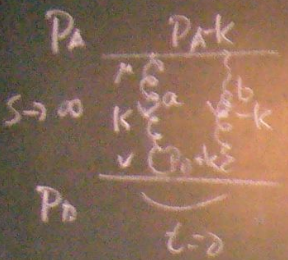
BFKL-Derivation.



- eikonal (helicity cons.)
- $g_{\mu\nu} \to \frac{2 p'^{\mu}_B p'^{\nu}_A}{s}$

$k = \alpha p'_A + \beta p'_B + k_\perp$

$-\frac{k_\perp^2}{s} < \alpha < 1$

$-\frac{k_\perp^2}{s} \quad -\beta < 1$

for β: $(p_A - k)^2 = m_A^2$

$k^2 = s\alpha\beta + k_\perp^2, \quad k_\perp^2 = -\vec{k}^2$

$(p_A - k)^2 = (1-\alpha)\left(\frac{m_A^2}{s} - \beta\right)s + k_\perp^2 + i\epsilon$

$(p_B + k)^2 = (1+\beta)\left(\frac{m_B^2}{s} + \alpha\right)s + k_\perp^2 + i\epsilon$

$k^2 = s\alpha\beta + k_\perp^2 + i\epsilon$

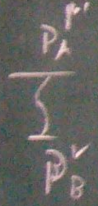
$0 < \alpha < 1$

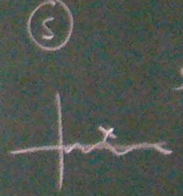
$\beta - \cancel{\frac{m_A^2}{s}} = \frac{k_\perp^2}{s} - \cancel{\frac{m_A^2}{s}}$

$\beta = \frac{k_\perp^2}{s}, \quad s\alpha\beta \ll k_\perp^2$

$T = -g^4 (2s)^2 \,\text{color}\, \frac{s}{2} \int \frac{d^2k}{(2\pi)^4} \frac{1}{s^2} \ln\frac{-s}{k_\perp^2} \cdot \frac{1}{(k_\perp^2)^2}$

$\int_0^1 \frac{d\alpha}{\alpha s + \cancel{\frac{m_B^2}{s}} s + k_\perp^2 - \cancel{m_B^2}} = \int_{-\frac{k_\perp^2}{s}}^{1} \frac{d\alpha}{\alpha s + k_\perp^2} = \frac{1}{s} \ln \frac{-s}{k_\perp^2}$

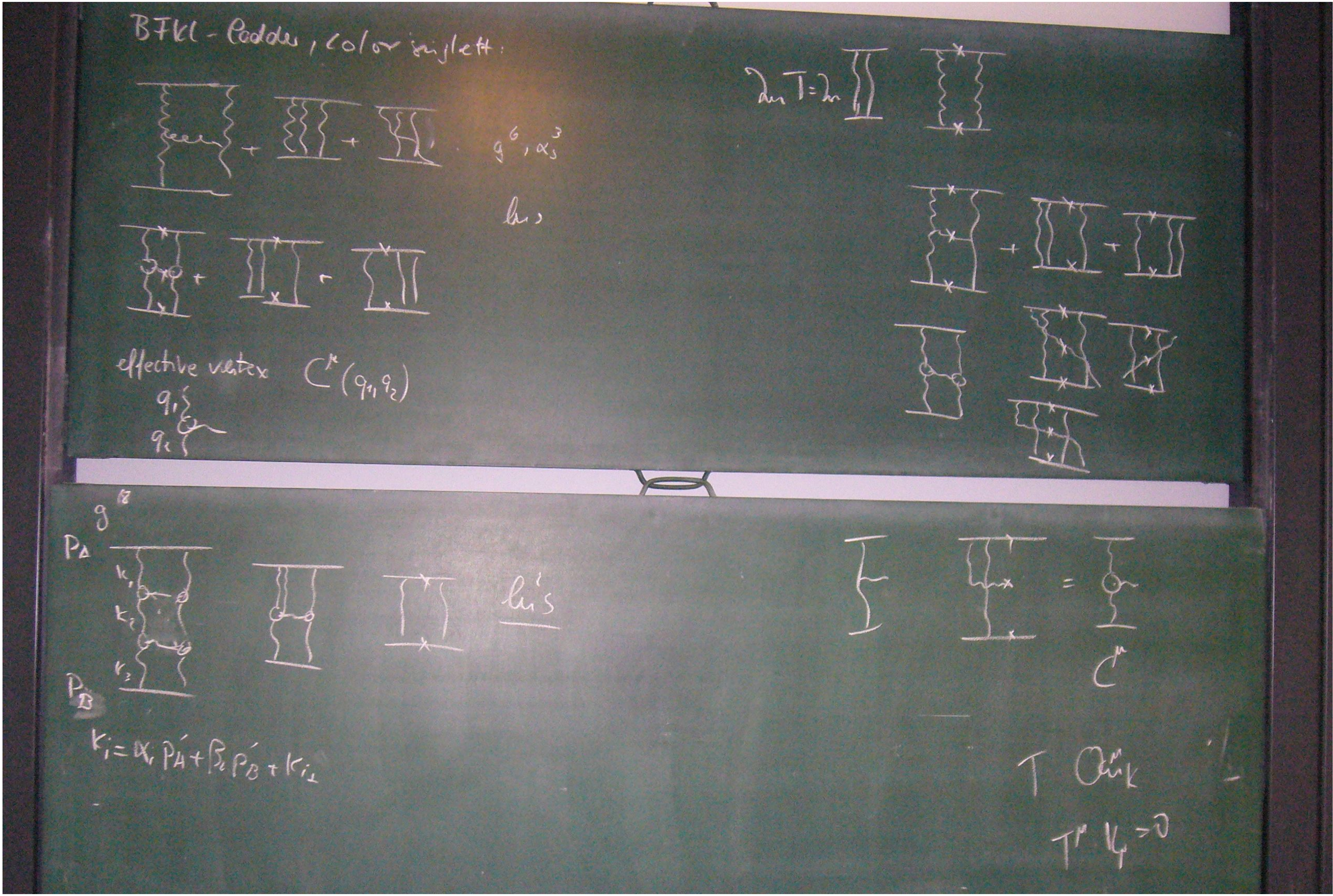




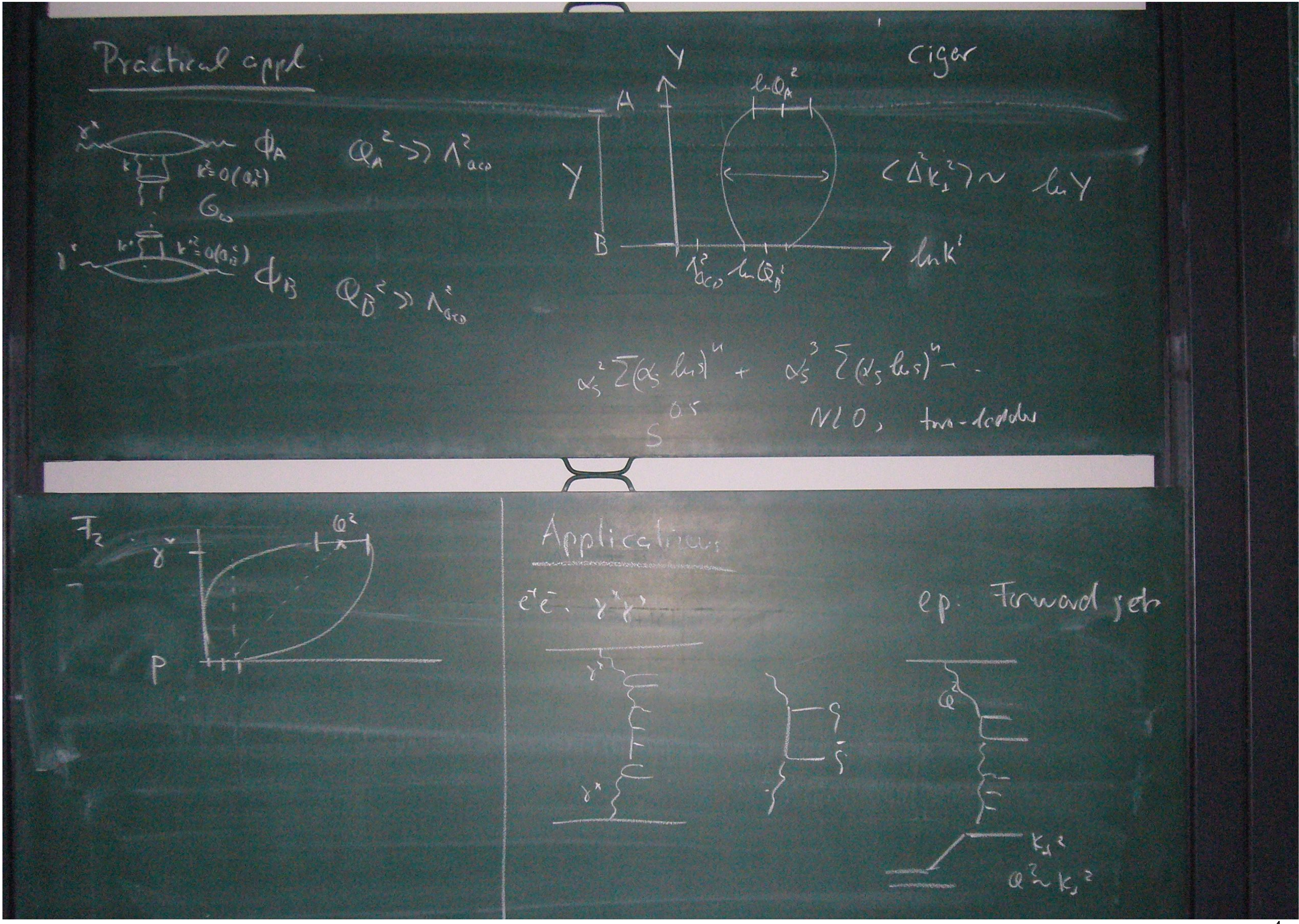
$= -s \frac{4\alpha_s^2}{\pi} \,\text{color}\, \ln\left(-\frac{s}{m_A^2}\right) \int \frac{d^2 k_\perp}{k_\perp^2 \, k_\perp^2}$

$\ln(-s) = \ln s - i\pi$

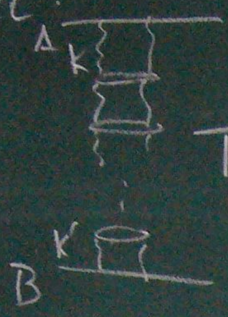
color: $(\lambda^a \lambda^b) \otimes (\lambda^a \lambda^b)$


BFKL - ladder, color singlett:
g^6, α_s^3
effective vertex C^μ(q_1, q_2)
q_1
q_2
g^18
P_A
P_B
k_1
k_2
k_3
k_i = α_i P_A' + β_i P_B' + k_i⊥
C^μ

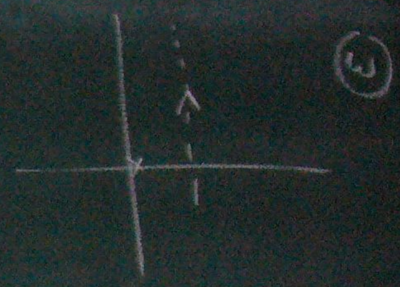

BFKL - ladder, color singlett:
g^6, α_s^3
effective vertex C^μ(q_1, q_2)
q_1
q_2
Im T = Im
g^8
P_A
P_B
k_1
k_2
k_3
k_i = α_i p'_A + β_i p'_B + k_{i⊥}
C^μ
T^μ k_μ = 0


Practical appl
ϕ_A
$Q_A^2 \gg \Lambda_{QCD}^2$
$k^2 = O(Q_A^2)$
G_ω
$k^2 = O(Q_B^2)$
ϕ_B
$Q_B^2 \gg \Lambda_{QCD}^2$
Y
A
B
$\ln Q_A^2$
Λ_{QCD}^2
$\ln Q_B^2$
$\ln k^2$
ciger
$\langle \Delta^2 k_\perp^2 \rangle \sim \ln Y$
$\alpha_s^2 \sum (\alpha_s \ln s)^n + \alpha_s^3 \sum (\alpha_s \ln s)^n + \ldots$
$s^{0.5}$
NLO, two-ladder
F_2
γ^*
Q^2
P
Applications
e^+e^-: $\gamma^*\gamma^*$
γ^*
γ^*
ep: Forward jets
Q^2
$k_\perp^2$
$Q^2 \sim k_\perp^2$

BFKL: $\left(\frac{s}{m^2}\right)^{\omega}$

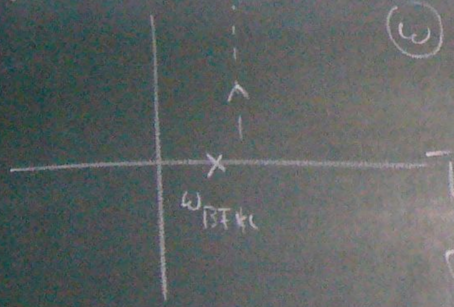


$$T_1 = is \int d^2k\, d^2k' \int \frac{d\omega}{2\pi i}\, \Phi_A(k)\, \underline{G_\omega(k,k')}\, \Phi_B(k') \left(\frac{s}{m^2}\right)^{\omega}$$



$$G_\omega = G_0 + \underline{K} \otimes G_\omega$$

$$G_\omega = \frac{1}{kk'} \sum_n \int_{-\infty}^{\infty} d\nu \left(\frac{k^2}{k'^2}\right)^{i\nu} \frac{1}{\omega - \underline{\chi(\nu,n)}\,\bar{\alpha}_s}$$



$$T \sim s^{\omega_{BFKL}+1}$$

$$\sigma_{tot} \sim s^{\omega_{BFKL}}$$