Exercises week 4: problem 1:

$$
\begin{aligned}
& \left\{\begin{array}{c}
\lambda \\
\mu_{r}
\end{array}\right\}=\frac{1}{2} g^{\lambda \rho}\left\{\frac{\partial g_{v \rho}}{\partial \times r^{\prime}}+\frac{\partial g_{\mu \nu}}{\partial x_{\nu}}-\frac{\partial g_{\mu \nu}}{\partial \times \rho}\right\} \\
& \left\{\begin{array}{l}
\lambda \\
\mu_{v}
\end{array}\right\}^{\prime}=\frac{1}{2} g^{\lambda \rho^{\prime}}\{
\end{aligned}
$$

$$
=\frac{\partial g^{\prime}}{\partial x^{\prime}}
$$

$$
=\left[\frac{\partial}{\partial x^{\prime}}\left(\frac{\partial x}{\partial^{\prime}} \frac{\partial x}{\partial^{\prime}}\right) g\right] \times 3
$$

first term: $\quad g^{\lambda} J^{\prime}\left(\frac{\partial}{\partial x^{\prime \prime}} \frac{\partial x^{\prime} \partial x^{\beta}}{\partial x^{\prime} \frac{\overline{\partial x}}{} x^{\prime} \rho}\right) g_{\alpha \beta}$

$$
\begin{aligned}
g^{\prime \prime} \rho\left(\begin{array}{ll}
\frac{\partial x^{\alpha}}{\partial x^{\prime v}} \partial x^{\prime} \Gamma & \frac{\partial x^{\beta} \beta}{\partial y^{\prime} \rho}+\frac{\partial x^{\alpha}}{\partial x^{\prime \prime}}
\end{array} \frac{\frac{\partial x^{\beta} \beta}{\partial x^{\prime} \rho} \partial x^{\prime}}{}\right. \\
g_{\alpha \beta} \beta
\end{aligned}
$$

Symunetiseton' $\left(g^{\prime}\right)^{\prime} \quad\left(\frac{\partial x^{\alpha}}{\partial x^{\prime \prime} \partial \times 1} \frac{\partial x \beta}{\partial x^{\prime} \rho}\right) g_{\alpha \beta}$
problem 2:

$$
\frac{d^{2} x^{\mu}}{d \tau^{2}}+\Gamma_{\alpha \beta}^{\beta} \frac{\partial x^{\alpha} \frac{\partial x}{\partial \tau}}{\partial \tau}
$$

use: $\left(\frac{d x}{L T}\right)^{\prime}=\frac{\partial x^{\prime} \mu}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial T}$

$$
\begin{aligned}
& \frac{\partial x^{\prime} \mu}{\partial x^{\alpha}}\left(\frac{\partial^{2} x^{\alpha}}{(\partial T)^{2}}+T_{\rho \sigma}^{\alpha} \frac{\partial x^{\rho}}{\partial T} \frac{\partial x^{\sigma}}{\partial T}\right) \\
& +\frac{\partial x^{\prime \mu}}{\partial x^{\prime} \partial x^{\gamma}} \frac{\partial x^{\alpha}}{\partial T} \frac{\partial x^{\alpha}}{\partial T}+\frac{\partial x^{\prime}}{\partial x^{\rho}} \frac{\partial x^{\rho}}{\partial x^{\prime \prime} \partial x^{\prime} \beta} \frac{\partial x^{\prime \alpha}}{\partial T} \frac{\partial x^{\prime} \beta}{\partial T} \\
& =0 \text { due to } \\
& \frac{\partial^{2} x^{\prime} \mu}{\partial x^{\prime} x^{\beta}}=-\frac{\partial x^{\prime k}}{\partial x^{\alpha}} \frac{\partial x^{\prime \lambda}}{\partial x^{\beta}} \frac{\partial x^{\prime \mu}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial x^{\prime k} x^{\prime \lambda}}
\end{aligned}
$$

problem 3:

$$
\begin{aligned}
& \left(\nabla_{\mu} v^{k}\right)^{\prime}=\frac{\partial}{\partial x \mu^{\prime}}\left(\frac{\partial x^{\prime} k_{\alpha}}{\partial \alpha^{2}} v^{2}\right) \\
& +\Gamma^{\prime} k_{\mu v} V^{\prime \prime} \\
& =\frac{\partial x^{\prime k}}{\partial x^{\beta}} \frac{\partial x^{\alpha}}{\partial x^{\prime}} \mu \nabla_{\alpha} v^{\beta} \\
& +\frac{\partial x \beta^{\beta}}{\partial x^{\prime} \mu} \frac{\partial^{2} x^{\prime} k}{\partial x^{k} \partial x \beta} V^{\alpha} \\
& +\frac{\partial x^{\prime k} \frac{\partial x \rho}{\partial x \rho} \partial x^{\prime \prime \partial} x^{\prime v} \frac{\partial x^{\prime v}}{\partial x^{\sigma}} V^{\sigma}}{} \\
& =\frac{\partial x^{\prime k}}{\partial x^{\beta}} \frac{\partial x^{\alpha}}{\partial x^{\prime} \mu^{\mu}} P_{a} V^{\beta}
\end{aligned}
$$

problem 4:

$$
\begin{aligned}
& \left(\nabla_{k} V_{\alpha}\right)^{\prime}=\frac{\partial x^{\beta}}{\partial x^{*}} \frac{\partial}{\partial x^{\beta}}\left(\frac{\partial x^{\beta}}{\partial x^{\prime}}{ }^{\beta} V_{\rho}\right) \\
& -\Gamma_{k_{\alpha}}^{\prime \beta} V_{\beta}^{\prime} \\
& =\frac{\partial x^{\rho}}{\partial^{\prime}}{ }^{k} \frac{\partial x}{\partial x^{\prime}}{ }^{\alpha} \nabla_{\rho} V_{\beta} \\
& +\quad \frac{\partial x^{\beta}}{\partial x^{\prime \alpha} \partial x^{\prime k}} V_{\beta} \\
& -\frac{\partial x^{\beta}}{}{ }^{\beta} \frac{\partial \times \rho}{x^{\prime} \delta} x^{\prime \alpha} \quad V_{\beta}^{\prime} \\
& =\frac{\partial \times \rho}{\partial x^{\prime} K} \frac{\partial x^{\beta}}{\partial x^{\prime}} \alpha \nabla_{\rho} V_{\beta}
\end{aligned}
$$

problem 5:

$$
\begin{aligned}
& \nabla_{\mu} g_{\alpha \beta}=0 \\
& 0=\partial_{\mu} g_{\alpha \beta}-\Gamma_{\mu \beta}^{\beta} g_{\rho \alpha}-\Gamma_{\lambda \alpha}^{\rho} g_{\rho \beta}
\end{aligned}
$$

$$
\begin{aligned}
& =\quad \frac{\partial g_{\alpha \beta} \beta}{\partial x} \\
& =\partial_{\mu} g_{\alpha \beta}-\partial_{\mu} g_{\alpha \beta}=0
\end{aligned}
$$

