QCD and Collider Physics III: Hadronization

- fragmentation function
  - heavy quarks
  - light quarks
- hadronization models
  - independent, cluster & string models
  - color flows in proton – proton interactions

Literature:
- Andersson et al., Parton fragmentation and string dynamics, PhysRep 97 (1983) 31
- Andersson, The Lund Model
- Barger/Phillips: Collider Physics
- Dissertori, Knowles, Schmelling: QCD - High Energy Exp and Theory
- Ellis, Stirling, Webber: QCD and Collider Physics
- Field: Applications of perturbative QCD

http://www-h1.desy.de/~jung/qcd_collider_physics_wise_2006
Scaling violations of Frag. Fcts.

- Similarity with evolution of parton density functions

\[ t \frac{\partial}{\partial t} D_i(x, t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ji}(z, \alpha_s) D_j \left( \frac{x}{z}, t \right) \]

- with splitting functions:

\[ P_{ji}(x, \alpha_s) = P_{ji}^{(0)} + \frac{\alpha_s}{2\pi} P_{ji}^{(1)} \]

- lowest order splitting functions are the same as for PDF case

- higher order \( P_{gg}, P_{qg} \) are more singular than in PDFs

\[ \Rightarrow \text{resummation of small } x \text{ enhanced terms have different behavior...} \]
Scaling violations in inclusive $e^+e^-$ annihilation spectra

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The gross features of the amplitude for a fast moving heavy quark $Q$ fragmentation into a hadron $H = (Q\bar{q})$ and light quark $q$ (Fig. 3) are determined by the value of the energy transfer $\Delta E = E_H + E_q - E_Q$ in the breakup process,

$$\text{amplitude } \propto \Delta E^{-1}. \quad (2)$$

Expanding the energies about the (transverse) particle masses ($m_H \approx m_Q$ for simplicity),

$$\Delta E = (m_Q^2 + z^2 p^2)^{1/2} + (m_q^2 + (1-z)^2 p^2)^{1/2} - (m_Q^2 + p^2)^{1/2}$$

$$\propto 1 - (1/z) - (\epsilon_Q / (1-z)) \quad (3)$$

and taking a factor $z^{-1}$ for longitudinal phase space, we suggest the following ansatz for the fragmentation function of heavy quarks $Q$

$$D_Q^H(z) = \frac{N}{z[1-(1/z)-(\epsilon_Q/(1-z))^2]} \quad (4)$$

FIG. 3. The fragmentation of a heavy quark $Q$ into a meson $H(Q\bar{q})$. Dashed lines are time slices used in the derivation of Eq. (3).
Heavy Quark Fragmentation

- transition from heavy quark to observable hadron by fragmentation function FF

\[ D_Q(z) = \frac{N}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right]^{-2} \]
Heavy Quark Fragmentation

- Transition from heavy quark to observable hadron by fragmentation function $FF$


$$D_Q(z) = \frac{N}{z} \left[ 1 - \frac{1}{z} - \frac{\epsilon_Q}{1 - z} \right]^{-2}$$

- **Kartvelishvili**: (V. Kartvelishvili, A.Likhoded, V.Petrov, PLB78 (1978) 61)

$$D_Q(z) = N z^\alpha (1 - z)$$

- **Lund FF**: (B. Andersson et al, Phys.Rep 97, 31 (1983))

$$D_Q(z) = N \left( \frac{1 - z}{z} \right)^a \exp \left[ -\frac{bm^2}{z} \right]$$
Heavy Quark FF

- Results from CLEO (hep-ex/0402040) ➔ compared to Lund FF
parametrisations by: Kniehl, Kramer & Poetter NPB582 (2000) 514

use

\[ e^+ e^- \rightarrow (\gamma, Z) \rightarrow h + X \]

starting distribution:

\[ D_Q(z) = N z^\alpha (1 - z)^\beta \]

scaling violations… evolution in \( Q^2 \)
Fragmentation Models

- describe transition from quarks to hadrons
  - quarks fragment independently
  - gluon are split: \( g \rightarrow q\bar{q} \)
  - fragmentation depends on momentum (energy), but not on virtuality
  - not Lorentz invariant
  - with 4 parameters can describe broad features of 2-jet and 3-jet
  - for qq is similar to independent fragmentation
  - **BUT** is covariant and has no leftover
  - constraints on fragmentation function: \( q\bar{q} \) symmetric
  - transverse momentum distribution from tunneling effect
- **Cluster Fragmentation** (Webber NPB 238 (1984) 492)
  - pre-confinement of color
  - gluon split \( g \rightarrow q\bar{q} \)
**Fragmentation: simple example**

- process \( e^+ e^- \rightarrow q\bar{q} \)
- \( \frac{d\sigma}{d \cos \theta d\phi} = \frac{\alpha_{em}^2}{4s} \left( 1 + \cos^2 \theta \right) \)
- **BUT** what about fragmentation/ hadronization???
- use concept of local parton-hadron duality

linear confinement potential: \( V(r) \sim -1/r + \kappa r \)  
with \( \kappa \sim 1 \text{ GeV/fm} \)  
qq connected via color flux tube of transverse size of hadrons (~1 fm)  
color tube: uniform along its length \( \rightarrow \) linearly rising potential  
\[ \rightarrow \text{Lund string fragmentation} \]
in a color neutral qq-pair, a color force is created in between

color lines of the force are concentrated in a narrow tube connecting q and q, with a string tension of:

\[ \kappa \approx 1 \text{GeV/fm} \approx 0.2 \text{ GeV}^2 \]

as q and q are moving apart in qq rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)

viewed in a moving system, the string is boosted
Fragmentation in the String Model

- hadronization: iterative process
- string breaks in $q\bar{q}$ pairs (still respecting color flow)
- select transverse motion with $m=m_{qq}$ (and flavor)

$$P \sim \exp \left( -\frac{\pi m_t^2}{\chi} \right) = \exp \left( -\frac{\pi m^2}{\chi} \right) \exp \left( -\frac{\pi p_t^2}{\chi} \right)$$

- suppression of heavy quark production
  $u : d : s : c \sim 1 : 1 : 0.37 : 10^{-10}$
- actually leave it as a free parameter
- longitudinal fragmentation
  symmetric fragmentation function (from either $q$ or $\bar{q}$)
  $$f(z) \sim \frac{1}{z} (1 - z)^a \exp \left[ -\frac{bm_1^2}{z} \right]$$
  - harder spectrum for heavy quarks
- start from $q$ or $\bar{q}$
- repeat until cutoff is reached
- heavy use of random numbers and importance sampling method
Hadronization: particle mass and decays

- particle masses
  - taken from PDG, where known, otherwise from constituent masses
- particle widths
  - in hard scattering production process short lived particles ($\rho, \Delta$) have nominal mass, without mass broadening
  - in hadronization use Breit-Wigner:
    \[
    \mathcal{P}(m) dm \propto \frac{1}{(m - m_0)^2 + \Gamma^2/4}
    \]
- lifetimes
  - related to widths ... but for practical purpose separated
  - \[ P(\tau) d\tau \sim \exp(-\tau/\tau_0) d\tau \]
  - calculate new vertex position \( v' = v + \tau \frac{p}{m} \)
- decays
  - taken from PDG, where known
  - assume momentum distribution given by phase space only
  - exceptions, like \( \omega, \phi \rightarrow \pi^+ \pi^- \pi^0 \), or \( D \rightarrow K \pi, D^* \rightarrow K \pi \pi \)
  and some semileptonic decays use matrix elements
PRECONFINEMENT AS A PROPERTY OF PERTURBATIVE QCD

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The first important point to realize is that, in the axial gauge and at the leading log level we are working in all relevant graphs are planar [2]. It follows that the final quanta can be ordered, as shown in fig. 1. Furthermore, there is a natural way to group them (fig. 1) into sets C_i of adjacent partons each consisting of a quark, an antiquark and a number of gluons. These systems contain a dominant singlet component and,

Fig. 1. Planar diagrams contributing to the leading log evolution of e⁺e⁻ jets in the axial gauge. Final partons are O(Q_0^3) off-shell and naturally group into ordered colour singlets C_i.
Cluster Fragmentation

- Pre-confinement of color
- Gluon split $g \rightarrow q\bar{q}$
Gluons in string fragmentation

- process $e^+ e^- \rightarrow q\bar{q}g$
- watch out color flow !!!
- gluons act as kinks on strings
- string effect seen in experiment

![Diagram showing jet1, jet2, jet3, and the corresponding experimental data for $e^+ e^- \rightarrow q\bar{q}g$ process.](image)

TPC (PEP) H. Aihara, ZPC 28, 31 (1985)
quarks carry color
anti-quarks carry anticolor
gluons carry color – anticolor
connect to color singlet systems
watch out $pp$ or $p\bar{p}$

$pp \rightarrow b\bar{b} + X$
Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson

Process: $gg \rightarrow q\bar{q}$

Diagrams:

Amplitudes:

$t$: $-ig^2 T^b_{\alpha\gamma} T^a_{\beta\delta} \bar{u}^a(q_4) \frac{\delta^\mu}{\hat{s}} \frac{\delta^{\nu}}{\hat{t}} \frac{\delta^{\rho}}{\hat{t}} C_{\alpha\lambda\mu}(q_1, q_2, -q_1 - q_2) \nu^c(q_3)$

$u$: $-ig^2 T^b_{\alpha\gamma} T^a_{\beta\delta} \bar{u}^a(q_4) \frac{\delta^\mu}{\hat{s}} \frac{\delta^{\nu}}{\hat{t}} \frac{\delta^{\rho}}{\hat{t}} C_{\alpha\lambda\mu}(q_1, q_2, -q_1 - q_2) \nu^c(q_3)$

Colour flows:

String configurations:

Colour factors: A: $T^a_{\alpha\gamma} T^a_{\beta\delta}$; B: $T^b_{\alpha\gamma} T^b_{\beta\delta}$

Amplitudes:

$A$: $-ig^2 \bar{u}^a(q_4) \left[ \epsilon_1 \frac{\delta_1 - \delta_4}{\hat{t}} \epsilon_2 - \epsilon_1 \frac{\delta_1 \gamma^\lambda}{\hat{s}} \right] C_{\alpha\lambda\mu}(q_1, q_2, -q_1 - q_2) \nu^c(q_3)$

$B$: $-ig^2 \bar{u}^a(q_4) \left[ \epsilon_2 \frac{\delta_1 - \delta_4}{\hat{t}} \epsilon_1 + \epsilon_2 \frac{\delta_1 \gamma^\lambda}{\hat{s}} \right] C_{\alpha\lambda\mu}(q_1, q_2, -q_1 - q_2) \nu^c(q_3)$

Cross-sections:

$A$: $\frac{\alpha_s^2}{\hat{s}} \frac{1}{4} \left( \frac{\hat{u}}{\hat{t}} - 2 \frac{\hat{u}^2}{\hat{s}^2} \right)$; $B$: $\frac{\alpha_s^2}{\hat{s}} \frac{1}{4} \left( \frac{\hat{u}}{\hat{t}} - 2 \frac{\hat{u}^2}{\hat{s}^2} \right)$
Color Flow in pp

Process: $gg \rightarrow gg$

Diagrams:

Amplitudes:

\[ s: \quad -ig^2 \frac{1}{3} f^{abc} f^{cde} \varepsilon_{1}^{\lambda} \varepsilon_{2}^{\mu} \varepsilon_{3}^{\nu} C_{\lambda \mu \tau} (-q_1 - q_2, q_1 + q_2) C_{\nu \tau \sigma} (-q_3, q_4, -q_3 - q_4) \]

\[ t: \quad -ig^2 \frac{1}{3} f^{abc} f^{bde} \varepsilon_{1}^{\lambda} \varepsilon_{2}^{\mu} \varepsilon_{3}^{\nu} C_{\lambda \mu \tau} (q_4 - q_3, q_1 - q_4) C_{\nu \tau \sigma} (-q_2, q_3, q_2, q_3) \]

\[ u: \quad -ig^2 \frac{1}{3} f^{abc} f^{bde} \varepsilon_{1}^{\lambda} \varepsilon_{2}^{\mu} \varepsilon_{3}^{\nu} C_{\lambda \mu \tau} (q_3, -q_1, q_1 - q_3) C_{\nu \tau \sigma} (-q_2, q_4, q_2 - q_4) \]

\[ 4: \quad -ig^2 f^{abc} f^{cde} (g_{\kappa \mu} g_{\lambda \nu} - g_{\kappa \nu} g_{\lambda \mu}) + ig^2 f^{abc} f^{bde} (g_{\kappa \lambda} g_{\mu \nu} - g_{\kappa \nu} g_{\lambda \mu}) \]

Colour flows:
**Color Flow in**  \( p\bar{p} \rightarrow b\bar{b} + X \)

\[ p\bar{p} \rightarrow b\bar{b} + X \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure9_45.png}
\caption{Example of a string configuration in a \( p\bar{p} \) collision. (a) Graph of the process, with brackets denoting the final color singlet subsystems. (b) Corresponding momentum space picture, with dashed lines denoting the strings.}
\end{figure}
Due to color connection of produced b-quark with beam remnants, the rapidity distribution of b-quarks and B-hadrons is different.

Asymmetry of $\overline{B}^0 B^0$

**Figure 9.47:** Bottom production at the Tevatron. (a) Rapidity distribution of bottom quarks (full) and the B hadrons produced from them (dashed). (b) The asymmetry $A = \frac{\sigma(B^0) - \sigma(\overline{B}^0)}{\sigma(B^0) + \sigma(\overline{B}^0)}$ as a function of rapidity. For simplicity, only pair production is included.

HowTo connect this to factorised fragmentation functions?
Summary

- Fragmentation functions from longitudinal phase space model
- different behavior for light and heavy quarks
- Hadronization models (iterative procedure)
- **NEEDED** if more than single inclusive quantities are investigated
  - different models available
    - independent fragmentation (do not use anymore !!!)
    - LUND string fragmentation (PYTHIA / JETSET)
    - cluster fragmentation (HERWIG)
      - dedicated effects observable:
        - string effect
        - beam drag effect
- Important to respect all color informations !!!!!