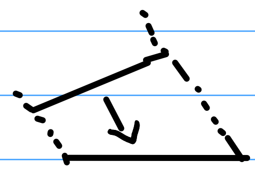
solutions exercises 1:

problem 1:

In the observer frame, the longer stick is Lorentz-contracted. Hence the two sticks can appear on top of each other (=start and end-points coincide).

The frame of the longer stick is more involved. Notice that the smaller stick moves diagonally and is Lorentz-contracted. So there is no way the two sticks can be on top of each other. However, there will be (different) times where the start and end-points of the two sticks coincide. This is achieved because in this frame the small stick actually has a tilt. (which is why the end points coincide after the start points coincide.)



problem 2:

The twin that flies away has to change frame. This breaks the symmetry between the twins. The twin at home is older when they meet, which can be calculated by using the proper time of the twins that passed.

Notice that this problem has nothing to do with GR, since the younger twin could change its direction by e.g. EM forces.

problem 3:

If the three indices $\beta\gamma\delta$ are different, the expression

$$\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta}$$

will produce 6 terms that amount to

$$2\left(\partial_\beta F_{\gamma\delta} + \partial_\delta F_{\beta\gamma} + \partial_\gamma F_{\delta\beta}\right)$$

If two of the indices are the same, the expression

$$\partial_\beta F_{\gamma\delta} + \partial_\delta F_{\beta\gamma} + \partial_\gamma F_{\delta\beta}$$

vanishes due to antisymmetry of F.

problem 4:

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

$$\Rightarrow \quad \epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = \epsilon^{\alpha\beta\gamma\delta}\left(\partial_\beta\partial_\gamma A_\delta - \partial_\beta\partial_\delta A_\gamma\right)$$
$$= 0$$

The equations

$$\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$$

amount to

$$\vec{\nabla}\vec{B} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

for arbitrary $\vec{E}(\vec{x},t)$, the second equation can be integrated.

Then $\vec{\nabla}\vec{B} = 0$ is automatically fulfilled (or only has to be imposed on the initial conditions of B)

Hence F has 3 degrees of freedom if (4) is imposed.

A_μ has 4 degrees of freedom, so $F_{\mu\nu}$ can only depend on 3 of them. The one that does not change F is the gauge degree of freedom.

problem 5:

When receiver and source approach each other, the problem is best described in the frame where the receiver is stationary.

The motion reduces the wavelength by a factor

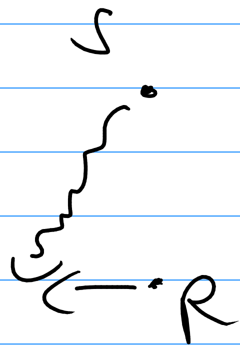
$$1-\beta$$

while the source is time dilated. The received frequency is hence

$$f_r = \frac{1}{\gamma}\,\frac{1}{1-v} = \sqrt{\frac{1+v}{1-v}}$$

When the receiver moves transverse to the source, there are actually several cases and one has to specify when exactly the signal is send/received.

Consider first the signal is send when the receiver/source are closest to each other. This is best described in the rest frame of the receiver.

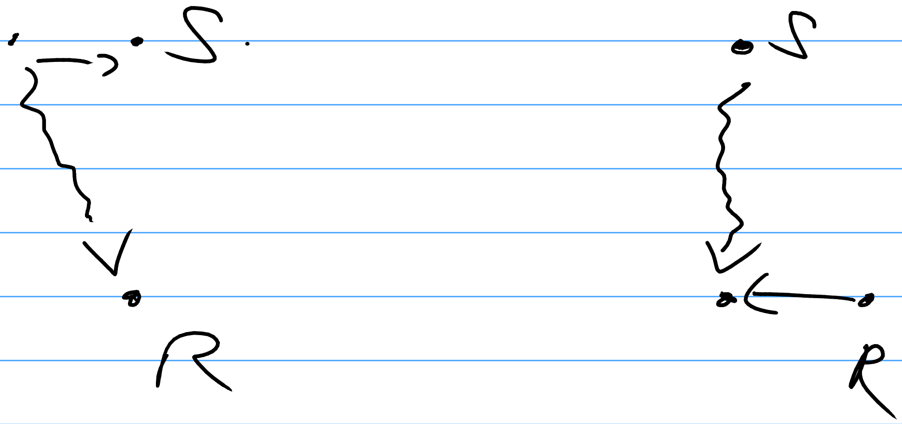


In this frame only the time dilation of the source is relevant.

$$f_R = \frac{1}{\gamma} f_S$$

In the rest frame of the source, there is a combination of time dilation of the receiver and a longitudinal Doppler effect of order v^2.
The result is the same.

In case source and receiver are closest
when the signal is received, one best describes
the problem in the rest frame of the source



The observed frequency then only
is affected by the time dilation of the receiver

$$f_R = \gamma f_S$$

The same result is obtained in the rest frame
of the receiver, where again a combination
of time dilatation and longitudinal Doppler
effect gives the same result.