tau_{\rm bks}^{-1} \quad \text{(actually a polynomial in } \gamma^{2N}\text{)}$$

===> Trouble at high energy!

# Spin matching:

To minimise depolarisation:

Minimise appropriate bits of $\mathbf{G_{2\times 6}}$ for appropriate stretches of ring
===> lots of independent quadrupole circuits.

Spin transparency!!! – not the trivial kind to which this term is applied at certain labs.

## More on spin–orbit resonances

(1) Linear orbit motion with linearised spin motion (SLIM/SLICK/SLICKTRACK):
just first order spin–orbit resonances.

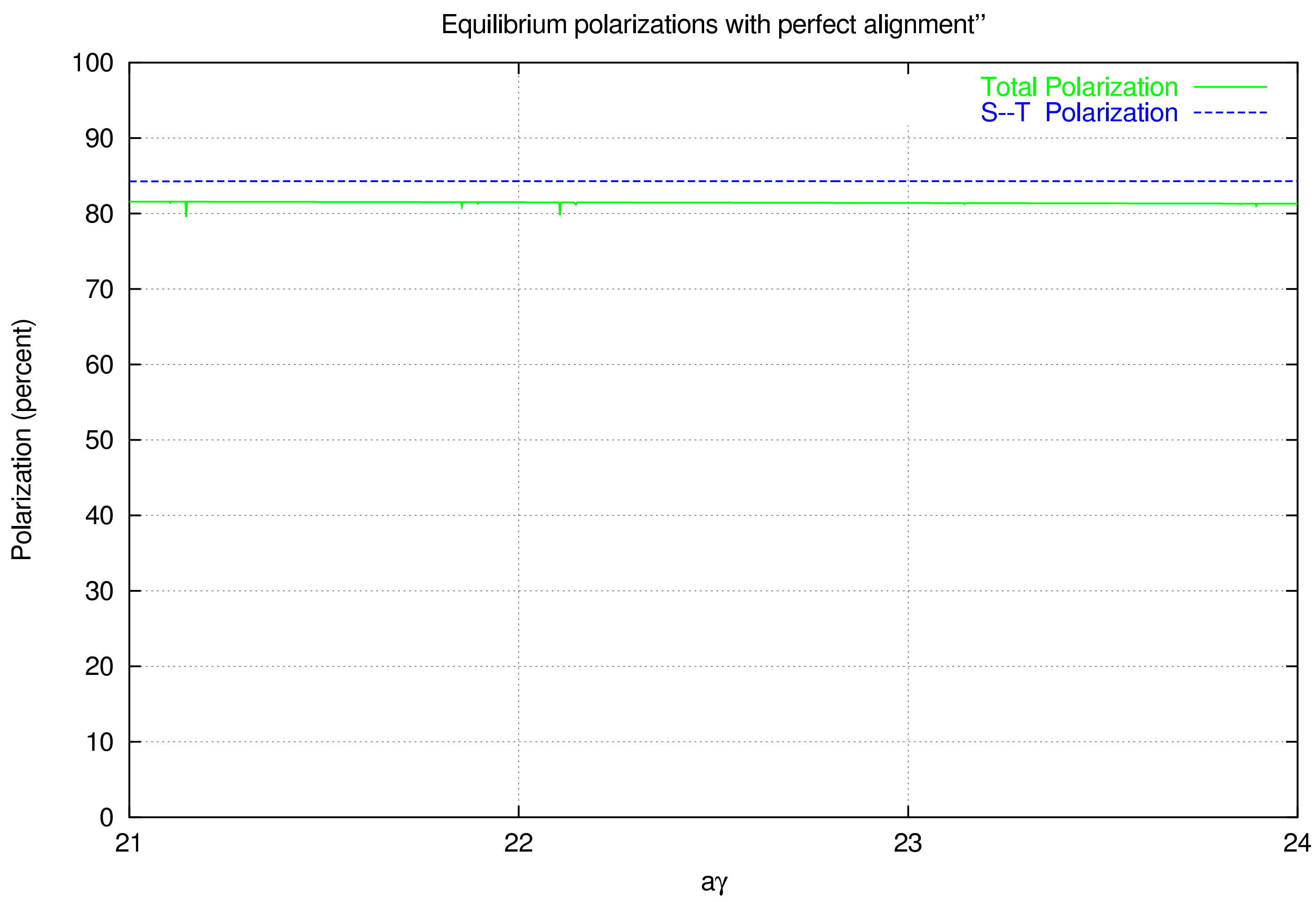
(2) Linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

(3) Non–linear orbit motion with linearised spin motion:
orders of spin–orbit resonances just reflecting the orbital spectrum.

(4) Non–linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

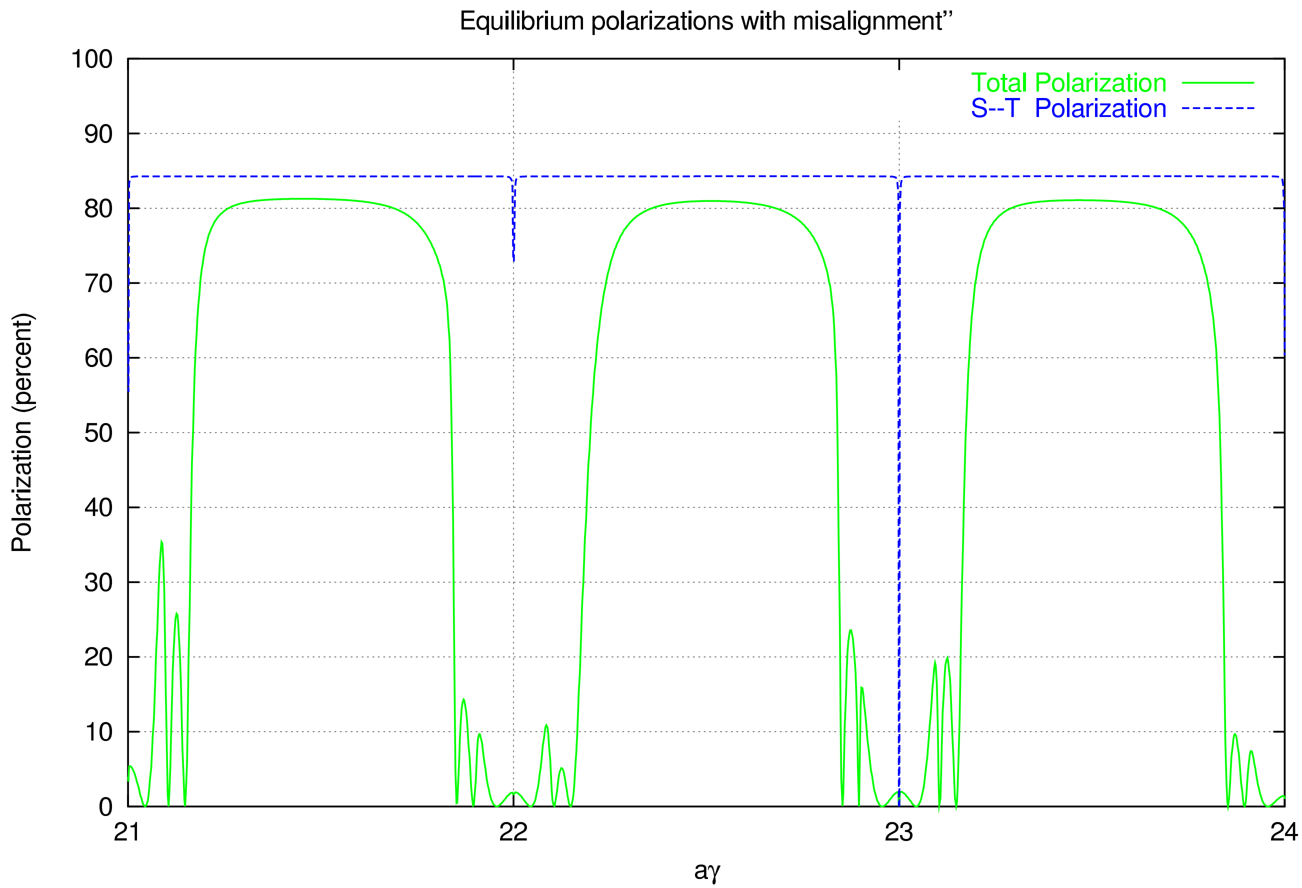
**Diagnostics : With (1) and (3) we use spin motion to Fourier analyse the orbital motion!**

# eRHIC


Equilibrium polarizations with perfect alignment"
Total Polarization
S--T Polarization
Polarization (percent)
100
90
80
70
60
50
40
30
20
10
0
21
22
23
24
aγ

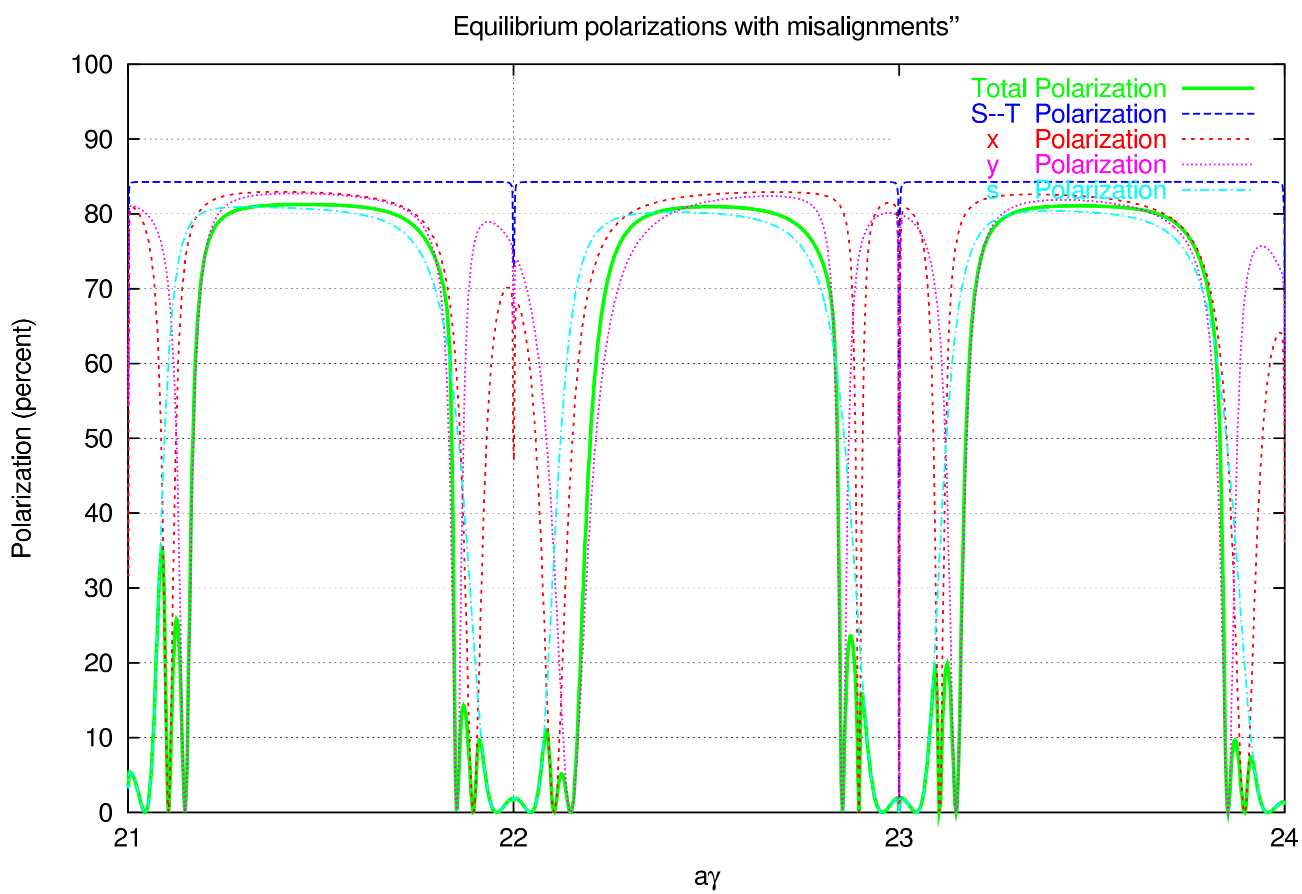
# eRHIC

## All monitors on: just ONE example!



# eRHIC

## All monitors on: DIAGNOSTICS



## Spin–orbit maps for sections

For linearised spin motion (SLIM/SLICK/SLICKTRACK):

$$\hat{\mathbf{M}} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

The $\mathbf{G_{2\times 6}} \times (\mathbf{x}, \mathbf{x}', \mathbf{y}, \mathbf{y}', \mathbf{\Delta l}, \delta)^{\mathbf{T}}$ delivers changes to the 2 small angles $\alpha$ and $\beta$

..........................................................................................................................

For full 3–D spin motion:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \\ \gamma \end{pmatrix} (s_2) = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 3}} \\ \mathbf{G_{3\times 6}} & \mathbf{D_{3\times 3}} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \\ \gamma \end{pmatrix} (s_1)$$

The $\mathbf{G_{3\times 6}} \times (\mathbf{x}, \mathbf{x}', \mathbf{y}, \mathbf{y}', \mathbf{\Delta l}, \delta)^{\mathbf{T}}$ delivers 3 (small) rotations $(\alpha, \beta, \gamma)$ around $(\hat{n}_0,\ \hat{m}_0,\ \hat{l}_0)$

The beam–beam (non–linear) kicks are applied at single points.

## First result of 3-D spin motion

Sidebands of parent first order betatron resonances: a useful **approximation**

$$\tau_{dep}^{-1} \propto \frac{A}{(\nu_0 \pm Q_y)^2} \quad \rightarrow \quad \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{A\, B(\xi; m_s)}{(\nu_0 \pm Q_y \pm m_s\, Q_s)^2}$$

$A$ is an energy dependent factor

$B(\xi; m_s)$'s: *enhancement factors*, contain modified Bessel functions $I_{|m_s|}(\xi)$ and $I_{|m_s|+1}(\xi)$ depending on the *modulation index*

$$\xi = (\frac{a\gamma\ \sigma_\delta}{Q_s})^2$$

in a flat ring.

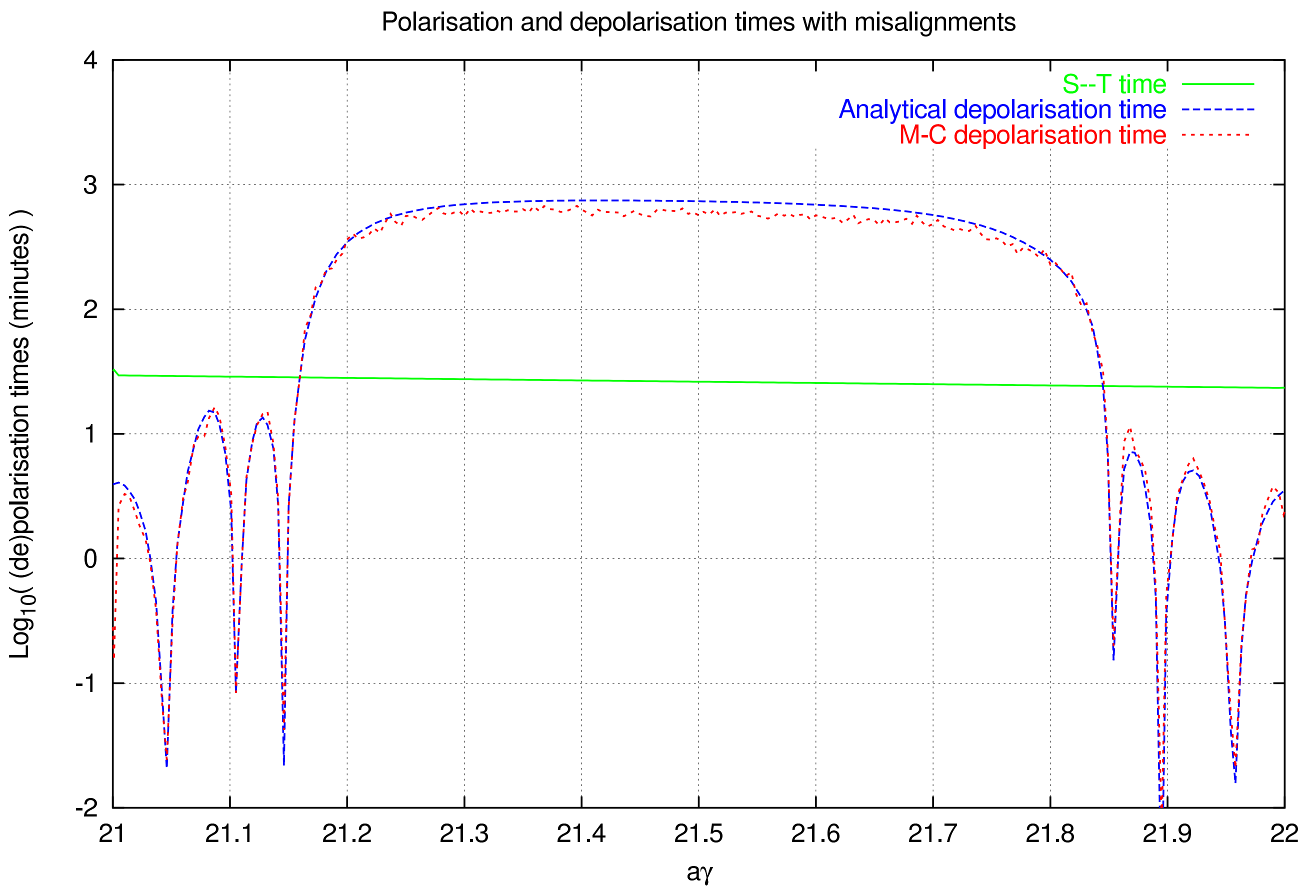
===> very strong effects at high energy — dominant source of trouble

Recall the limitations at LEP!!

Analogous formula for sidebands of first order synchrotron resonances.

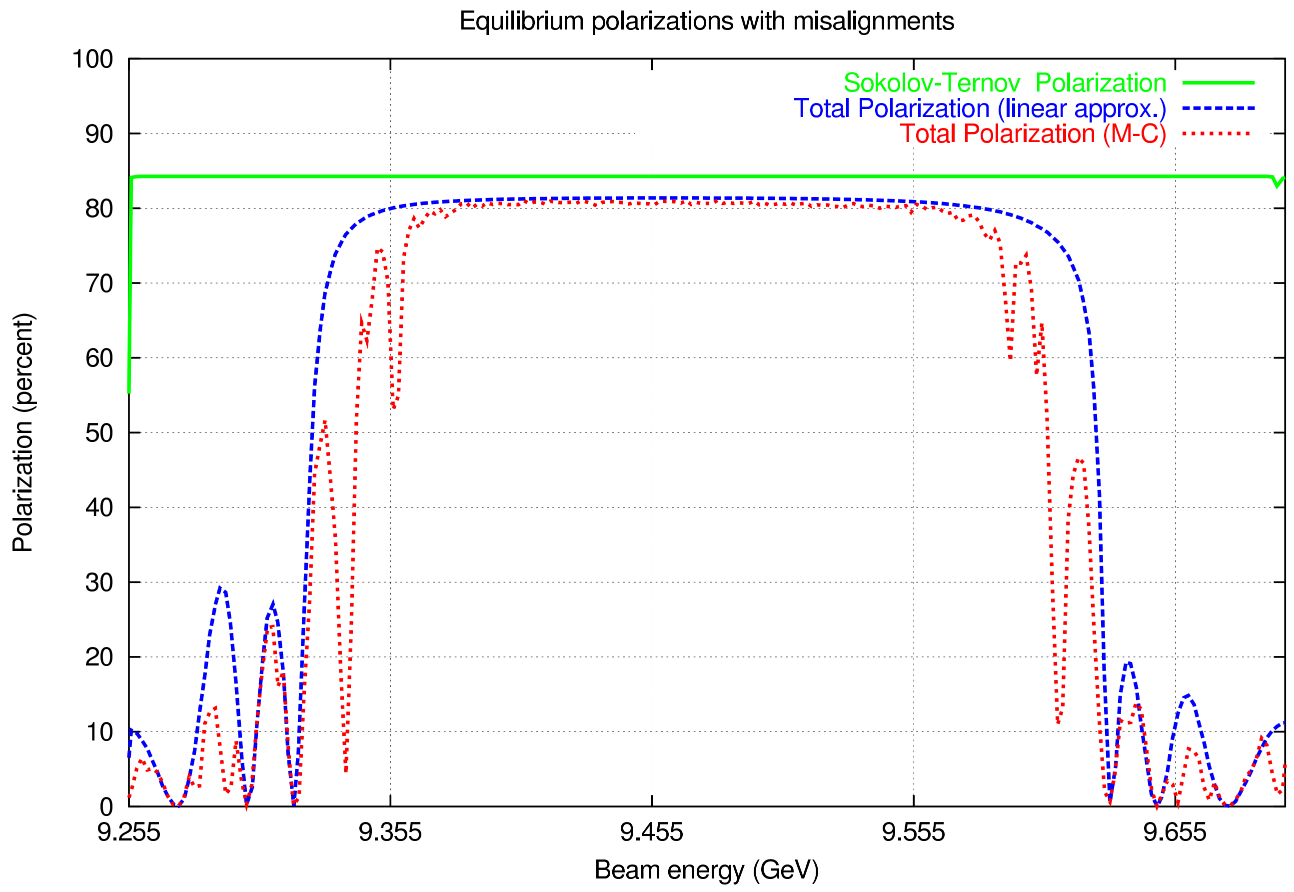
# eRHIC

## Calibrating the (first order) M-C software structure against SLICK

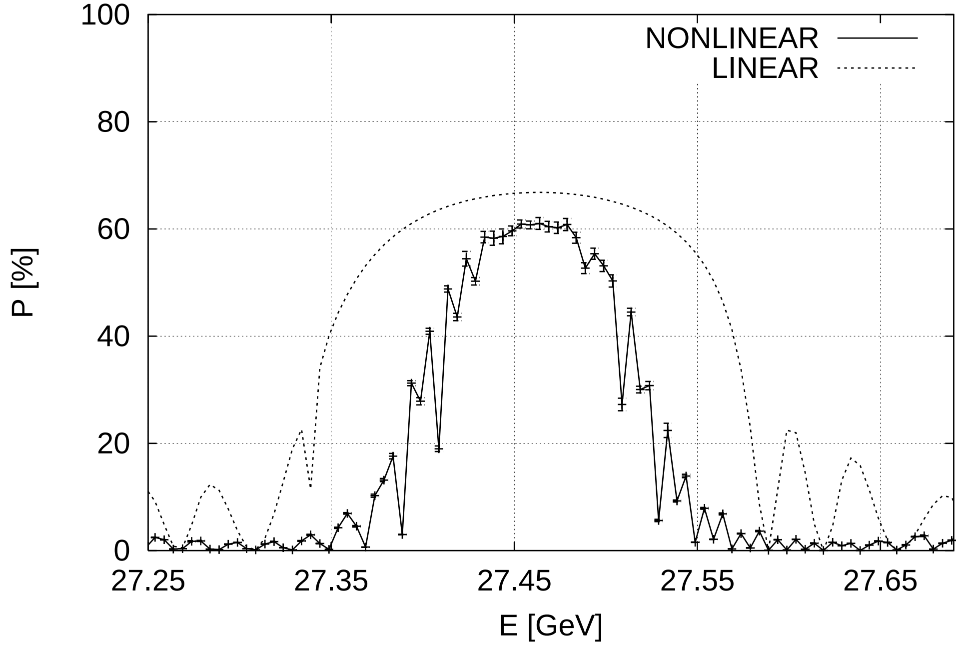


# eRHIC

## Full 3–D spin motion



## Full 3–D spin motion: HERA-II ≈ 27.5 GeV




**SITROS: M. Berglund, DESY–THESIS 2001–044 (2001)**

## Diagnostics! Diagnostics! Diagnostics!

**Switch spin-orbit coupling off/on to see what does what.**

# SPEAR

SPEAR around 1980

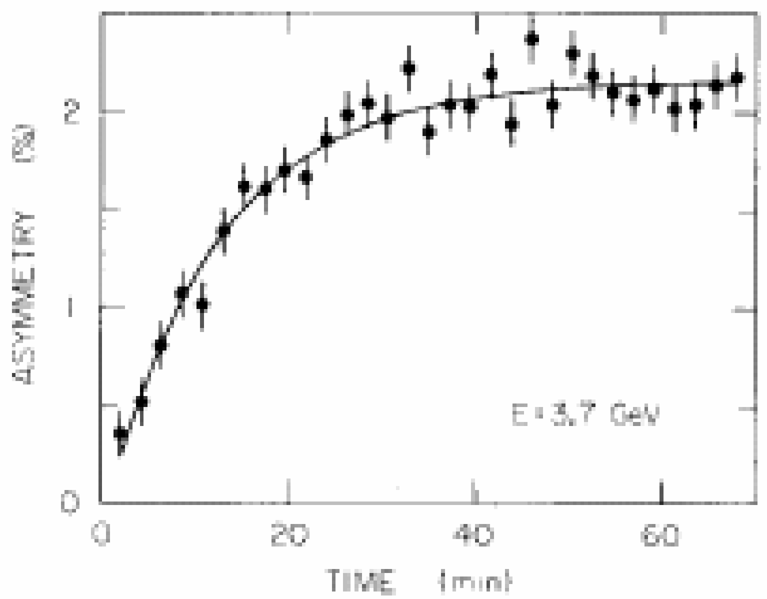



**Fig. 3. Measured asymmetry versus time, with 3.7 GeV positrons only circulating in SPEAR.**

# SPEAR around 1980

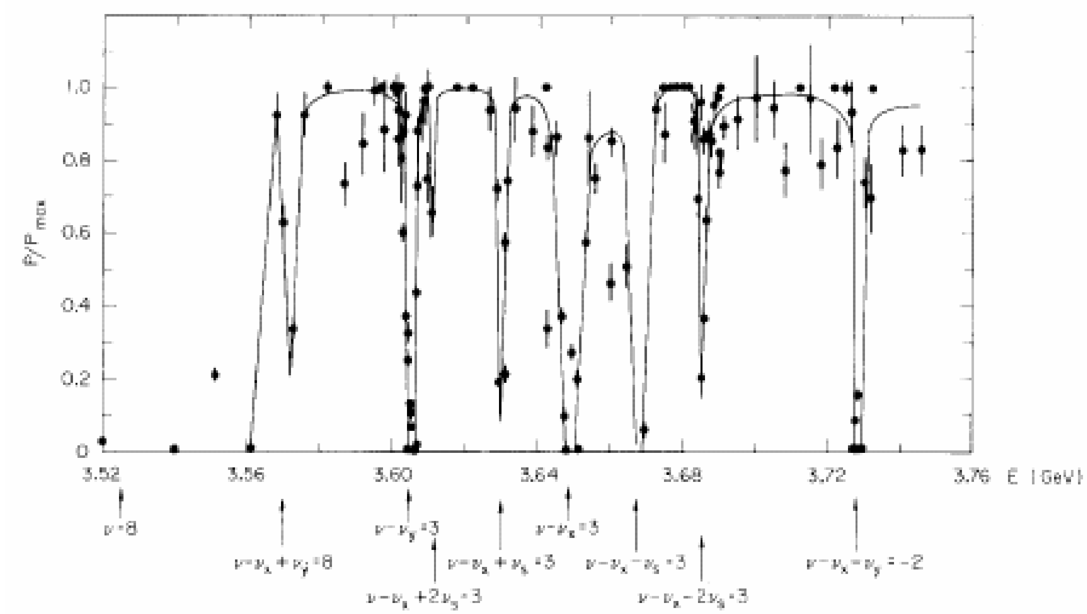
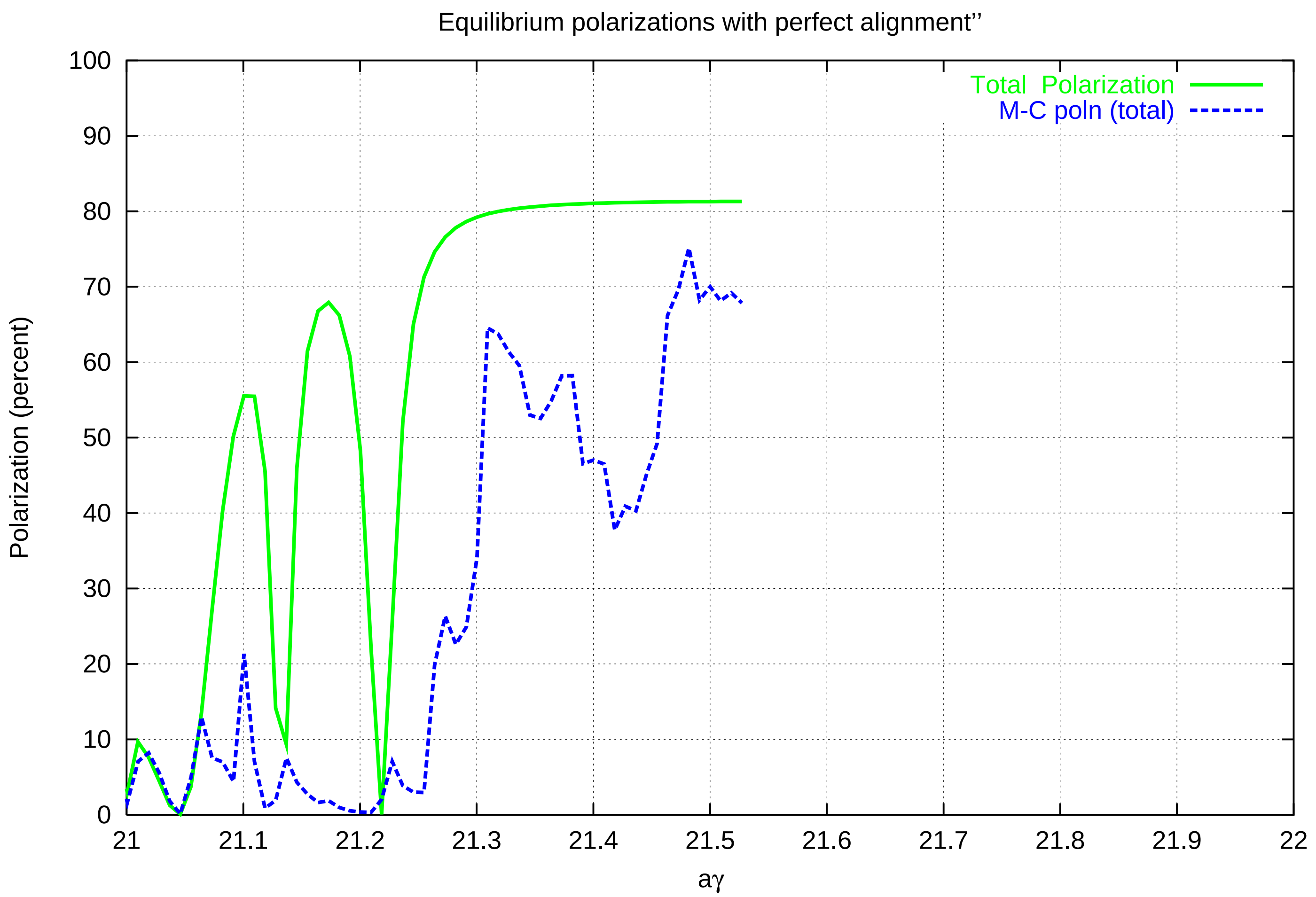



Fig. 8. Compilation of results from all single-beam $e^+$ energy scans in the region of 3.50 to 3.75 GeV. The solid line is hand-drawn to guide the eye. The arrows below the horizontal axis indicate observed depolarizing resonances.

# BEAM-BEAM FORCES

# eRHIC

## Effect of beam-beam forces – preliminary



# FIELDS IN THE DETECTOR

???????????????


By Brian Montague during the lead-up to LEP and HERA polarisation.