Exercises, week 1

1 Sticks

Imagine two sticks with (intrinsic) lengths L and l that are both aligned with the x-axis and $l < L$. The sticks fly in the x (length L) and y (length l) directions respectively. The observer is at rest and observes that at some point in time the two sticks are exactly on top of each other (same beginning and end points). How is this possible (remember $l < L$)? How would the pass-by look in the frame of the stick with length L?

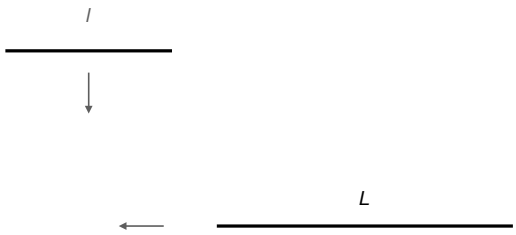

Figure 1: sketch of the setup

2 Twin paradox

Two twins separate and one of them flies in a rocket with relativistic speed ($v \sim c$) to Andromeda and back. Which of the twins is younger when they reunite? Why?

3 Bianchi identity

Show that the constraint

$$\epsilon^{\alpha\beta\gamma\delta}\frac{\partial}{\partial x^\beta}F_{\gamma\delta} = 0\,, \tag{1}$$

is equivalent to

$$\frac{\partial}{\partial x^\beta}F_{\gamma\delta} + \frac{\partial}{\partial x^\delta}F_{\beta\gamma} + \frac{\partial}{\partial x^\gamma}F_{\delta\beta} = 0\,, \tag{2}$$

in case F is an antisymmetric tensor. The significance of this relation is that only the second form will have a generalization in GR.

4 Gauge potential

Show that the Ansatz

$$F_{\gamma\delta}(x^\mu) = \frac{\partial}{\partial x^\gamma} A_\delta(x^\mu) - \frac{\partial}{\partial x^\delta} A_\gamma(x^\mu)\,, \tag{3}$$

implies

$$\epsilon^{\alpha\beta\gamma\delta} \frac{\partial}{\partial x^\beta} F_{\gamma\delta} = 0\,, \tag{4}$$

How many degrees of freedom does the (antisymmetric) tensor F encode if it fulfills (4)? How many degrees of freedom are in A_δ? Show that F is invariant under the gauge transformation

$$A_\beta(x^\mu) \to A_\beta(x^\mu) + \frac{\partial}{\partial x^\beta} \alpha(x^\mu)\,. \tag{5}$$

5 Relativistic Doppler effect

Consider a light source with frequency f_s and a receiver moving relative to each other. What is the frequency f_r observed by the receiver when

- The receiver moves towards the source
- The receiver moves away from the source
- The receiver moves transverse to the source