Phenomenology of Physics beyond the Standard Model

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Contents

Chapter 1

The Standard Model in a nutshell

1.1 Preliminaries

- What specifies the theory? Action S or (more or less equivalently) Lagrangian (density) $\mathcal{L}$. The action is dimensionless and simply the integral over the Lagrangian density, $S = \int d^4x\mathcal{L}$. Here, dimension refers to the mass dimension in natural units, where $[m] = [E] = [p] = [x^{-1}] = [t^{-1}]$. The Lagrangian (density) therefore has dimension 4. So what are possible terms? Look at the mass dimension of fields: The dimension of fields in 4 dimensions is

$$[\phi] = 1, \qquad [\psi] = \frac{3}{2}, \qquad [A_\mu] = 1. \tag{1.1.1}$$

- In addition there can be coupling constants with mass dimension.
- renormalisable theory: no coupling constants with negative mass dimension (e.g. Standard Model). Only limited number of terms possible, such as $m^2\phi^2 + \phi^4$... (we will discuss effective theories later, where also non-renormalisable terms are present).
- How to arrive at the Lagrangian? Input: Field content and symmetries.
- Given the Lagrangian can read off Feynman rules and perturbatively calculate any process of interest.

1.1.1 Symmetries

We know nature likes symmetries. Symmetry principles underlie most of the dynamics of the Standard Model. We are familiar with two basic types of symmetry, spacetime and internal.

1.1.1.1 Lorentz symmetry

Since Einstein we know that we live in a four dimensional spacetime and that for systems where gravity is only a small effect this can be approximately described by Minkowski space, subject to Lorentz symmetry. The Minwoski space metric $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$ is invariant under $SO(3,1)$ Lorentz transformations of the coordinates. These transformations are the Lorentz transformations and are the foundation of special relativity. Different kinds of fields transform differently under Lorentz transformation, they live in different **representations** of the Lorentz group.

For example, under a Lorentz transformation given by an $SO(3,1)$ matrix M^μ_ν , a vector field A_μ transforms as

$$A^\mu_{\text{new}} = M^\mu_\nu A^\nu_{\text{old}} \tag{1.1.2}$$

Other representations are the spinor representation that describes fermions such as the quarks and leptons and the (trivial) scalar representation which describes the Higgs. In general there are different representations for different spin (and for the case of spinors there are two different irreducible representations, the left- and right- handed spinors).

The Lagrangian has to be invariant under Lorentz transformations, so fields which transform non-trivially have to be combined in the right way. For the case of Dirac spinors this combination turns out to be $\bar{\psi}\psi \equiv \psi^\dagger \gamma^0 \psi$, with γ^0 one of the Dirac gamma matrices which are subject to the Clifford algabra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} , \tag{1.1.3}$$

with $\eta^{\mu\nu}$ the Minkowski metric. For the case of particles vectors the Lorentz indices have to be contracted, e.g. $\eta_{\mu\nu} A^\nu A^\mu \equiv A_\mu A^\mu$.

1.1.1.2 Internal symmetries

A second type of symmetry are internal symmetries. These may be either global symmetries (such as the approximate $SU(2)$ isospin symmetry of the strong interactions) or local (gauge) symmetries. An example of these is the $SU(3)$ gauge symmetry of the strong interactions which transforms quark states of different colour into one another. Internal symmetries do not change the Lorentz indices of a particle: an $SU(3)_c$ rotation changes the colour of a quark, but not its spin. Again, the Lagrangian has to be a singlet, so only particular combinations of fields are allowed.

Gauge theories

The theory of a free Dirac fermion

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x) \tag{1.1.4}$$

with $\partial\!\!\!/ \equiv \gamma^\mu \partial_\mu$ is a Lorentz scalar and invariant under a global phase transformation $\psi(x) \to e^{i\alpha}\psi(x)$. To make it invariant under a *local* (= gauge) transformation, $\psi(x) \to e^{i\alpha(x)}\psi(x)$, need to introduce a **gauge field** transforming as

$$A_\mu(x) \to A_\mu(x) + \frac{1}{g}\partial_\mu \alpha(x) . \tag{1.1.5}$$

We can then write down a gauge invariant Lagrangian:

$$\mathcal{L} = \bar{\psi}(x)(i\not{D} - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x), \tag{1.1.6}$$

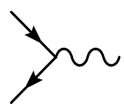
where the **covariant derivative** is

$$D_\mu = \partial_\mu - igA_\mu(x) \tag{1.1.7}$$

and the **field strength tensor**

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\,. \tag{1.1.8}$$

(1.1.6) is the Lagrangian of QED. A_μ is the massless photon. $g = eQ$ where e is the elementary charge and Q the electric charge of fermion ψ in units of e. It contains in particular the fermion-gauge vertex

 $= ieQ\gamma^\mu\,.$

The local transformation $\psi(x) \to e^{i\alpha}\psi(x)$ defines a group, the Abelian unitary group $U(1)$, so QED is also called an Abelian gauge theory. Generalizing this to more complicated groups, one obtains the **non-Abelian gauge theories** or Yang-Mills theories. Now ψ is an n-dimensional vector corresponding to an n-dimensional representation of a compact Lie group. Focusing on the special unitary groups $SU(N)$, one can write the gauge transformation as

$$\psi_i(x) \to e^{i\alpha^a T^a_{ij}}\psi_j(x) \tag{1.1.9}$$

where T^a are the **generators**, (traceless hermitian) matrices fulfilling the commutation relations

$$[T^a, T^b] = if^{abc}T^c \tag{1.1.10}$$

where f^{abc} are the (totally antisymmetric) **structure constants** that vanish for an Abelian group. A gauge invariant Lagrangian can be obtained by introducing the gauge fields A^a_μ transforming as

$$A^a_\mu(x) \to A^a_\mu(x) + \frac{1}{g}\partial_\mu\alpha^a(x) + f^{abc}A^b_\mu(x)\alpha^c(x)\,. \tag{1.1.11}$$

The covariant derivative reads

$$D_\mu = \partial_\mu - igA^a_\mu(x)T^a\,. \tag{1.1.12}$$

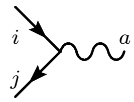
and the field strength tensor

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g\,f^{abc}A^b_\mu A^c_\nu\,. \tag{1.1.13}$$

The resulting **Yang-Mills Langrangian** reads

$$\mathcal{L}_{\text{YM}} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a\,. \tag{1.1.14}$$

It contains not only the fermion-gauge vertex



$$= ig\gamma^\mu T^a_{ij}\,,$$

but also 3- and 4-gauge boson vertices.

Chiral gauge theories

A four-component **Dirac spinor** ψ can be decomposed into two Weyl spinors ψ_L and ψ_R (they are the irreducible pieces wrt Lorentz transformations)

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{1.1.15}$$

ψ_L and ψ_R really are independent fields and can carry different gauge quantum numbers. This is exactly the case in the Standard Model. In this case the Dirac mass term

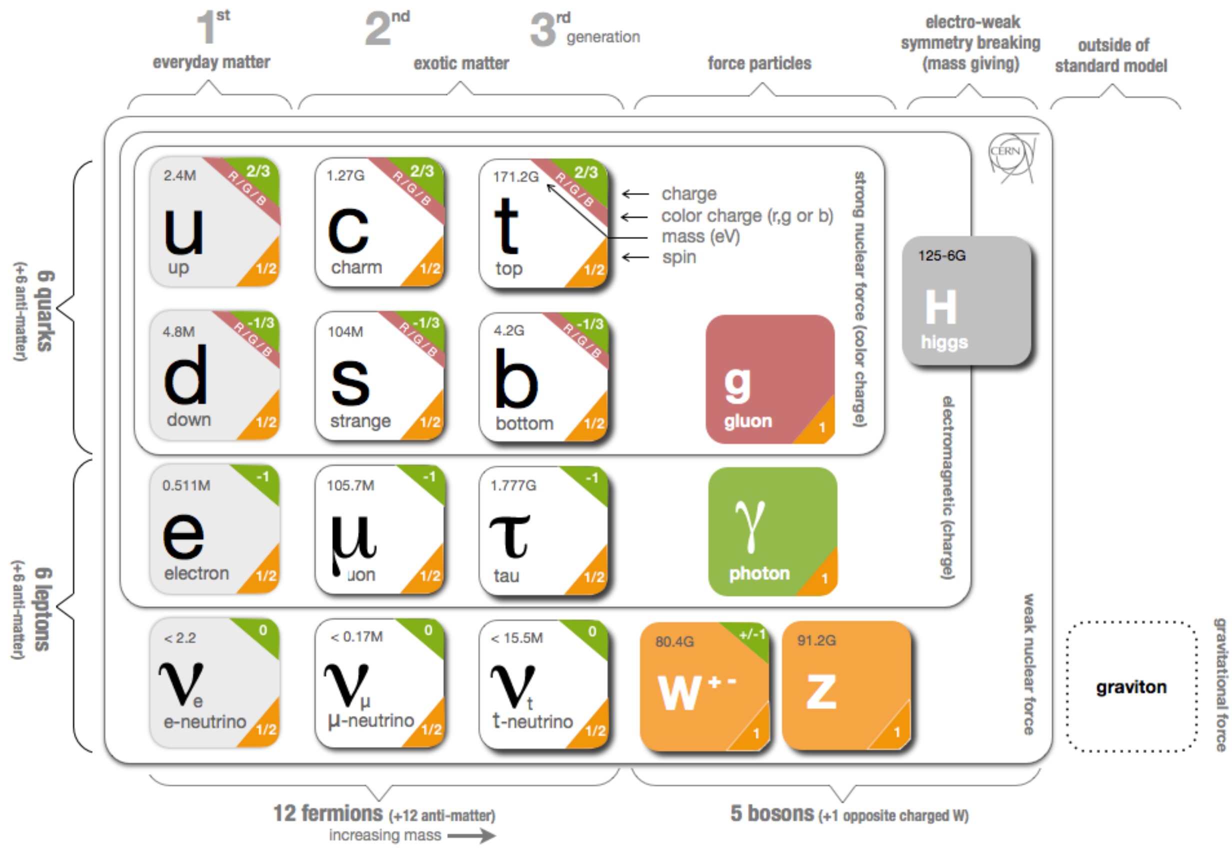
$$-m\,\bar{\psi}\psi = -m\,(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) = -m\,\bar{\psi}_L\psi_R + \text{h.c.} \tag{1.1.16}$$

is forbidden by gauge invariance. We were slightly sloppy here and take $\psi_{L,R}$ to mean

$$\psi_L = P_L\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \quad \psi_R = P_R\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \tag{1.1.17}$$

1.2 Express review of the Standard Model

1.2.1 SM particle content



1.2.2 SM Lagrangian

The SM is a quantum field theory that is defined by its gauge symmetries and its matter content. The gauge symmetry group is

$$G_{\rm SM} = SU(3)_c \times SU(2)_L \times U(1)_Y. \tag{1.2.1}$$

The matter content contains the fermions (**quarks** and **leptons**) with the following quantum numbers under $G_{\rm SM}$,

$$\begin{aligned} q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2)_{\frac{1}{6}}, \qquad & u_R \sim (3,1)_{\frac{2}{3}}, \\ d_R \sim (3,1)_{-\frac{1}{3}}, \qquad & l_L \sim (1,2)_{-\frac{1}{2}}, \qquad e_R \sim (1,1)_{-1}, \end{aligned} \tag{1.2.2}$$

where the numbers in brackets refer to the dimension of the $SU(N)$ representations and subscript corresponds to weak **hypercharge** Y. All fermion fields come in three copies or

flavours. Finally, there is the scalar **Higgs field**,

$$H \sim (1,2)_{\frac{1}{2}} \,. \tag{1.2.3}$$

We can now write down the most general renormalizable G_{SM} invariant Lagrangian with these fields,

$$\begin{aligned}
\mathcal{L}_{\text{SM}} &= \mathcal{L}_{\text{g}} + \mathcal{L}_{\text{f}} + \mathcal{L}_{\text{H}} + \mathcal{L}_{\text{Y}} \,, && (1.2.4)\\
\mathcal{L}_{\text{g}} &= -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \frac{1}{4} W_a^{\mu\nu} W_{\mu\nu}^a - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \,, && (1.2.5)\\
\mathcal{L}_{\text{f}} &= \bar{q}_L^i \, i\not{D}\, q_L^i + \bar{u}_R^i \, i\not{D}\, u_R^i + \bar{d}_R^i \, i\not{D}\, d_R^i + \bar{l}_L^i \, i\not{D}\, l_L^i + \bar{e}_R^i \, i\not{D}\, e_R^i \,, && (1.2.6)\\
\mathcal{L}_{\text{H}} &= |D_\mu H|^2 - V(H) \,, && (1.2.7)\\
\mathcal{L}_{\text{Y}} &= -Y_u^{ij} \, \bar{q}_L^i \, \tilde{H} \, u_R^j - Y_d^{ij} \, \bar{q}_L^i \, H \, d_R^j - Y_l^{ij} \, \bar{l}_L^i \, H \, e_R^j + \text{h.c.} \,, && (1.2.8)
\end{aligned}$$

where we have introduced $\tilde{H} = i\sigma_2 H^*$ in the last line. The covariant derivative of the quark doublet reads[1]

$$D_\mu q_L = \left(\partial_\mu - i g_s G_\mu^a t^a - i g W_\mu^a \tfrac{\sigma^a}{2} - i g' Y B_\mu\right) q_L \,, \tag{1.2.9}$$

where t^a is the generator of $SU(3)$ in the fundamental representation, σ^a a Pauli matrix, and Y the hypercharge, see (1.2.2). In the covariant derivatives of the lepton and Higgs doublet, the first term in (1.2.9) is absent, for the quark singlets the second one is absent, and the covariant derivative of the lepton singlet only contains the last term.

Actually, there is one additional term compatible with the gauge symmetries that was omitted in (1.2.4). The term

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} G_{\mu\nu}^a G_{\alpha\beta}^a \,, \tag{1.2.10}$$

although a total divergence, has a physical effect due to the non-trivial vacuum structure of non-Abelian gauge theory. In practice, it would induce CP violation in the strong interactions, leading e.g. to an electric dipole moment for the neutron. Since this has not been observed experimentally, one obtains a bound $\theta \lesssim 10^{-9}$. Within the SM, there is no explanation for the absence of this term (i.e. the smallness of this parameter): this is the **strong CP problem**. An interesting solution is the Peccei-Quinn mechanism that requires the existence of an **axion**. We will discuss this later in the lecture.

1.2.3 Electroweak symmetry breaking

The Higgs potential is given by

$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4 \tag{1.2.11}$$

[1] Other common notations for the gauge couplings are $g_s \equiv g_3$, $g \equiv g_2$, $g' \equiv g_1$. Another common notation is $g_1 = \sqrt{5/3} g'$, which is the normalization obtained in GUTs (see below). Note that the signs before the i in (1.2.9) are a convention; this convention differs between textbooks and leads to different signs in the Feynman rules. So be careful never to mix Feynman rules from two different sources!For a comparison of conventions in popular textbooks, see table 2 of [1].

$\lambda > 0$ required for consistency to have a bounded potential. For $\mu^2 < 0$ the minimum of the potential will be at zero. For $\mu^2 > 0$ on the other hand, the Higgs potential has a minimum for a non-zero field value of H. This means that the Higgs field develops a **vacuum expectation value** (VEV); since the Higgs field is charged under the electroweak (EW) gauge symmetry $SU(2)_L \times U(1)_Y$, its VEV leads to **spontaneous symmetry breaking** and gives masses to the EW gauge bosons.

In general the Higgs field (which has four real components) can be written as

$$H(x) = e^{i\pi^a(x)\frac{\sigma^a}{2}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi(x) \end{pmatrix} . \tag{1.2.12}$$

A gauge transformation (to unitary gauge) can now be used to remove the (unphysical) **Nambu-Goldstone bosons** π^a from the spectrum (they are eaten by the W and Z gauge bosons). The potential for the real scalar ϕ is then minimized for

$$\langle\phi\rangle \equiv v = \sqrt{\frac{\mu^2}{\lambda}} . \tag{1.2.13}$$

Let us call the excitation around this vev $h(x)$, so

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.2.14}$$

Lets plug this back into the SM Lagrangian and start with the covariant derivative of the Higgs field, which completely determines the coupling to the gauge fields:

$$|D_\mu H|^2 \equiv (D_\mu H)^\dagger (D^\mu H) \supset \frac{1}{2}(0 \quad v) \left(g W_\mu^a \tau^a + \frac{1}{2} g' B_\mu \right) \left(g W_b^\mu \tau^b + \frac{1}{2} g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.2.15}$$

$$\frac{1}{2}\frac{v^2}{4} \left[g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-g W_\mu^3 + g' B_\mu)^2 \right] \tag{1.2.16}$$

$$= \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{(g^2 + g'^2) v^2}{4} Z_\mu Z^\mu \tag{1.2.17}$$

$$\equiv m_W^2 W_\mu^+ W^{-\mu} + m_Z^2 Z_\mu Z^\mu , \tag{1.2.18}$$

where

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) , \tag{1.2.19}$$

$$Z_\mu = c_w W_\mu^3 - s_w B_\mu , \qquad A_\mu = s_w W_\mu^3 + c_w B_\mu , \tag{1.2.20}$$

$$s_w \equiv \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}} , \qquad c_w \equiv \cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}} . \tag{1.2.21}$$

Comparing with the measured W and Z masses, the electroweak vev is $v = 246$ GeV. The photon remains massless and corresponds to the unbroken part of the EW gauge symmetry, with gauge coupling $e = g s_w$. The electric charge is given by $Q = T_3 + Y$, where T_3 is the third component of weak isospin and is $\pm\frac{1}{2}$ for the two components of an $SU(2)_L$ doublet and 0 for $SU(2)_L$ singlets.

1.2.4 Quark masses and mixing

Since the SM fermions are chiral, i.e. left- and right-handed chiralities have different gauge quantum numbers, fermion masses are forbidden by the SM gauge symmetries. Quark and charged lepton masses are only generated in the course of EWSB from the Yukawa couplings in (1.2.8). In the broken phase, one obtains the mass terms for the quarks

$$\mathcal{L}_{m_q} = -\frac{v}{\sqrt{2}}\bar{u}_L^i Y_u^{ij} u_R^j - \frac{v}{\sqrt{2}}\bar{d}_L^i Y_d^{ij} d_R^j \,. \tag{1.2.22}$$

In general, $Y_{u,d}$ can be arbitrary complex 3×3 matrices. To obtain the physical quark fields, we have to go to the **mass eigenstate basis**. This can be achieved by performing unitary field rotations,

$$u_L \to V_L^u u_L, \qquad u_R \to V_R^u u_R, \qquad d_L \to V_L^d d_L, \qquad d_R \to V_R^d d_R. \tag{1.2.23}$$

The rotation matrices can always be chosen such that

$$\frac{v}{\sqrt{2}}(V_L^u)^\dagger Y_u V_R^u = M_u \,, \qquad \frac{v}{\sqrt{2}}(V_L^d)^\dagger Y_d V_R^d = M_d \,, \tag{1.2.24}$$

are diagonal, i.e.

$$M_u = \mathrm{diag}(m_u, m_c, m_t) \,, \qquad M_d = \mathrm{diag}(m_d, m_s, m_b) \,. \tag{1.2.25}$$

To see if the rotations (1.2.23) have any physical impact, we have to insert them into $\mathcal{L}_{\mathrm{SM}}$ and see if the rotation matrices drop out everywhere. In fact it is clear that all flavour diagonal couplings will be unchanged, since the matrices are unitary, e.g.

$$\bar{u}_L \gamma^\mu u_L \to \bar{u}_L (V_u^L)^\dagger \gamma^\mu V_u^L u_L = \bar{u}_L \gamma^\mu u_L \tag{1.2.26}$$

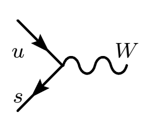
This applies to

- kinetic terms
- neutral currents

However, they do not cancel in the W vertices:

$$\bar{u}_L \gamma_\mu d_L \to \bar{u}_L (V_u^L)^\dagger \gamma^\mu V_d^L d_L = \bar{u}_L \gamma^\mu V_{\mathrm{CKM}} d_L \,. \tag{1.2.27}$$

This can lead to flavour changing transitions such as



V_{CKM} is the Cabibbo-Kobayashi-Maskawa quark mixing matrix that parametrizes quark **flavour violation** in the SM,

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

Note that there are unitarity relations between the different entries, such as $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. How many physical parameters are contained in $V_{\rm CKM}$?
Consider N generations: $N \times N$ unitary: $2N^2 - N^2$ (from the condition $U^\dagger U = \mathbb{1}$). An orthogonal matrix $O(N)$ has $N(N-1)/2$ angles, i.e. the $N(N+1)/2$ are phases!But $2N$ fermions can absorb $2N-1$ phases (A phase rotation which is equal for all fermions cancels out), so in total we have $(N-1)(N-2)/2$ phases.

N	angles	phases
2	1	0
3	3	1
4	6	3

A complex phase is equivalent to CP violation. Need three generations to have CP violation in general.

There are many different physically equivalent ways to parametrize the CKM matrix. A convenient parametrization is the **Wolfenstein parametrization** that is based on an expansion in the **Cabibbo angle** $\theta_{\rm C} \equiv \theta_{12}$ or in $\lambda = s_{12} \approx 0.22$,

$$V_{\rm CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)\,. \tag{1.2.28}$$

1.2.5 The Higgs

We have seen that 3 out of 4 degrees of freedom were absorbed by the W and Z bosons. Plugging

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.2.29}$$

back into the Higgs potential and covariant derivative, we also obtain the Lagrangian of a real, massive scalar, the **Higgs boson**,

$$\mathcal{L}_{\rm H} = \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 - \frac{\lambda}{2} m_h h^3 - \frac{1}{4}\lambda h^4 + \text{terms with gauge fields}\,. \tag{1.2.30}$$

where $m_h = \sqrt{2}\mu = \sqrt{\lambda/2}v$. We see that roughly the Higgs boson mass is given by the electroweak vev $v = 246$GeV, with an additional unknown coupling. It could be much lighter for smaller coupling (and has been searched for in rare meson decays in the 70s...) As we all know it has now been measured to be $m_h \sim 125$ GeV.

Properties of the Higgs

Given the Higgs mass, the couplings of the Higgs to all SM states are completely determined. A natural question to ask is therefore how well does the observed particle match these predictions, or in other words is it really *the* Higgs?

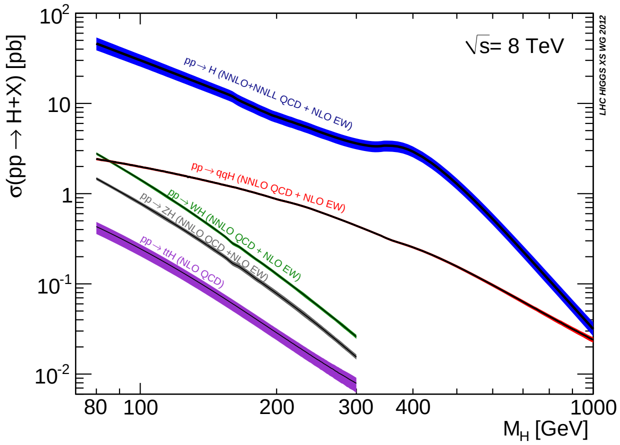


Figure 1.1: Production cross section at 8 TeV LHC.

The Higgs couplings to gauge bosons and fermions are directly proportional to the masses of the particles. Since the pole masses of the gauge bosons and fermions are known [the electron and light quark masses are too small to be relevant]

$$\begin{aligned} &M_Z = 91.187 \text{ GeV} \ , \ M_W = 80.425 \text{ GeV} \ , \ m_\tau = 1.777 \text{ GeV} \ , \ m_\mu = 0.106 \text{ GeV} \, , \\ &m_t = 173.3 \pm 1.2 \text{ GeV} \ , \ m_b = 4.88 \pm 0.07 \text{ GeV} \ , \ m_c = 1.64 \pm 0.07 \text{ GeV} \end{aligned} \tag{1.2.31}$$

the production cross section and all partial widths for the Higgs decays into these particles can be predicted.

1.2.5.1 Production cross section

The initial state at the LHC is protons. Higgs has tiny couplings to valence quarks. As the largest coupling of the Higgs is to the top quark, the main production channel at the LHC turns out to be: gluon fusion. The overall production cross section is about 20pb, i.e. about half a million Higgses have been produced in the last LHC run.

1.2.5.2 Decays

In the Born approximation, the partial width of the Higgs boson decay into fermion pairs is given by

$$\Gamma_{\text{Born}}(H \to f\bar{f}) = \frac{G_\mu N_c}{4\sqrt{2}\pi} \, M_H \, m_f^2 \, \beta_f^3 \tag{1.2.32}$$

with $\beta = (1 - 4m_f^2/M_H^2)^{1/2}$ being the velocity of the fermions in the final state and N_c the color factor $N_c = 3\,(1)$ for quarks (leptons). The Higgs boson will have the tendency to decay into the heaviest ones allowed by phase space. The W and Z bosons are too heavy to both be on-shell. As the Higgs carries no electric charge, the decay into photons is a loop process and hence suppressed (but still the most important channel for discovery due to small background).

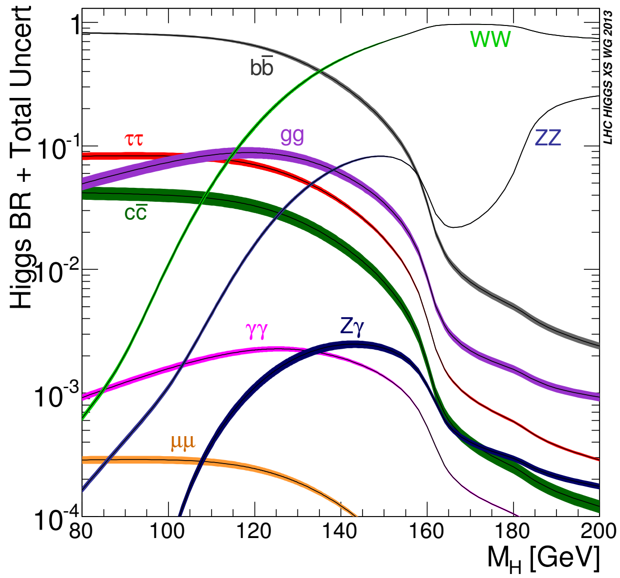


Figure 1.2: Branching ratios of the SM Higgs as a function of mass.

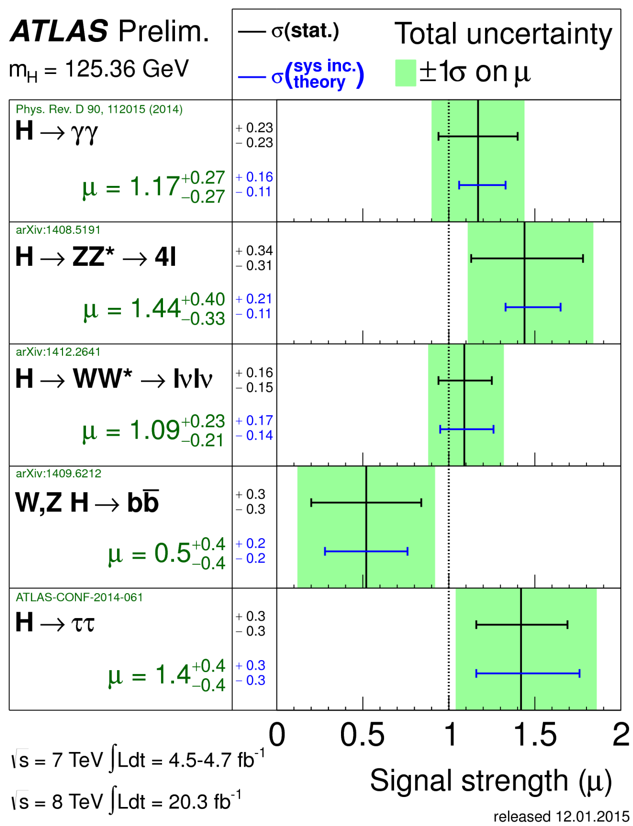


Figure 1.3: Branching ratios of the measured boson compared with the SM expectation.

1.2.6 Accidental & approximate symmetries of the SM

While the gauge symmetry of the SM is seen as *fundamental*, there are also **accidental symmetries** in the SM that only arise as a consequence of imposing gauge invariance and renormalizability with a given particle content. These accidental symmetries can be exact or approximate. They are very powerful when constraining physics beyond the SM since the new physics can violate the accidental symmetries, but the violation is typically subject to strong experimental constraints. Typically, this means that new physics either has to contain these symmetries as accidental symmetries as well, or they have to be promoted to fundamental symmetries.

1.2.6.1 Baryon & lepton number

The classical SM Lagrangian is exactly invariant under baryon number $U(1)_B$, where (anti-) quarks carry $B = \frac{1}{3}$ $(-\frac{1}{3})$ and leptons have $B = 0$. There are strong bounds on B violation from proton decay.

Likewise, individual lepton family numbers L_e, L_μ, and L_τ are conserved in the SM with massless neutrinos. This implies the absence of processes such as $\mu \to e\gamma$. Since neutrinos are massive, the lepton family numbers are no longer exactly conserved, but they still constitute very good approximate symmetries due to the smallness of neutrino masses.

At the quantum level, B and L are violated by sphaleron processes. This is completely irrelevant for low-energy physics but might have played a role in the early universe. The combination $B - L$ is still exactly conserved.

1.2.6.2 Custodial symmetry

The Higgs potential

$$V_{\mathrm{H}} = -\mu^2|H|^2 + \lambda|H|^4 \tag{1.2.33}$$

is not only invariant under local $SU(2)_L \times U(1)_Y$ gauge transformations, but in fact has a larger *global* symmetry group. This can be seen by writing the four real components of H,

$$H = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{1.2.34}$$

as a vector, $\vec{\phi} = (\phi_1, \phi_2, \phi_3, \phi_4)^T$. Then, one has

$$|H|^2 = H^\dagger H = \vec{\phi} \cdot \vec{\phi}\,, \tag{1.2.35}$$

which is obviously invariant under four-dimensional rotations that leave the norm of $\vec{\phi}$ invariant, i.e. $SO(4)$ transformations. The same is then true for the potential.

In the broken phase, one has $\phi_4 = (v + h)/\sqrt{2}$, so

$$|H|^2 = \vec{\phi}_3 \cdot \vec{\phi}_3 + \frac{1}{2}(h + v)^2\,, \tag{1.2.36}$$

where $\vec{\phi}_3 = (\phi_1, \phi_2, \phi_3)^T$, which is only invariant under $SO(3)$.

The relation of these symmetries to $SU(2)_L$ can be understood by noting that $SO(4) \sim SU(2) \times SU(2)$ and $SO(3) \sim SU(2)$.

The group $SU(2)_L \times SU(2)_R$ has been broken to the *diagonal subgroup* $SU(2)_V$, the **custodial symmetry** group.

Under the custodial symmetry $SU(2)_V$, the physical Higgs h transforms as a singlet and the $SU(2)_L$ gauge fields $(W_\mu^1, W_\mu^2, W_\mu^3)$ as a triplet. If it were exact, it would thus predict $m_W = m_Z$. In reality, custodial symmetry is violated by g' so that at tree-level, we have

$$\frac{m_W^2}{m_Z^2 c_w^2} \equiv \rho = 1\,. \tag{1.2.37}$$

Exact custodial symmetry would further predict $Y_u = Y_d$; thus, the ρ parameter deviates from 1 also due to effects involving the difference between Yukawa couplings.

Models in which custodial symmetry is not a good approximate symmetry typically lead to large deviations from $\rho = 1$.

1.2.6.3 Flavour symmetry

In the absence of fermion masses there is a large (accidental) global symmetry,

$$\begin{aligned} q_L^i &\rightarrow U_{q_L}^{ij} q_L^j \\ u_R^i &\rightarrow U_{u_R}^{ij} u_R^j \\ d_R^i &\rightarrow U_{d_R}^{ij} d_R^j \\ l_L^i &\rightarrow U_{l_L}^{ij} l_L^j \\ e_R^i &\rightarrow U_{e_R}^{ij} e_R^j\,. \end{aligned}$$

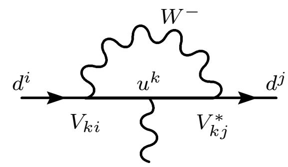
This symmetry is accidental in the sense that it is not imposed, but rather follows from the fermion content and gauge symmetries of the standard model. Since there are five independent $U(3)$ symmetries, the global flavor symmetry of the matter Lagrangian is $[U(3)]^5$. These symmetries are only approximate in the presence of Yukawa couplings Only a very small subgroup of $[U(3)]^5$ is not violated, corresponding to baryon and lepton number.

The Yukawa couplings of the SM show a very hierarchical pattern: only the top quark mass is of the order of the electroweak scale, $m_t \approx v/\sqrt{2}$, while the other quark and charged lepton masses are a factor of 50 to a million smaller. Also the CKM mixing is very hierarchical. This means that the SM exhibits an approximate flavour symmetry

$$G_{\text{fl}} = U(2)_{q_L} \times U(2)_{u_R} \times U(3)_{d_R}\,, \tag{1.2.38}$$

which is exact in the limit of switching of all Yukawa couplings but the top one.

An important consequence of the smallness of quark masses is the suppression of **flavour-changing neutral currents** (FCNCs). While they are forbidden at tree level, they can be induced at 1-loop level through loops involving W bosons. Consider the transition $d_i \rightarrow d_j$,



The resulting amplitude will be of the form

$$\begin{aligned}\sum V_{ki}V_{kj}^* \, F(m_{u^k}) &= V_{ui}V_{uj}^* F(m_u) + V_{ci}V_{cj}^* F(m_c) + V_{ti}V_{tj}^* F(m_t) \\ &\approx \left(V_{ui}V_{uj}^* + V_{ci}V_{cj}^*\right) F(0) + V_{ti}V_{tj}^* F(m_t) \\ &= V_{ti}V_{tj}^* \left[F(m_t) - F(0)\right] \end{aligned} \tag{1.2.39}$$

where we have used the unitarity of the CKM matrix and the smallness of the up and charm masses. Obviously, the amplitude would vanish for degenerate quark masses. The vanishing of FCNCs for degenerate (or massless) quarks is a general and fundamental property called the **GIM mechanism** (Glashow-Iliopoulos- Maiani). In this example, GIM is violated by the large top quark mass.

Further reading: section 3 of [2]

1.3 Why physics beyond the SM?

1.3.1 Observational hints for BSM

1.3.1.1 Dark matter

There is overwhelming evidence for the existence of dark matter over a very large range of astrophysical scales, ranging from galactic scales to the largest observable scales in the Universe. On galactic scales for instance it solves the puzzle of galactic rotation curves: while the rotational velocity of an object circling the galactic center should scale like $v(r) \propto \sqrt{M(r)/r}$ inside the galaxy and $\propto 1/\sqrt{r}$ outside of it, one observes that the rotation curves remain constant up to the outermost visible stars. This can be solved by postulating and invisible halo with mass density $\rho(r) \propto 1/r^2 \Rightarrow M(r) \propto r$.

There are many more ways to infer the existence of dark matter, from e.g. the anisotropies of the cosmic microwave background and the large scale structure of visible matter in the universe to galaxy cluster collisions such as the "bullet cluster", a collision of two clusters of galaxies where the visible matter (hot gas) decelerated in the course of the collision, but gravitational lensing was used to show the the bulk of the total mass continued ballistically.

The crucial point about dark matter is that one can infer from primordial nucleosynthesis that it is non baryonic. The only candidate within the standard model are neutrinos, but they turn out not to work. So it has to be BSM physics.

There are many candidates, including

- axions,
- sterile (G_{SM} singlet) neutrinos,

- WIMPs.

An example for a WIMP is the lightest SUSY particle (LSP) in SUSY models with conserved R-parity. We will come back to dark matter in much more detail later.

1.3.1.2 Neutrino oscillations

The SM Lagrangian (1.2.4) does not give masses to neutrinos. The observational evidence for neutrino oscillations has however shown conclusively that neutrinos are massive. The simplest possibility is to add an additional field $\nu_R \sim (1,1)_0$ and write down another Yukawa coupling,

$$\mathcal{L}_{Y_\nu} = -Y_\nu^{ij}\, \bar{l}_L^i\, \tilde{H}\, \nu_R^j + \text{h.c.} \tag{1.3.1}$$

This would not explain the fact that neutrinos are at least one million times lighter than charged leptons. But actually since ν_R does not carry any quantum numbers, an additional term is allowed,

$$\mathcal{L}_{M_\nu} = -\frac{1}{2} M_\nu^{ij} \overline{(\nu_R^i)^c} \nu_R^j + \text{h.c.} \tag{1.3.2}$$

This is a **Majorana mass** term. Since it is not protected by any symmetry, it could be very large. The physical neutrino masses would then be (for a single generation) the eigenvalues of the matrix

$$\begin{pmatrix} 0 & \frac{v}{\sqrt{2}} Y_\nu \\ \frac{v}{\sqrt{2}} Y_\nu & M_\nu \end{pmatrix}. \tag{1.3.3}$$

For $M_\nu \gg v$, there is a heavy state with approximate mass M_ν and one with approximate mass

$$m_\nu \approx \frac{v^2 Y_\nu^2}{2 M_\nu}\,, \tag{1.3.4}$$

for each generation. You can see that the light neutrino mass decreases as the Majorana mass increases. This is why this mechanism is called the **see-saw mechanism**. For $Y_\nu \sim 1$, one needs $M_\nu \sim 10^{14}$ GeV to be in the right ballpark for sub-eV neutrino masses.

Further reading: [3]

1.3.1.3 Baryon asymmetry

The universe contains much more matter than antimatter. If there was a period of inflation, the initial state is symmetric however, so the asymmetry hsa to be generated dynamically. This requires among other things the violation of CP symmetry. Although CP is violated in the SM by the CKM phase, this effect is too small to explain the BAU. Thus there have to exist other sources of CPV beyond the standard model.

1.3.2 Theoretical motivation for BSM

1.3.2.1 SM Charges

There is no *a priori* reason in the SM why hypercharge should come in fractional numbers (or why electric charge should come in multiples of $e/3$). It turns out that the cancellation of gauge anomalies requires charge to be quantized. [2]

Charge quantization would be explained if the $U(1)_Y$ gauge symmetry were the remnant of a larger, spontaneously broken non-Abelian gauge symmetry. This can happen e.g. in theories of **grand unification** of the strong and EW interactions.

More generally the quantum numbers of standard model matter points to some unifying symmetry. In particular the $G_{\rm SM}$ quantum numbers of the SM fermions (+ 1 right-handed neutrino) fit exactly to a single 16-dimensional representation of $SO(10)$.

1.3.2.2 Gauge coupling unification

Extrapolating the three gauge couplings of the SM to higher energies, they come tantalizingly close at a scale $\sim 10^{16}$ GeV. Could this be a hint of a unification of all forces at or close to this scale?

1.3.2.3 Flavour puzzle

The SM with massless neutrinos has 18 parameters (not counting the QCD vacuum angle θ):

- 3 gauge couplings
- Higgs mass and VEV
- 6 quark masses
- 3 charged lepton masses
- 3 CKM angles, 1 phase.

That is, 13 of the 18 parameters come from the Yukawa Lagrangian $\mathcal{L}_{\rm Y}$. The values of these parameters are completely unexplained in the SM. Their values show a very hierarchical pattern:

$$(m_u, m_c, m_t) \approx (2\times 10^{-3}, 1\times 10^{0}, 2\times 10^{2})\ \text{GeV}\,, \tag{1.3.5}$$

$$(m_d, m_s, m_b) \approx (5\times 10^{-3}, 1\times 10^{-1}, 4\times 10^{0})\ \text{GeV}\,, \tag{1.3.6}$$

$$(m_e, m_\mu, m_\tau) \approx (5\times 10^{-4}, 1\times 10^{-1}, 2\times 10^{0})\ \text{GeV}\,, \tag{1.3.7}$$

[2] Interestingly, explaining neutrino masses by the addition of the state $\nu_R \sim (1,1)_0$, anomaly cancellation then guarantees charge quantization.

$$|V_{\mathrm{CKM}}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \quad \text{with } \lambda \sim 0.2\,. \tag{1.3.8}$$

The origin of these hierarchies is a big mystery.

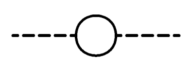
1.3.2.4 How to include gravity?

General relativity has a coupling constant with negative mass dimension $G_N \sim 1/M_P^2$, so it is non-renormalisable. It can therefore not be simply treated as the other forces. It can still be treated as an effective theory, but new physics need to come in at the Planck scale (at the latest).

1.3.2.5 Hierarchy problem

Since the Higgs boson discovery, we know that the Higgs boson exists and that its mass is $m_h \approx 125\,\mathrm{GeV}$. In principle, this makes the SM consistent up to very high energy scales (Maybe the Higgs quartic coupling runs negative below the Planck scale, so new physics would be needed already below 10^{12} GeV or so.).

However, assuming that there is new physics at high energy scales, there is a fundamental problem with the Higgs mass. The problem is that m_H^2 receives enormous quantum corrections from the virtual effects of every particle that couples, directly or indirectly, to the Higgs field.



This corresponds to a correction to m_H^2 from a loop containing a Dirac fermion f with mass m_f. If the Higgs field couples to f with a term in the Lagrangian $-\lambda_f H \bar{f} f$, then the Feynman diagram yields a correction

$$\Delta m_H^2 \;=\; -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\mathrm{UV}}^2 + \ldots\,. \tag{1.3.9}$$

Here Λ_{UV} is an ultraviolet momentum cutoff used to regulate the loop integral; the ellipses represent terms proportional to m_f^2, which grow at most logarithmically.

The total mass is now the bare mass plus the radiative correction,

$$m_h^2 = m_0^2 + \Delta m_H^2\,. \tag{1.3.10}$$

For $\Lambda_{\mathrm{UV}} \gg 100$ GeV a precise cancellation is required. As long as we only consider the Standard Model, there is no problem, as it is a renormalisable QFT and Λ does not have any meaning. But as we said the SM cannot be valid up to infinite energies

- gravity will come into the game at the Planck scale at around 10^{19} GeV
- if grand unification, NP at 10^{16} GeV ('optional')

- also Landau pole of g_1 (very high scales, but also can't trust perturbation theory at large couplings...)

This implies that the SM should be treated as an EFT, and the cutoff is 'physical'.

This is called the **naturalness problem** because it is considered highly unnatural to have such an enormously precise cancellation without a symmetry reason. Related is the **hierarchy problem**, the question why the electroweak scale is so much smaller than the Planck scale.

Technically, the problem is that the Higgs mass renormalizes additively as opposed to multiplicatively. The latter is what happens for fermion and vector masses: the correction to the mass is then proportional to the mass itself. Fermions and vectors don't have a naturalness problem even if they are light because their mass terms violate a symmetry – chiral or gauge symmetry, respectively.

This "hierarchy problem" has occupied theorists for a long time, and there are many potential solutions, including:

- Perhaps Λ is actually not much larger than v, so there is no hierarchy after all. Theories with extra dimensions are a recent attempt along this direction.
- Perhaps M really is much larger than v, and low-energy supersymmetry makes this natural. In an exactly supersymmetric theory, scalar masses renormalize multiplicatively like fermion masses and are therefore safe from the naturalness problem.
- Perhaps there are no fundamental scalars in nature after all, and the electroweak symmetry is broken some other way. For example, Technicolor models break the electroweak symmetry via a fermion-antifermion condensate.

Further reading: [4–6]

What is the scale of new physics?

Since the fine-tuning increases quadratically with $(\Lambda/m_h)^2$, the hierarchy problem indicates the presence of a new scale not far above the electroweak scale.

Note however that this is quite different from the situation of the Standard Model without the Higgs boson: Before the start of the LHC we knew that something new had to be discovered that unitarises WW scattering with a mass below a TeV. Now that we have found the Higgs, the situation is a bit different: We still know that there has to be physics beyond the SM, but the scale is much less clear (and depends on the somewhat subjective view as to how much fine tuning is acceptable).

Chapter 2

Effective field theories

Effective field theory (EFT) is a powerful tool in QFT that allows to separate physics operating at very different energy scales. In EFTs, we "integrate out" physics at short distances (high energies) and replace it by non-renormalizable interactions among the "light" fields in our low-energy theory.

This has two important applications:

1. Avoiding large logarithms in multi-scale problems.

   Using dimensional regularization with minimal subtraction to perform a calculation of a process at energy E, one typically chooses the renormalization scale μ close to this scale E to avoid large logarithms of the form $\ln E/\mu$. However, if particles with vastly different masses m contribute to the process (e.g. in loop integrals), one can run into large logs of the form $\ln m/\mu$ that invalidate perturbation theory.

2. Parametrizing the ignorance about short-distance physics.

   At energies much lower than the mass scale of new physics, indirect new physics effects on low-energy observables can be described by new and/or modified interactions among the light (known) particles.

2.1 Integrating out fields

Consider a theory with Lagrangian $\mathcal{L}_{\text{full}} = \mathcal{L}_H(\phi_H, \phi_L) + \mathcal{L}(\phi_L)$, where ϕ_H are fields describing particles with masses bigger than some scale Λ and ϕ_L all the lighter particles. The basic idea of EFTs is that there are two equivalent descriptions of physics at energies much lower than Λ:

1. The **full theory** described by $\mathcal{L}_{\text{full}}$;

2. The **effective theory** described by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}(\phi_L) + \sum_k \frac{C_i}{\Lambda^{k-4}} \mathcal{O}_i^{(k)}(\phi_L) \tag{2.1.1}$$

where C_i is a dimensionless coefficient and $\mathcal{O}_i^{(k)}(\phi_L)$ are **local operators** of mass dimension k built out of the light fields.

Here, dimension refers to the mass dimension in natural units, where $[m] = [E] = [p] = [x^{-1}] = [t^{-1}]$. The Lagrangian (density) has units of energy per length^3, so its dimension is 4. The dimension of fields in 4 dimensions is

$$[\phi] = 1, \qquad [\psi] = \frac{3}{2}, \qquad [A_\mu] = 1, \qquad [F_{\mu\nu}] = 2. \tag{2.1.2}$$

There are two unsettling facts about (2.1.1). First, it involves an infinite sum over operators with arbitrarily high dimension. The reason this is not a problem is because the contribution of an operator of dimension k to a process at energies $E \ll \Lambda$ will scale as

$$C_i \left(\frac{E}{\Lambda}\right)^{k-4} = \begin{cases} O(1) & \text{if } k = 4 \text{ (“marginal”)} \\ \ll 1 & \text{if } k > 4 \text{ (“irrelevant”)} \\ \gg 1 & \text{if } k < 4 \text{ (“relevant”)} \end{cases} \tag{2.1.3}$$

If we are interested in describing physics to a given accuracy, we can therefore truncate the series and consider only operators up to a certain dimension. The number of operators with fixed dimension is always finite.

The second unsettling fact is that the operators with $k > 4$ lead to **non-renormalizable** interactions. This is a general feature of EFTs: at low energies, the effect of the heavy fields is replaced by non-renormalizable interactions of the light fields. This is not a problem because, as we just saw, there is always a finite number of non-renormalizable operators that contributes to a given process if probed to a given accuracy.

In fact, this implies that the low-energy limit of any fundamental theory will always be a renormalizable theory, since the non-renormalizable interactions are suppressed by powers of E/Λ. (From this point of view, the fact that the SM is renormalizable is neither surprising nor fundamental!)

If one knows the full theory, one can compute the **Wilson coefficients** C_i in terms of the parameters of the full theory. This is done via the procedure of **matching**. By construction, the full and the effective theories describe the same physics at low energies. So one simply computes an observable in both theories and equates the results. This is best illustrated by an example.

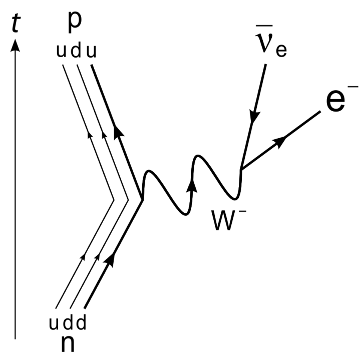


Figure 2.1: Feynman diagram for β decay.

2.2 Fermi theory of beta decay

In the SM, the beta decay of a neutron is based on the quark-level transition $d \to ue^-\bar{\nu}$. Let's compute the diagram in unitary gauge.

$$i\mathcal{M}_{\text{full}} = (\bar{u}_u \frac{ig}{\sqrt{2}} V_{ud}\gamma^\mu P_L u_d) \tag{2.2.1}$$

$$\times \frac{-i}{k^2 - m_W^2 + i\epsilon}\left[g_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right](\bar{u}_e \frac{ig}{\sqrt{2}}\gamma^\nu P_L v_\nu) \tag{2.2.2}$$

$$\xrightarrow{m_W^2 \gg k^2} -i\frac{g^2}{2m_W^2} V_{ud}(\bar{u}_u\gamma^\mu P_L u_d)(\bar{u}_e\gamma_\mu P_L v_\nu) \tag{2.2.3}$$

Now let's construct an effective theory containing Fermi's 4-fermion contact interaction. It reads

$$\mathcal{L}_{\text{eff}} = \frac{C_{4f}}{\Lambda^2}(\bar{u}\gamma^\mu P_L d)(\bar{e}\gamma_\mu P_L \nu)\,. \tag{2.2.4}$$

We can easily compute the amplitude in the effective theory,

$$i\mathcal{M}_{\text{eff}} = i\frac{C_{4f}}{\Lambda^2}(\bar{u}_u\gamma^\mu P_L u_d)(\bar{u}_e\gamma_\mu P_L v_\nu)\,. \tag{2.2.5}$$

From the **matching condition** $\mathcal{M}_{\text{eff}} = \mathcal{M}_{\text{full}}$, we can then read off

$$\frac{C_{4f}}{\Lambda^2} = -\frac{g^2}{2m_W^2}V_{ud} \equiv \frac{4G_F}{\sqrt{2}}V_{ud} \tag{2.2.6}$$

where G_F is **Fermi's constant**.

2.3 Standard Model as an effective theory

We know that the SM is not the final theory, but we know that it gives a good description of "low energy" physics. Treating it as an effective field theory, we can get an understanding of possible new physics effects in low energy experiments by adding non-renormalizable operators invariant under the SM gauge symmetry to the SM Lagrangian.

Given a Lagrangian for an extension of the SM we want to construct the effective Lagrangian

$$\mathcal{L}(\varphi_{SM}; \chi_{BSM}) \longrightarrow \mathcal{L}_{eff} = \mathcal{L}_{SM}(\varphi_{SM}) + \sum_i a_i\, O_i(\varphi_{SM}), \tag{2.3.1}$$

where we collectively denoted the SM fields as φ_{SM} and the heavy fields as χ_{BSM}. All the information about the original Lagrangian and its parameters is now encoded in the coefficients a_i of the higher-dimensional operators O_i. The operators O_i are independent of any hypothetical SM extension because they are constructed from the known SM fields. Note that a possible caveat is that the BSM fields are actually not heavy. It is conceivable that we haven't seen any BSM fields yet not due to their heaviness, but because they are so weakly coupled. This case would not be captured by the EFT approach.

Dimension 2

There is only a single "relevant" operator: the dimension-2 Higgs mass term. Let's for simplicity consider a single real scalar with mass term $\frac{1}{2}m^2\phi^2$. Without a symmetry reason forbidding it, loops involving physics above the cutoff should lead to corrections of order Λ^2. Another way to put this is to add an operator $C\Lambda^2\phi^2$. Without a symmetry reason, one should expect that C is $O(1)$. We see that if we want our scalar to be much lighter than Λ, we need to perform a fine-tuning. This is the hierarchy problem in EFT language!

Dimension 3

While the SM Lagrangian does not contain explicit fermion mass terms since they are forbidden by the gauge symmetry, what would happen if a mass term for a fermion, $m\bar{\psi}\psi$, were present? Although fermion masses have dimension 3 and are thus relevant, they do not suffer from a naturalness problem. The reason is that the fermion mass term breaks chiral symmetry under which $\psi \to e^{i\alpha\gamma_5}\psi$. Since this symmetry is obeyed by the kinetic term, the radiative correction to m has to vanish for $m = 0$. This fixes the scaling of the mass correction to $\Delta m \propto m(a + b\ln \mu/m)$.

Dimension 4

The kinetic terms, gauge interactions, Yukawa couplings, and Higgs quartic couplings are all dimension-4 operators in the SM.

(Dimension 0)

We could also add a dimension-0 operator to the SM Lagrangian: $\mathcal{L} \supset C\Lambda^4$. This is a **vacuum energy**. Astrophysical observations show that $C\Lambda^4 \approx (\text{meV})^4$, in stark disagreement with the EFT expectation.

Dimension 5

It turns out that there is only a single dimension-5 operator invariant under the SM gauge symmetry. It reads

$$\mathcal{L}_5 = -\frac{C_5^{ij}}{\Lambda} (\tilde{H}^\dagger \, \overline{(l_L^i)^c})(\tilde{H}^\dagger \, l_L^j) \tag{2.3.2}$$

In the broken phase and in unitary gauge, it reads

$$\mathcal{L}_5 = -\frac{C_5^{ij}}{\Lambda}\frac{v^2}{2}\overline{(\nu_L^i)^c}\nu_L^j + \text{terms with } h \tag{2.3.3}$$

This is a Majorana mass term for left-handed neutrinos! If $\Lambda \gg v$ and $C_5^{ij} \sim O(1)$, we automatically get very small neutrino masses.

Dimension 6

At the dimension-6 level, a large number of operators invariant under the SM gauge symmetries can be written down. Ignoring the flavour structure and assuming baryon number conservation, the number of operators is 59. Taking into account the flavour structure, the total number of free parameters (real and imaginary parts of Wilson coefficients) is 2499.

Among these operators, the most interesting ones for phenomenology are the ones that violate the approximate and/or accidental symmetries of the SM discussed in section 1.2.6, because they can be probed in precision experiments:

- operators violating custodial symmetry are probed by precision measurements at the Z pole by LEP;
- operators violating baryon number can lead to proton decay, with extremely stringent experimental bounds;
- flavour-violating operators can contribute to flavour-changing neutral currents.

Further reading: [7–10]

Chapter 3

Precision tests of the Standard Model

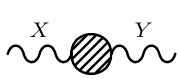
3.1 Electroweak precision tests

The idea of electroweak precision tests is to constrain the presence of new heavy particles via their influence on standard model observables which are well measured. This task of analyzing SM extensions and comparing with experiments is in principle straightforward. One needs to calculate all the observables, including the contributions of the proposed new particles, and needs to make sure that the results agree with the experiments within errors. In practice, this can be quite tedious. When the new particles are heavy compared to the energies at which the PEW measurements were made, one can integrate the new particles out and construct an effective theory in terms of the SM fields only as we have done in the last section. The PEW experiments can be used to constrain the coefficients of the effective theory. This can be, and has been, done once and for all (or at least until there is new data). Various SM extensions can then be constrained by comparing with the bounds on the effective coefficients instead of comparing to the experimental data. This is often much simpler.

3.1.1 Oblique corrections

One type of new physics contribution is particularly constrained by the PEW measurements, corrections to the electroweak gauge boson self energies. The corresponding operators are those that do not contain any fermion fields. Such operators originate whenever heavy fields directly couple only to the SM gauge fields and the Higgs doublet. Sometimes such operators are referred to as "oblique".

Historically this has not been formulated in the EFT language but in terms of electroweak gauge boson **vacuum polarizations**. For completeness let us take a brief detour and follow the historic path. The XY gauge boson vacuum polarization can be written as



$$= i\Pi_{XY}(q^2)\, g^{\mu\nu} + i\Pi^q_{XY}(q^2)\, q^\mu q^\nu \,. \qquad (3.1.1)$$

The form is fixed by Lorentz invariance. The second term only correct the longitudinal components of the gauge boson propagators and are suppressed. Let's take two external Z bosons as an example. The corrected Z propagator will then read

$$iG_Z^{\mu\nu}(q) = \frac{-ig_{\mu\nu}}{q^2 - m_Z^2}\left(1 + i\mathit{\Pi}_{ZZ}\,\frac{-i}{q^2 - m_Z^2} + \ldots\right) + q^\mu q^\nu \text{ terms,} \tag{3.1.2}$$

which can be resummed to yield

$$iG_Z^{\mu\nu}(q) = \frac{-ig_{\mu\nu}}{q^2 - m_Z^2 - \mathit{\Pi}_{ZZ}(q^2)} + q^\mu q^\nu \text{ terms.} \tag{3.1.3}$$

The physical mass is given by the pole in the propagator,

$$m_{Z,\text{pole}}^2 = m_Z^2 + \text{Re}[\mathit{\Pi}_{ZZ}(m_Z^2)]\,. \tag{3.1.4}$$

Assuming that the new particles are heavy allows for the expansion of the new physics contributions to the self-energies in powers of q^2/M_{new}^2 about $q^2 = 0$, where M_{new} represents the heavy scale of the new interactions. Keeping only constant and linear terms in q^2 thus yields

$$\begin{aligned}
\Pi_{\gamma\gamma}(q^2) &= q^2\Pi'_{\gamma\gamma}(0) + \cdots \\
\Pi_{\rm Z\gamma}(q^2) &= q^2\Pi'_{\rm Z\gamma}(0) + \cdots \\
\Pi_{\rm ZZ}(q^2) &= \Pi_{\rm ZZ}(0) + q^2\Pi'_{\rm ZZ}(0) + \cdots \\
\Pi_{\rm WW}(q^2) &= \Pi_{\rm WW}(0) + q^2\Pi'_{\rm WW}(0) + \cdots
\end{aligned} \tag{3.1.5}$$

for the part of the vacuum polarization functions which arise solely from the new interactions. This approximation thus allows us to express the new contributions in terms of six parameters. In the electroweak sector, the SM has three undetermined parameters that is the gauge coupling constants g and g' and the electroweak vev v. Three most precisely measured quantities, $\alpha\,, G_F$, and M_Z are used to determine the parameters of the SM. These three measurements cannot be therefore used to constrain new physics. This leaves three measurable parameters. A convenient parameterization which describes potential new physics contributions to electroweak radiative corrections is given by the $S\,,T\,,U$ formalism of Peskin and Takeuchi. These parameters are defined such that they vanish for a reference point in the SM (i.e., a specific value for the top-quark and Higgs masses) and deviations from zero would then signal the existence of new physics. They are given by

$$\begin{aligned}
\alpha S &= 4s_w^2c_w^2\left[\Pi'_{\rm ZZ}(0) - \frac{c_w^2 - s_w^2}{s_w c_w}\Pi'_{\rm Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right]\,, \\
\alpha T &= \frac{\Pi_{\rm WW}(0)}{M_W^2} - \frac{\Pi_{\rm ZZ}(0)}{M_Z^2}\,, \\
\alpha U &= 4s_w^2\left[\Pi'_{\rm WW}(0) - c_w^2\Pi'_{\rm ZZ}(0) - 2s_w c_w\Pi'_{\rm Z\gamma}(0) - s_w^2\Pi'_{\gamma\gamma}(0)\right]\,.
\end{aligned} \tag{3.1.6}$$

This definition ensures that the parameters T and U vanish if the new physics observes custodial isospin symmetry. In fact T represents the shift of the ρ parameter due to new physics,

$$\rho = 1 + \delta\rho_{\rm SM} + \alpha T\,. \tag{3.1.7}$$

U is typically very suppressed in BSM models, so for simplicity we'll put it to zero.

The electroweak observables can be expressed in terms of the parameters $S\,,T$ in a straightforward fashion. For example the gauge boson masses are

$$\begin{aligned} M_Z^2 &= M_{Z0}^2 \frac{1-\alpha T}{1-G_F M_{Z0}^2 S/2\sqrt{2}\pi}\,, \\ M_W^2 &= M_{W0}^2 \frac{1}{1-G_F M_{W0}^2 S/2\sqrt{2}\pi}\,, \end{aligned} \tag{3.1.8}$$

where $M_{Z0}\,, M_{W0}$ are the Standard Model expressions.

3.1.2 EFT language

3.1.2.1 T parameter

The T parameter has a very simple interpretation in the SM EFT discussed in chapter 2. Up to an overall factor, it is the Wilson coefficient of the operator $\mathcal{O}_T = |H^\dagger D_\mu H|^2$,

$$\frac{C_T}{\Lambda^2} = -\frac{2\alpha}{v^2} T\,. \tag{3.1.9}$$

To see this, we can expand $\mathcal{O}_T$ in the unitary gauge,

$$\mathcal{O}_T = |H^\dagger D_\mu H|^2 = \frac{e^2 v^4}{16 c_w^2 s_w^2} Z_\mu Z^\mu + \text{ terms with } h \tag{3.1.10}$$

$$= \frac{v^2}{2}\frac{m_Z^2}{2} Z_\mu Z^\mu + \text{ terms with } h\,, \tag{3.1.11}$$

so we get a contribution to the Z mass, but not the W mass.

3.1.2.2 S parameter

S is the Wilson coefficient of the operator $\mathcal{O}_S = H^\dagger \sigma^i H W^i_{\mu\nu} B^{\mu\nu}$,

$$\frac{C_S}{\Lambda^2} = \frac{\alpha}{4 s_w c_w v^2} S\,. \tag{3.1.12}$$

3.1.2.3 U parameter

The operator for the U parameter is

$$\mathcal{O}_U = (H^\dagger W^{\mu\nu} H)(H^\dagger W_{\mu\nu} H) \tag{3.1.13}$$

which corresponds to a dimension 8 operator and is therefore suppressed by $\frac{1}{\Lambda^4}$.

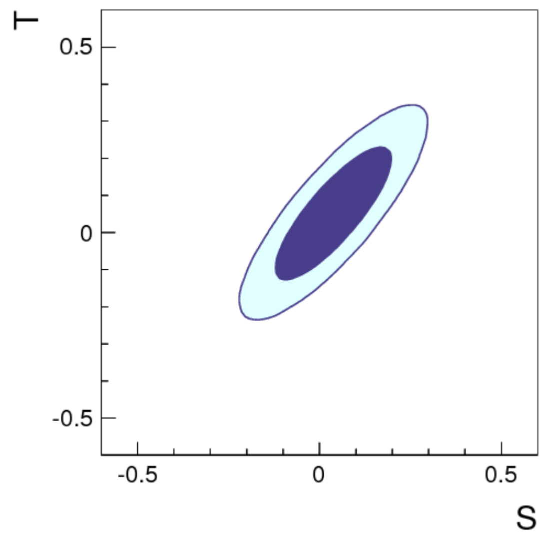


Figure 3.1: Experimentally allowed region in the plane of the oblique parameters S and T. Taken from [11].

3.1.2.4 Experimental bounds

A global χ^2 analysis, that includes all data and correlations, is possible using the EFT methods. All of the data is included in bounding the effective parameters S and T. One needs to consider the two-dimensional allowed range for S and T instead of the independent bounds on these parameters, as the experimentally allowed range for S is strongly correlated with T, see fig. 3.1.

Using naive dimensional analysis, from $|T| \lesssim 0.2$ we can now estimate that $\Lambda \gtrsim 5\,\mathrm{TeV} \times \sqrt{C_T}$, where C_T should be of $O(1)$ for a theory *not* respecting custodial symmetry.

The electroweak precision observables also depend on the Higgs mass. A global analysis of SM data yielded $m_H \lesssim 200$ GeV, so perfectly compatible with the measured value.

3.1.3 Example: heavy quark doublet

As a concrete example, let's introduce a 4th generation of quarks: a doublet $Q_L = (T_L, B_L)^T$ and two singlets T_R, B_R with Lagrangian

$$\mathcal{L}_Q = i\bar{Q}_L \not{D} Q_L + i\bar{T}_R \not{D} T_R + i\bar{B}_R \not{D} B_R - (Y_T\,\bar{Q}_L \tilde{H} T_R + Y_B\,\bar{Q}_L H B_R + \text{h.c.}) \qquad (3.1.14)$$

For simplicity, we do not introduce any tree-level mixing with the SM quarks. Just as in the SM, after EWSB the heavy quarks will get masses $M_{T,B} = vY_{T,B}/\sqrt{2}$.

Let us get a rough estimate of how the S and T parameters will depend on the Lagrangian parameters. Computing the T parameter in the unbroken phase, we need a loop with 4 external Higgs legs. Let's assume $M_T \gg M_B$ and focus on the T contribution. Accounting for the colour factor, loop factor, mass suppression and Yukawa couplings, one expects

$$\frac{C_T}{\Lambda^2} \sim \frac{N_c}{16\pi^2}\frac{Y_T^4}{M_T^2} = \frac{N_c}{4\pi^2}\frac{M_T^2}{v^4}\,. \tag{3.1.15}$$

A remarkable fact about this contribution is that it does not decouple for $M_T \to \infty$. This is because the T parameter measures the violation of custodial symmetry, which grows with the T-B mass difference. For the S parameter, we need a loop with an external W_μ, B_μ and two Higgses. Taking $M = M_T = M_B$, we can then estimate

$$\frac{C_S}{\Lambda^2} \sim \frac{N_c}{16\pi^2}\frac{gg'Y_T^2}{M^2} = \frac{N_c}{8\pi^2}\frac{gg'}{v^2}\,. \tag{3.1.16}$$

We see that S becomes independent of v and the masses, so it essentially "counts" the number of new $SU(2)_L$ doublets in the theory.

Doing the exact computation, one finds

$$T = \frac{N_c}{\alpha 16\pi^2 v^2}\left(M_T^2 + M_B^2 - \frac{2M_T^2 M_B^2}{M_T^2 - M_B^2}\ln\frac{M_T^2}{M_B^2}\right)\,, \tag{3.1.17}$$

$$S = \frac{N_c}{6\pi}\left[1 + 2Y\ln\left(\frac{M_B^2}{M_T^2}\right)\right]\,. \tag{3.1.18}$$

As expected, T vanishes for $M_T \to M_B$. S becomes mass-independent in this limit.

Further reading: [12]

Chapter 4

Introduction to supersymmetry

A SUSY transformation transforms a fermionic state into a bosonic one and vice versa,

$$Q|\text{fermion}\rangle = |\text{boson}\rangle\,, \qquad Q|\text{boson}\rangle = |\text{fermion}\rangle\,. \tag{4.0.1}$$

Q must be a spinor. Q and its hermitian conjugate $Q^\dagger$ are the generators of SUSY transformations. They carry spin $\frac{1}{2}$, so clearly SUSY is a spacetime symmetry, i.e. an extension of Poincaré symmetry.

4.1 Interlude: spacetime symmetries and spinors

The **Poincaré group** is the symmetry group of Minkowski spacetime, which is the Lorentz group plus spacetime translations.

- The proper orthochronous Lorentz group $SO^+(1,3)$ describes boosts and rotations;
- The Lorentz group $O(1,3)$ contains in addition time reversal and parity;
- The Poincaré group contains in addition spacetime translations.

Under a Lorentz transformation

$$x^\mu \to (x')^\mu = \Lambda^\mu{}_\nu x^\nu \tag{4.1.1}$$

where $\Lambda^\mu{}_\nu$ satisfies

$$\Lambda^\mu{}_\sigma \eta^{\sigma\tau} \Lambda^\nu{}_\tau = \eta^{\mu\nu} \tag{4.1.2}$$

Limiting ourselves for the moment to transformations in the neighbourhood of the identity (i.e. no time reversal or parity), an infinitesimal Lorentz transformation can be written as

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu = (\delta^\mu{}_\nu + \omega^\mu{}_\nu) x^\nu \tag{4.1.3}$$

where $\omega_{\mu\nu} = -\omega_{\nu\mu}$. We can write the $\omega^\mu{}_\nu$ as a linear combination of six basis matrices $M^{\rho\sigma}$,

$$\omega^\mu{}_\nu = \frac{1}{2}\Omega_{\rho\sigma}(M^{\rho\sigma})^\mu{}_\nu \tag{4.1.4}$$

where $\Omega_{\rho\sigma}$ are just six numbers which parametrise the Lorentz transformation. A finite transformation is then given by $\Lambda_M = \exp(-\frac{i}{2}\Omega_{\rho\sigma}M^{\rho\sigma})$, where the basis matrices are called generators. The group multiplication law is encoded in the commutation relations of the generators,

$$[P_\mu, P_\nu] = 0, \tag{4.1.5}$$
$$[M_{\mu\nu}, P_\rho] = i(g_{\nu\rho}P_\mu - g_{\mu\rho}P_\nu), \tag{4.1.6}$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = i(g^{\nu\rho}M^{\mu\sigma} - g^{\mu\rho}M^{\nu\sigma} - g^{\nu\sigma}M^{\mu\rho} + g^{\mu\sigma}M^{\nu\rho}) \tag{4.1.7}$$

where we have suppressed the matrix indices. This can also be written in terms of rotations J and boosts K,

$$J^k = \frac{1}{2}\epsilon^{ijk}M^{ij} \quad K^i = M^{0i} = -M^{i0} \tag{4.1.8}$$

with $i, j, k = 1, 2, 3$. The commutator relations then become

$$\begin{aligned} [J_i, J_j] &= i\epsilon_{ijk}J_k \\ [K_i, K_j] &= -i\epsilon_{ijk}J_k \\ [J_i, K_j] &= i\epsilon_{ijk}K_k \ . \end{aligned}$$

The J_i are Hermitian, and the K_i are anti-Hermitian. The J_i satisfy the algebra of the rotation group, $SU(2)$. To disentangle the algebra, define the Hermitian generators

$$\begin{aligned} A_i &= \frac{1}{2}(J_i + iK_i) \\ B_i &= \frac{1}{2}(J_i - iK_i) \ . \end{aligned}$$

which satisfy the algebra

$$\begin{aligned} [A_i, A_j] &= i\epsilon_{ijk}A_k \\ [B_i, B_j] &= i\epsilon_{ijk}B_k \\ [A_i, B_j] &= 0 \ . \end{aligned}$$

The algebra for the A_i and the B_i is that of $SU(2)$, and the two algebras are independent. We have thus shown that the Lorentz group, $SO(3, 1)$, is locally isomorphic to $SU(2) \times SU(2)$.

Fields transform under irreducible **representations** of this group, that is

$$\phi^a(x) \to D[\Lambda]^a{}_b\phi^b(\Lambda^{-1}x) \tag{4.1.9}$$

where the matrices $D[\Lambda]$ form a representation of the Lorentz group, meaning that

$$D[\Lambda_1]D[\Lambda_2] = D[\Lambda_1\Lambda_2], \quad D[\Lambda^{-1}] = D[\Lambda]^{-1}, \quad D[1] = 1 \ . \tag{4.1.10}$$

Irreducible representations of this algebra are classified according to a pair of half-integer numbers, one for each of the $SU(2)$. The $(0, 0)$ representation corresponds to a scalar. The $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations are called **spinors** and are needed to describe fermion fields.

Under Lorentz transformations, a $(\frac{1}{2},0)$ spinor χ transforms as $\chi \to S[\Lambda]\chi$, where

$$S[\Lambda] = \exp(-\tfrac{i}{2}\Omega_{\mu\nu}\sigma^{\mu\nu}), \qquad \sigma^{\mu\nu} = \tfrac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu), \tag{4.1.11}$$

$$\sigma^\mu = (1,\vec{\sigma}), \qquad \bar{\sigma}^\mu = (1,-\vec{\sigma}) \tag{4.1.12}$$

with

$$\vec{\sigma} = (\sigma_1,\sigma_2,\sigma_3), \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4.1.13}$$

Here the $\sigma^{\mu\nu}$ satisfy the Lorentz algebra. A $(0,\frac{1}{2})$ spinor $\eta^\dagger$ (the dagger is just a convention) instead transforms as $\eta^\dagger \to S[\Lambda]^*\eta^\dagger$. Consequently, when representing spinors belonging to these two representations by two-column vectors, care has to be taken because their indices cannot be contracted to form a Lorentz invariant quantity. It is conventional to use dotted indices for the ones transforming as $(0,\frac{1}{2})$ and undotted ones for those transforming as $(\frac{1}{2},0)$. The $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ representations are related by hermitian conjugation. That is, if χ_α is a $(\frac{1}{2},0)$ fermion, then $(\chi_\alpha)^\dagger$ transforms as a $(0,\frac{1}{2})$ fermion.

$$(\chi_\alpha)^\dagger \equiv \chi^\dagger_{\dot{\alpha}} \qquad (\eta^\dagger_{\dot{\alpha}})^\dagger \equiv \eta_\alpha \tag{4.1.14}$$

The indices can be raised and lowered using the epsilon symbol

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta \qquad \eta^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\eta_{\dot{\beta}} \tag{4.1.15}$$

where

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1 \tag{4.1.16}$$

both for dotted and undotted indices. As a convention, repeated spinor indices contracted like

$$^{\alpha}{}_{\alpha} \qquad \text{or} \qquad {}_{\dot{\alpha}}{}^{\dot{\alpha}} \tag{4.1.17}$$

can be suppressed. The components of the spinors are anticommuting **Grassmann numbers** so one has e.g.

$$\chi\eta \equiv \chi^\alpha\eta_\alpha = \chi^\alpha\epsilon_{\alpha\beta}\eta^\beta = -\eta^\beta\epsilon_{\alpha\beta}\chi^\alpha = \eta^\beta\epsilon_{\beta\alpha}\chi^\alpha = \eta^\beta\chi_\beta \equiv \eta\chi. \tag{4.1.18}$$

Likewise, for dotted indices one can define $\chi^\dagger\eta^\dagger = \chi^\dagger_{\dot{\alpha}}\eta^{\dagger\dot{\alpha}} = \eta^\dagger\chi^\dagger$. These two-component spinors are called **Weyl spinors**.

To connect to the four component notation for Dirac spinors, note that in the Weyl basis

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}. \tag{4.1.19}$$

We can combine the two types of spinor into a four-component **Dirac spinor**

$$\psi = \begin{pmatrix} \chi_\alpha \\ \eta^{\dagger\dot{\alpha}} \end{pmatrix} \tag{4.1.20}$$

and we have $\bar{\psi} = \psi^\dagger \gamma^0 = (\eta^\alpha \ \chi^\dagger_{\dot\alpha})$. We can also define left- and right-handed spinors (four component spinors with only one Weyl spinor inside),

$$\psi_L = P_L \psi = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix}, \qquad \psi_R = P_R \psi = \begin{pmatrix} 0 \\ \eta^{\dagger\dot\alpha} \end{pmatrix}, \tag{4.1.21}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, as well as **Majorana spinors**

$$\psi_{M,L} = \begin{pmatrix} \chi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}, \qquad \psi_{M,R} = \begin{pmatrix} \eta_\alpha \\ \eta^{\dagger\dot\alpha} \end{pmatrix}. \tag{4.1.22}$$

We see that a Majorana spinor is a Dirac spinor with $\chi = \eta$. A **charge conjugated** spinor field is defined as

$$\psi^c = C\bar{\psi}^T = \begin{pmatrix} \eta_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}, \tag{4.1.23}$$

where

$$C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta} \end{pmatrix}, \tag{4.1.24}$$

so it is obtained from the Dirac spinor by interchanging $\eta \leftrightarrow \chi$. For a Majorana spinor, $\psi^c_M = \psi_M$. For a chiral spinor, $(\psi^c)_L = P_L\psi^c = (\psi_R)^c$, that is charge conjugation and chiral projection don't commute.

There are now several equivalent ways to write a Dirac mass term for the field ψ:

$$-m\,\bar{\psi}\psi = -m\,(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) = -m\,\bar{\psi}_L\psi_R + \text{h.c.} \tag{4.1.25}$$

$$= -m\,\bar{\psi}_{M,L}\psi_{M,R} \tag{4.1.26}$$

$$= -m\,(\chi\eta + \chi^\dagger\eta^\dagger) = -m\,\chi\eta + \text{h.c.} \tag{4.1.27}$$

A Majorana mass term for the left-handed component can be written as

$$-\frac{1}{2}M\,\bar{\psi}_{M,L}\psi_{M,L} = -\frac{1}{2}M\,\left[\overline{(\psi_L)^c}\psi_L + \bar{\psi}_L(\psi_L)^c\right] \tag{4.1.28}$$

$$= -\frac{1}{2}M\,(\chi\chi + \chi^\dagger\chi^\dagger) = -\frac{1}{2}M\,\chi\chi + \text{h.c.} \tag{4.1.29}$$

4.2 SUSY algrebra

Historically, there is a theorem by Coleman and Mandula stating that the only continuous symmetries of the S matrix, satisfying a Lie algebra, are the generators of Poincaré transformations, plus generators that commute with Poincaré transformations, i.e. internal symmetries such as gauge symmetries,

$$G = G_{\text{Poincaré}} \times G_{\text{internal}} \tag{4.2.1}$$

That is there is no non-trivial way to combine space time and internal symmetries.

The loophole is that there can be symmetry generators not forming an ordinary Lie agebra with commutation relations, but a **graded Lie agebra** with anti-commutation relations. This is what happens for the SUSY generators. They satisfy the **SUSY algebra**

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \,, \tag{4.2.2}$$

$$\{Q_\alpha, Q_\beta\} = 0 \,, \qquad \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \,, \tag{4.2.3}$$

$$[Q_\alpha, P^\mu] = 0 \,, \qquad [Q^\dagger_{\dot\alpha}, P^\mu] = 0 \,, \tag{4.2.4}$$

$$[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta \,, \tag{4.2.5}$$

$$[Q_\alpha, \text{gauge symmetry}] = 0 \,. \tag{4.2.6}$$

Q_α ($Q^\dagger_{\dot\alpha}$) is a left- (right-) handed spinor.

In a supersymmetric theory, particles fall into irreducible representations of the SUSY algebra called **supermultiplets**. They must contain both fermionic and bosonic states which are known as **superpartners** of each other. Consider a fermionic state $|f\rangle$ with mass m. There will be a corresponding bosonic state $|b\rangle = Q_\alpha|f\rangle$. We have

$$P^2|b\rangle = P^2 Q_\alpha|f\rangle = Q_\alpha P^2|f\rangle = Q_\alpha m^2|f\rangle = m^2|b\rangle \,. \tag{4.2.7}$$

Since the mass operator P^2 commutes with $Q, Q^\dagger$, superpartners must be degenerate in mass. Since $Q, Q^\dagger$ also commute with gauge generators, the superpartners must also transform under the same representation of the gauge group.

Consider the operator $(-1)^{2s}$ where s is the spin angular momentum. By the spin-statistics theorem, this operator has eigenvalue $+1$ acting on a bosonic state and eigenvalue -1 acting on a fermionic state. Any fermionic operator will turn a bosonic state into a fermionic state and vice versa. Therefore $(-1)^{2s}$ must anticommute with every fermionic operator in the theory, and in particular with Q and $Q^\dagger$. Now, consider a supermultiplet with mass m. As P^μ commutes with the SUSY generators, any combination of Q or $Q^\dagger$ acting on $|i\rangle$ must give another state $|i'\rangle$ with the same four-momentum eigenvalue. Now one can take a trace over all such states of the operator $(-1)^{2s}P^\mu$:

$$\begin{aligned}\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle &\sim \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s}Q^\dagger Q|i\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|Q(-1)^{2s}Q^\dagger|i\rangle \\ &= \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle - \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle \\ &= 0.\end{aligned} \tag{4.2.8}$$

Now $\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle = p^\mu \,\mathrm{Tr}[(-1)^{2s}]$ is just proportional to the number of bosonic degrees of freedom n_B minus the number of fermionic degrees of freedom n_F in the trace, so that

$$n_B = n_F \tag{4.2.9}$$

must hold for a given $p^\mu \neq 0$ in each supermultiplet. Each supermultiplet therefore contains an equal number of fermion and boson degrees of freedom.

So what are possible supermultiplets? There are two minimal possibilities. The first one contains a single Weyl fermion with two degrees of freedom and a complex scalar field. This is called a **chiral supermultiplet**. The second one contains a massless vector boson with two degrees of freedom and a Weyl fermion. This is called a **vector supermultiplet**. In a renormalizable theory, the spin-1 field is a gauge boson that transforms in the adjoint representation of a gauge group. The fermion thus also transforms in the adjoint representation and it is called a **gaugino**. Since the adjoint representation is a real representation, the gaugino cannot be a chiral fermion, i.e. its left- and right handed components transform in the same way under the gauge group, unlike all the fermions in the SM.

4.3 Chiral supermultiplets

The simplest SUSY model contains a single non-interacting chiral supermultiplet, with a single left-handed two-component Weyl fermion ψ and a complex scalar field ϕ. The simplest action we can write down for these fields just consists of kinetic energy terms for each:

$$S = \int d^4x \left(\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{fermion}}\right), \tag{4.3.1}$$

$$\mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi, \qquad \mathcal{L}_{\text{fermion}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi. \tag{4.3.2}$$

This is called the massless, non-interacting *Wess-Zumino model*.

A supersymmetry transformation should turn the scalar boson field ϕ into something involving the fermion field ψ_α. The simplest possibility for the transformation of the scalar field is

$$\delta_\epsilon \phi = \epsilon\psi, \qquad \delta_\epsilon \phi^* = \epsilon^\dagger \psi^\dagger, \tag{4.3.3}$$

where ϵ^α is an infinitesimal, anticommuting, two-component Weyl fermion object parameterizing the supersymmetry transformation. For global supersymmetry, ϵ^α is a constant, satisfying $\partial_\mu \epsilon^\alpha = 0$. [1] Using eq. (4.3.3), we find that the scalar part of the Lagrangian transforms as

$$\delta_\epsilon \mathcal{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \delta_\epsilon \phi - \partial^\mu \delta_\epsilon \phi^* \partial_\mu \phi = -\epsilon \partial^\mu \psi \, \partial_\mu \phi^* - \epsilon^\dagger \partial^\mu \psi^\dagger \, \partial_\mu \phi. \tag{4.3.4}$$

We would like for this to be canceled by $\delta_\epsilon \mathcal{L}_{\text{fermion}}$, at least up to a total derivative, so that the action will be invariant under the supersymmetry transformation. For this to have any chance of happening, $\delta_\epsilon \psi$ should be linear in $\epsilon^\dagger$ and in ϕ, and should contain one spacetime derivative.

$$\delta_\epsilon \psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \, \partial_\mu \phi, \qquad \delta_\epsilon \psi^\dagger_{\dot\alpha} = i(\epsilon \sigma^\mu)_{\dot\alpha} \, \partial_\mu \phi^*. \tag{4.3.5}$$

[1] The relation between the SUSY generators and this transformation is $\delta_\epsilon X = (\epsilon Q + \epsilon^\dagger Q^\dagger) X$. Because the ϵ parameter is anticommuting, the SUSY algebra takes the form $[\epsilon Q, \epsilon^\dagger Q^\dagger] = 2\epsilon \sigma^\mu \epsilon^\dagger P_\mu$.

With this guess, one immediately obtains

$$\delta_\epsilon \mathcal{L}_{\text{fermion}} = -\epsilon\sigma^\mu\overline{\sigma}^\nu\partial_\nu\psi\,\partial_\mu\phi^* + \psi^\dagger\overline{\sigma}^\nu\sigma^\mu\epsilon^\dagger\,\partial_\mu\partial_\nu\phi\,. \tag{4.3.6}$$

This can be simplified by employing the Pauli matrix identities

$$\left[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu\right]_\alpha{}^\beta = -2\eta^{\mu\nu}\delta_\alpha^\beta, \tag{4.3.7}$$

$$\left[\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu\right]^{\dot\beta}{}_{\dot\alpha} = -2\eta^{\mu\nu}\delta_{\dot\alpha}^{\dot\beta}, \tag{4.3.8}$$

and using the fact that partial derivatives commute ($\partial_\mu\partial_\nu = \partial_\nu\partial_\mu$). Equation (4.3.6) then becomes

$$\begin{aligned}\delta_\epsilon \mathcal{L}_{\text{fermion}} &= \epsilon\partial^\mu\psi\,\partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\,\partial_\mu\phi \\ &\quad -\partial_\mu\left(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\,\partial_\nu\phi^* + \epsilon\psi\,\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\,\partial^\mu\phi\right).\end{aligned} \tag{4.3.9}$$

The first two terms here just cancel against $\delta_\epsilon\mathcal{L}_{\text{scalar}}$, while the remaining contribution is a total derivative. So we arrive at

$$\delta_\epsilon S = \int d^4x\;\; (\delta_\epsilon\mathcal{L}_{\text{scalar}} + \delta_\epsilon\mathcal{L}_{\text{fermion}}) = 0, \tag{4.3.10}$$

We are not quite finished in showing that the theory described by eq. (4.3.1) is supersymmetric. We must also show that the supersymmetry algebra closes; in other words, that the commutator of two supersymmetry transformations parameterized by two different spinors ϵ_1 and ϵ_2 is another symmetry of the theory. Using eq. (4.3.5) in eq. (4.3.3), one finds

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi \;\equiv\; \delta_{\epsilon_2}(\delta_{\epsilon_1}\phi) - \delta_{\epsilon_1}(\delta_{\epsilon_2}\phi) \;=\; i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\phi. \tag{4.3.11}$$

This is a remarkable result; in words, we have found that the commutator of two supersymmetry transformations gives us back the derivative of the original field.

All of this will be for nothing if we do not find the same result for the fermion ψ. Using eq. (4.3.3) in eq. (4.3.5), we get

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \;=\; -i(\sigma^\mu\epsilon_1^\dagger)_\alpha\,\epsilon_2\partial_\mu\psi + i(\sigma^\mu\epsilon_2^\dagger)_\alpha\,\epsilon_1\partial_\mu\psi. \tag{4.3.12}$$

This can be put into a more useful form by applying some identities

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \;=\; i(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\psi_\alpha + i\epsilon_{1\alpha}\,\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi - i\epsilon_{2\alpha}\,\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi.$$

The last two terms vanish on-shell; that is, if the equation of motion $\overline{\sigma}^\mu\partial_\mu\psi = 0$ following from the action is enforced. The remaining piece is exactly the same spacetime translation that we found for the scalar field.

The fact that the supersymmetry algebra only closes on-shell (when the classical equations of motion are satisfied) might be somewhat worrisome, since we would like the symmetry to hold even quantum mechanically. This can be fixed by a trick. We invent a new

	ϕ	ψ	F
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	2	4	2

Table 4.1: Counting of real degrees of freedom in the Wess-Zumino model.

complex scalar field F, which does not have a kinetic term. Such fields are called *auxiliary*, and they are really just book-keeping devices that allow the symmetry algebra to close off-shell. The Lagrangian density for F and its complex conjugate is simply

$$\mathcal{L}_{\text{auxiliary}} = F^* F \,. \tag{4.3.13}$$

The dimensions of F are [mass]^2, unlike an ordinary scalar field, which has dimensions of [mass]. Equation (4.3.13) implies the not-very-exciting equations of motion $F = F^* = 0$. However, we can use the auxiliary fields to our advantage by including them in the supersymmetry transformation rules, to make the theory supersymmetric even off-shell.

We arrive at the Lagrangian of the free Wess-Zumino model,

$$\mathcal{L}_{\text{free}} = |\partial_\mu \phi|^2 + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + |F|^2 \tag{4.3.14}$$

It is invariant under the following set of transformations,

$$\delta\phi = \epsilon\psi \,, \tag{4.3.15}$$

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon)_\alpha \partial_\mu \phi + \epsilon_\alpha F \,, \tag{4.3.16}$$

$$\delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi. \tag{4.3.17}$$

In retrospect, one can see why we needed to introduce the auxiliary field F in order to get the supersymmetry algebra to work off-shell. On-shell, the complex scalar field ϕ has two real propagating degrees of freedom, matching the two spin polarization states of ψ (related to the fact, that the equations of motion are only first order in time derivatives). Off-shell, however, the Weyl fermion ψ is a complex two-component object, so it has four real degrees of freedom. (Going on-shell eliminates half of the propagating degrees of freedom for ψ, because the Lagrangian is linear in time derivatives, so that the canonical momenta can be re-expressed in terms of the configuration variables without time derivatives and are not independent phase space coordinates.) To make the numbers of bosonic and fermionic degrees of freedom match off-shell as well as on-shell, we had to introduce two more real scalar degrees of freedom in the complex field F, which are eliminated when one goes on-shell. This counting is summarized in Table 4.1.

4.3.1 Interactions of chiral multiplets

Now we add interactions to our model. We start with $\mathcal{L}_{\text{free}}$, but adding several copies ("flavours") of the fields ϕ, ψ, F labelled by an index i. For the interaction terms, we restrict ourselves to *renormalizable* interactions, i.e. allowing only terms with mass dimension up to 4, the most general possiblity is (for a more complete discussion see e.g. [13])

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} W^{ij}\psi_i\psi_j + W^i F_j + \text{h.c.} \tag{4.3.18}$$

where W^{ij}, W^i are polynomials of the scalar fields. Applying the above SUSY transformations to $\mathcal{L}_{\text{int}}$ – note that $\mathcal{L}_{\text{free}}$ was already supersymmetric by itself – one finds that it is supersymmetric if and only if the conditions

- W^{ij} is symmetry in i, j,
- the variation $\delta W^{ij}/\delta\phi_k$ is totally symmetric in i, j, k,
- the variation $\delta W^{ij}/\delta\phi_k^*$ vanishes, i.e. W^{ij} does not contain the complex conjugate of the scalar field, i.e. it is a **holomorphic** function,
- $W^{ij} = \delta W^i/\delta\phi_j$,

are all satisfied. In that case, one can write W_i and W_{ij} as functional derivatives of the **superpotential**, which reads in general[2]

$$W = \frac{1}{2} M^{ij}\,\phi_i\phi_j + \frac{1}{6} y_{ijk}\,\phi_i\phi_j\phi_k\,, \tag{4.3.19}$$

where $W^i = \delta W/\delta\phi_i$, $W^{ij} = \delta^2 W/\delta\phi_i\delta\phi_j$.

We can now write down the Lagrangian of the interacting Wess-Zumino model. Only looking at the terms containing the auxiliary field F,

$$\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} \supset F_i F^{i*} + W^i F_i + W_i^* F^{i*} \tag{4.3.20}$$

we note that we can use the equations motion

$$F_i = -W_i^*, \qquad\qquad F^{i*} = -W^i, \tag{4.3.21}$$

to remove the auxiliary fields and we arrive at

$$\mathcal{L}_{\text{chiral}} = |\partial_\mu\phi_i|^2 + i\psi_i^\dagger\bar\sigma^\mu\partial_\mu\psi_i - \frac{1}{2}(W^{ij}\psi_i\psi_j + \text{h.c.}) + W^i W_i^*\,. \tag{4.3.22}$$

Sometimes supersymmetry is formulated in terms of superfields. A chiral multiplet is then written as

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F \tag{4.3.23}$$

with θ a constant Grassmann spinor. In this notation the Lagrangian is then given by

$$\mathcal{L} = \int d^2\theta W(\Phi) \tag{4.3.24}$$

where the integral basically projects onto the θ^2 component of the superpotential. We will mostly stick to component fields for the rest of this lecture.

[2]Up to a possible term linear in ϕ, which however is not relevant for the MSSM.

	A_μ	λ	D
on-shell ($n_B = n_F = 2$)	2	2	0
off-shell ($n_B = n_F = 4$)	3	4	1

Table 4.2: Counting of real degrees of freedom for each gauge supermultiplet.

4.4 Supersymmetric gauge interactions

A gauge supermultiplet contains a set of gauge fields A^a_μ and the associated gaugino fields λ^a, both of which transform under the adjoint representation of the gauge group,

$$A^a_\mu \to A^a_\mu - \partial_\mu \Lambda^a + g\, f^{abc} A^b_\mu \Lambda^c, \tag{4.4.1}$$

$$\lambda^a \to \lambda^a + g\, f^{abc} \lambda^b \Lambda^c, \tag{4.4.2}$$

where the structure constants f^{abc} vanish for an Abelian gauge group. As in the chiral case, the counting of degrees of freedom is subtle. On shell, we have two bosonic and two fermionic degrees of freedom. But off-shell, the gauge boson has three and the gaugino four degrees of freedom. We thus need to introduce an auxiliary field D^a transforming in the adjoint as well,

$$D^a \to D^a + g\, f^{abc} D^b D^c. \tag{4.4.3}$$

We arrive at the Lagrangian

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i\lambda^{\dagger a} \bar{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2} D^a D^a, \tag{4.4.4}$$

where

$$D_\mu \lambda^a = \partial_\mu \lambda^a - g\, f^{abc} A^b_\mu \lambda^c\,. \tag{4.4.5}$$

(Showing that this Lagrangian is indeed supersymmetric and deriving the required SUSY transformation properties is subtle and slightly tedious. We will skip it here.)

4.4.1 Coupling chiral and gauge multiplets

To describe supersymmetric gauge interactions of the members of chiral supermultiplets, we proceed in two steps. First, we replace the ordinary derivatives in the chiral supermultiplet Lagrangian by covariant derivatives,

$$\mathcal{L}_{\text{chiral}} = |D_\mu \phi_i|^2 + i\psi_i^\dagger \bar{\sigma}^\mu D_\mu \psi_i + \ldots \tag{4.4.6}$$

where

$$D_\mu \phi_i = \partial_\mu \phi_i + igA^a_\mu (T^a \phi)_i\,, \tag{4.4.7}$$

$$D_\mu \psi_i = \partial_\mu \psi_i + igA^a_\mu (T^a \psi)_i\,. \tag{4.4.8}$$

Second, we have to add additional renormalizable terms compatible with the gauge symmetry and describing the interactions of the chiral supermultiplet with the gaugino and auxiliary fields,

$$\mathcal{L} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{gauge}} - \sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi)D^a. \tag{4.4.9}$$

One can show that this Lagrangian is supersymmetric with somewhat generalised SUSY transformations. In particular the supersymmetry transformation laws for the matter fields are modified to include gauge-covariant rather than ordinary derivatives. Note that the interaction strengths are fixed to be gauge couplings by the requirements of supersymmetry, even though they are not gauge interactions from the point of view of an ordinary field theory.

Finally, one can remove the auxiliary field D^a by using its equation of motion,

$$D^a = -g(\phi^* T^a \phi). \tag{4.4.10}$$

4.5 Summary

A renormalizable supersymmetric theory is specified by just three ingredients,

- the gauge group,
- the gauge representations of the chiral supermultiplets, and
- the superpotential.

All interactions and masses are then fixed. This has many interesting implications. For example, the scalar potential reads

$$V(\phi, \phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^a D^a = W_i^* W^i + \frac{1}{2}\sum_a g_a^2(\phi^* T^a \phi)^2, \tag{4.5.1}$$

so it is fixed once one has fixed the gauge and Yukawa interactions. This will play an important role in the Higgs sector of the MSSM.

One also finds that many vertices are related. In terms of the fermion-sfermion Yukawa couplings y^{ijk} and the supersymmetric sfermion mass term M_{ij}, the quartic and trilinear sfermion vertices are given by $y^{ijn}y^*_{kln}$ and $M^*_{in}y^{jkn}$, respectively. This is precisely what eliminates the quadratically divergent corrections to scalar masses that lead to the hierarchy problem in non-SUSY theories.

Further reading: [13]

4.6 Soft SUSY breaking

The spectrum of known particles is manifestly non-supersymmetric. To turn the MSSM intro a realistic theory, we thus have to add SUSY breaking. From a theoretical point of view, it would be most attractive to have a fundamentally supersymmetric theory, with supersymmetry being spontaneously broken at low energies. How this might come about is a very active field of research, but there is no unambiguous, preferred mechanism. For low energy phenomenology, one can simply parametrize the ignorance about the exact mechanism of SUSY breaking by writing down all possible SUSY breaking terms involving the low energy fields. An important clue as to the nature of supersymmetry breaking can be obtained by returning to the motivation provided by the hierarchy problem. Supersymmetry forced us to introduce two complex scalar fields for each Standard Model Dirac fermion, which is just what is needed to enable a cancellation of the quadratically divergent ($\Lambda_{\rm UV}^2$) pieces for the higgs mass correction. This sort of cancellation also requires that the associated dimensionless couplings should be related (for example $\lambda_S = |\lambda_f|^2$). The necessary relationships between couplings indeed occur in unbroken supersymmetry. Now, if broken supersymmetry is still to provide a solution to the hierarchy problem even in the presence of supersymmetry breaking, then the relationships between dimensionless couplings that hold in an unbroken supersymmetric theory must be maintained. Otherwise, there would be quadratically divergent radiative corrections to the Higgs scalar masses of the form

$$\Delta m_H^2 = \frac{1}{8\pi^2}(\lambda_S - |\lambda_f|^2)\Lambda_{\rm UV}^2 + \dots . \tag{4.6.1}$$

We are therefore led to consider "soft" supersymmetry breaking. This means that the effective Lagrangian of the MSSM can be written in the form

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}, \tag{4.6.2}$$

where $\mathcal{L}_{\rm SUSY}$ contains all of the gauge and Yukawa interactions and preserves supersymmetry invariance, and $\mathcal{L}_{\rm soft}$ violates supersymmetry but contains only mass terms and coupling parameters with *positive* mass dimension. Without further justification, soft supersymmetry breaking might seem like a rather arbitrary requirement. Fortunately, we will see later that theoretical models for supersymmetry breaking do indeed yield effective Lagrangians with just such terms for $\mathcal{L}_{\rm soft}$.

The possible soft supersymmetry-breaking terms in the Lagrangian of a general theory are

$$\mathcal{L}_{\rm soft} = -\left(\frac{1}{2}M_a\,\lambda^a\lambda^a + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + \text{c.c.} - (m^2)^i_j\phi^{j*}\phi_i, \tag{4.6.3}$$

$$\mathcal{L}_{\text{maybe soft}} = -\frac{1}{2}c_i^{jk}\phi^{*i}\phi_j\phi_k + \text{c.c.} \tag{4.6.4}$$

They consist of gaugino masses M_a for each gauge group, scalar squared-mass terms $(m^2)_i^j$ and b^{ij}, and (scalar)^3 couplings a^{ijk} and c_i^{jk}, and "tadpole" couplings t^i. The last of these requires ϕ_i to be a gauge singlet, and so t^i does not occur in the MSSM. One might wonder why we have not included possible soft mass terms for the chiral supermultiplet fermions, like $\mathcal{L} = -\frac{1}{2}m^{ij}\psi_i\psi_j + \text{c.c.}$ Including such terms would be redundant; they can always be absorbed into a redefinition of the superpotential and the terms $(m^2)^i_j$ and c_i^{jk}.

In the special case of a theory that has chiral supermultiplets that are singlets or in the adjoint representation of a simple factor of the gauge group, then there are also possible soft supersymmetry-breaking Dirac mass terms between the corresponding fermions ψ_a and the gauginos

$$\mathcal{L} \;=\; -M^a_{\rm Dirac}\lambda^a\psi_a + \text{c.c.} \tag{4.6.5}$$

This is not relevant for the MSSM with minimal field content, which does not have adjoint representation chiral supermultiplets.

The terms in $\mathcal{L}_{\rm soft}$ clearly do break supersymmetry, because they involve only scalars and gauginos and not their respective superpartners. In fact, the soft terms in $\mathcal{L}_{\rm soft}$ are capable of giving masses to all of the scalars and gauginos in a theory, even if the gauge bosons and fermions in chiral supermultiplets are massless (or relatively light). The gaugino masses M_a are always allowed by gauge symmetry. The $(m^2)^i_j$ terms are allowed for i, j such that ϕ_i, ϕ^{j*} transform in complex conjugate representations of each other under all gauge symmetries; in particular this is true of course when $i = j$, so every scalar is eligible to get a mass in this way if supersymmetry is broken. The remaining soft terms may or may not be allowed by the symmetries. The a^{ijk}, b^{ij}, and t^i terms have the same form as the y^{ijk}, M^{ij}, and L^i terms in the superpotential, so they will each be allowed by gauge invariance if and only if a corresponding superpotential term is allowed.

Chapter 5

Minimal Supersymmetric Standard Model

5.1 Field content and superpotential

Since we are looking for the supersymmetric extension of the SM with the minimal field content, we start by fixing the gauge group: it is the same as in the SM, $SU(3)_c \times SU(2)_L \times U(1)_Y$. We thus require three vector supermultiplets containing the following gauge and gaugino fields:

- The gluon g and the **gluino** $\widetilde{g}$ which transform as $(8,1)_0$;
- The $W^{\pm,0}$ and the **winos** $\widetilde{W}^{\pm,0}$ which transform as $(1,3)_0$;
- The B^0 and the **bino** $\widetilde{B}^0$ which transform as $(1,1)_0$.

After EWSB, the W^0 and B^0 fields mix into the massive Z^0 and the massless photon. Their superpartners mix as well into the neutralinos and charginos, but since supersymmetry is not an exact symmetry in the MSSM, this mixing depends on SUSY breaking as we will discuss later.

Having fixed the gauge sector, we need to specify the chiral supermultiplets needed to accomodate the SM particle content. A first relevant observation is that none of the fermions transform in the adjoint of any of the gauge group factors, so cannot be identified with any of the gauginos. It turns out that the simplest thing to do is to postulate a seperate chiral supermultiplet for each of the SM fermion and scalar fields.

It is conventional to write all fields as left-handed (Remember that hermitian conjugation turns right handed fields into left handed ones). This is natural if one thinks about an underlying GUT, where the complete GUT multiplet is left-handed. Notice that the subscripts L and R are just labels indicating the chirality of the field's fermionic superpartner. Even though one speaks about "left- and right-handed squarks", scalars of course don't have chirality. The sleptons and squarks are collectively denoted as **sfermions**.

Finally, the Higgs doublet is part of a chiral supermultiplet containing two $SU(2)_L$ doublet Weyl fermions called **higgsinos**.

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\, \mathbf{2}\,,\, \frac{1}{6})$
($\times 3$ families)	$\bar{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\bar{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
($\times 3$ families)	$\bar{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, higgsinos	H_d	$(H^0_d \;\; H^-_d)$	$(\widetilde{H}^0_d \;\; \widetilde{H}^-_d)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
	H_u	$(H^+_u \;\; H^0_u)$	$(\widetilde{H}^+_u \;\; \widetilde{H}^0_u)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, +\frac{1}{2})$

Table 5.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

Having specified the field content, we can proceed to write down the most general superpotential compatible with all the symmetries. We will write it in terms of superfields, to avoid notational clutter. You can equally well replace each superfield by its scalar component.

$$W = L_i\,\Phi_i + \frac{1}{2}M^{ij}\,\Phi_i\Phi_j + \frac{1}{6}y_{ijk}\,\Phi_i\Phi_j\Phi_k\,, \tag{5.1.1}$$

We can write it in terms of squark, slepton, and Higgs fields as

$$\begin{aligned} W = &-Y_d^{ij}\,H\,Q^i\bar{d}^j - Y_l^{ij}\,H\,L^i\bar{e}^j \\ &+ \frac{1}{2}\lambda^{ijk}\,L_iL_j\bar{e}_k + \lambda'^{ijk}\,L_iQ_j\bar{d}_k + \frac{1}{2}\lambda''^{ijk}\,\bar{u}_i\bar{d}_j\bar{d}_k\,, \end{aligned} \tag{5.1.2}$$

where all of the $SU(3)_C$ colour and $SU(2)_L$ weak isospin gauge indices are suppressed. i, j are family indices and labels the three generations. This superpotential has two immediate and serious problems.

Firstly, the superpotential gives rise to Yukawa couplings for charged leptons and down-type quarks. But there is no corresponding term for the up-type quarks. Recall the Yukawa Lagrangian of the SM: we had

$$\mathcal{L}_\mathrm{Y} = -Y_u^{ij}\,\bar{q}^i_L\,\tilde{H}\,u^j_R - Y_d^{ij}\,\bar{q}^i_L\,H\,d^j_R - Y_l^{ij}\,\bar{l}^i_L\,H\,e^j_R + \text{h.c.}\,, \tag{5.1.3}$$

where $\tilde{H} = i\sigma_2 H^*$. The root of our problem is that we cannot use $\tilde{H}$ in the superpotential as it has to be a holomorphic function and must not contain the complex conjugates of the scalar fields if we want our theory to be supersymmetric. We can remedy this problem by doubling the Higgs sector of the SM. We introduce H_u and H_d as in the table above.

Now the superpotential reads,

$$\begin{aligned} W = &\,Y_u^{ij}\,H_uQ^i\bar{u}^j - Y_d^{ij}\,H_dQ^i\bar{d}^j - Y_l^{ij}\,H_dL^i\bar{e}^j + \mu\,H_uH_d \\ &+ \mu'^i L_iH_u + \frac{1}{2}\lambda^{ijk}\,L_iL_j\bar{e}_k + \lambda'^{ijk}\,L_iQ_j\bar{d}_k + \frac{1}{2}\lambda''^{ijk}\,\bar{u}_i\bar{d}_j\bar{d}_k\,. \end{aligned} \tag{5.1.4}$$

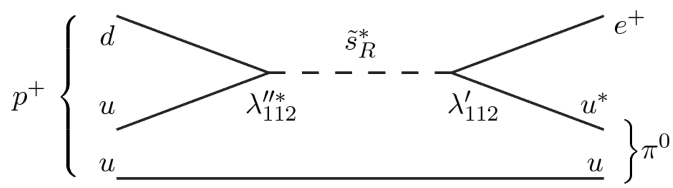


Figure 5.1: Proton decay induced by non-zero coefficients λ^{ijk}.

Note that by introducing a second Higgs doublet, we have also solved a problem that was hidden so far: while the gauginos transform under a real representation of the gauge group and thus have no impact on the cancellation of gauge anomalies in the SM, the Higgsinos are chiral fermions. A single higgsino transforming as $(1,2)_{\frac{1}{2}}$ would in fact spoil the anomaly cancellation that is so delicately balanced among quarks and leptons in the SM. Adding the second Higgs (and higgsino) doublet with opposite hypercharge, all gauge anomalies cancel between the two doublets and the theory is consistent again.

The second problem is due to unwanted terms. We can assign baryon and lepton number to the squarks and sleptons just as to the quarks and leptons, i.e. $B = \frac{1}{3}$ for Q and $B = -\frac{1}{3}$ for U and D, $L = 1$ for L and $L = -1$ for E. Then the first two terms in the second line violate lepton number and the last term violates baryon number. When constructing the Lagrangian from the superpotential, by supersymmetry we will unavoidably generate baryon and lepton number violating interactions involving quarks and leptons. This is very problematic because both B and L are (accidentally) conserved to a very high precision in the SM and their is no experimental sign of a violation of any of them.

To avoid these problematic terms, in the MSSM one postulates that the theory be invariant under a discrete symmetry called **R-parity** defined for each particle as

$$P_{\mathrm{R}} = (-1)^{3(B-L)+2s} \tag{5.1.5}$$

where s is the particle's spin. Demanding invariance under this symmetry, the second line in (5.1.4) is forbidden, while the first line is allowed.

$$W_{\mathrm{MSSM}} = Y_u^{ij}\, H_u Q^i \bar{u}^j - Y_d^{ij}\, H_d Q^i \bar{d}^j - Y_l^{ij}\, H_d L^i \bar{e}^j + \mu\, H_u H_d \tag{5.1.6}$$

R-parity has far-reaching consequences for phenomenology. It turns out that $P_{\mathrm{R}} = +1$ for all SM fields (gauge bosons, quarks, leptons, and the Higgs), while $P_{\mathrm{R}} = -1$ for the squarks, sleptons, gauginos, and Higgsinos. Consquently, in a theory conserving R-parity, every interaction vertex has to contain an even number of "sparticles". This implies that

- The lightest SUSY particle (LSP) is *stable* (and could form the dark matter!);
- All other sparticles have to decay to final states involving an odd number of LSPs;
- In collider experiments, sparticles can only be produced in pairs.

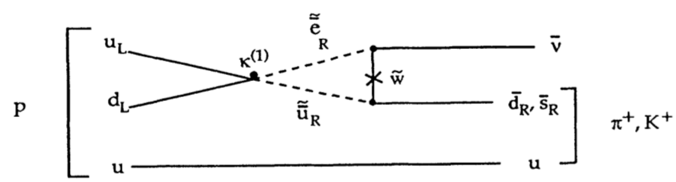


Figure 5.2: Proton decay induced by a non-zero coefficient κ_{ijkl}.

The dimensionful couplings in the supersymmetric part of the MSSM Lagrangian are all dependent on μ. μ provides for higgsino fermion mass terms

$$-\mathcal{L}_{\text{higgsino mass}} = \mu(\widetilde{H}_u^+ \widetilde{H}_d^- - \widetilde{H}_u^0 \widetilde{H}_d^0) + \text{c.c.}, \tag{5.1.7}$$

as well as Higgs squared-mass terms in the scalar potential

$$-\mathcal{L}_{\text{supersymmetric Higgs mass}} = |\mu|^2 \big(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2\big). \tag{5.1.8}$$

Since eq. (5.1.8) is non-negative with a minimum at $H_u^0 = H_d^0 = 0$, we cannot understand electroweak symmetry breaking without including a negative supersymmetry-breaking squared-mass soft term for the Higgs scalars. However, we can already see a puzzle: we expect that μ should be roughly of order 10^2 or 10^3 GeV, in order to allow a Higgs VEV of order 174 GeV without too much miraculous cancellation between $|\mu|^2$ and the negative soft squared-mass terms that we have not written down yet. But why should $|\mu|^2$ be so small compared to, say, $M_{\rm P}^2$, and in particular why should it be roughly of the same order as $m^2_{\rm soft}$? The scalar potential of the MSSM seems to depend on two types of dimensionful parameters that are conceptually quite distinct, namely the supersymmetry-respecting mass μ and the supersymmetry-breaking soft mass terms. Yet the observed value for the electroweak breaking scale suggests that without miraculous cancellations, both of these apparently unrelated mass scales should be within an order of magnitude or so of 100 GeV. This puzzle is called "the μ problem". Several different solutions to the μ problem have been proposed, involving extensions of the MSSM of varying intricacy. They all work in roughly the same way; the μ term is required or assumed to be absent at tree-level before symmetry breaking, and then it arises from the VEV(s) of some new field(s). These VEVs are in turn determined by minimizing a potential that depends on soft supersymmetry-breaking terms. In this way, the value of the effective parameter μ is no longer conceptually distinct from the mechanism of supersymmetry breaking; if we can explain why $m_{\rm soft} \ll M_{\rm P}$, we will also be able to understand why μ is of the same order.

5.1.1 MSSM as an effective theory

So far, we have discussed only renormalizable supersymmetric Lagrangians. However, when any realistic supersymmetric theory is extended to include gravity, the resulting supergravity theory is non-renormalizable as a quantum field theory. It is therefore clear that, in principle, non-renormalizable interactions must be present in any low-energy effective description of the MSSM. Fortunately, these can be neglected for most phenomenological purposes, because non-renormalizable interactions have couplings of negative mass dimension, proportional to powers of $1/M_\mathrm{P}$ (or perhaps $1/\Lambda_\mathrm{UV}$, where Λ_UV is some other cutoff scale associated with new physics). This means that their effects at energy scales E ordinarily accessible to experiment are typically suppressed by powers of E/M_P as we have seen before.

Although these terms are very suppressed at low energies, they can still dominate very rare processes (like proton decay). In general this is indeed the case. One operator of dimension 5 which is particularly dangerous is

$$\mathcal{L}_{5D} = \kappa_{ijkl} Q_i Q_j Q_k L_l \tag{5.1.9}$$

which lead to proton decay. The coefficient needs to be smaller than $10^{-8}/M_\mathrm{P}$ for the proton to be stable enough. R parity does not forbid this term, which is very problematic. One needs a more powerful symmetry. One possibility is a larger discrete R symmetry. An R symmetry is a symmetry which does not commute with SUSY, implying that the different component fields have different charges and that the superpotential has R charge 2. This is the exception to the rule that all internal symmetries commute with SUSY. The maximal R symmetry for $N = 1$ SUSY is a global $U(1)$. Particularly interesting is a Z_4 R symmetry under which all matter superfields have charge 1 and a Higgs chiral multiplet has charge 0.

One can immediately see that this symmetry forbids the QQQL term as well as all other R parity violating terms (in fact R parity is a subgroup). This symmetry also forbids the μ term. After SUSY breaking this is broken to R parity (the soft gaugino mass terms break any R symmetry down to R parity) and a μ term can be induced. So effectively at low scales we do have R parity - at high scales however the symmetry is larger, which is necessary for a viable theory.

5.1.2 Soft supersymmetry breaking in the MSSM

To complete the description of the MSSM, we need to specify the soft supersymmetry breaking terms. For the MSSM, we have:

$$\begin{aligned}
\mathcal{L}_\mathrm{soft}^\mathrm{MSSM} \;=\; & -\frac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B} + \mathrm{c.c.}\right) \\
& - \left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \mathrm{c.c.}\right) \\
& - \widetilde{Q}^\dagger\,\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u} H_u^* H_u - m^2_{H_d} H_d^* H_d - (bH_u H_d + \mathrm{c.c.})\,.
\end{aligned} \tag{5.1.1}$$

In eq. (5.1.1), M_3, M_2, and M_1 are the gluino, wino, and bino mass terms. Here, and from now on, we suppress the adjoint representation gauge indices on the wino and gluino fields, and the gauge indices on all of the chiral supermultiplet fields. The second line in eq. (5.1.1) contains the (scalar)^3 couplings [of the type a^{ijk} in eq. (4.6.3)]. Each of $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ is a complex 3×3 matrix in family space, with dimensions of [mass]. They are in one-to-one correspondence with the Yukawa couplings of the superpotential. The third line of eq. (5.1.1) consists of squark and slepton mass terms of the $(m^2)_i^j$ type in eq. (4.6.3). Each of $\mathbf{m^2_Q}$, $\mathbf{m^2_{\overline{u}}}$, $\mathbf{m^2_{\overline{d}}}$, $\mathbf{m^2_L}$, $\mathbf{m^2_{\overline{e}}}$ is a 3×3 matrix in family space that can have complex entries, but they must be hermitian so that the Lagrangian is real. (To avoid clutter, we do not put tildes on the $\mathbf{Q}$ in $\mathbf{m^2_Q}$, etc.) Finally, in the last line of eq. (5.1.1) we have supersymmetry-breaking contributions to the Higgs potential; $m^2_{H_u}$ and $m^2_{H_d}$ are squared-mass terms of the $(m^2)_i^j$ type, while b is the only squared-mass term of the type b^{ij} in eq. (4.6.3) that can occur in the MSSM.[1] We expect roughly

$$M_1,\, M_2,\, M_3,\, \mathbf{a_u},\, \mathbf{a_d},\, \mathbf{a_e} \sim m_{\text{soft}}, \tag{5.1.2}$$

$$\mathbf{m^2_Q},\, \mathbf{m^2_L},\, \mathbf{m^2_{\overline{u}}},\, \mathbf{m^2_{\overline{d}}},\, \mathbf{m^2_{\overline{e}}},\, m^2_{H_u},\, m^2_{H_d},\, b \sim m^2_{\text{soft}}, \tag{5.1.3}$$

with a characteristic mass scale m_{soft} that is not much larger than 1000 GeV. The expression eq. (5.1.1) is the most general soft supersymmetry-breaking Lagrangian of the form eq. (4.6.3) that is compatible with gauge invariance and matter parity conservation in the MSSM.

Unlike the supersymmetry-preserving part of the Lagrangian, the above $\mathcal{L}^{\text{MSSM}}_{\text{soft}}$ introduces many new parameters that were not present in the ordinary Standard Model. A careful count reveals that there are 105 masses, phases and mixing angles in the MSSM Lagrangian that cannot be rotated away by redefining the phases and flavour basis for the quark and lepton supermultiplets, and that have no counterpart in the ordinary Standard Model. Thus, in principle, supersymmetry *breaking* (as opposed to supersymmetry itself) appears to introduce a tremendous arbitrariness in the Lagrangian.

5.1.3 Hints of an Organizing Principle

Fortunately, there is already good experimental evidence that some powerful organizing principle must govern the soft supersymmetry breaking Lagrangian. This is because most of the new parameters in eq. (5.1.1) imply flavor mixing or CP violating processes of the types that are severely restricted by experiment.

For example, suppose that $\mathbf{m^2_{\overline{e}}}$ is not diagonal in the basis $(\widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R)$ of sleptons whose superpartners are the right-handed parts of the Standard Model mass eigenstates e, μ, τ. In that case, slepton mixing occurs, so the individual lepton numbers will not be conserved, even for processes that only involve the sleptons as virtual particles. A particularly strong limit on this possibility comes from the experimental bound on the process $\mu \to e\gamma$, which could arise from the one-loop diagram shown in Figure 5.3a. The symbol "×" on the slepton line represents an insertion coming from $-(\mathbf{m^2_{\overline{e}}})_{21}\widetilde{\mu}^*_R\widetilde{e}_R$ in $\mathcal{L}^{\text{MSSM}}_{\text{soft}}$. The result of calculating this diagram gives approximately,

$$BR(\mu \to e\gamma) \;\simeq\; \left(\frac{|m^2_{\widetilde{\mu}^*_R\widetilde{e}_R}|}{m^2_{\tilde{\ell}_R}}\right)^2 \left(\frac{100\text{ GeV}}{m_{\tilde{\ell}_R}}\right)^4 10^{-6} \tag{5.1.1}$$

for $m_{\tilde{B}} \simeq m_{\tilde{\ell}_R}$. This result is to be compared to the present experimental upper limit $BR(\mu \to e\gamma)_{\text{exp}} < 1.2 \times 10^{-11}$. So, if the right-handed slepton squared-mass matrix $\mathbf{m^2_{\overline{e}}}$ were

[1]The parameter called b here is often seen elsewhere as $B\mu$ or m^2_{12} or m^2_3.

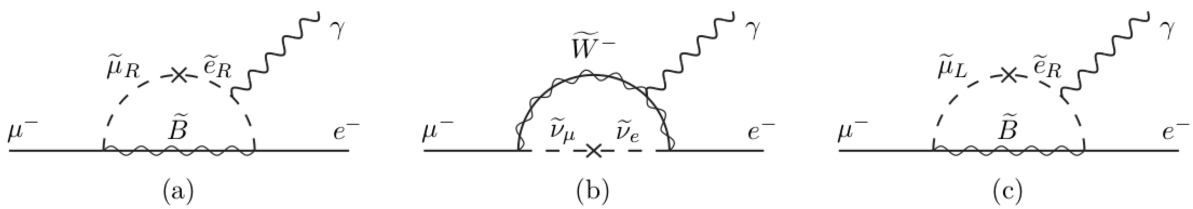


Figure 5.3: Some of the diagrams that contribute to the process $\mu^- \to e^-\gamma$ in models with lepton flavor-violating soft supersymmetry breaking parameters (indicated by $\times$). Diagrams (a), (b), and (c) contribute to constraints on the off-diagonal elements of $\mathbf{m^2_{\bar{e}}}$, $\mathbf{m^2_L}$, and $\mathbf{a_e}$, respectively.

"random", with all entries of comparable size, then the prediction for $BR(\mu \to e\gamma)$ would be too large even if the sleptons and bino masses were at 1 TeV. There are even stronger flavour constraints than this.

All of these potentially dangerous flavor-changing and CP-violating effects in the MSSM can be evaded if one assumes (or can explain!) that supersymmetry breaking is suitably "universal". Consider an idealized limit in which the squark and slepton squared-mass matrices are flavour-blind, each proportional to the 3×3 identity matrix in family space:

$$\mathbf{m^2_Q} = m^2_Q \mathbf{1}, \quad \mathbf{m^2_{\bar{u}}} = m^2_{\bar{u}} \mathbf{1}, \quad \mathbf{m^2_{\bar{d}}} = m^2_{\bar{d}} \mathbf{1}, \quad \mathbf{m^2_L} = m^2_L \mathbf{1}, \quad \mathbf{m^2_{\bar{e}}} = m^2_{\bar{e}} \mathbf{1}. \tag{5.1.2}$$

Supersymmetric contributions to flavor-changing neutral current processes will be very small in such an idealized limit. There are other options, e.g. all scalar superpartners could be very heavy ('split SUSY'), but then one gives up on the solution to the hierarchy problem. Making the further assumption that the (scalar)^3 couplings are each proportional to the corresponding Yukawa coupling matrix,

$$\mathbf{a_u} = A_{u0}\, \mathbf{y_u}, \qquad \mathbf{a_d} = A_{d0}\, \mathbf{y_d}, \qquad \mathbf{a_e} = A_{e0}\, \mathbf{y_e}, \tag{5.1.3}$$

will ensure that only the squarks and sleptons of the third family can have large (scalar)^3 couplings.

Finally, one can avoid disastrously large CP-violating effects by assuming that the soft parameters do not introduce new complex phases. This is automatic for $m^2_{H_u}$ and $m^2_{H_d}$, and for m^2_Q, $m^2_{\bar{u}}$, etc. if eq. (5.1.2) is assumed; if they were not real numbers, the Lagrangian would not be real. One can also fix μ in the superpotential and b in eq. (5.1.1) to be real, by appropriate phase rotations of fermion and scalar components of the H_u and H_d supermultiplets. If one then assumes that

$$\mathrm{Im}(M_1), \mathrm{Im}(M_2), \mathrm{Im}(M_3), \mathrm{Im}(A_{u0}), \mathrm{Im}(A_{d0}), \mathrm{Im}(A_{e0}) \;=\; 0, \tag{5.1.4}$$

then the only CP-violating phase in the theory will be the usual CKM phase found in the ordinary Yukawa couplings.

The MSSM with these flavour- and CP-preserving relations imposed has far fewer parameters than the most general case. Besides the usual Standard Model gauge and Yukawa coupling parameters, there are 3 independent real gaugino masses, only 5 real squark and slepton squared mass parameters, 3 real scalar cubic coupling parameters, and 4 Higgs mass parameters (one of which can be traded for the known electroweak breaking scale).

The soft-breaking universality relations can be presumed to be the result of some specific model for the origin of supersymmetry breaking, although there is no consensus among theorists as to what the specific model should actually be. In any case, they are indicative of an assumed underlying simplicity or symmetry of the Lagrangian at some very high energy scale Q_0. If we used this Lagrangian to compute masses and cross-sections and decay rates for experiments at ordinary energies near the electroweak scale, the results would involve large logarithms of order $\ln(Q_0/m_Z)$ coming from loop diagrams. As is usual in quantum field theory, the large logarithms can be conveniently resummed using renormalization group (RG) equations, by treating the couplings and masses appearing in the Lagrangian as running parameters. Therefore, eqs. (5.1.2)-(5.1.4) should be interpreted as boundary conditions on the running soft parameters at the scale Q_0, which is likely very far removed from direct experimental probes. We must then RG-evolve all of the soft parameters, the superpotential parameters, and the gauge couplings down to the electroweak scale where humans perform experiments.

At the electroweak scale, eqs. (5.1.2) and (5.1.3) will no longer hold, even if they were exactly true at the input scale Q_0. However, to a good approximation, key flavor- and CP-conserving properties remain. This is because, as we will see in section **??** below, the dominant RG corrections due to gauge interactions will respect the form of eqs. (5.1.2) and (5.1.3), while RG corrections due to Yukawa interactions are quite small.

One good reason to believe that a high scale is indeed present in the theory is the celebrated apparent unification of gauge couplings in the MSSM. The 1-loop RG equations for the Standard Model gauge couplings g_1, g_2, g_3 are

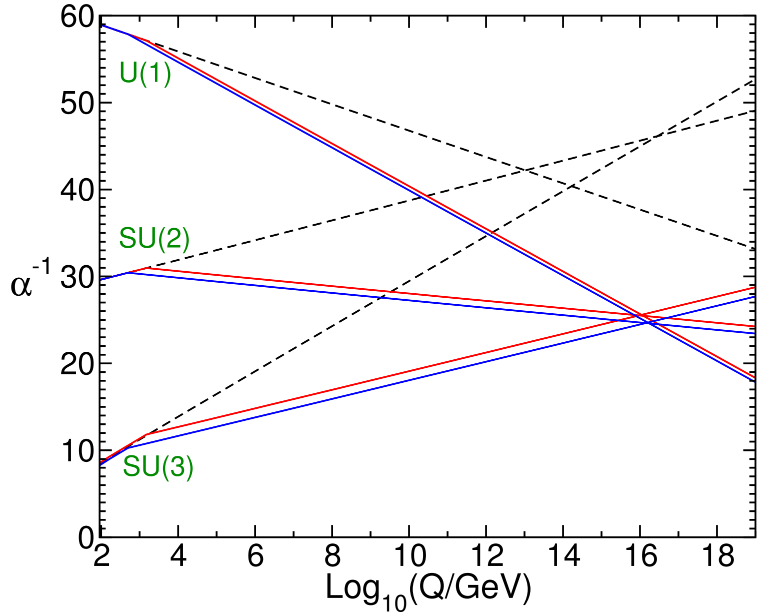
$$\beta_{g_a} \equiv \frac{d}{dt} g_a = \frac{1}{16\pi^2} b_a g_a^3, \qquad (b_1, b_2, b_3) = \begin{cases} (41/10,\ -19/6,\ -7) & \text{Standard Model} \\ (33/5,\ 1,\ -3) & \text{MSSM} \end{cases} \tag{5.1.5}$$

where $t = \ln(Q/Q_0)$, with Q the RG scale. The MSSM coefficients are larger because of the extra MSSM particles in loops. The normalization for g_1 here is chosen to agree with the canonical covariant derivative for grand unification of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ into $SU(5)$ or $SO(10)$. Thus in terms of the conventional electroweak gauge couplings g and g' with $e = g \sin\theta_W = g' \cos\theta_W$, one has $g_2 = g$ and $g_1 = \sqrt{5/3} g'$. The quantities $\alpha_a = g_a^2/4\pi$ have the nice property that their reciprocals run linearly with RG scale at one-loop order:

$$\frac{d}{dt}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3) \tag{5.1.6}$$

Figure 5.4 compares the RG evolution of the α_a^{-1}, including two-loop effects, in the Standard Model (dashed lines) and the MSSM (solid lines). Unlike the Standard Model, the MSSM

Figure 5.4: Two-loop renormalization group evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle masses are treated as a common threshold varied between 500 GeV and 1.5 TeV, and $\alpha_3(m_Z)$ is varied between 0.117 and 0.121.



includes just the right particle content to ensure that the gauge couplings can unify, at a scale $M_U \sim 2 \times 10^{16}$ GeV. This unification is of course not perfect; α_3 tends to be slightly smaller than the common value of $\alpha_1(M_U) = \alpha_2(M_U)$ at the point where they meet, which is often taken to be the definition of M_U. However, this small difference can easily be ascribed to threshold corrections due to whatever new particles exist near M_U. While the apparent approximate unification of gauge couplings at M_U might be just an accident, it may also be taken as a strong hint in favor of a grand unified theory (GUT).

More generally, one needs to evolve superpotential parameters and soft terms using their renormalization group (RG) equations, in order to translate a set of predictions at an input scale into physically meaningful quantities that describe physics near the electroweak scale. More details can be found in the SUSY primer.[2]

5.2 Theory of supersymmetry breaking

In the MSSM, supersymmetry breaking is simply introduced explicitly. But this is only an effective description of an underlying model in which supersymmetry is spontaneously broken. By definition, this means that the vacuum state $|0\rangle$ is not invariant under supersymmetry transformations, so $Q_\alpha|0\rangle \neq 0$ and $Q^\dagger_{\dot\alpha}|0\rangle \neq 0$. Now, in global supersymmetry, the Hamiltonian operator H is related to the supersymmetry generators through the algebra,

$$H = P^0 = \frac{1}{4}(Q_1Q_1^\dagger + Q_1^\dagger Q_1 + Q_2Q_2^\dagger + Q_2^\dagger Q_2). \tag{5.2.1}$$

If supersymmetry is unbroken in the vacuum state, it follows that $H|0\rangle = 0$ and the vacuum has zero energy. Conversely, if supersymmetry is spontaneously broken in the vacuum state,

[2]In practice there are a number of spectrum calculators on the market, which do the work for you...

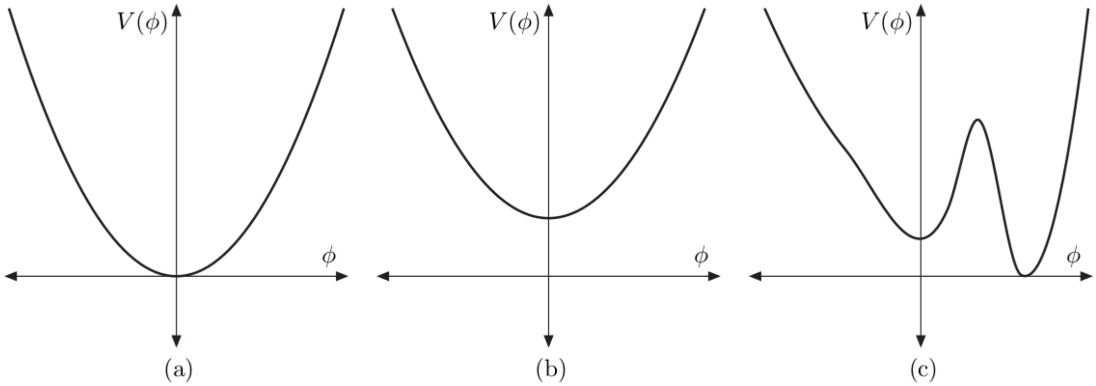


Figure 5.5: Scalar potentials for (a) unbroken supersymmetry, (b) spontaneously broken supersymmetry, and (c) metastable supersymmetry breaking, as a function of an order parameter ϕ.

then the vacuum must have positive energy, since

$$\langle 0|H|0\rangle = \frac{1}{4}\Big(\|Q_1^\dagger|0\rangle\|^2 + \|Q_1|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2\Big) > 0 \tag{5.2.2}$$

If fermion condensates can be neglected, then $\langle 0|H|0\rangle = \langle 0|V|0\rangle$, where V is the scalar potential. Therefore, supersymmetry will be spontaneously broken if the expectation value of F_i and/or D^a does not vanish in the vacuum state.

If any state exists in which all F_i and D^a vanish, then it will have zero energy, implying that supersymmetry is not spontaneously broken in the true ground state. Conversely, one way to guarantee spontaneous supersymmetry breaking is to look for models in which the equations $F_i = 0$ and $D^a = 0$ cannot all be simultaneously satisfied for *any* values of the fields. Then the true ground state necessarily has broken supersymmetry, as does the vacuum state we live in (if it is different). However, another possibility is that the vacuum state in which we live is not the true ground state (which may preserve supersymmetry), but is instead a higher energy metastable supersymmetry-breaking state with lifetime at least of order the present age of the universe. Finite temperature effects can indeed cause the early universe to prefer the metastable supersymmetry-breaking local minimum of the potential over the supersymmetry-breaking global minimum. Scalar potentials for the three possibilities are illustrated qualitatively in Figure 5.5.

Regardless of whether the vacuum state is stable or metastable, the spontaneous breaking of a global symmetry always implies a massless Nambu-Goldstone mode with the same quantum numbers as the broken symmetry generator. In the case of global supersymmetry, the broken generator is the fermionic charge Q_α, so the Nambu-Goldstone particle ought to be a massless neutral Weyl fermion, called the *goldstino*. The goldstino is the fermionic component of the supermultiplet whose auxiliary field obtains a VEV.

The above remarks apply to the breaking of global supersymmetry. However, taking into account gravity, supersymmetry must be promoted to a local symmetry. The resulting locally supersymmetric theory is called *supergravity.* In supergravity, the spin-2 graviton has a spin-3/2 fermion superpartner called the gravitino, which we will denote $\widetilde{\Psi}_\mu^\alpha$. The gravitino has odd R-parity ($P_R = -1$). As long as supersymmetry is unbroken, the graviton and the gravitino are both massless, each with two spin helicity states. Once supersymmetry is spontaneously broken, the gravitino acquires a mass by absorbing ("eating") the goldstino, which becomes its longitudinal (helicity $\pm 1/2$) components. This is called the *super-Higgs* mechanism, and it is analogous to the ordinary Higgs mechanism for gauge theories.

Importantly the scalar potential in supergravity is modified by Planck suppressed contributions, but does not necessarily have a positive vacuum energy once supergravity is broken. This is of course crucial with regards to the cosmological constant.

5.2.1 Fayet-Iliopoulos (D-term) supersymmetry breaking

Supersymmetry breaking with a non-zero D-term VEV can occur through the Fayet-Iliopoulos mechanism. If the gauge symmetry includes a $U(1)$ factor, one can introduce a term linear in the auxiliary field of the corresponding gauge supermultiplet,

$$\mathcal{L}_{\rm FI} = -\kappa D, \tag{5.2.1}$$

where κ is a constant with dimensions of [mass]^2. This term is gauge-invariant and supersymmetric by itself. If we include it in the Lagrangian, then D may be forced to get a non-zero VEV.

$$V = \kappa D - \frac{1}{2}D^2 - gD\sum_i q_i|\phi_i|^2. \tag{5.2.2}$$

Here the q_i are the charges of the scalar fields ϕ_i under the $U(1)$ gauge group in question. The presence of the Fayet-Iliopoulos term modifies the equation of motion to

$$D = \kappa - g\sum_i q_i|\phi_i|^2. \tag{5.2.3}$$

Now suppose that the scalar fields ϕ_i that are charged under the $U(1)$ all have non-zero superpotential masses m_i. Then the potential will have the form

$$V = \sum_i |m_i|^2|\phi_i|^2 + \frac{1}{2}(\kappa - g\sum_i q_i|\phi_i|^2)^2. \tag{5.2.4}$$

Since this cannot vanish, supersymmetry must be broken

In the MSSM, one might imagine that the D term for $U(1)_Y$ has a Fayet-Iliopoulos term as the principal source of supersymmetry breaking. Unfortunately, this cannot work, because the squarks and sleptons do not have superpotential mass terms. So, at least some of them would just get non-zero VEVs in order to make eq. (5.2.3) vanish. That would break color and/or electromagnetism, but not supersymmetry.

One could instead attempt to trigger supersymmetry breaking with a Fayet-Iliopoulos term for some other $U(1)$ gauge symmetry, however this is also problematic phenomenologically.

5.2.2 O'Raifeartaigh (F-term) supersymmetry breaking

Models where spontaneous supersymmetry breaking is ultimately due to a non-zero F-term VEV are called O'Raifeartaigh models. The idea is to pick a set of chiral supermultiplets $\Phi_i \supset (\phi_i, \psi_i, F_i)$ and a superpotential W in such a way that the equations $F_i = -\delta W^*/\delta\phi^{*i} = 0$ have no simultaneous solution.

The simplest example with a supersymmetry breaking global minimum has three chiral supermultiplets $\Phi_{1,2,3}$, with superpotential

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2. \qquad (5.2.1)$$

Note that W contains a linear term, with k having dimensions of [mass]^2. Such a term is allowed if the corresponding chiral supermultiplet is a gauge singlet. In fact, a linear term is necessary to achieve F-term breaking at tree-level in renormalizable superpotentials, since otherwise setting all $\phi_i = 0$ will always give a supersymmetric global minimum with all $F_i = 0$. The scalar potential following from eq. (5.2.1) is

$$V_{\text{tree-level}} = |F_1|^2 + |F_2|^2 + |F_3|^2, \qquad (5.2.2)$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2}, \qquad F_2 = -m\phi_3^*, \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*. \qquad (5.2.3)$$

Clearly, $F_1 = 0$ and $F_2 = 0$ are not compatible, so supersymmetry must indeed be broken.

The O'Raifeartaigh superpotential determines the mass scale of supersymmetry breaking $\sqrt{F_1}$ in terms of a dimensionful parameter k put in by hand. This appears somewhat artificial, since k will have to be tiny compared to $M_{\rm P}^2$ in order to give the right order of magnitude for the MSSM soft terms. It may be more plausible to have a mechanism that can instead generate such scales naturally. This can be done in models of dynamical supersymmetry breaking, in which the small mass scales associated with supersymmetry breaking arise by dimensional transmutation. In other words, they generally feature a new asymptotically free non-Abelian gauge symmetry with a gauge coupling g that is perturbative at $M_{\rm P}$ and gets strong in the infrared at some smaller scale $\Lambda \sim e^{-8\pi^2/|b|g_0^2} M_{\rm P}$, where g_0 is the running gauge coupling at $M_{\rm P}$ with negative beta function $-|b|g^3/16\pi^2$. Just as in QCD, it is perfectly natural for Λ to be many orders of magnitude below the Planck scale. Supersymmetry breaking may then be best described in terms of the effective dynamics of the strongly coupled theory. Supersymmetry is still broken by the VEV of an F field, but it may be the auxiliary field of a composite chiral supermultiplet built out of fields that are charged under the new strongly coupled gauge group.

5.2.3 The need for a separate supersymmetry-breaking sector

It is now clear that spontaneous supersymmetry breaking (dynamical or not) requires us to extend the MSSM. The ultimate supersymmetry-breaking order parameter cannot belong to any of the MSSM supermultiplets; a D-term VEV for $U(1)_Y$ does not lead to an acceptable spectrum, and there is no candidate gauge singlet whose F-term could develop a VEV. Therefore one must ask what effects *are* responsible for spontaneous supersymmetry

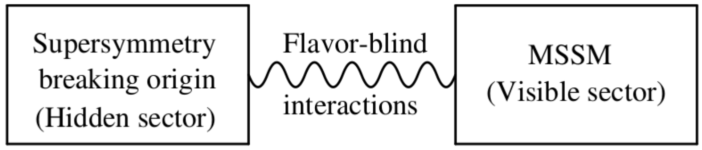


Figure 5.6: The presumed schematic structure for supersymmetry breaking.

breaking, and how supersymmetry breakdown is "communicated" to the MSSM particles. It is very difficult to achieve the latter in a phenomenologically viable way working only with renormalizable interactions at tree-level, even if the model is extended to involve new supermultiplets including gauge singlets. First, on general grounds it would be problematic to give masses to the MSSM gauginos, because supersymmetry never has any (scalar)-(gaugino)-(gaugino) couplings that could turn into gaugino mass terms when the scalar gets a VEV. Second, at least some of the MSSM squarks and sleptons would have to be unacceptably light, and should have been discovered already. This can be understood from the existence of sum rules. In particular the *supertrace* of the tree-level squared-mass eigenvalues, defined in general by a weighted sum over all particles with spin j:

$$\mathrm{STr}(m^2) \equiv \sum_j (-1)^{2j}(2j+1)\mathrm{Tr}(m_j^2), \tag{5.2.1}$$

satisfies the sum rule $\mathrm{STr}(m^2) = 0$ for spontaneously broken SUSY. This would imply e.g. (assuming no flavour violation) that

$$m_{\tilde{e}_1}^2 + m_{\tilde{e}_2}^2 = 2m_e^2, \tag{5.2.2}$$

which is of course ruled out by experiment.

For these reasons, we expect that the MSSM soft terms arise indirectly or radiatively, rather than from tree-level renormalizable couplings to the supersymmetry-breaking order parameters. Supersymmetry breaking evidently occurs in a "hidden sector" of particles that have no (or only very small) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some interactions that are responsible for mediating supersymmetry breaking from the hidden sector to the visible sector, resulting in the MSSM soft terms. (See Figure 5.6.) In this scenario, the tree-level squared mass sum rules need not hold, even approximately, for the physical masses of the visible sector fields, so that a phenomenologically viable superpartner mass spectrum is, in principle, achievable. As a bonus, if the mediating interactions are flavor-blind, then the soft terms appearing in the MSSM will automatically obey conditions like eqs. (5.1.2), (5.1.3) and (5.1.4).

There have been two main competing proposals for what the mediating interactions might be, gravity and gauge mediation.

5.2.4 Planck-scale-mediated supersymmetry breaking

The most studied possibility is that the mediating interactions are gravitational. More precisely, they are associated with the new physics, including gravity, that enters near the Planck scale. In this "gravity-mediated", or *Planck-scale-mediated supersymmetry breaking* (PMSB) scenario, if supersymmetry is broken in the hidden sector by a VEV $\langle F\rangle$, then the soft terms in the visible sector should be roughly

$$m_{\rm soft} \;\sim\; \langle F\rangle/M_{\rm P}\;. \tag{5.2.3}$$

This framework should be analysed with an effective theory with Planck scale suppressed higher dimensional operators. To specify such a non-renormalisable theory, one needs in addition to the superpotential the real *Kähler potential* K, and the holomorphic *gauge kinetic function* f_{ab}. Let X be some chiral superfield whose F term auxiliary field breaks supersymmetry and talks to the MSSM only via these Planck suppressed operators. With

$$W \;=\; W_{\rm MSSM} - \frac{1}{M_{\rm P}}\left(\frac{1}{6}y^{Xijk}X\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{Xij}X\Phi_i\Phi_j\right) + \ldots, \tag{5.2.4}$$

$$K \;=\; \Phi^{*i}\Phi_i + \frac{1}{M_{\rm P}}\big(n_i^jX + \overline{n}_i^jX^*\big)\Phi^{*i}\Phi_j - \frac{1}{M_{\rm P}^2}k_i^jXX^*\Phi^{*i}\Phi_j + \ldots, \tag{5.2.5}$$

$$f_{ab} \;=\; \frac{\delta_{ab}}{g_a^2}\Big(1 - \frac{2}{M_{\rm P}}f_aX + \ldots\Big). \tag{5.2.6}$$

One can then generally write the resulting supersymmetry-breaking Lagrangian as

$$\begin{aligned}\mathcal{L}_{\rm soft} \;=\; & -\frac{F}{2M_{\rm P}}f_a\lambda^a\lambda^a - \frac{F}{6M_{\rm P}}y^{Xijk}\phi_i\phi_j\phi_k - \frac{F}{2M_{\rm P}}\mu^{Xij}\phi_i\phi_j - \frac{F}{M_{\rm P}}n_i^j\phi_jW^i_{\rm MSSM} + \text{c.c.}\\ & -\frac{|F|^2}{M_{\rm P}^2}(k_j^i + n_p^i\overline{n}_j^p)\phi^{*j}\phi_i,\end{aligned} \tag{5.2.7}$$

where ϕ_i and λ^a are the scalar and gaugino fields in the MSSM sector, y^{Xijk}, k_i^j, n_i^j, $\overline{n}_i^j$ and f_a are dimensionless couplings while μ^{Xij} has the dimension of mass. This translates into the MSSM soft terms as follows,

$$M_a \;=\; \frac{F}{M_{\rm P}}f_a, \tag{5.2.8}$$

$$a^{ijk} \;=\; \frac{F}{M_{\rm P}}(y^{Xijk} + n_p^iy^{pjk} + n_p^jy^{pik} + n_p^ky^{pij}), \tag{5.2.9}$$

$$b^{ij} \;=\; \frac{F}{M_{\rm P}}(\mu^{Xij} + n_p^i\mu^{pj} + n_p^j\mu^{pi}), \tag{5.2.10}$$

$$(m^2)_j^i \;=\; \frac{|F|^2}{M_{\rm P}^2}(k_j^i + n_p^i\overline{n}_j^p). \tag{5.2.11}$$

Note that couplings of the form $\mathcal{L}_{\text{maybe soft}}$ in eq. (4.6.4) do not arise.

In principle, the parameters f_a, k^i_j, n^j_i, y^{Xijk} and μ^{Xij} ought to be determined by the fundamental underlying theory. In general, this need not lead to flavour blind terms. However, it has been popular to assume a "minimal" form for the normalization of kinetic terms and gauge interactions such that the soft terms in $\mathcal{L}_{\rm soft}^{\rm MSSM}$ are all determined by just four parameters:

$$m_{1/2} = f\frac{\langle F\rangle}{M_{\rm P}}, \qquad m_0^2 = (k+n^2)\frac{|\langle F\rangle|^2}{M_{\rm P}^2}, \qquad A_0 = (\alpha + 3n)\frac{\langle F\rangle}{M_{\rm P}}, \qquad B_0 = (\beta + 2n)\frac{\langle F\rangle}{M_{\rm P}}. \tag{5.2.12}$$

In terms of these, the parameters appearing in eq. (5.1.1) are:

$$M_3 = M_2 = M_1 = m_{1/2}, \tag{5.2.13}$$

$$\mathbf{m^2_Q} = \mathbf{m^2_{\overline{u}}} = \mathbf{m^2_{\overline{d}}} = \mathbf{m^2_L} = \mathbf{m^2_{\overline{e}}} = m_0^2\,\mathbf{1}, \qquad m^2_{H_u} = m^2_{H_d} = m_0^2, \tag{5.2.14}$$

$$\mathbf{a_u} = A_0\mathbf{y_u}, \qquad \mathbf{a_d} = A_0\mathbf{y_d}, \qquad \mathbf{a_e} = A_0\mathbf{y_e}, \tag{5.2.15}$$

$$b = B_0\mu, \tag{5.2.16}$$

at a renormalization scale $Q \approx M_{\rm P}$. It is a matter of some controversy whether the assumptions going into this parameterization are well-motivated on purely theoretical grounds, but from a phenomenological perspective they are clearly very nice and extraordinarily predictive. These soft terms should be viewed as RG boundary conditions at the scale $M_{\rm P}$. The RG evolution of the soft parameters down to the electroweak scale will then allow us to predict the entire MSSM spectrum in terms of just five parameters $m_{1/2}$, m_0^2, A_0, B_0, and μ (plus the already-measured gauge and Yukawa couplings of the MSSM). A popular approximation is to start this RG running from the unification scale $M_U \approx 2 \times 10^{16}$ GeV instead of $M_{\rm P}$. The reason for this is more practical than principled; the apparent unification of gauge couplings gives us a strong hint that we know something about how the RG equations behave up to M_U, but unfortunately gives us little guidance about what to expect at scales between M_U and $M_{\rm P}$. The errors made in neglecting these effects are proportional to a loop suppression factor times $\ln(M_{\rm P}/M_U)$. These corrections hopefully can be partly absorbed into a redefinition of m_0^2, $m_{1/2}$, A_0 and B_0 at M_U, but in many cases will lead to other important effects that are difficult to anticipate.

The framework described in the previous two paragraphs has been the subject of the bulk of phenomenological and experimental studies of supersymmetry, and has become a benchmark scenario for experimental collider search limits. It is sometimes referred to as the *minimal supergravity* (MSUGRA) or *Constrained Minimal Supersymmetric Standard Model* (CMSSM) scenario for the soft terms.

5.2.5 Gauge-mediated supersymmetry breaking models

A second possibility is that the flavor-blind mediating interactions for supersymmetry breaking are the ordinary electroweak and QCD gauge interactions. In this *gauge-mediated supersymmetry breaking* (GMSB) scenario, the MSSM soft terms come from loop diagrams involving some *messenger* particles. The messengers are new chiral supermultiplets that

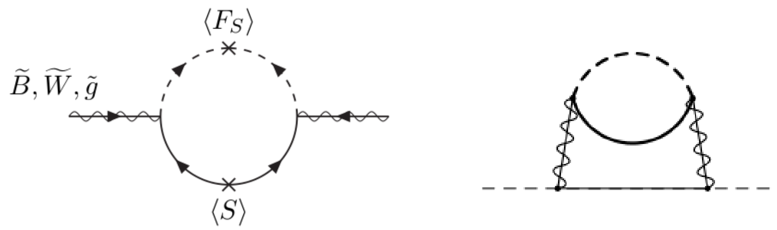


Figure 5.7: loop induced soft masses.

couple to a supersymmetry-breaking VEV $\langle F \rangle$, and also have $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions, which provide the necessary connection to the MSSM. In contrast to Planck-scale mediation, GMSB can be understood entirely in terms of loop effects in a renormalizable framework. Gaugino masses arise at one-loop while scalar masses arise at two-loop. Roughly, one estimates for the MSSM soft terms

$$m_{\text{soft}} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{\text{mess}}} \tag{5.2.17}$$

where the $\alpha_a/4\pi$ is a loop factor for Feynman diagrams involving gauge interactions, and M_{mess} is a characteristic scale of the masses of the messenger fields. So if M_{mess} and $\sqrt{\langle F \rangle}$ are roughly comparable, then the scale of supersymmetry breaking can be as low as about $\sqrt{\langle F \rangle} \sim 10^4$ GeV (much lower than in the gravity-mediated case!) to give m_{soft} of the right order of magnitude. There is still gravitational communication between the MSSM and the source of supersymmetry breaking, of course, but that effect is now relatively unimportant compared to the gauge interaction effects.

These soft terms are automatically falvour blind. The soft masses should now be interpreted as RG boundary conditions at the messenger scale, which will typically be much below the GUT scale. Messengers with masses far below the GUT scale will affect the running of gauge couplings and might therefore be expected to ruin the apparent unification shown in Figure 5.4. However, if the messengers come in complete multiplets of the $SU(5)$ global symmetry that contains the Standard Model gauge group, and are not very different in mass, then approximate unification of gauge couplings will still occur when they are extrapolated up to the same scale M_U (but with a larger unified value for the gauge couplings at that scale).

5.2.5.1 The gravitino

We have seen that extending SUSY to a local symmetry implied the existence of the gravitino, the superpartner of the graviton. We also learned that in supergravity the cosmological constant can be zero even for broken supersymmetry. Requiring that the cosmological constant be zero (at tree level), the gravitino mass $m_{3/2}$ is given by

$$m_{3/2} \simeq \frac{\langle F \rangle}{\sqrt{3} M_{\text{P}}} . \tag{5.2.18}$$

It is therefore directly proportional to the SUSY breaking scale and quite different for gravity or gauge mediation. In the Planck-scale-mediated supersymmetry breaking case, the gravitino mass is comparable to the masses of the MSSM sparticles. Therefore $m_{3/2}$ is expected to be at least of order 100 GeV or so. Its interactions will be of gravitational strength, so the gravitino will not play any role in collider physics, but it can be important in cosmology. If it is the LSP, then it is stable and its primordial density could easily exceed the critical density, causing the universe to become matter-dominated too early. Even if it is not the LSP, the gravitino can cause problems unless its density is diluted by inflation at late times, or it decays sufficiently rapidly.

In contrast, gauge-mediated supersymmetry breaking models predict that the gravitino is much lighter than the MSSM sparticles as long as $M_{\rm mess} \ll M_{\rm P}$. This can be seen by comparing eqs. (5.2.17) and (5.2.18). The gravitino is almost certainly the LSP in this case, and all of the MSSM sparticles will eventually decay into final states that include it. Naively, one might expect that these decays are extremely slow. However, this is not necessarily true, because the gravitino inherits the non-gravitational interactions of the goldstino it has absorbed. This means that the gravitino, or more precisely its longitudinal (goldstino) components, can play an important role in collider physics experiments.

Chapter 6

Mass spectrum of the MSSM

6.1 Electroweak symmetry breaking

Let us now consider electroweak symmetry breaking in the MSSM. There are two complex Higgs doublets $H_u = (H_u^+,\, H_u^0)$ and $H_d = (H_d^0,\, H_d^-)$ rather than just one in the ordinary Standard Model. The classical scalar potential for the Higgs scalar fields in the MSSM is given by

$$\begin{aligned} V &= (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ &\quad + [b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 + \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2. \end{aligned} \quad (6.1.1)$$

The terms proportional to $|\mu|^2$ come from F-terms while the terms proportional to g^2 and g'^2 are the D-term contributions. Finally, the terms proportional to $m_{H_u}^2$, $m_{H_d}^2$ and b are soft terms. The full scalar potential of the theory also includes many terms involving the squark and slepton fields that we can ignore here, since they do not (should not) get VEVs, which is likely because they have large positive squared masses[1].

We now have to demand that the minimum of this potential should break electroweak symmetry down to electromagnetism $SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$. We can use the freedom to make gauge transformations to simplify this analysis. First, the freedom to make $SU(2)_L$ gauge transformations allows us to rotate away a possible VEV for one of the weak isospin components of one of the scalar fields, so without loss of generality we can take $H_u^+ = 0$ at the minimum of the potential. Then one can check that a minimum of the potential satisfying $\partial V / \partial H_u^+ = 0$ must also have $H_d^- = 0$. This is good, because it means that at the minimum of the potential electromagnetism is necessarily unbroken, since the charged components of the Higgs scalars cannot get VEVs. After setting $H_u^+ = H_d^- = 0$, we are left

[1] There are in fact regions in the MSSM parameter space in which CCB vacua can occur, in particular for large A-terms.

to consider the scalar potential

$$
\begin{aligned}
V &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + \text{c.c.}) \\
&\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \qquad (6.1.2)
\end{aligned}
$$

CP cannot be spontaneously broken by the Higgs scalar potential, since the VEVs and b can be simultaneously chosen real, as a convention. This means that the Higgs scalar mass eigenstates can be assigned well-defined eigenvalues of CP, at least at tree-level.

In order for the MSSM scalar potential to be viable, we must first make sure that the potential is bounded from below for arbitrarily large values of the scalar fields, so that V will really have a minimum. The scalar quartic interactions in V will stabilize the potential for almost all arbitrarily large values of H_u^0 and H_d^0. However, for the special directions in field space $|H_u^0| = |H_d^0|$, the quartic contributions to V are identically zero. Such directions in field space are called D-flat directions, because along them the part of the scalar potential coming from D-terms vanishes. In order for the potential to be bounded from below, we need the quadratic part of the scalar potential to be positive along the D-flat directions. This requirement amounts to

$$
2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \qquad (6.1.3)
$$

Requiring further that one linear combination of H_u^0 and H_d^0 has a negative squared mass near $H_u^0 = H_d^0 = 0$ gives

$$
\det \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & -m_{12}^2 \\ -m_{12}^2 & |\mu|^2 + m_{H_d}^2 \end{pmatrix} = \left(|\mu|^2 + m_{H_u}^2\right)\left(|\mu|^2 + m_{H_d}^2\right) - b^2 < 0\,. \qquad (6.1.4)
$$

If this inequality is not satisfied, then $H_u^0 = H_d^0 = 0$ will be a stable minimum of the potential (or there will be no stable minimum at all), and electroweak symmetry breaking will not occur.

In models derived from the MSUGRA or GMSB boundary conditions, $m_{H_u}^2 = m_{H_d}^2$ is supposed to hold at tree level at the input scale. This has to be evolved down to the electroweak scale. The RG equations for the soft Higgs squared-mass parameters $m_{H_u}^2$ and $m_{H_d}^2$ are

$$
\begin{aligned}
16\pi^2 \frac{d}{dt} m_{H_u}^2 &= 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 + \frac{3}{5}g_1^2 S, \qquad (6.1.5) \\
16\pi^2 \frac{d}{dt} m_{H_d}^2 &= 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2 - \frac{3}{5}g_1^2 S \qquad (6.1.6)
\end{aligned}
$$

with

$$
\begin{aligned}
X_t &= 2|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\overline{u}_3}^2) + 2|a_t|^2, \qquad (6.1.7) \\
X_b &= 2|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\overline{d}_3}^2) + 2|a_b|^2, \qquad (6.1.8) \\
S &= m_{H_u}^2 - m_{H_d}^2 + \text{Tr}[\mathbf{m_Q^2} - \mathbf{m_L^2} - 2\mathbf{m_{\overline{u}}^2} + \mathbf{m_{\overline{d}}^2} + \mathbf{m_{\overline{e}}^2}]. \qquad (6.1.9)
\end{aligned}
$$

So the RG evolution of $m^2_{H_u}$ naturally pushes it to negative or small values $m^2_{H_u} < m^2_{H_d}$ at the electroweak scale, due to the large top Yukawa. So in these models electroweak symmetry breaking is actually driven by quantum corrections; this mechanism is therefore known as *radiative electroweak symmetry breaking*.

Having established the conditions necessary for H_u^0 and H_d^0 to get non-zero VEVs, we can now require that they are compatible with the observed phenomenology of electroweak symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$. Let us write[2]

$$v_u = \langle H_u^0 \rangle, \qquad v_d = \langle H_d^0 \rangle. \tag{6.1.10}$$

These VEVs are related to the known mass of the Z^0 boson and the electroweak gauge couplings:

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\text{GeV})^2. \tag{6.1.11}$$

The ratio of the VEVs is traditionally written as

$$\tan\beta \equiv v_u/v_d. \tag{6.1.12}$$

The value of $\tan\beta$ is not fixed by present experiments, but it depends on the Lagrangian parameters of the MSSM in a calculable way. Now one can write down the conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ under which the potential eq. (6.1.2) will have a minimum satisfying eqs. (6.1.11) and (6.1.12):

$$m^2_{H_u} + |\mu|^2 - b\cot\beta - (m_Z^2/2)\cos(2\beta) \;=\; 0, \tag{6.1.13}$$
$$m^2_{H_d} + |\mu|^2 - b\tan\beta + (m_Z^2/2)\cos(2\beta) \;=\; 0. \tag{6.1.14}$$

These conditions can be rewritten as:

$$\sin(2\beta) \;=\; \frac{2b}{m^2_{H_u} + m^2_{H_d} + 2|\mu|^2}, \tag{6.1.15}$$
$$m_Z^2 \;=\; \frac{|m^2_{H_d} - m^2_{H_u}|}{\sqrt{1-\sin^2(2\beta)}} - m^2_{H_u} - m^2_{H_d} - 2|\mu|^2 \tag{6.1.16}$$
$$\;=\; -2(m^2_{H_u} + |\mu|^2) + \mathcal{O}(1/\tan^2\beta). \tag{6.1.17}$$

As an aside, this highlights the "μ problem" already mentioned earlier. Without miraculous cancellations, all of the input parameters ought to be within an order of magnitude or two of m_Z^2. However, in the MSSM, μ is a supersymmetry-respecting parameter appearing in the superpotential, while b, $m^2_{H_u}$, $m^2_{H_d}$ are supersymmetry-breaking parameters. This has lead to a widespread belief that the MSSM must be extended at very high energies to include a mechanism that relates the effective value of μ to the supersymmetry-breaking mechanism in some way.

[2] often a factor $1/\sqrt{2}$ is included in this definition, with $v = 246$ GeV.

Even if the value of μ is set by soft supersymmetry breaking, the cancellation needed by eq. (6.1.17) is often remarkable when evaluated in specific model frameworks, after constraints from direct searches for the Higgs bosons and superpartners are taken into account. Typical viable solutions for the MSSM have $-m_{H_u}^2$ and $|\mu|^2$ each much larger than m_Z^2, so that significant cancellation is needed.

To quantify the measure of fine-tuning, one possibility is to consider the sensitivity of the weak scale, or equivalently the Z mass, to changes in the fundamental parameters a_i of the theory (in MSUGRA $a_i = m_{1/2}, m_0, ...$),

$$\Delta = \max_i \frac{\partial \ln m_Z^2}{\partial \ln a_i} = \frac{a_i}{m_Z^2}\frac{\partial m_Z^2}{\partial a_i} \, . \tag{6.1.18}$$

The larger the parameter Δ, the larger the fine-tuning in the parameters a_i required to reproduce the correct Z mass. Note that the question which value of Δ is deemed "acceptable" is completely subjective. Also it of course depends on what one thinks are the fundamental parameters and correlations can be very important.

Typically the fundamental parameters are defined at the high scale and running can be very important. The μ parameter is exceptional because it does not run much. We can therefore take $\mu_{\rm EW} \simeq \mu_{\rm GUT}$ and find $\Delta\mu \simeq 2\mu^2/m_Z^2$. This implies

$$|\mu| \lesssim \begin{cases} 200\,\text{GeV} & \text{for } \Delta < 10\,, \\ 650\,\text{GeV} & \text{for } \Delta < 100\,. \end{cases} \tag{6.1.19}$$

This bound should be taken with a grain of salt since there could be correlations between μ and the other soft terms. Barring this caveat, naturalness implies that the Higgsino-like fermions should be among the lightest sparticles. Doing a scan over the viable parameter space in MSUGRA requiring the correct Higgs mass etc. the fine tuning is around $\Delta \sim 10^2 - 10^3$. To obtain a more natural theory one would need to go beyond this, and we will look at the NMSSM as an example.

6.1.0.2 The Higgs sector

The Higgs scalar fields in the MSSM consist of two complex $SU(2)_L$-doublet, or eight real, scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bosons G^0, $G^\pm$, which become the longitudinal modes of the Z^0 and $W^\pm$ massive vector bosons. The remaining five Higgs scalar mass eigenstates consist of two CP-even neutral scalars h^0 and H^0, one CP-odd neutral scalar A^0, and a charge $+1$ scalar H^+ and its conjugate charge -1 scalar H^-. (Here we define $G^- = G^{+*}$ and $H^- = H^{+*}$. Also, by convention, h^0 is lighter than H^0.) The gauge-eigenstate fields can be expressed in terms of the mass eigenstate fields as:

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}} R_{\beta_0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \tag{6.1.20}$$

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_{\beta_\pm} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \tag{6.1.21}$$

where the orthogonal rotation matrices

$$R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \tag{6.1.22}$$

$$R_{\beta_0} = \begin{pmatrix} \sin\beta_0 & \cos\beta_0 \\ -\cos\beta_0 & \sin\beta_0 \end{pmatrix}, \qquad R_{\beta_\pm} = \begin{pmatrix} \sin\beta_\pm & \cos\beta_\pm \\ -\cos\beta_\pm & \sin\beta_\pm \end{pmatrix}, \tag{6.1.23}$$

are chosen so that the quadratic part of the potential has diagonal squared-masses:

$$\begin{aligned} V &= \frac{1}{2}m_{h^0}^2(h^0)^2 + \frac{1}{2}m_{H^0}^2(H^0)^2 + \frac{1}{2}m_{G^0}^2(G^0)^2 + \frac{1}{2}m_{A^0}^2(A^0)^2 \\ &\quad + m_{G^\pm}^2|G^+|^2 + m_{H^\pm}^2|H^+|^2 + \ldots, \end{aligned} \tag{6.1.24}$$

Then, provided that v_u, v_d minimize the tree-level potential,[3] one finds that $\beta_0 = \beta_\pm = \beta$, and $m_{G^0}^2 = m_{G^\pm}^2 = 0$, and

$$m_{A^0}^2 = 2b/\sin(2\beta) = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2 \tag{6.1.25}$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 - m_Z^2)^2 + 4m_Z^2 m_{A^0}^2 \sin^2(2\beta)}\Big), \tag{6.1.26}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2. \tag{6.1.27}$$

The masses of A^0, H^0 and $H^\pm$ can in principle be arbitrarily large since they all grow with $b/\sin(2\beta)$. In contrast, the mass of h^0 is bounded above. At tree-level

$$m_{h^0} < m_Z|\cos(2\beta)| \tag{6.1.28}$$

If this tree-level inequality were robust, the lightest Higgs boson of the MSSM would have been discovered at LEP2. However, the tree-level formula for the squared mass of h^0 is subject to quantum corrections that are relatively drastic. The largest such contributions typically come from top and stop loops, as shown in fig. 6.1.0.2.

In the simple limit of top squarks that have a small mixing in the gauge eigenstate basis and with masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ much greater than the top quark mass m_t, one finds a large positive one-loop radiative correction to eq. (6.1.26):

$$\Delta(m_{h^0}^2) = \frac{3}{4\pi^2}\cos^2\!\alpha\; y_t^2 m_t^2 \ln\left(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2\right). \tag{6.1.29}$$

[3] It is often more useful to expand around VEVs v_u, v_d that do not minimize the tree-level potential, for example to minimize the loop-corrected effective potential instead. In that case, β, β_0, and $\beta_\pm$ are all slightly different.

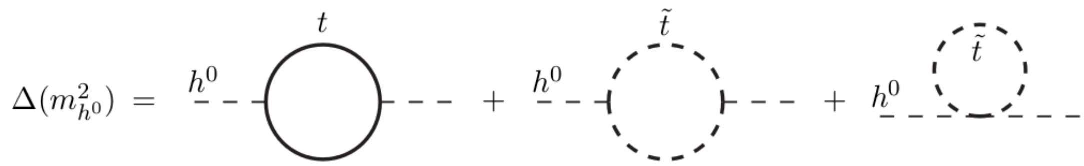


Figure 6.1: Leading corrections to MSSM higgs mass.

Top-squark mixing (to be discussed in section 6.1.3) can result in a further large positive contribution to $m^2_{h^0}$. This shows that m_{h^0} can exceed the LEP bounds. Including these and other important corrections one can obtain only a weaker, but still very interesting, bound

$$m_{h^0} \lesssim 135\,\text{GeV} \tag{6.1.30}$$

in the MSSM. This assumes that all of the sparticles that can contribute to $m^2_{h^0}$ in loops have masses that do not exceed 1 TeV.

An interesting case, often referred to as the "decoupling limit", occurs when $m_{A^0} \gg m_Z$. Then m_{h^0} can saturate the upper bounds just mentioned, with $m^2_{h^0} \approx m^2_Z \cos^2(2\beta)+$ loop corrections. The particles A^0, H^0, and $H^\pm$ will be much heavier and nearly degenerate, forming an isospin doublet that decouples from sufficiently low-energy experiments. The angle α is very nearly $\beta - \pi/2$, and h^0 has the same couplings to quarks and leptons and electroweak gauge bosons as would the physical Higgs boson of the ordinary Standard Model without supersymmetry.

In the MSSM, the masses and CKM mixing angles of the quarks and leptons are determined not only by the Yukawa couplings of the superpotential but also the parameter $\tan\beta$. This is because the top, charm and up quark mass matrix is proportional to $v_u = v\sin\beta$ and the bottom, strange, and down quarks and the charge leptons get masses proportional to $v_d = v\cos\beta$. At tree-level,

$$m_t = y_t v \sin\beta, \qquad m_b = y_b v \cos\beta, \qquad m_\tau = y_\tau v \cos\beta. \tag{6.1.31}$$

These relations hold for the running masses rather than the physical pole masses, which are significantly larger for t, b. Including those corrections, one can relate the Yukawa couplings to $\tan\beta$ and the known fermion masses and CKM mixing angles. It is now clear why we have not neglected y_b and y_τ, even though $m_b, m_\tau \ll m_t$. To a first approximation, $y_b/y_t = (m_b/m_t)\tan\beta$ and $y_\tau/y_t = (m_\tau/m_t)\tan\beta$, so that y_b and y_τ cannot be neglected if $\tan\beta$ is much larger than 1. In fact, there are good theoretical motivations for considering models with large $\tan\beta$. For example, models based on the GUT gauge group $SO(10)$ can unify the running top, bottom and tau Yukawa couplings at the unification scale; this requires $\tan\beta$ to be very roughly of order m_t/m_b.

Note that if one tries to make $\sin\beta$ too small, y_t will be nonperturbatively large. Requiring that y_t does not blow up above the electroweak scale, one finds that $\tan\beta \gtrsim 1.2$ or so, depending on the mass of the top quark, the QCD coupling, and other details. In principle, there is also a constraint on $\cos\beta$ if one requires that y_b and y_τ do not become nonperturbatively large. This gives a rough upper bound of $\tan\beta \lesssim 65$.

6.1.1 The gluino

The gluino is a color octet fermion, so it cannot mix with any other particle in the MSSM, even if R-parity is violated. In this regard, it is unique among all of the MSSM sparticles. The one-loop RG equations for the three gaugino mass parameters in the MSSM are determined by the same quantities $b_a^{\rm MSSM}$ that appear in the gauge coupling RG,

$$\beta_{M_a} \equiv \frac{d}{dt} M_a \;=\; \frac{1}{8\pi^2} b_a g_a^2 M_a \qquad (b_a = 33/5,\ 1,\ -3) \tag{6.1.32}$$

for $a = 1, 2, 3$. In models with MSUGRA boundary conditions a rough prediction is

$$M_3 : M_2 : M_1 \approx 6 : 2 : 1 \tag{6.1.33}$$

near the TeV scale. It is therefore reasonable to suspect that the gluino is considerably heavier than the lighter neutralinos and charginos.

6.1.2 Neutralinos and charginos

The higgsinos and electroweak gauginos mix with each other because of the effects of electroweak symmetry breaking (as can be seen in the mass matrix below). The neutral higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$, $\widetilde{W}^0$) combine to form four mass eigenstates called *neutralinos*. The charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) and winos ($\widetilde{W}^+$ and $\widetilde{W}^-$) mix to form two mass eigenstates with charge ± 1 called *charginos*. We will denote[4] the neutralino and chargino mass eigenstates by $\widetilde{N}_i$ ($i = 1, 2, 3, 4$) and $\widetilde{C}_i^{\pm}$ ($i = 1, 2$). By convention, these are labeled in ascending order, so that $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$ and $m_{\widetilde{C}_1} < m_{\widetilde{C}_2}$. The lightest neutralino, $\widetilde{N}_1$, is usually assumed to be the LSP, unless there is a lighter gravitino or unless R-parity is not conserved, because it is the only MSSM particle that can make a good dark matter candidate (the sneutrino is already excluded by direct detection). Whether it really is the LSP of course depends on the RG boundary conditions whicha are assumed.

In the gauge-eigenstate basis $\psi^0 = (\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$, the neutralino mass part of the Lagrangian is

$$\mathcal{L}_{\text{neutralino mass}} = -\frac{1}{2}(\psi^0)^T \mathbf{M}_{\widetilde{N}} \psi^0 + \text{c.c.}, \tag{6.1.34}$$

where

$$\mathbf{M}_{\widetilde{N}} \;=\; \begin{pmatrix} M_1 & 0 & -g' v_d/\sqrt{2} & g' v_u/\sqrt{2} \\ 0 & M_2 & g v_d/\sqrt{2} & -g v_u/\sqrt{2} \\ -g' v_d/\sqrt{2} & g v_d/\sqrt{2} & 0 & -\mu \\ g' v_u/\sqrt{2} & -g v_u/\sqrt{2} & -\mu & 0 \end{pmatrix}. \tag{6.1.35}$$

[4]Other common notations use $\widetilde{\chi}_i^0$ or $\widetilde{Z}_i$ for neutralinos, and $\widetilde{\chi}_i^{\pm}$ or $\widetilde{W}_i^{\pm}$ for charginos.

The entries M_1 and M_2 in this matrix come directly from the MSSM soft Lagrangian, while the entries $-\mu$ are the supersymmetric higgsino mass terms. The terms proportional to g, g' are the result of Higgs-higgsino-gaugino couplings with the Higgs scalars replaced by their VEVs.

The mass matrix $\mathbf{M}_{\widetilde{N}}$ can be diagonalized by a unitary matrix $\mathbf{N}$ to obtain mass eigenstates:

$$\widetilde{N}_i = \mathbf{N}_{ij}\psi_j^0, \tag{6.1.36}$$

so that

$$\mathbf{N}^*\mathbf{M}_{\widetilde{N}}\mathbf{N}^{-1} = \begin{pmatrix} m_{\widetilde{N}_1} & 0 & 0 & 0 \\ 0 & m_{\widetilde{N}_2} & 0 & 0 \\ 0 & 0 & m_{\widetilde{N}_3} & 0 \\ 0 & 0 & 0 & m_{\widetilde{N}_4} \end{pmatrix} \tag{6.1.37}$$

has real positive entries on the diagonal.

In general, the parameters M_1, M_2, and μ in the equations above can have arbitrary complex phases. A redefinition of the phases of $\widetilde{B}$ and $\widetilde{W}$ always allows us to choose a convention in which M_1 and M_2 are both real and positive. The phase of μ within that convention is then really a physical parameter and cannot be rotated away. However, if μ is not real, then there can be potentially disastrous CP-violating effects in low-energy physics. Therefore, it is usual to assume that μ is real in the same set of phase conventions that make M_1, M_2, b, $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ real and positive.

In models with gaugino mass unification, the neutralino masses and mixing angles depend on only three unknown parameters as $M_2 \sim 2M_1$.

Often electroweak symmetry breaking effects can be viewed as a small perturbation on the neutralino mass matrix. If

$$m_Z \ll |\mu \pm M_1|, |\mu \pm M_2|, \tag{6.1.38}$$

then the neutralino mass eigenstates are very nearly a "bino-like" $\widetilde{N}_1 \approx \widetilde{B}$; a "wino-like" $\widetilde{N}_2 \approx \widetilde{W}^0$; and "higgsino-like" $\widetilde{N}_3, \widetilde{N}_4 \approx (\widetilde{H}_u^0 \pm \widetilde{H}_d^0)/\sqrt{2}$, with mass eigenvalues:

$$m_{\widetilde{N}_1} = M_1 + \ldots \tag{6.1.39}$$

$$m_{\widetilde{N}_2} = M_2 + \ldots \tag{6.1.40}$$

$$m_{\widetilde{N}_3}, m_{\widetilde{N}_4} = |\mu| + \ldots \tag{6.1.41}$$

The above labeling of $\widetilde{N}_1$ and $\widetilde{N}_2$ assumes $M_1 < M_2 \ll |\mu|$. This limit, leading to a bino-like neutralino LSP, often emerges from MSUGRA boundary conditions. So while this seems very natural from this perspective, current collider bounds push the neutralino masses into a regime in which it is hard to obtain the correct relic abundance of dark matter for a bino LSP. We will come back to this later.

Analogously to the neutralinos, we can write the part of the Lagrangian involving charginos as

$$\mathcal{L} \supset -\frac{1}{2}(\psi^+)^T \begin{pmatrix} M_2 & gv_u/\sqrt{2} \\ gv_d/\sqrt{2} & \mu \end{pmatrix} \psi^- + \text{h.c.} \tag{6.1.42}$$

where $\psi^+ = (\widetilde{W}^+, \tilde{H}_u^+,)$ and $\psi^- = (\widetilde{W}^-, \tilde{H}_d^-)$. Diagonalizing this mass matrix with a bi-unitary rotation, we obtain two chargino mass eigenstates. In the limit of heavy sparticles, one chargino is higgsino-like with mass close to $|\mu|$ the other one wino-like with mass close to $|M_2|$. In this limit, both charginos will be close in mass to three of the neutralinos.

6.1.3 The squarks and sleptons

As before the low energy particle spectrum is determined by evolving the mass parameters from the high to the low scale and then diagonalising the mass matrices. The squark RGEs look e.g. like

$$16\pi^2 \frac{d}{dt} m_{Q_3}^2 = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2 + \frac{1}{5} g_1^2 S. \tag{6.1.43}$$

In principle, any scalars with the same electric charge, R-parity, and color quantum numbers can mix with each other. This means that with completely arbitrary soft terms, the mass eigenstates of the squarks and sleptons of the MSSM should be obtained by diagonalizing three 6×6 squared-mass matrices for up-type squarks, down-type squarks, and charged sleptons. Fortunately, the general hypothesis of flavor-blind soft parameters predicts that most of these mixing angles are very small, as long as the Yukawa couplings are small.

Let us therefore first consider the spectrum of first- and second-family squarks and sleptons. In this case only the terms proportional to gauge couplings are relevant in the running, increasing the mass at low energies because of the minus sign. In many models, including both MSUGRA, their running squared masses can be conveniently parameterized, to a good approximation, as:

$$m_{Q_1}^2 = m_{Q_2}^2 = m_0^2 + K_3 + K_2 + \frac{1}{36} K_1, \tag{6.1.44}$$

$$m_{\overline{u}_1}^2 = m_{\overline{u}_2}^2 = m_0^2 + K_3 + \frac{4}{9} K_1, \tag{6.1.45}$$

$$m_{\overline{d}_1}^2 = m_{\overline{d}_2}^2 = m_0^2 + K_3 + \frac{1}{9} K_1, \tag{6.1.46}$$

$$m_{L_1}^2 = m_{L_2}^2 = m_0^2 + K_2 + \frac{1}{4} K_1, \tag{6.1.47}$$

$$m_{\overline{e}_1}^2 = m_{\overline{e}_2}^2 = m_0^2 + K_1. \tag{6.1.48}$$

A key point is that the same K_3, K_2 and K_1 appear everywhere, since all of the chiral supermultiplets couple to the same gauginos with the same gauge couplings. The different coefficients in front of K_1 just correspond to the various values of weak hypercharge squared for each scalar.

If the input scale is approximated by the apparent scale of gauge coupling unification $Q_0 = M_U \approx 2 \times 10^{16}$ GeV, one finds that numerically

$$K_1 \approx 0.15 m_{1/2}^2, \qquad K_2 \approx 0.5 m_{1/2}^2, \qquad K_3 \approx 5 m_{1/2}^2. \tag{6.1.49}$$

for Q near the electroweak scale. Here $m_{1/2}$ is the common gaugino mass parameter at the unification scale. Note that $K_3 \gg K_2 \gg K_1$; this is a direct consequence of the relative sizes of the gauge couplings g_3, g_2, and g_1. In general, one therefore expects that the squarks should be considerably heavier than the sleptons

For the third generation squarks and sleptons the Yukawa couplings and the a terms are non-negligible and tend to make these states a bit lighter because of the plus sign. These terms also induce mixing between the left- and right handed states. Putting these all together, we have a squared-mass matrix for the top squarks, which in the gauge-eigenstate basis $(\widetilde{t}_L, \widetilde{t}_R)$ is given by

$$\mathcal{L}_{\text{stop masses}} = -\begin{pmatrix} \widetilde{t}_L^* & \widetilde{t}_R^* \end{pmatrix} \mathbf{m^2_{\widetilde{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{6.1.50}$$

where

$$\mathbf{m^2_{\widetilde{t}}} = \begin{pmatrix} m^2_{Q_3} + m_t^2 + \Delta_{\tilde{u}_L} & v(a_t^* \sin\beta - \mu y_t \cos\beta) \\ v(a_t \sin\beta - \mu^* y_t \cos\beta) & m^2_{\bar{u}_3} + m_t^2 + \Delta_{\tilde{u}_R} \end{pmatrix}. \tag{6.1.51}$$

This hermitian matrix can be diagonalized by a unitary matrix to give mass eigenstates $\widetilde{t}_1$ and $\widetilde{t}_2$.

6.1.4 Summary: the MSSM sparticle spectrum

Figure 6.2 shows the RG running of scalar and gaugino masses in a typical model based on the MSUGRA boundary conditions imposed at $Q_0 = 2 \times 10^{16}$ GeV. [The parameter values used for this illustration were $m_0 = 200$ GeV, $m_{1/2} = -A_0 = 600$ GeV, $\tan\beta = 10$, and sign(μ)= +.] The running gaugino masses are solid lines labeled by M_1, M_2, and M_3. The dot-dashed lines labeled H_u and H_d are the running values of the quantities $(\mu^2 + m^2_{H_u})^{1/2}$ and $(\mu^2 + m^2_{H_d})^{1/2}$, which appear in the Higgs potential. The other lines are the running squark and slepton masses, with dashed lines for the square roots of the third family parameters $m^2_{\bar{d}_3}$, $m^2_{Q_3}$, $m^2_{\bar{u}_3}$, $m^2_{L_3}$, and $m^2_{\bar{e}_3}$ (from top to bottom), and solid lines for the first and second family sfermions. Note that $\mu^2 + m^2_{H_u}$ runs negative because of the effects of the large top Yukawa coupling as discussed above, providing for electroweak symmetry breaking. At the electroweak scale, the values of the Lagrangian soft parameters can be used to extract the physical masses, cross-sections, and decay widths of the particles, and other observables such as dark matter abundances and rare process rates. In practice this can be done with the help of spectrum calculators.

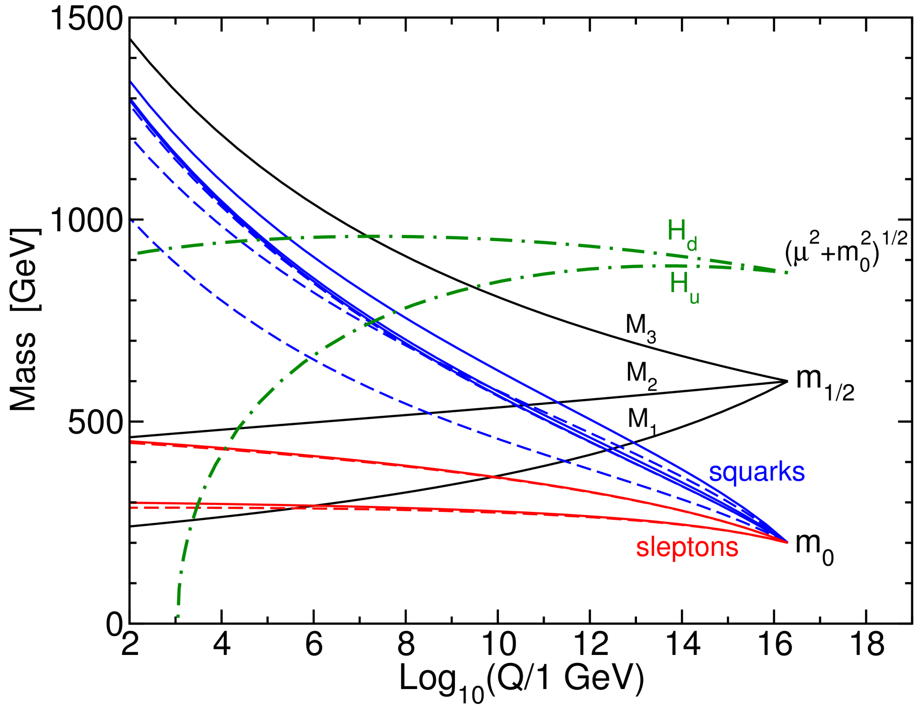


Figure 6.2: RG evolution of scalar and gaugino mass parameters in the MSSM with MSUGRA boundary conditions imposed at $Q_0 = 2 \times 10^{16}$ GeV. The parameter $\mu^2 + m_{H_u}^2$ runs negative, provoking electroweak symmetry breaking.

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	$+1$	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	-1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	-1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	-1	$\widetilde{B}^0\ \ \widetilde{W}^0\ \ \widetilde{H}_u^0\ \ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	-1	$\widetilde{W}^\pm\ \ \widetilde{H}_u^+\ \ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \ \widetilde{C}_2^\pm$
gluino	1/2	-1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	-1	$\widetilde{G}$	(same)

Table 6.1: The particles in the Minimal Supersymmetric Standard Model (with sfermion mixing for the first two families assumed to be negligible).

Chapter 7

Direct SUSY searches

While the indirect searches for sparticles are sensitive in principle to very high mass scales, they typically do not allow to put firm bounds on sparticle masses. First, they are always sensitive to a combination of masses and couplings; Second, there can be several loop contributions from different sparticles that can interfere and even cancel each other. Moreover, after a discovery of new physics in one of the precision observables indirectly sensitive to sparticles, it might be difficult to identify the particle responsible for the deviation. *Direct* searches refer to the production and subsequent decay of sparticles in collider experiments. While they are limited in reach by the available center-of-mass energy, they would allow a more unambiguous identification of new particles.

7.1 LEP bounds

The LEP e^+e^- collider operated at CERN until the year 2000 with a maximum center-of-mass energy of 209 GeV. Compared to hadron colliders like the LHC, there are two main advantages of SUSY searches at lepton colliders:

- the full center-of-mass energy is available in a single "partonic" collision;
- the initial state is known precisely, making for a much cleaner experimental environment.

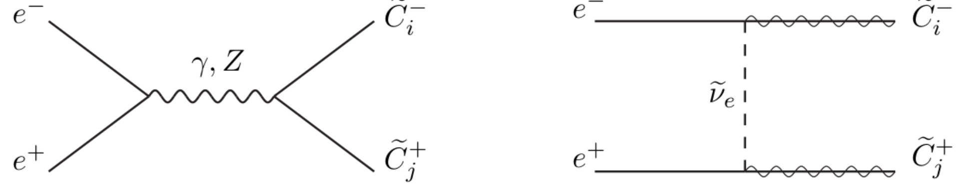


Figure 7.1: Feynman diagrams for chargino production at LEP (or ILC).

For the production of sparticles at colliders, we first need to recall two crucial consequences of R-parity:

- sparticles can only be produced in pairs;
- the final decay products of all sparticles (the decays can proceed in several steps) always contain 1 sparticle, namely the LSP. It is often assumed (and we will assume it as well in the following) that the LSP is the lightest neutralino.

Keeping these conditions in mind, there are two ways how sparticles can be produced at a lepton collider:

- s-channel exchange of a Z boson (for all sparticles with electroweak quantum numbers, i.e. all but the gluino) or a photon (for all charged sparticles, i.e. charginos, sfermions, and the charged Higgs);
- t-channel exchange of a sparticle (slepton, sneutrino, chargino, or neutralino).

For the t-channel exchange to be relevant, the exchanged sparticle should not be too heavy.

Concerning the decays, the implication of R-parity is that the final state always contains exactly two LSPs (which we assume to be the lightest neutralino). Since it is electrically neutral and stable, they escape the detector without being detected. Still, they can be traced indirectly by looking at the kinematics of all "visible" particles. Assuming all particles have been detected and their energies and 3-momenta measured correctly, energy and momentum conservation implies

$$\sum_i E_i = \sqrt{s}\,, \qquad \sum_i \vec{p}_i = \vec{0}\,, \tag{7.1.1}$$

in the lab system. Consequently, one can define **missing energy** $\not{E} = \sqrt{s} - \sum_i E_i$ and **missing momentum** $\not{\vec{p}} = -\sum_i \vec{p}_i$ which, if nonzero, indicate the presence of particles that have escaped detection. This occurs also in the SM, in decays involving neutrinos, but the missing energy in SUSY decay chains is typically much larger.

An example is the decay of two pair-produced sleptons: they could decay to two leptons and two LSPs, leading to a signature with two leptons and missing energy. Charginos can decay e.g. to an LSP and an off-shell W boson which could further decay to two leptons, leading to a signature with four leptons and missing energy. If the W boson decays to quarks, what is actually observed experimentally is a **jet** of hadrons, since quarks cannot appear freely.

Roughly speaking, the SUSY searches at LEP have constrained all charginos, squarks and sleptons to have masses above about half the center-of-mass energy, i.e. above 100 GeV. These limits can only be avoided in special cases, e.g. a sparticle that is nearly degenerate with the LSP, such that the visible decay products are very "soft".

7.2 LHC bounds

Since 2010, the LHC experiments ATLAS and CMS have strongly improved the lower bounds on sparticle masses[1]. The LHC is a *pp* collider with a center-of-mass energy of 7 TeV (2010–2011), 8 TeV (2012), and, starting now (2015), 13 TeV[2].

A complication with respect to a lepton collider is that the colliding hadrons are composite particles. What we are actually interested in are reactions with large momentum transfers where new particles are produced. Fortunately, such processes can be factorized into the hard scattering of a pair of **partons** (quarks, antiquarks, or gluons), convoluted with **parton distribution functions** (pdfs) describing the structure of the proton. The cross section for the production of a high invariant mass final state Y in pp collisions can then be written as

$$d\sigma(p(P_1) + p(P_2) \to Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{i_1,i_2} f_{i_1}(x_1) f_{i_2}(x_2) d\sigma(i_1(x_1P_1) + i_2(x_2P_2) \to Y) \quad (7.2.1)$$

where X includes the hadronized remnants of the colliding protons, $i_{1,2}$ are the colliding partons, $x_{1,2}$ their **momentum fractions** in terms of the original proton momenta $P_{1,2}$, and $f_i(x)$ are the pdfs. The invariant mass of the state Y (which can be one or several particles) is given by the parton center-of-mass energy $\sqrt{\hat{s}}$, given by $\hat{s} = x_1 x_2 s$ in terms of the pp center-of-mass energy $\sqrt{s}$.

An important feature is that the largest momentum fractions are carried by the **valence quarks** u, d and the gluons. Consequently, the production of a heavy particle not far from the energy threshold will have a larger cross section if it can be iniated by these partons.

Typical MSSM production cross sections at the 8 TeV LHC are shown in fig. 7.2. One can observe the following qualitative features:

- Production of (first generation) squarks and gluinos have the largest cross sections as these processes profit from the large pdfs at high momentum fraction of the valence quarks and gluons. They can proceed via s-channel exchange of quarks or gluons or t-channel exchange of squarks or gluinos and are enhanced by the strong coupling constant. In the case of squarks, this plot sums over all flavours but the stop.

- Production of third generation squarks is suppressed. This is mostly[3] due to the fact that the diagram with production from quark-antiquark annihilation via a t-channel gluino exchange is absent in the stop case (or, rather, extremely suppressed due to the small top quark pdf).

- Production of electroweakinos is even more suppressed due to the absence of gluon-initiated processes and due to the smallness of the weak gauge couplings.

[1] The most up-to-date bounds can be found on the public websites of ATLAS and CMS, which is also where the exclusion plots in this section are taken from.

[2] The original design energy is 14 TeV.

[3] In addition, the stop cross section is smaller in the plot compared to $\tilde{q}\tilde{q}^*$ because, in the latter case, a sum over all non-stop flavours is shown.

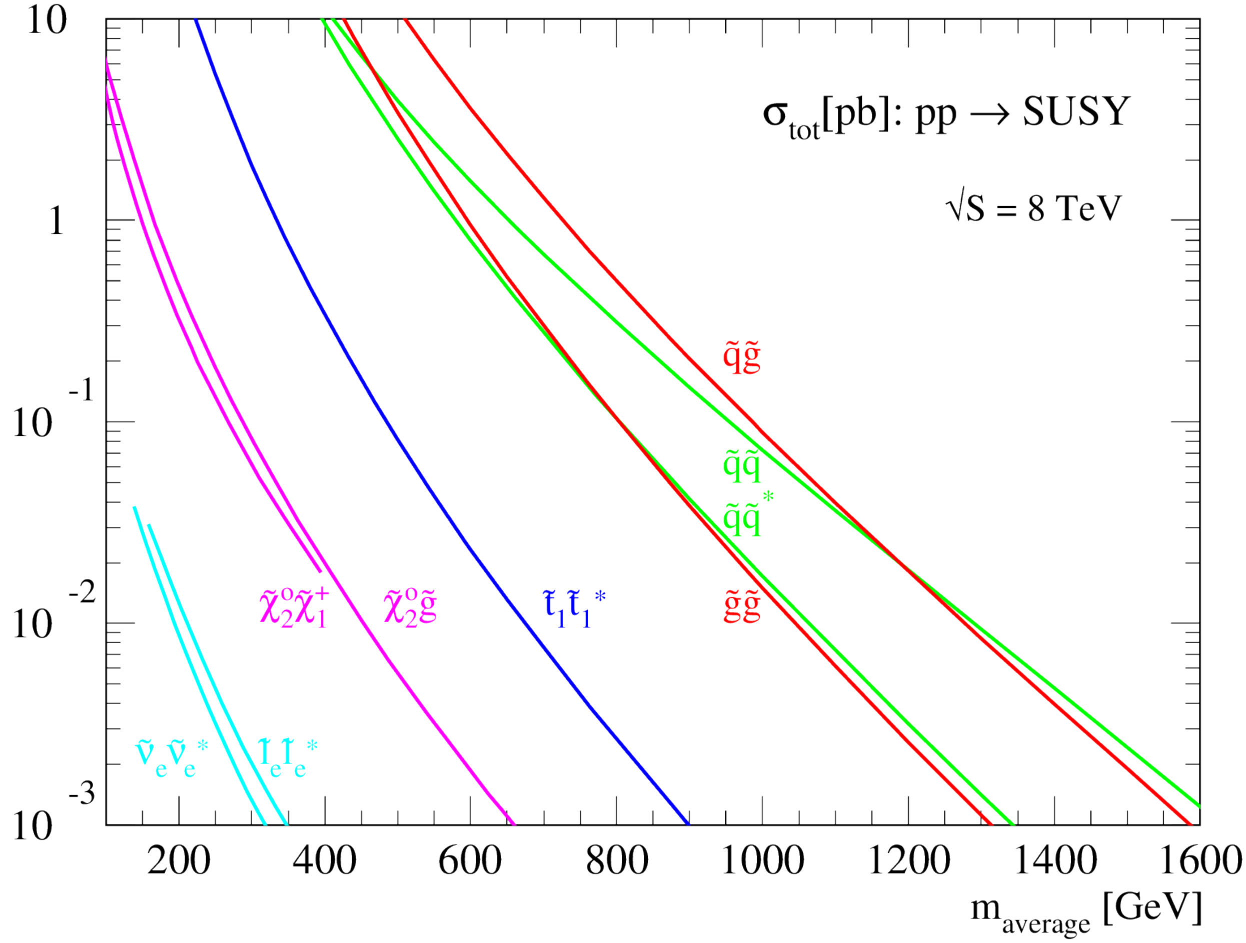


Figure 7.2: Sparticle production cross sections at the 8 TeV LHC (plot taken from the Prospino web site).

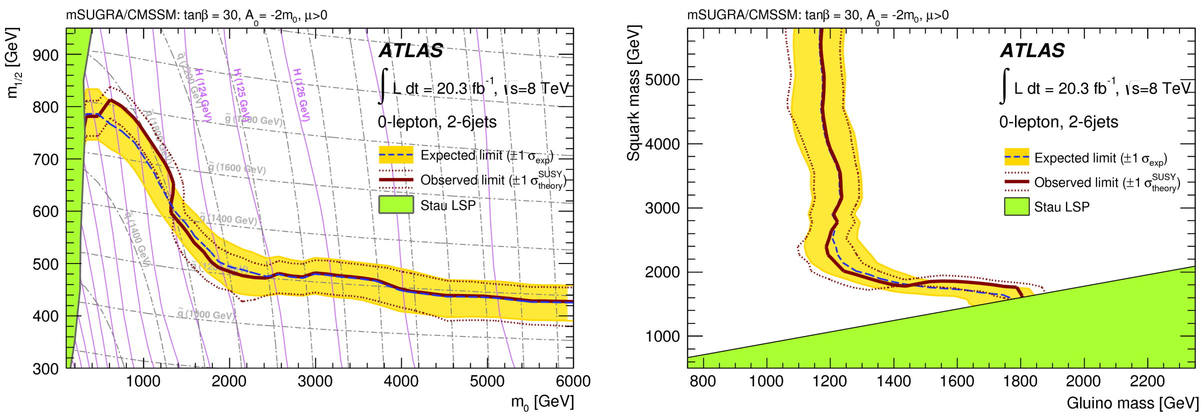


Figure 7.3: Combined bounds in the MSUGRA parameter plane $m_0 - m_{1/2}$ and translated into the gluino squark plane from different search channels.

- Slepton production is smallest. Charged sleptons or sneutrinos can only be produced via s-channel photon, Z, or W exchange.

As in the case of LEP, conservation of R-parity implies that all production and decay chains of sparticles end with two LSPs, leading to missing momentum. However, an important difference is that the total momentum *before* the collision is not known in a hadron collider, as what matters is the kinematics of the *partonic* collision. For this reason, one considers the **missing transverse momentum**,

$$\vec{\not{p}}_T = -\sum_i (\vec{p}_T)_i \tag{7.2.2}$$

where the transverse momentum vector $\vec{p}_T$ is the projecction of the 3-vector onto the plane orthogonal to the beam direction. Both in the partonic and in the hadronic system, the total transverse energy vanishes. Unfortunately, at the LHC, one often refers to the absolute value of this vector, $|\vec{\not{p}}_T|$, as **missing transverse energy**, denoted by $\not{E}_T$, E_T^{miss}, or simply MET, even though this quantity corresponds to an energy only in the special case of massless invisible particles (like neutrinos).

The observable signals for supersymmetry at hadron colliders are n leptons + m jets + $\not{E}_T$, where either n or m might be 0. In general there are many different search channels. There are important Standard Model backgrounds to these signals, especially from processes involving production of W and Z bosons that decay to neutrinos, which provide the $\not{E}_T$. Therefore it is important to identify specific signal region cuts for which the backgrounds can be reduced. Of course, the optimal choice of cuts depends on which sparticles are being produced and how they decay, facts that are not known in advance.

The latest limits on the CMSSM parameters are shown in figure 7.3 in the $m_0 - m_{1/2}$ plane and translated into the gluino squark plane. The bound does not depend strongly on the fixed parameters.

Thinking about the MSSM more generally, it is hard to give a clear-cut exclusion bound on a given sparticle since the cross-section and branching ratios depend on many of the

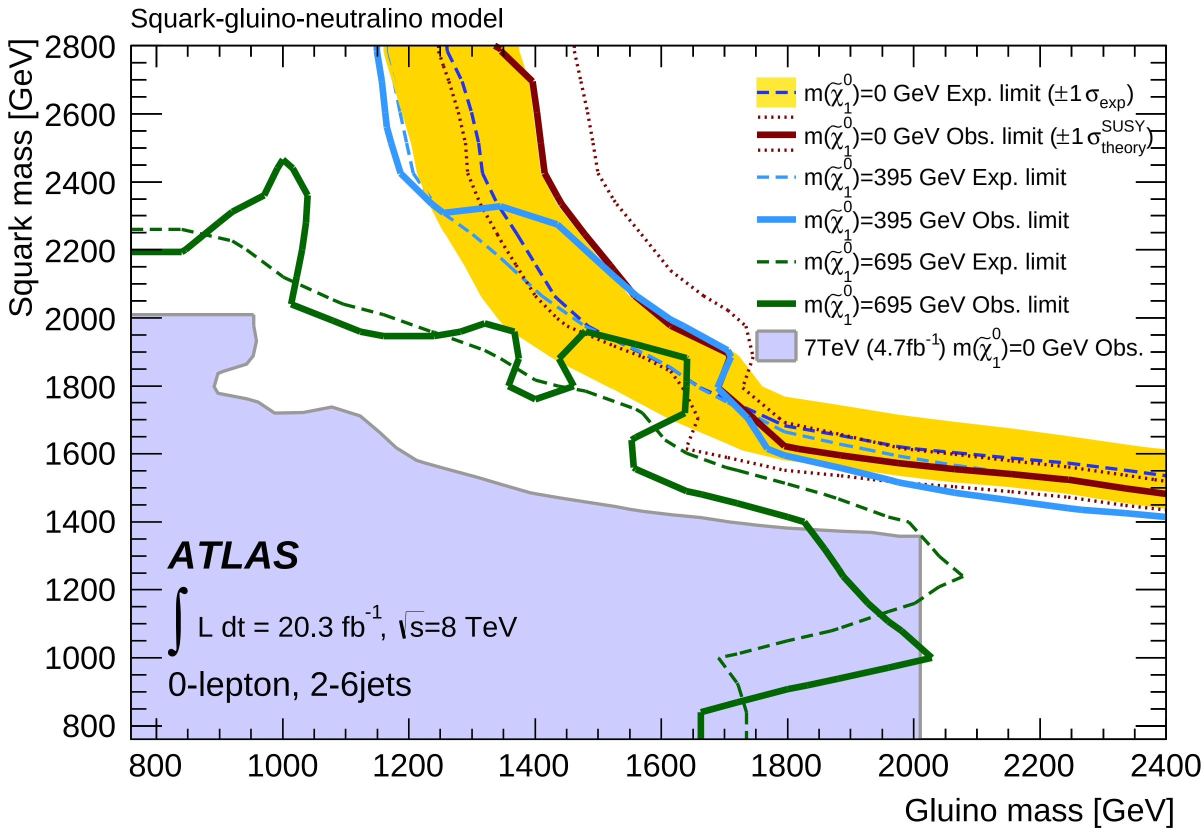


Figure 7.4: ATLAS bound on the gluino vs. first/second-generation squark mass in a simplified model.

other MSSM parameters like squark masses etc. For this reason, experimental results are often presented in terms of **simplified models**. In simplified models, typically only a single production channel and decay mode is considered and the branching ratio of the decay mode is assumed to be 100%. Then, exclusion bounds can be given as a function of a small numbers of parameters.

Exclusion limits for a simplified phenomenological MSSM scenario with only strong production of gluinos and first- and second-generation squarks (of common mass), with direct decays to quarks and lightest neutralinos. Three values of the lightest neutralino mass are considered: 0, 395 GeV and 695 GeV. Exclusion limits are obtained by using the signal region with the best expected sensitivity at each point.

7.2.1 Heavy Higgs searches

Searches for the additional Higgs states H^0, A^0 and $H^\pm$ – collectively referred to as "heavy" Higgses in the following – are different from the other sparticle searches since the Higgs bosons are even under R-parity, i.e. they can be singly produced. To understand the dominant production and decay modes, it is instructive to consider the couplings to quarks. At tree-level, one has

$$\mathcal{L} \supset \sum_{i=1}^{3} \frac{m_{u^i}}{\sqrt{2}v} \bar{u}^i u^i \left(\frac{\sin\alpha}{\sin\beta} H + \frac{1}{\tan\beta} iA \right) \tag{7.2.3}$$

$$+ \frac{m_{d^i}}{\sqrt{2}v} \bar{d}^i d^i \left(\frac{\cos\alpha}{\cos\beta} H + \tan\beta\, iA \right) + \text{h.c.}\,. \tag{7.2.4}$$

In the decoupling limit, $\cos\alpha/\cos\beta \to \tan\beta$ and $\sin\alpha/\sin\beta \to -1/\tan\beta$. We see that the couplings to down-type quarks are enhanced and the ones of up-type quarks suppressed with respect to the SM-like couplings of the light Higgs. The strongest bounds at LHC come from the process $gg \to \Phi^0 \to \tau^+\tau^-$, (with $\Phi^0 = H^0$ or A^0), where the gluons couple to the Higgses through a t or b quark loop.

The discussion of the production and subsequent decay of a heavy particle simplifies considerably if the particle is produced on-shell and its decay width (the inverse lifetime $\varGamma = 1/\tau$) is small compared to its mass. This is fulfilled to an excellent approximation for Higgs bosons. Then, one can approximate the cross section by a production cross section times a branching ratio,

$$\sigma(gg \to \Phi^0 \to \tau^+\tau^-) \approx \sigma(gg \to \Phi^0) \times \text{BR}(\Phi^0 \to \tau^+\tau^-)\,, \tag{7.2.5}$$

where $\text{BR}(i) = \Gamma_i/\Gamma$. This is called the **narrow width approximation**. Concerning the production cross section, at large $\tan\beta$ it is dominated by a b quark loop, since the Φ^0 coupling is proportional to $\tan\beta$, as discussed above. Thus, the production cross section is $\propto \tan^2\beta$. While the coupling to τ leptons is also proportional to $\tan\beta$, this does not lead to an additional enhancement, as what matters is the branching ratio. At large $\tan\beta$, the total width is dominated by the decays to $\tau^+\tau^-$ and $b\bar{b}$, both of which are $\propto \tan\beta$, so the enhancement factor cancels in the ratio $\Gamma_{\tau^+\tau^-}/\Gamma$.

The resulting bound in the m_{A^0}-$\tan\beta$ plane (recall $m_{A^0} \approx m_{H^0}$ in the decoupling limit) obtained by CMS is shown in fig. 7.5 for some choice of the other MSSM parameters (to which the bound is not very sensitive).

Further reading: section 3 of [14]

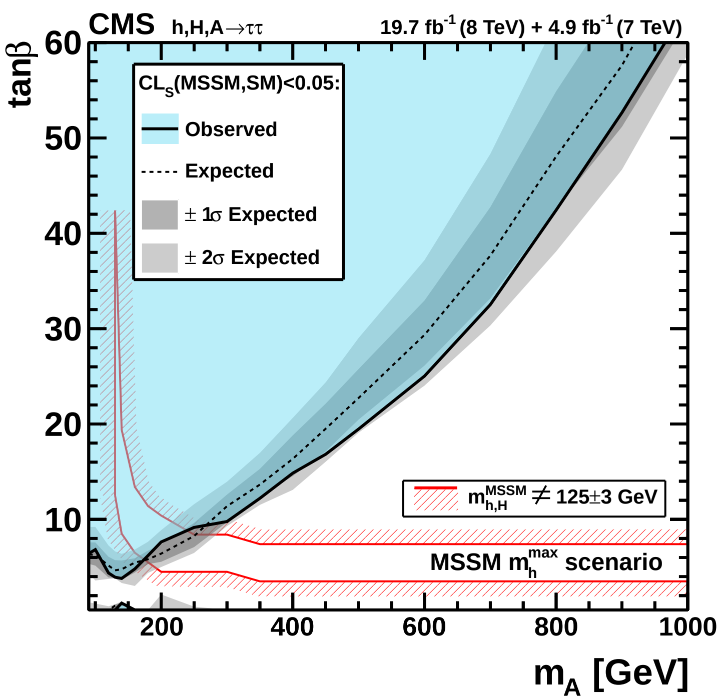


Figure 7.5: CMS bound on the mass of the neutral MSSM Higgs states as a function of $\tan\beta$.

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