QCD and Collider Physics III: Jets and Hadronization

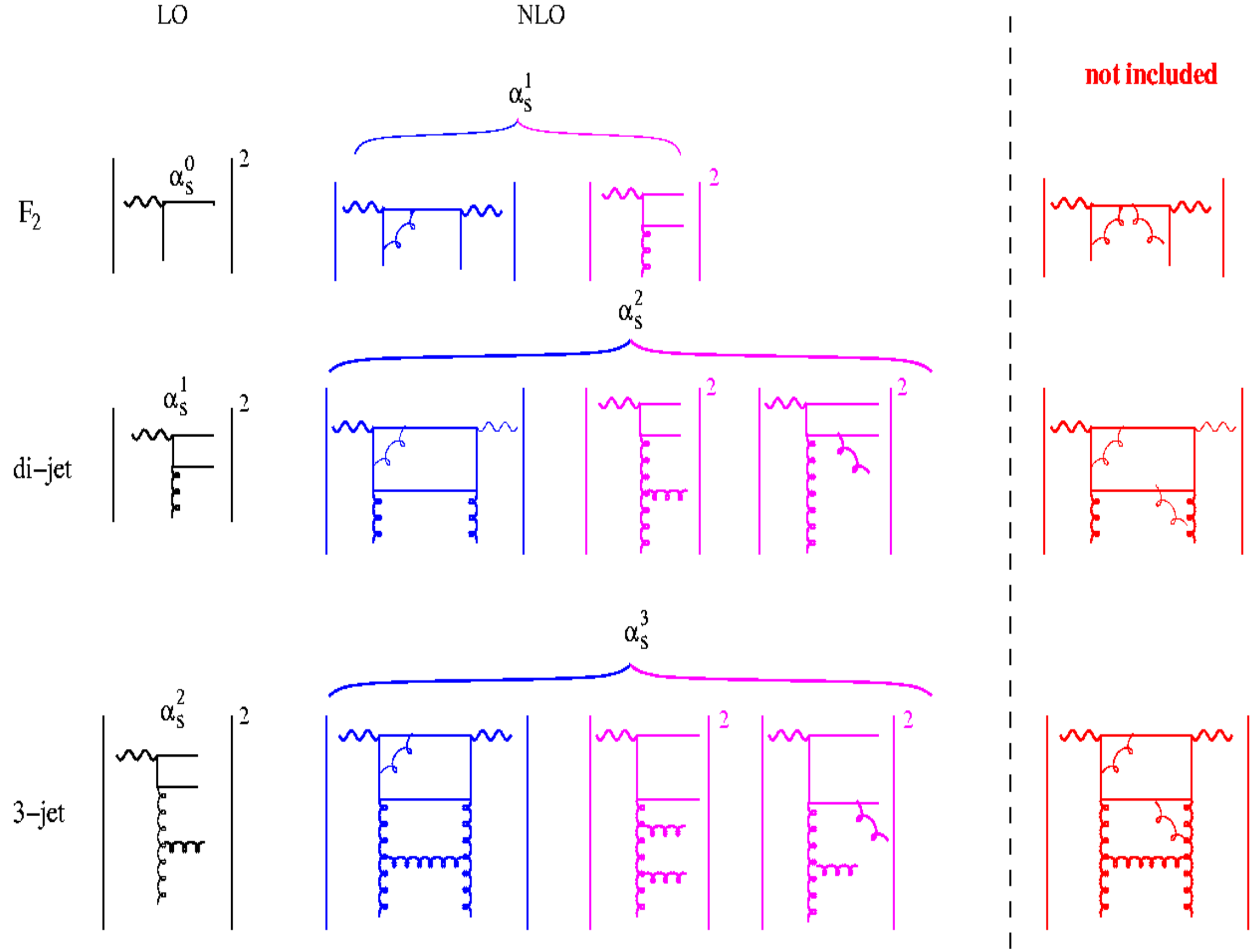
- evolution equations – how to solve ?
  - gluon density in DLL
  - jet evolution: particle multiplicities
- fragmentation function
  - heavy quarks
- hadronisation models
  - color flows in proton – proton interactions

- Literature:
  Ellis, Stirling, Webber: *QCD and Collider Physics*
  Dissertori, Knowles, Schmelling: *QCD - High Energy Exp and Theory*
  R. Field: *Applications of perturbative QCD*

http://www-h1.desy.de/~jung/qcd_collider_physics_wise_2006

From LO to NLO ...

- NLO for F_2: $O(\alpha_s)$
- NLO for dijets: $O(\alpha_s^{\,2})$
- NLO for 3-jets: $O(\alpha_s^{\,3})$



NOTE: NLO for 3-jets is **NOT** NNLO for dijets

Hadronization - Fragmentation

Hadronization

From Wikipedia, the free encyclopedia

In particle physics, hadronization is the process of the formation of hadrons out of quarks and gluons. This occurs after high-energy collisions in a particle collider in which free quarks or gluons are created. Due to colour confinement, these cannot exist individually. In the independent model they combine with quarks and antiquarks spontaneously created from the vacuum to form hadrons. The details of this process are not yet fully understood. Another model is the Lund string model.

The tight cone of particles created by the hadronization of a single quark is called a jet. Jets are observed in particle detectors, rather than quarks, whose existence must be inferred.

Fragmentation .... (but we use it differently !!!!)

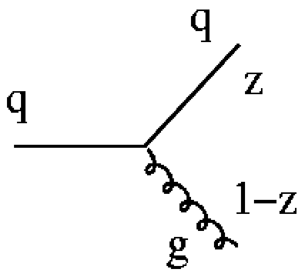
From Wikipedia, the free encyclopedia

Fragmentation is a term that occurs in several fields and describes a process of something breaking or being divided into pieces (fragments). See also divergence.

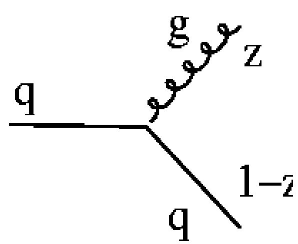
1Biology,2Computing,3Networking,4Economics,5Music,6Literature,7Urban sociology,

8Weaponry,9Mass spectrometry,10Waste management **BUT where is HEP ????**

Splitting functions in lowest order

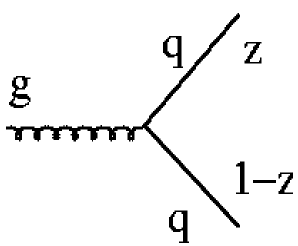


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

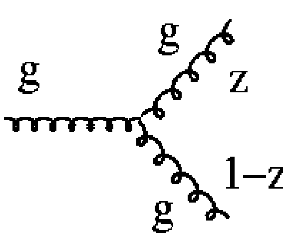


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to photon radiation from electron



$$P_{qg} = \frac{1}{2}\left(z^2+(1+z)^2\right)$$



$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

Solving integral equations

- Integral equation of *Fredholm type*:
- solve it by iteration (Neumann series):

$$\phi(x) = f(x) + \lambda \int_a^b K(x,y)\phi(y)dy$$

$$\begin{aligned}
\phi_0(x) &= f(x) \\
\phi_1(x) &= f(x) + \lambda \int_a^b K(x,y)f(y)dy \\
\phi_2(x) &= f(x) + \lambda \int_a^b K(x,y_1)f(y_1)dy_1 + \lambda^2 \int_a^b \int_a^b K(x,y_1)K(y_1,y_2)f(y_2)dy_2dy_1 \\
\phi_n(x) &= \sum_{i=0}^{n} \lambda^i u_i(x) \\
u_0(x) &= f(x) \\
u_1(x) &= \int_a^b K(x,y)f(y)dy \\
u_n(x) &= \int_a^b \int_a^b K(x,y_1)K(y_1,y_2)\cdots K(y_{n-1},y_n)f(y_n)dy_2\cdots dy_n
\end{aligned}$$

with the solution: $$\phi(x) = \lim_{n\to\infty} q_n(x) = \lim_{n\to\infty} \sum_{i=0}^{n} \lambda^i u_i(x)$$

Approximation at small x ?

- For $x \to 0$ only gluon splitting function matters:

$$P_{gg} = 6\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) = 6\left(\frac{1}{z} - 2 + z(1-z) + \frac{1}{1-z}\right)$$

$$P_{gg} \sim 6\frac{1}{z} \text{ for } z \to 0$$

- evolution equation is then:

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} g(\xi,\mu^2) P_{gg}\left(\frac{x}{\xi}\right)$$

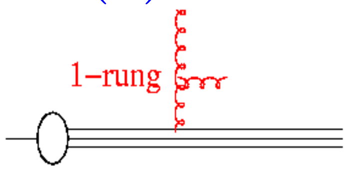
$$xg(x,t) = xg(x,t_0) + \frac{3\alpha_s}{\pi}\int_{t_0}^t d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t') \quad \text{with} \quad t = \mu^2$$

Estimates at small x: DLL

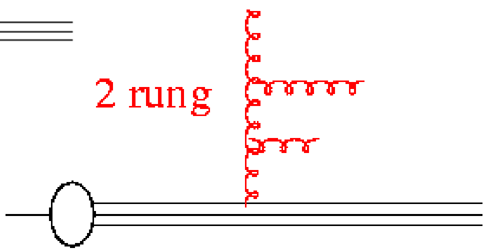
A.D. Martin,in Lectures at XXI International Meeting on Fundamental Physics, Miraflores de la Sierra, Madrid, 1993

$$xg(x,t) = xg(x,t_0) + \frac{3\alpha_s}{\pi}\int_{t_0}^{t} d\log t' \int_x^1 \frac{d\xi}{\xi}\xi g(\xi,t')$$ with $t = \mu^2$

- use constant starting distribution at small *t*: $u_0(x) = C$

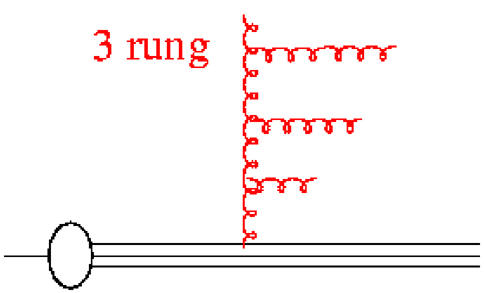


$$u_1(x,t) = \frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}C$$



$$u_2(x,t) = \left(\frac{3\alpha_s}{\pi}\frac{1}{2}\log\frac{t}{t_0}\frac{1}{2}\log\frac{1}{x}\right)^2 C$$

$$\vdots$$



$$u_n(x,t) = \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

$$xg(x,t) = \sum_n \frac{1}{n!}\frac{1}{n!}\left(\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}\right)^n C$$

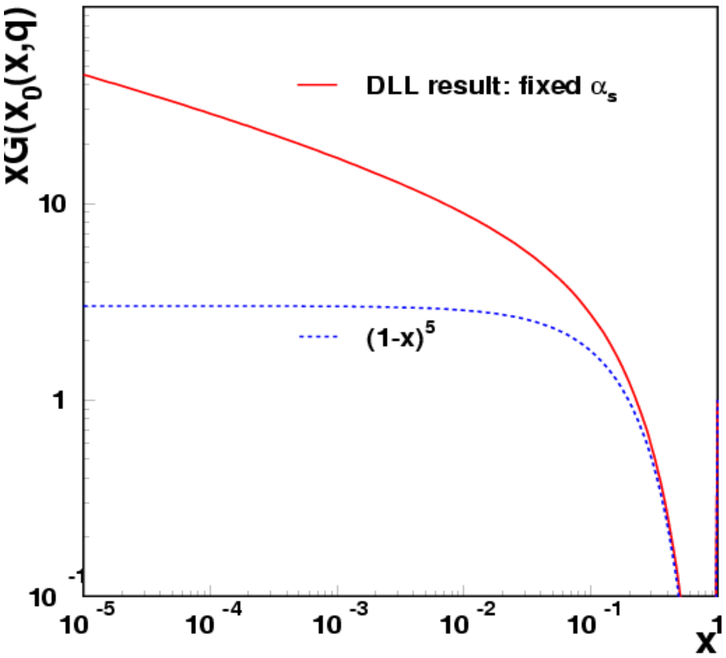
$$xg(x,t) \sim C\exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$

double leading log approximation (DLL)

Results from DLL approximation

$$xg(x,t) \sim C \exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\log\frac{t}{t_0}\log\frac{1}{x}}\right)$$

- DLL arise from taking small x limit of splitting fct:
  - $\log 1/x$ from small *x* limit of splitting fct
  - $\log t/t_0$ from *t* integration
  - strong ordering in *x* from small *x* limit
  - strong ordering in *t* from small *t* limit of ME...
- DLL gives rapid increase of gluon density from a flat starting distribution



consequences:
- rise continues forever ???
- what happens when too high gluon density ?

Quark and Gluon jet differences

- calculate average multiplicity in gluon and quark jets:
- consider only gluon emissions:

R. Field Appl. of pQCD, p79

$$\frac{< N_g >}{< N_q >} \sim \sqrt{\frac{C_A}{C_F}} \sim \frac{3}{2}$$

- comsider ANY type of emission:

Elllis,Stirling,Webber QCD & Collider physics, p219

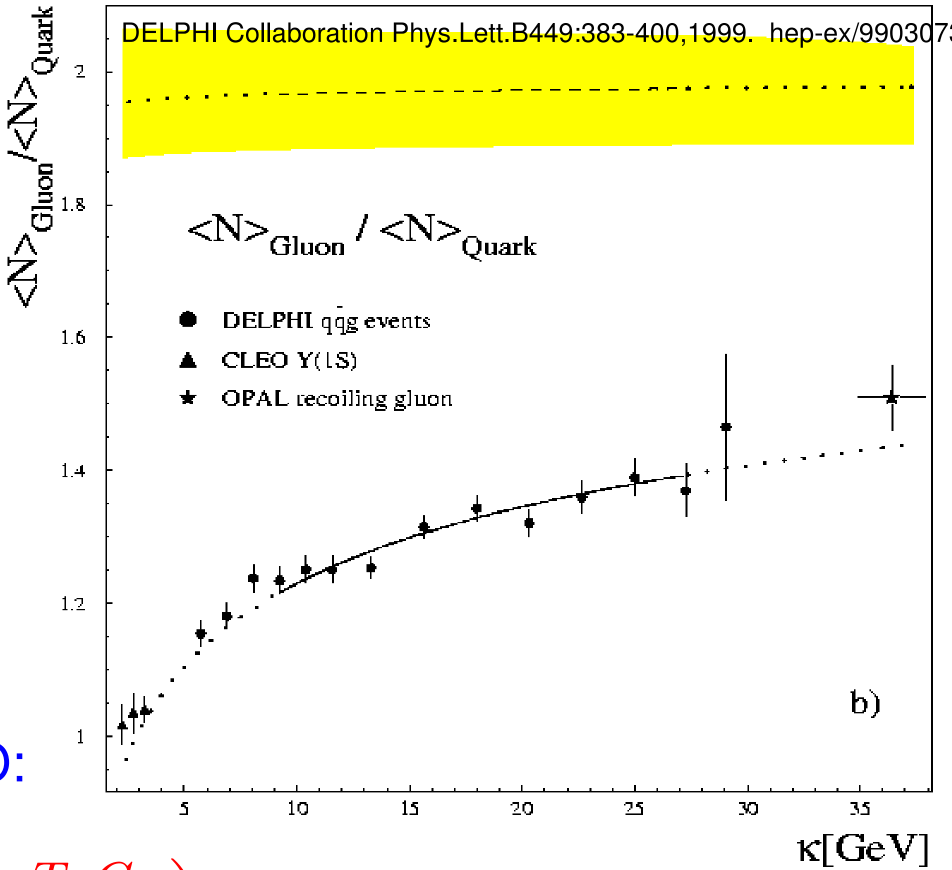
$$\frac{< N_g >}{< N_q >} \sim \frac{C_A}{C_F} \sim \frac{9}{4}$$

- including higher order corrections NNLO:

Dissertori, Knowles, Schmelling QCD, p376

$$\frac{< N_g >}{< N_q >} = \frac{C_A}{C_F}\left[1 - \left(1 + 2\frac{n_f T_F}{C_A} - 4\frac{n_f T_T C_F}{C_A^2}\right) \times \left(\sqrt{\frac{\alpha_s C_A}{18\pi}} + \left(\frac{25}{8} - \frac{3 n_f T_F}{2 C_A} - 2\frac{n_f T_F C_F}{C_A^2}\right)\frac{\alpha_s C_A}{18\pi}\right)\right]$$

$$\sim 1.7$$



- BUT still a difference to the measurement

Summary

- PDF evolution
- Jet evolution: resummation to all orders
  - evolution is suitable for itearative procedure
  - in some cases analytical calcs can be performed