QCD and Collider Physics III: Hadronization

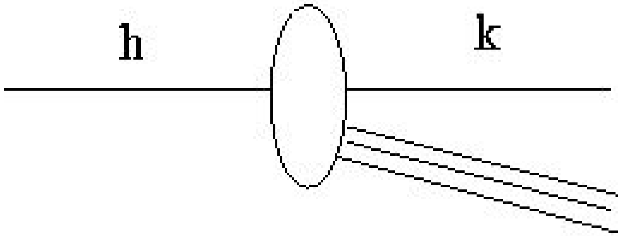
- fragmentation function
  - heavy quarks
  - light quarks
- hadronization models
  - independent, cluster & string models
  - color flows in proton – proton interactions

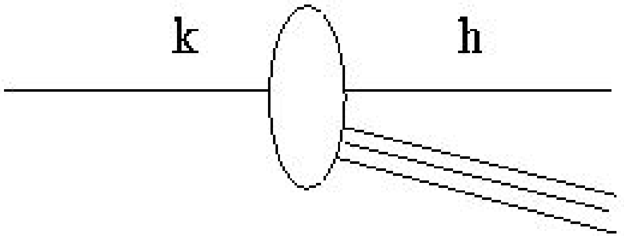
- Literature:

  Andersson et al., Parton fragmentation and string dynamics, PhysRep 97 (1983) 31
  Andersson, The Lund Model
  Barger/Phillips: *Collider Physics*
  Dissertori, Knowles, Schmelling: *QCD - High Energy Exp and Theory*
  Ellis, Stirling, Webber: *QCD and Collider Physics*
  Field: *Applications of perturbative QCD*

http://www-h1.desy.de/~jung/qcd_collider_physics_wise_2006

Scaling violations of Frag. Fcts.


partons in hadrons


hadrons in jets (partons)

- Similarity with evolution of parton density functions

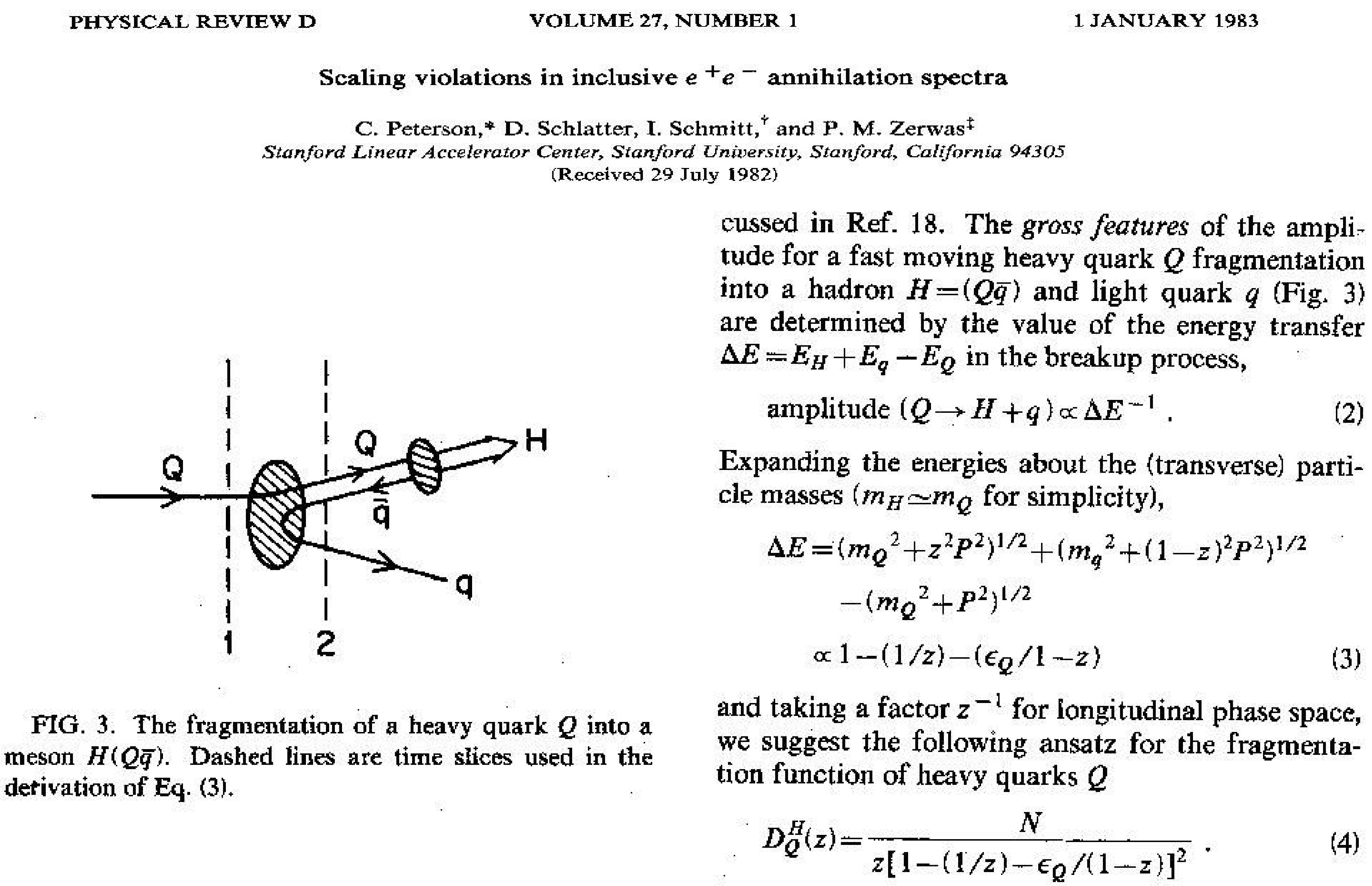
$$t\frac{\partial}{\partial t}D_i(x,t) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi}P_{ji}(z,\alpha_s)D_j\left(\frac{x}{z},t\right)$$

- with splitting functions: $P_{ji}(x,\alpha_s) = P_{ji}^{(0)} + \frac{\alpha_s}{2\pi}P_{ji}^{(1)}$

- lowest order splitting functions are the same as for PDF case
- higher order $P_{gg},\ P_{qg}$ are more singular than in PDFs

➔ resummation of small x enhanced terms have different behavior...

Peterson fragmentation



PHYSICAL REVIEW D VOLUME 27, NUMBER 1 1 JANUARY 1983

Scaling violations in inclusive e^+e^- annihilation spectra

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(Received 29 July 1982)

FIG. 3. The fragmentation of a heavy quark Q into a meson $H(Q\bar{q})$. Dashed lines are time slices used in the derivation of Eq. (3).

cussed in Ref. 18. The *gross features* of the amplitude for a fast moving heavy quark Q fragmentation into a hadron $H=(Q\bar{q})$ and light quark q (Fig. 3) are determined by the value of the energy transfer $\Delta E=E_H+E_q-E_Q$ in the breakup process,

$$\text{amplitude } (Q\to H+q)\propto \Delta E^{-1} . \qquad (2)$$

Expanding the energies about the (transverse) particle masses ($m_H\simeq m_Q$ for simplicity),

$$\begin{aligned}\Delta E&=(m_Q{}^2+z^2P^2)^{1/2}+(m_q{}^2+(1-z)^2P^2)^{1/2}\\&\quad-(m_Q{}^2+P^2)^{1/2}\\&\propto 1-(1/z)-(\epsilon_Q/1-z)\end{aligned} \qquad (3)$$

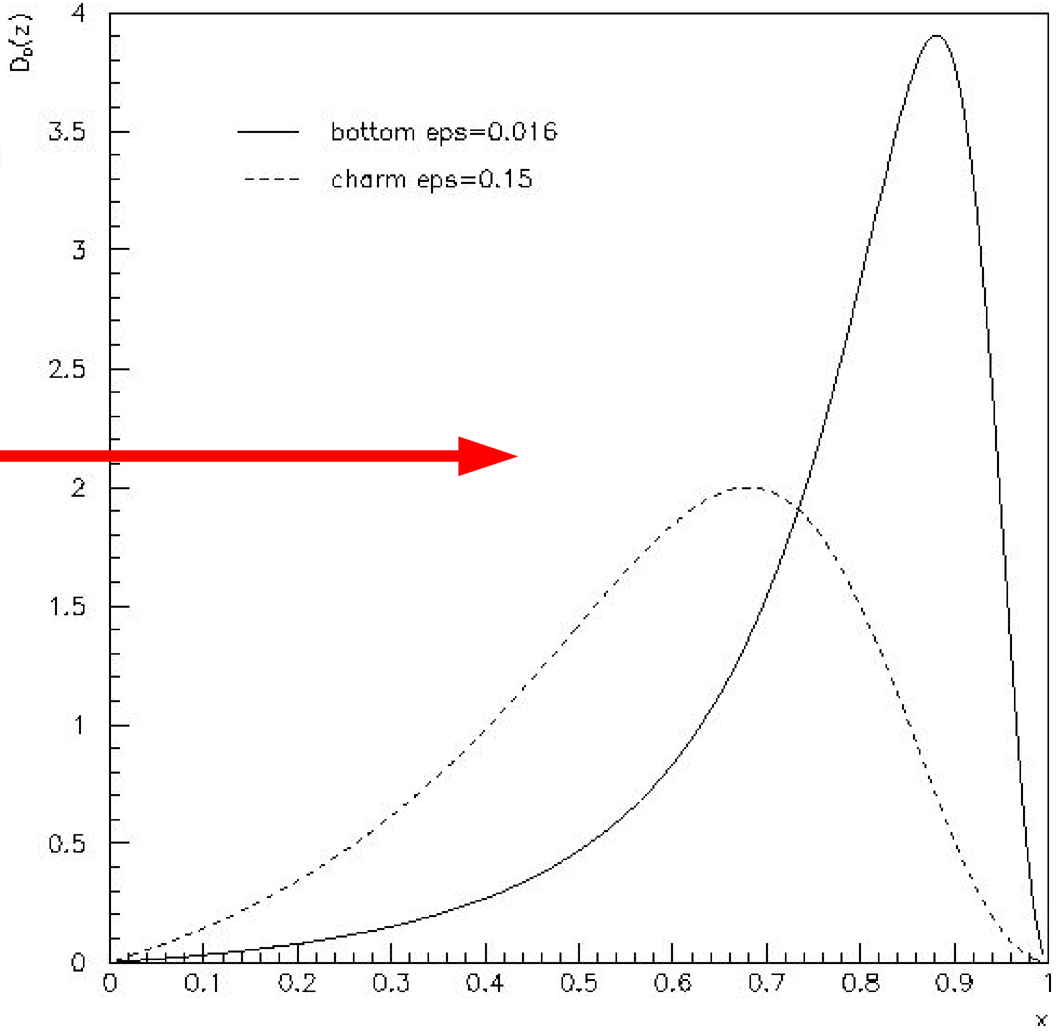
and taking a factor z^{-1} for longitudinal phase space, we suggest the following ansatz for the fragmentation function of heavy quarks Q

$$D_Q^H(z)=\frac{N}{z[1-(1/z)-\epsilon_Q/(1-z)]^2} . \qquad (4)$$

Heavy Quark Fragmentation

- transition from heavy quark to observable hadron by fragmentation function FF
- Peterson FF: (C. Peterson, D.Schlatter,I.Schmitt,P.Zerwas, PRD27 (1983) 105)

$$D_Q(z) = \frac{N}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}$$



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$$D_Q(z) = \frac{N}{z}\left[1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right]^{-2}$$

- Kartvelishvili (V. Kartvelishvili,A.Likhoded,V.Petrov PLB78 (1978) 61

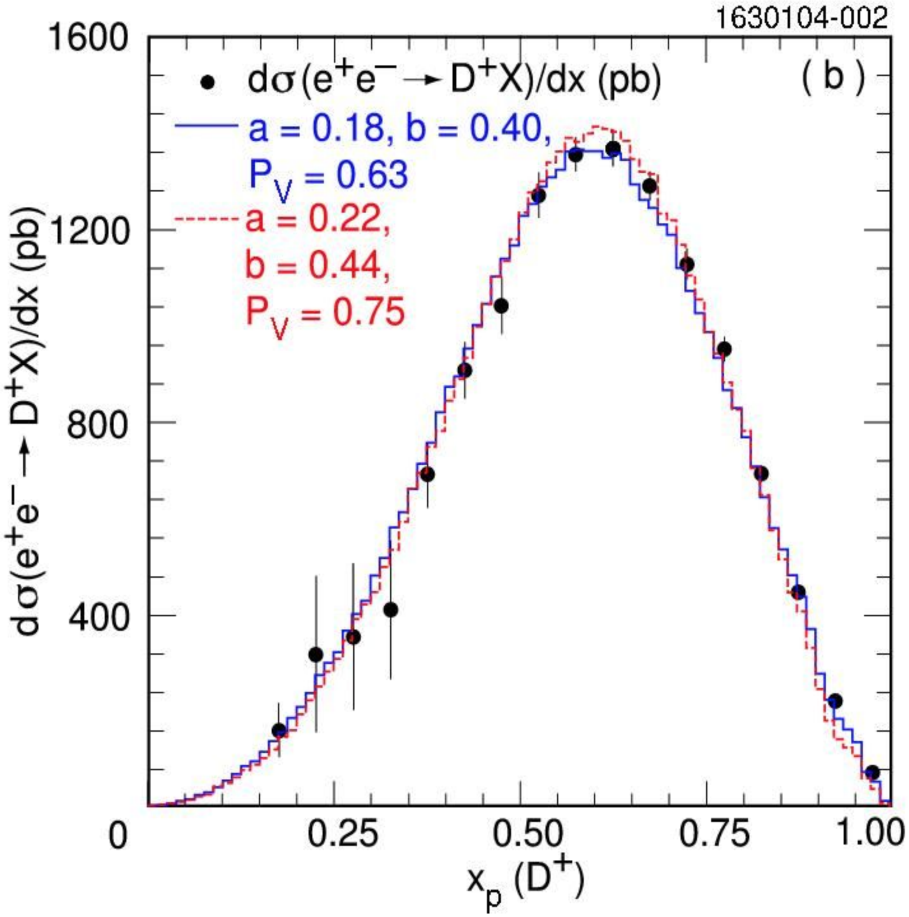
$$D_Q(z) = N z^{\alpha}(1-z)$$

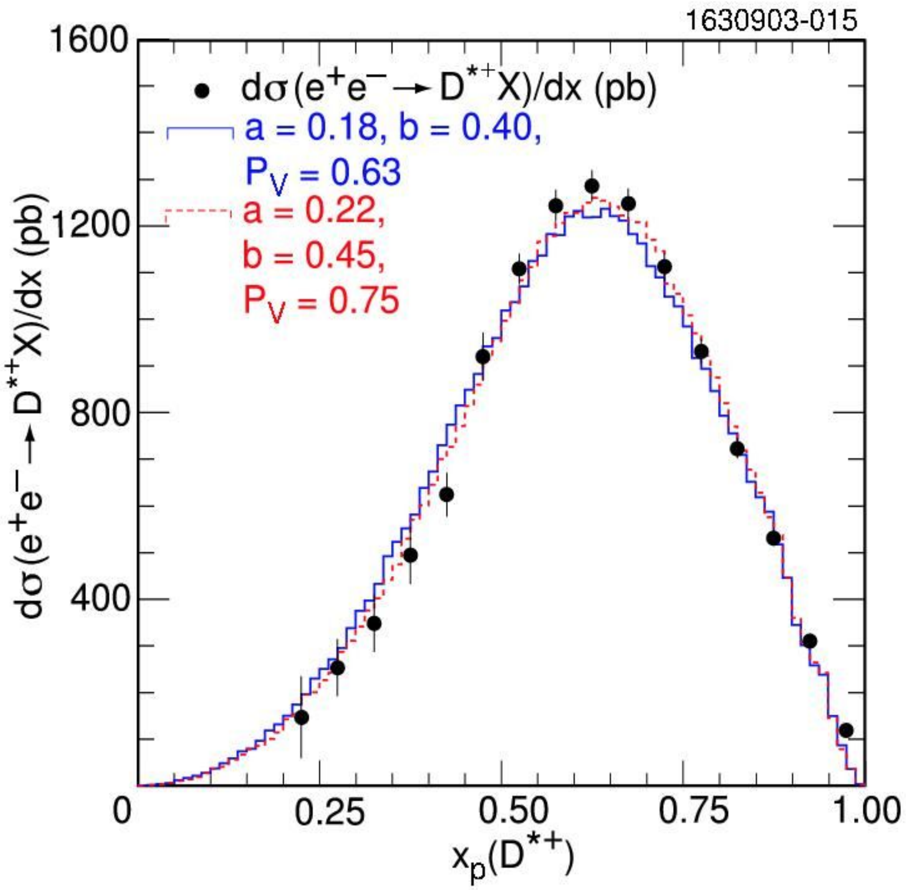
- Lund FF (B. Andersson et al, Phys.Rep 97, 31 (1983))

$$D_Q(z) = N\frac{(1-z)^a}{z}\exp\left[\frac{-bm_\perp^2}{z}\right]$$

Heavy Quark FF

- Results from CLEO (hep-ex/0402040) ➔ compared to Lund FF





Light Quark FF

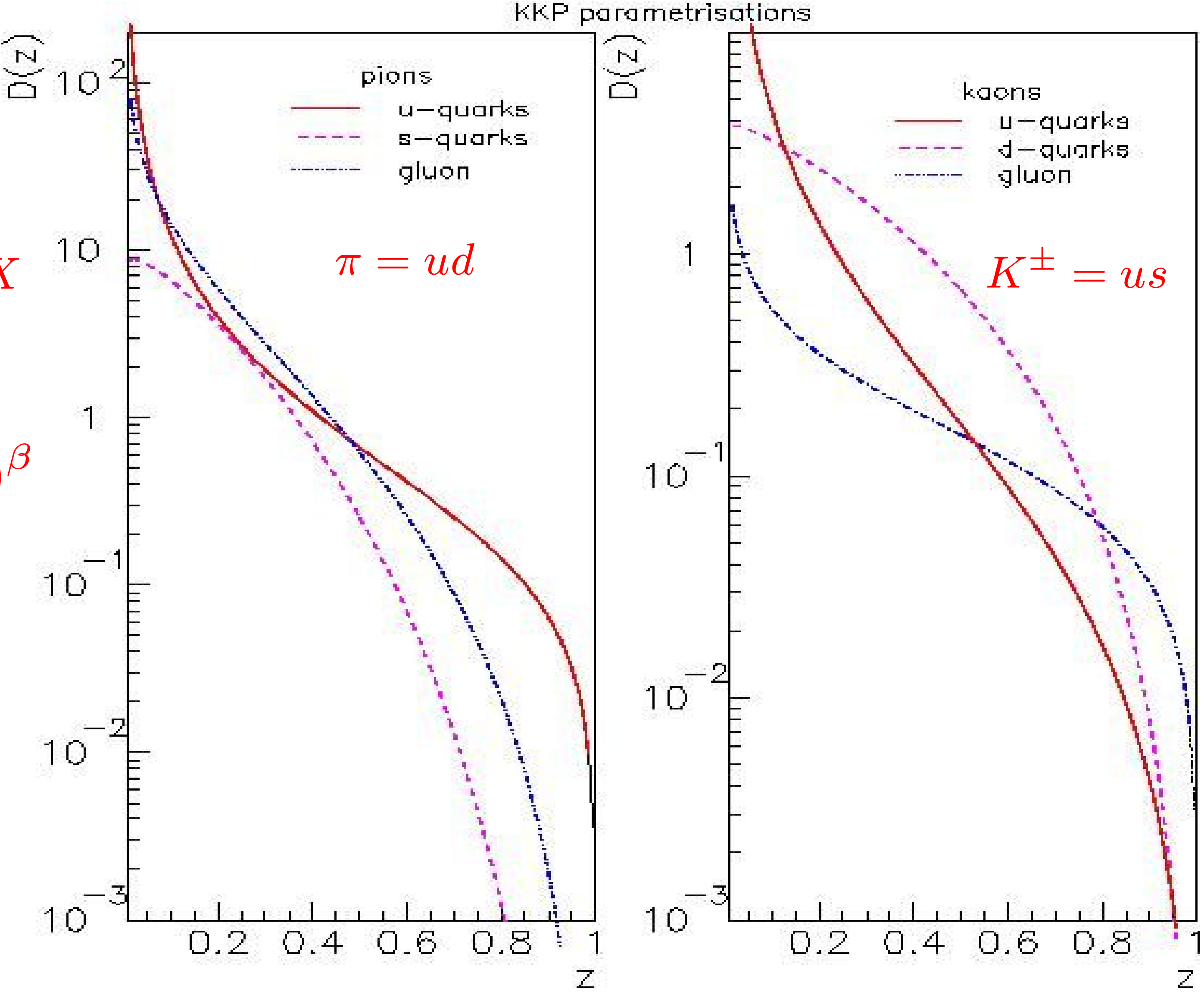
- parametrisations by:
  Kniehl,Kramer&Poetter NPB582 (2000) 514
- use

  $$e^+e^- \to (\gamma, Z) \to h + X$$

- starting distribution:

  $$D_Q(z) = N z^\alpha (1-z)^\beta$$

- scaling violations... evolution in Q^2

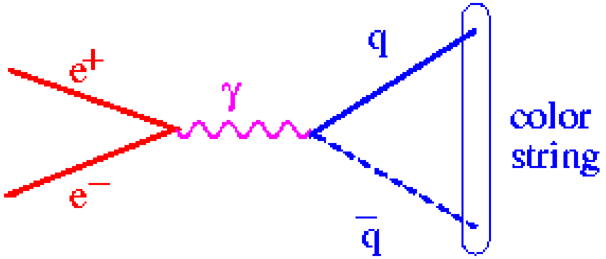


Fragmentation Models

- describe transition from quarks to hadrons
- Independent Fragmentation (Feynman & Field: Phys. Rev D15 (1977)2590, NPB 138 (1978) 1)
  - quarks fragment independently
  - gluon are split: $g \to q\bar{q}$
  - fragmentation depends on momentum (energy), but not on virtuality
  - not Lorentz invariant
  - with 4 parameters can describe broad features of 2-jet and 3-jet
- Lund String Fragmentation (Andersson, Gustafson, Ingelman, Sjostrand Phys. Rep 97 (1983) 33)
  - for qq is similar to independent fragmentation
    - ➔ **BUT** is covariant and has no leftover
    - ➔ constraints on fragmentation function: $q\bar{q}$ symmetric
    - ➔ transverse momentum distribution from tunneling effect
- Cluster Fragmentation (Webber NPB 238 (1984) 492)
  - pre-confinement of color
  - gluon split $g \to q\bar{q}$

Fragmentation: simple example

- process $e^+e^- \to q\bar{q}$
- $$\frac{d\sigma}{d\cos\theta d\phi} = \frac{\alpha_{em}^2}{4s}\left(1+\cos^2\theta\right)$$



- BUT what about fragmentation/ hadronization ???
- use concept of local parton-hadron duality

linear confinement potential: $V(r) \sim -1/r + \kappa r$

with $\kappa \sim 1$ GeV/fm

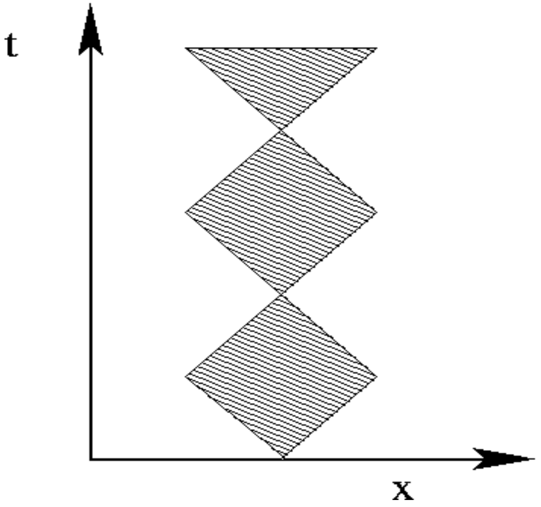
qq connected via color flux tube of transverse size of hadrons (~1 fm)

color tube: uniform along its length → linearly rising potential

→ Lund string fragmentation

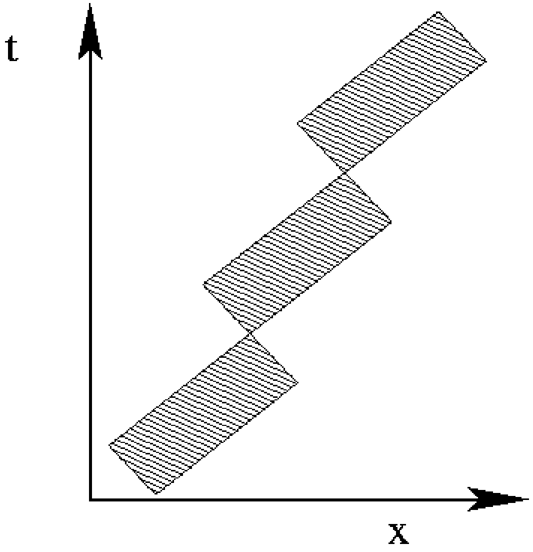
Lund string fragmentation

- in a color neutral qq-pair, a color force is created in between
- color lines of the force are concentrated in a narrow tube connecting q and q, with a string tension of:
  $\kappa \sim 1 GeV/fm \sim 0.2\ GeV^2$



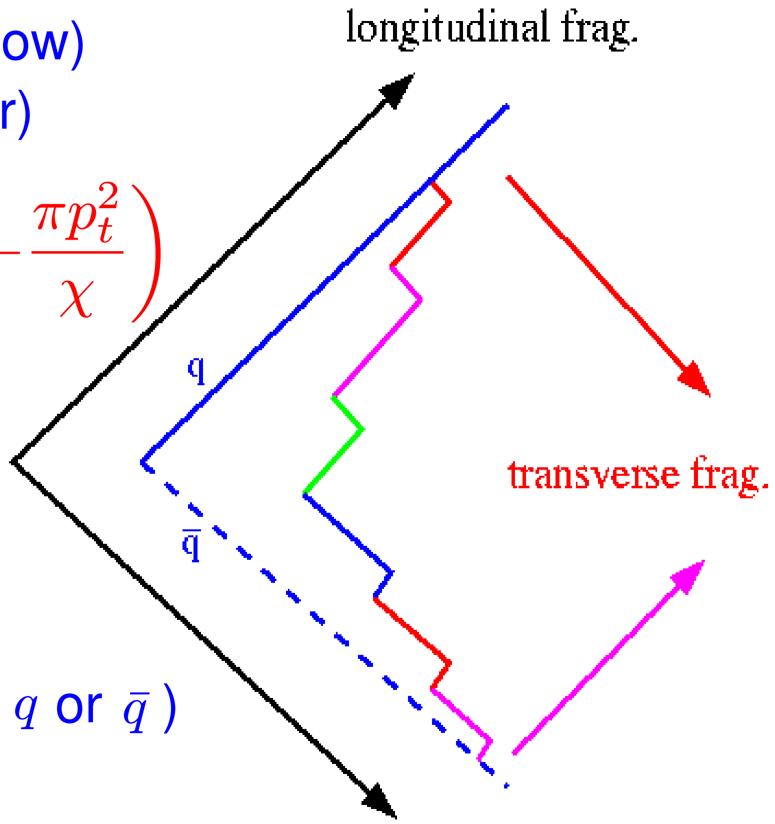
- as q and q are moving apart in qq rest frame, they are de-accelerated by string tension, accelerated back etc ... (periodic oscillation)

- viewed in a moving system, the string is boosted



Fragmentation in the String Model

- hadronization: iterative process
- string breaks in $q\bar{q}$ pairs (still respecting color flow)
- select transverse motion with $m = m_{qq}$ (and flavor)



$$P \sim \exp\left(-\frac{\pi m_t^2}{\chi}\right) = \exp\left(-\frac{\pi m^2}{\chi}\right) \exp\left(-\frac{\pi p_t^2}{\chi}\right)$$

- suppression of heavy quark production
  - $u : d : s : c \sim 1 : 1 : 0.37 : 10^{-10}$
  - actually leave it as a free parameter
- longitudinal fragmentation
  - symmetric fragmentation function (from either q or $\bar{q}$)

$$f(z) \sim \frac{1}{z}(1-z)^a \exp\left[-\frac{b m_\perp^2}{z}\right]$$

  - harder spectrum for heavy quarks
- start from q or q
- repeat until cutoff is reached
- heavy use of random numbers and importance sampling method

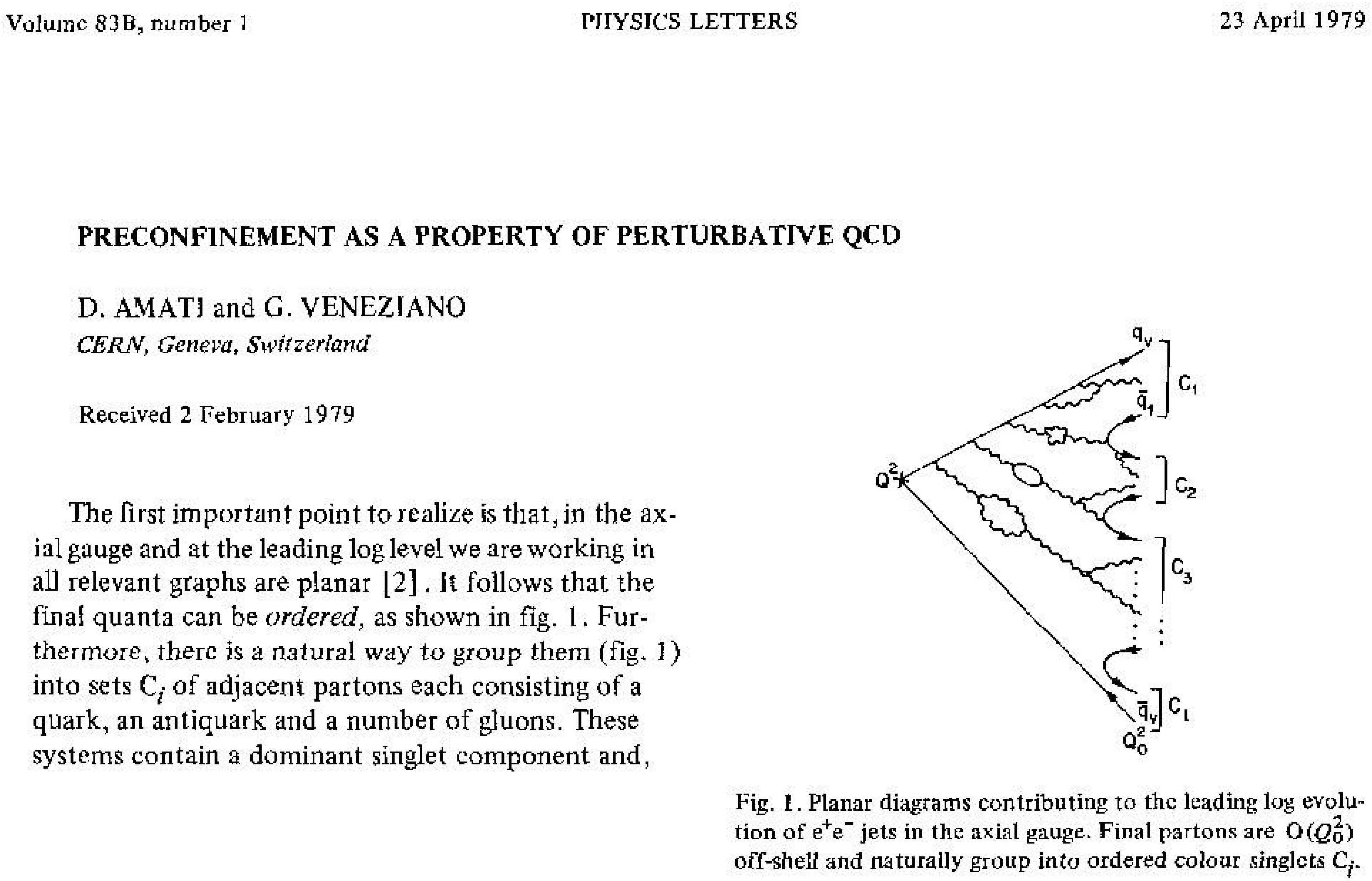
Hadronization: particle mass and decays

- particle masses
  - ➔ taken from PDG, where known, otherwise from constituent masses
- particle widths
  - ➔ in hard scattering production process short lived particles (ρ,Δ) have nominal mass, without mass broadening
  - ➔ in hadronization use Breit-Wigner:

$$\mathcal{P}(m)dm \propto \frac{1}{(m-m_0)^2+\Gamma^2/4}$$

- lifetimes
  - ➔ related to widths ... but for practical purpose separated
  - ➔ $P(\tau)d\tau \sim exp\,(-\tau/\tau_0)\,d\tau$
  - ➔ calculate new vertex position $v' = v + \tau\, p/m$
- decays
  - ➔ taken from PDG, where known
  - ➔ assume momentum distribution given by phase space only
  - ➔ exceptions, like $\omega, \phi \to \pi^+\pi^-\pi^0$, or $D \to K\pi,\ D^* \to K\pi\pi$ and some semileptonic decays use matrix elements

Cluster Fragmentation



Volume 83B, number 1 PHYSICS LETTERS 23 April 1979

PRECONFINEMENT AS A PROPERTY OF PERTURBATIVE QCD

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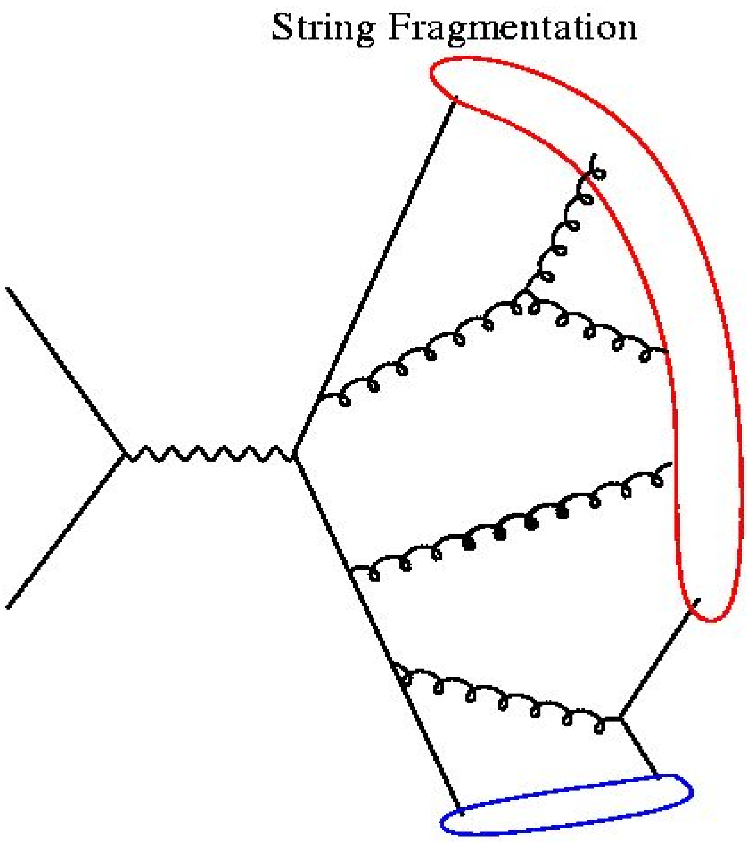
Received 2 February 1979

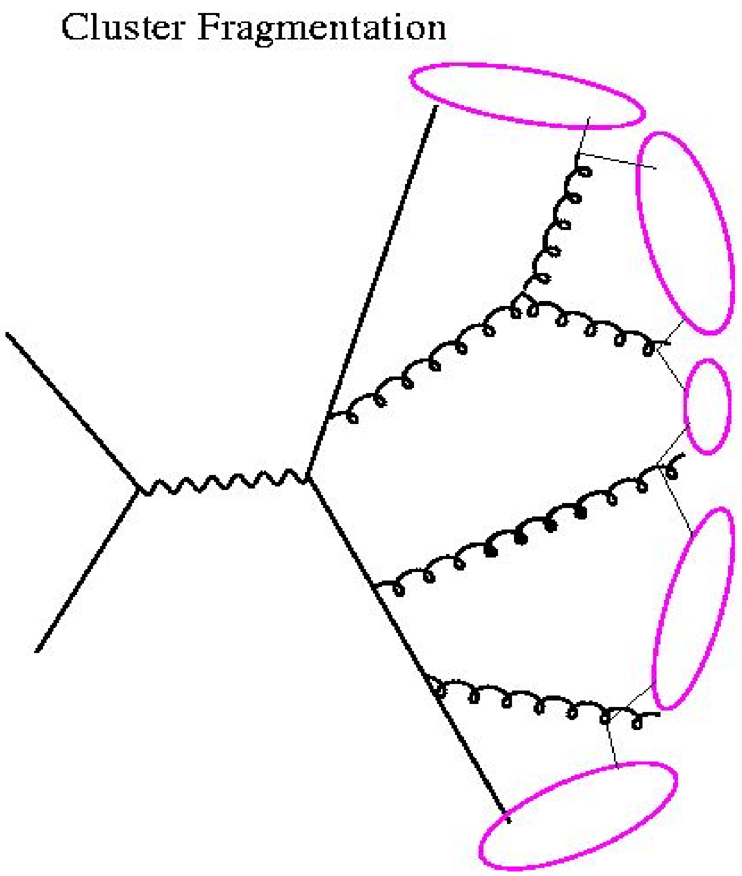
The first important point to realize is that, in the axial gauge and at the leading log level we are working in all relevant graphs are planar [2]. It follows that the final quanta can be *ordered*, as shown in fig. 1. Furthermore, there is a natural way to group them (fig. 1) into sets C_i of adjacent partons each consisting of a quark, an antiquark and a number of gluons. These systems contain a dominant singlet component and,

Fig. 1. Planar diagrams contributing to the leading log evolution of e^+e^- jets in the axial gauge. Final partons are $O(Q_0^2)$ off-shell and naturally group into ordered colour singlets C_i.

Cluster Fragmentation

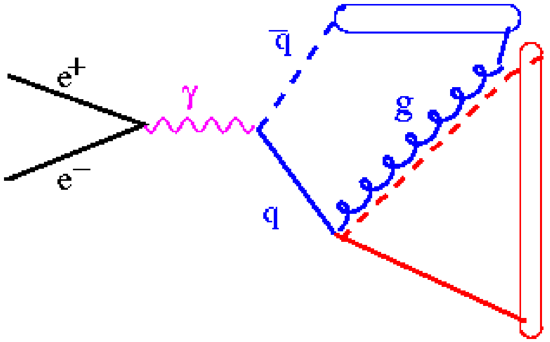
- Pre-confinement of color
- Gluon split $g \to q\bar{q}$

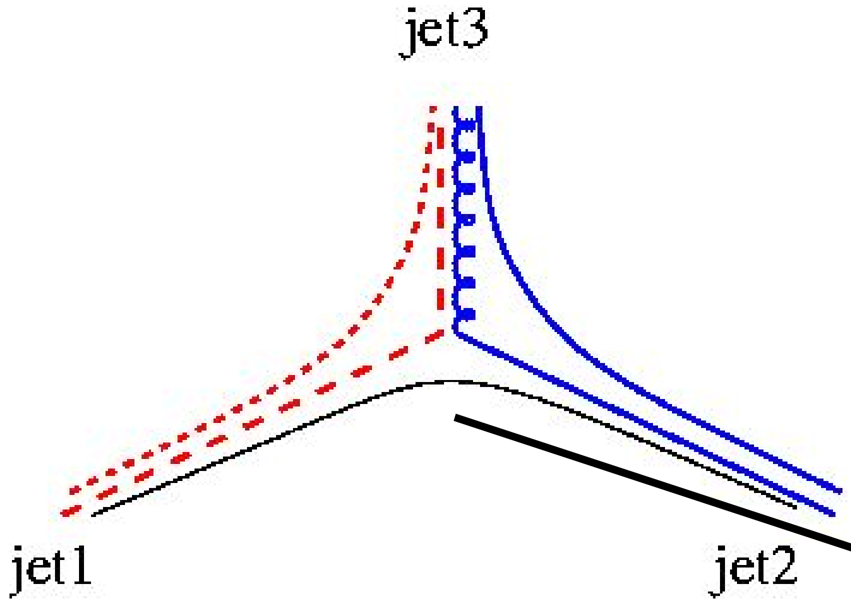




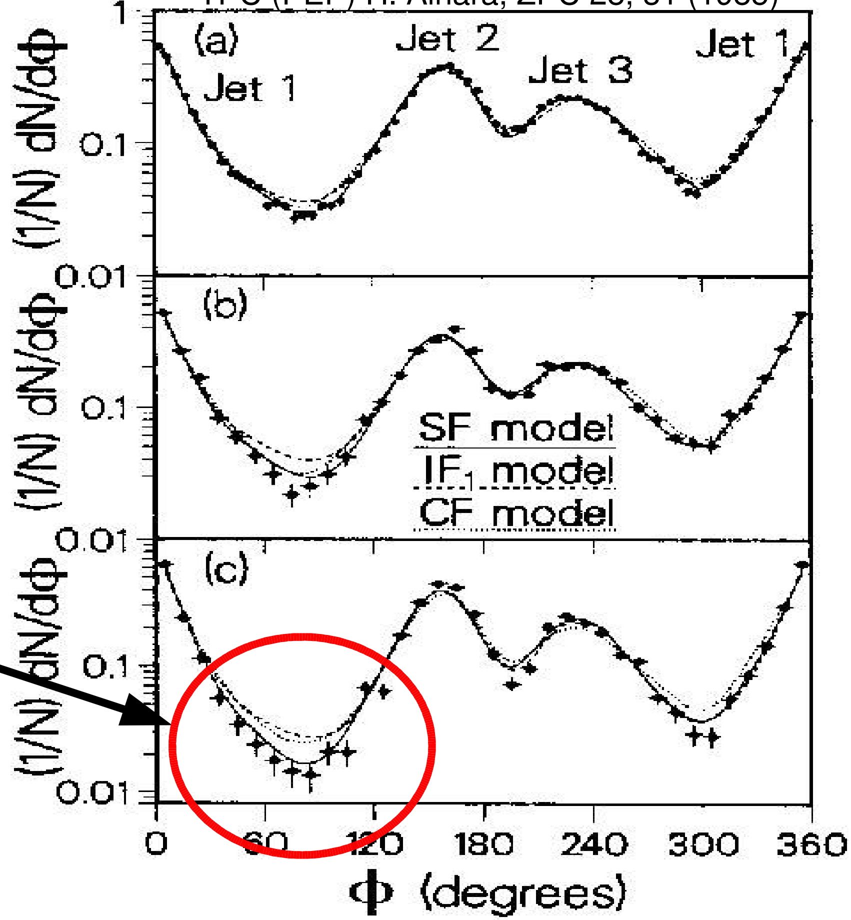
Gluons in string fragmentation

- process $e^+e^- \to q\bar{q}g$
- watch out color flow !!!
- gluons act as kinks on strings
- string effect seen in experiment





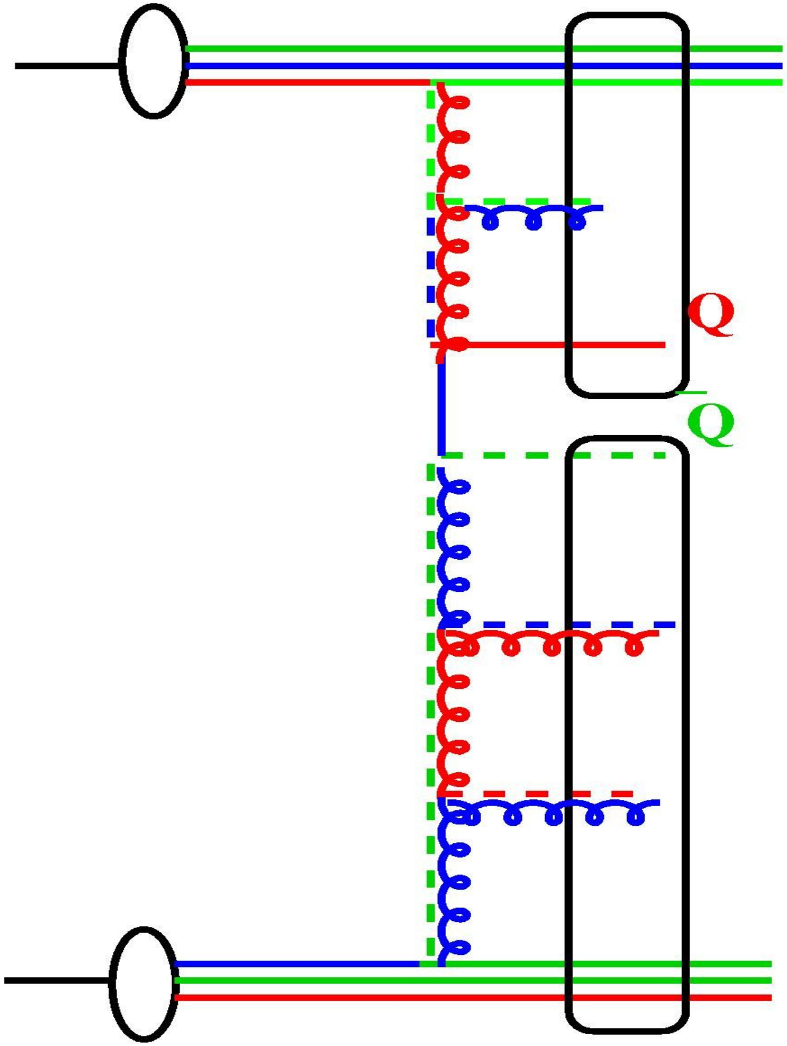
TPC (PEP) H. Aihara, ZPC 28, 31 (1985)



Color Flow in String Fragmentation

- quarks carry color
- anti-quarks carry anticolor
- gluons carry color – anticolor
  - connect to color singlet systems
  - watch out pp or $p\bar{p}$

$pp \to b\bar{b} + X$

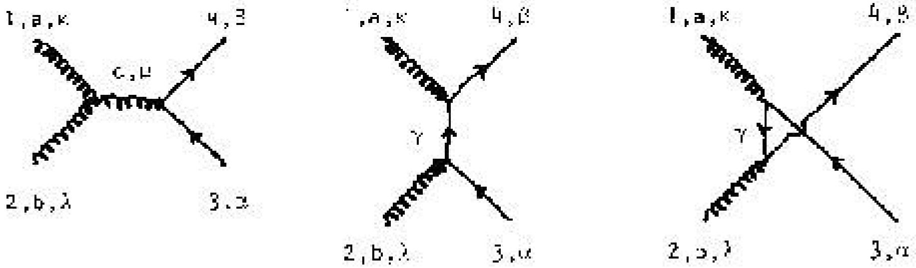


Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson
Comput.Phys.Commun.31:323,1984.

Process: $gg \to q_i\bar{q}_i$

Diagrams:



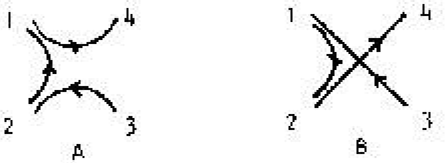
Amplitudes:

s: $g^2 f^{abc} T^c_{\alpha\beta} \bar{u}^\beta_i(q_4) \frac{\epsilon_1^\kappa \epsilon_2^\lambda \gamma^\mu}{\hat{s}} C_{\kappa\lambda\mu}(q_1, q_2, -q_1-q_2) v^\alpha_i(q_3)$

t: $-ig^2 T^b_{\alpha\gamma} T^a_{\gamma\beta} \bar{u}^\beta_i(q_4) \not\epsilon_1 \frac{\not q_1 - \not q_4}{\hat{t}} \not\epsilon_2 v^\alpha_i(q_3)$

u: $-ig^2 T^a_{\alpha\gamma} T^b_{\gamma\beta} \bar{u}^\beta_i(q_4) \not\epsilon_2 \frac{\not q_1 - \not q_3}{\hat{u}} \not\epsilon_1 v^\alpha_i(q_3)$

Colour flows:



String configurations:

Colour factors: A: $T^b_{\alpha\gamma} T^a_{\gamma\beta}$; B: $T^a_{\alpha\gamma} T^b_{\gamma\beta}$

Amplitudes:

A: $-ig^2 \bar{u}^\beta_i(q_4) \left[\not\epsilon_1 \frac{\not q_1 - \not q_4}{\hat{t}} \not\epsilon_2 - \frac{\epsilon_1^\kappa \epsilon_2^\lambda \gamma^\mu}{\hat{s}} C_{\kappa\lambda\mu}(q_1, q_2, -q_1-q_2) \right] v^\alpha_i(q_3)$

B: $-ig^2 \bar{u}^\beta_i(q_4) \left[\not\epsilon_2 \frac{\not q_1 - \not q_3}{\hat{u}} \not\epsilon_1 + \frac{\epsilon_1^\kappa \epsilon_2^\lambda \gamma^\mu}{\hat{s}} C_{\kappa\lambda\mu}(q_1, q_2, -q_1-q_2) \right] v^\alpha_i(q_3)$

Cross-sections:

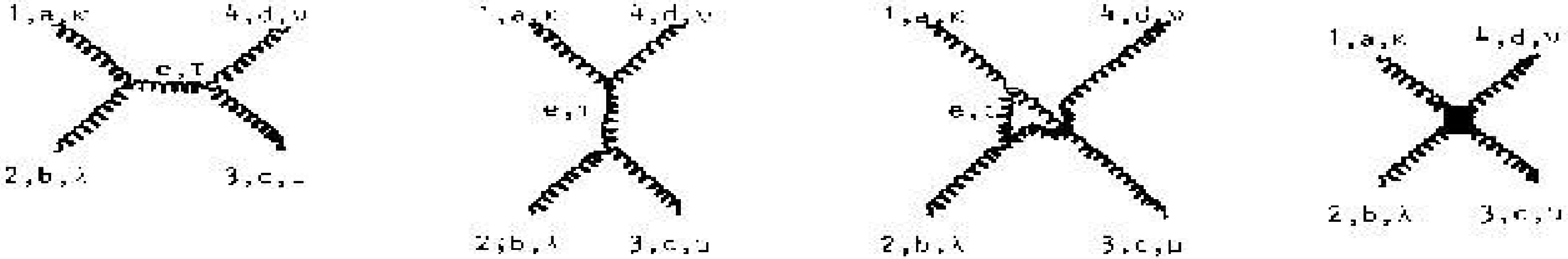
A: $\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{1}{6} \left(\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{u}^2}{\hat{s}^2} \right)$; B: $\frac{\pi\alpha_s^2}{\hat{s}^2} \frac{1}{6} \left(\frac{\hat{t}}{\hat{u}} - 2\frac{\hat{t}^2}{\hat{s}^2} \right)$

Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson
Comput.Phys.Commun.31:323,1984.

Process: $gg \to gg$

Diagrams:



Amplitudes:

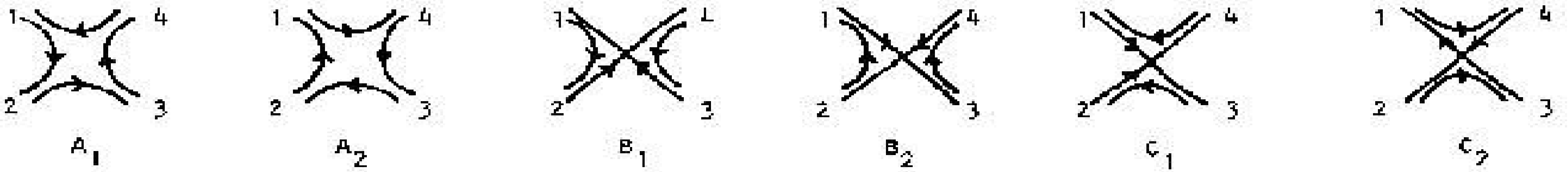
s: $-ig^2 \frac{1}{\hat{s}} f^{abe} f^{cde} \epsilon_1^\kappa \epsilon_2^\lambda \epsilon_3^\mu \epsilon_4^\nu C_{\kappa\lambda\tau}(-q_1, -q_2, q_1+q_2) C_{\mu\nu}{}^{\tau}(q_3, q_4, -q_3-q_4)$

t: $-ig^2 \frac{1}{\hat{t}} f^{dae} f^{bce} \epsilon_1^\kappa \epsilon_2^\lambda \epsilon_3^\mu \epsilon_4^\nu C_{\nu\kappa\tau}(q_4, -q_1, q_1-q_4) C_{\lambda\mu}{}^{\tau}(-q_2, q_3, q_2-q_3)$

u: $-ig^2 \frac{1}{\hat{u}} f^{cae} f^{bde} \epsilon_1^\kappa \epsilon_2^\lambda \epsilon_3^\mu \epsilon_4^\nu C_{\mu\kappa\tau}(q_3, -q_1, q_1-q_3) C_{\lambda\nu}{}^{\tau}(-q_2, q_4, q_2-q_4)$

4: $-ig^2 f^{abe} f^{cde}(g_{\kappa\mu} g_{\lambda\nu} - g_{\kappa\nu} g_{\lambda\mu}) - ig^2 f^{dae} f^{bce}$
$\times (g_{\kappa\mu} g_{\lambda\nu} - g_{\kappa\lambda} g_{\nu\mu}) + ig^2 f^{cae} f^{bde}(g_{\kappa\lambda} g_{\mu\nu} - g_{\kappa\nu} g_{\lambda\mu})$

Colour flows:



Color Flow in $p\bar{p} \to b\bar{b} + X$

B physics at the Tevatron: Run II and beyond
E. Norrbin, hep-ph/0201071,p522

$p\bar{p} \to b\bar{b} + X$

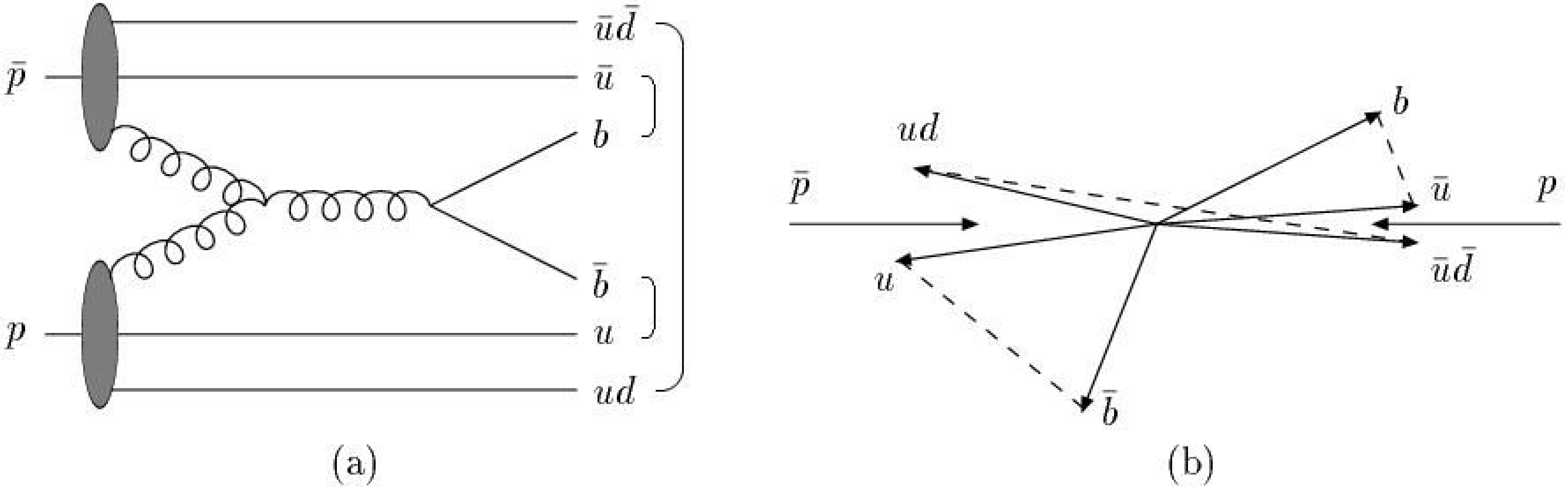


Figure 9.45: Example of a string configuration in a $p\bar{p}$ collision. (a) Graph of the process, with brackets denoting the final color singlet subsystems. (b) Corresponding momentum space picture, with dashed lines denoting the strings.

Beam - drag effect

- Due to color connection of produced b-quark with beam remnants, the rapidity distribution of b-quarks and B-hadrons is different.
- Asymmetry of $B^0 \bar{B}^0$

B physics at the Tevatron: Run II and beyond
E. Norrbin, hep-ph/0201071,p525

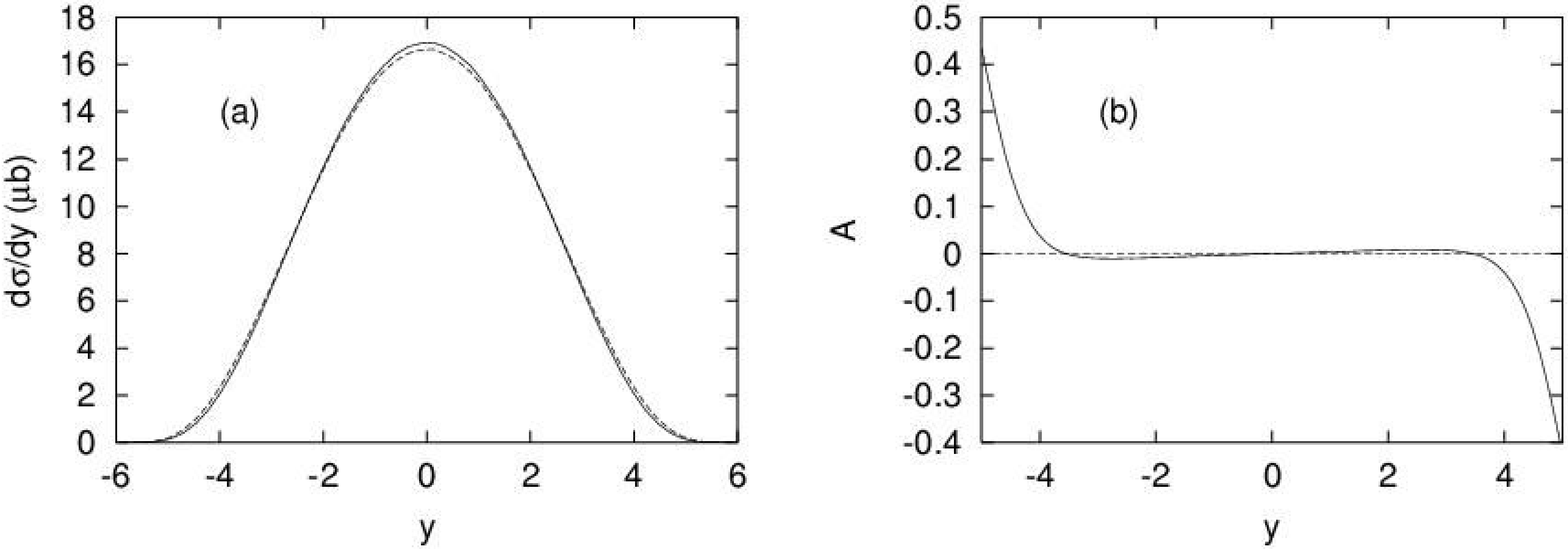


Figure 9.47: Bottom production at the Tevatron. (a) Rapidity distribution of bottom quarks (full) and the B hadrons produced from them (dashed). (b) The asymmetry $A = \frac{\sigma(B^0)-\sigma(\overline{B}^0)}{\sigma(B^0)+\sigma(\overline{B}^0)}$ as a function of rapidity. For simplicity, only pair production is included.

- **HowTo connect this to factorised fragmentation functions ?**

Summary

- Fragmentation functions from longitudinal phase space model
  - different behavior for light and heavy quarks
- Hadronization models (iterative procedure)
  - NEEDED if more than single inclusive quantities are investigated
  - different models available
    - independent fragmentation (do not use anymore !!!)
    - LUND string fragmentation (PYTHIA / JETSET)
    - cluster fragmentation (HERWIG)
      - dedicated effects observable:
      - string effect
      - beam drag effect
- Important to respect all color informations !!!!!