Exercises, week 12

1 Chirp mass - intro

In the lecture we have shown that toward the end of the merger of two black holes (BHs) the frequency dependence is

$$\omega \propto t^{-3/8} \tag{1}$$

However, we only considered two black holes with equal mass. If the two masses of the black holes are different, m_1 and m_2, the final result depends on the so-called chirp mass

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \,. \tag{2}$$

In the following we will derive this result in several steps.

2 Chirp mass - quadrupoles

Consider two BHs with masses m_1 and m_2 on spherical orbits. How is the ratio of the radii of the orbits r_1 and r_2 related to the masses?

What is the quadrupole moment of the system?

3 Chirp mass - power

What is the power of GWs emitted by the system? What is the time-dependence of the frequency assuming that the energy loss is due to GWs?