Exercises, week 10

1 Gauge fixing in ED

There is a strong analogy how gauge fixing works in gravitational waves compared to EM waves. Using the gauge potential A

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,, \tag{1}$$

show that the equation of motion (EoM) for A_μ derived from the Maxwell equation is of the form

$$\Box\, A_\mu - \partial_\nu \partial_\mu A^\nu = S_\mu \,. \tag{2}$$

How does the EoM look in Lorenz gauge ($\partial_\mu A^\mu = 0$)?

Now use a plane wave Ansatz for A_μ

$$A_\mu = \epsilon_\mu \, \exp(ik_\lambda x^\lambda) + c.c.\,. \tag{3}$$

What does the EoM imply for the momentum k? How does a gauge transformation change the polarization vector ϵ. Show that there is a residual gauge degree of freedom when the equation of motion is enforced. How many physical degrees of freedom are there?

2 Helicity in ED

The helicity of a degree of freedom can be determined by performing a rotation around the direction of flight. A degree of freedom f has helocity h when the rotation with angle α leads to a factor

$$f(x) \to \exp(i\alpha h) f(x) \tag{4}$$

Now consider the four-vector A_μ. Assume a plane wave Ansatz and a mode with momentum $\vec{k} \parallel z$. Please construct 4 linear combinations out of the 4 degrees of freedom A_μ with definite helicity.

3 Helicity in GR

Please construct the helicity eigenstates out of the modes $h_\times$ and h_+ in TT gauge.