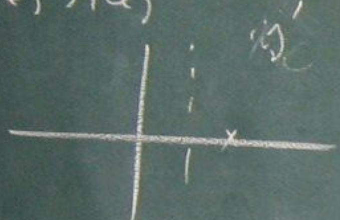
Addendum:

$$t\frac{\partial}{\partial t} q_{NS}(x,t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{d\xi}{\xi}\, P_{qq}\left(\frac{x}{\xi},\alpha_s\right) q_{NS}(\xi,t)$$

Re j > Re j'

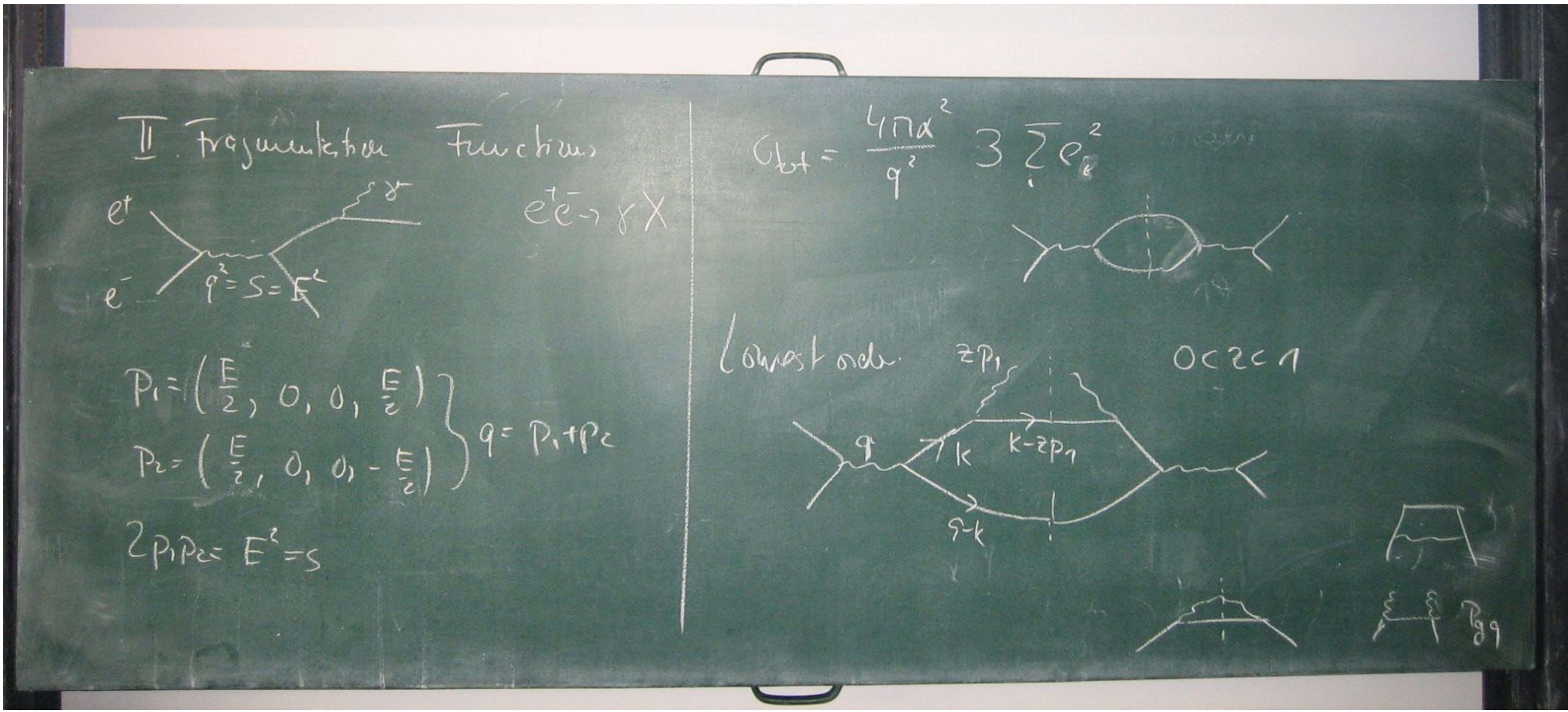


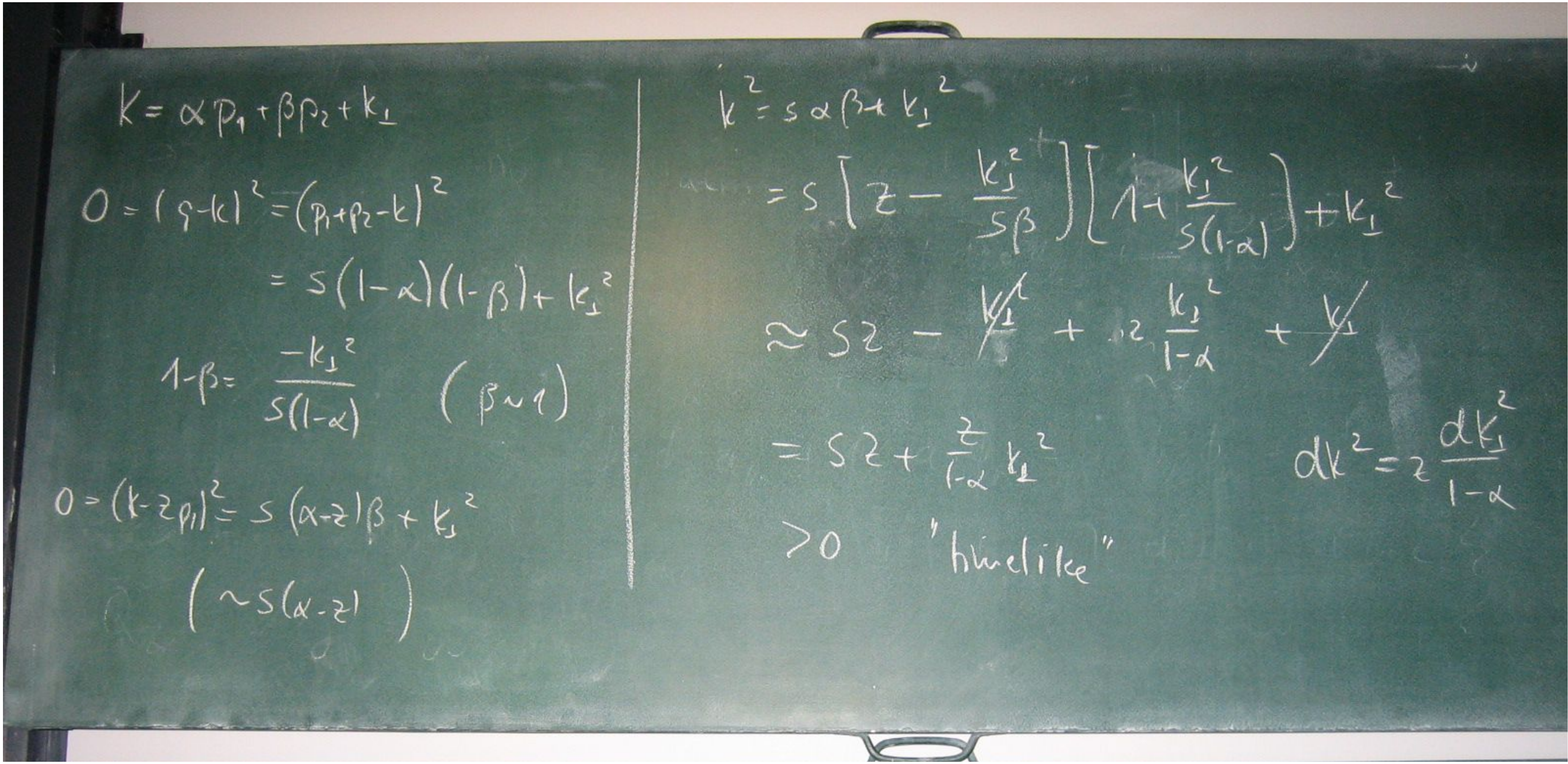
$$\int_0^1 \frac{d\xi}{\xi}\int_{-i\infty}^{+i\infty}\frac{dj'}{2\pi i}\,\xi^{j-j'}\,\tilde{q}_{NS}(j',t)$$

$$= \int \frac{dj'}{2\pi i}\,\frac{1}{j-j'}\, q_{NS}(j',t)$$

$$= q_{NS}(j,t)$$

$$\int_0^1 \frac{d\xi}{\xi}\,\xi^{j-j'} = \frac{1}{j-j'}$$


II. Fragmentation Functions
e^+
γ
e^-
$e^+e^- \to \gamma X$
$q^2 = S = E^2$
$P_1 = \left(\frac{E}{2}, 0, 0, \frac{E}{2}\right)$
$P_2 = \left(\frac{E}{2}, 0, 0, -\frac{E}{2}\right)$
$q = P_1 + P_2$
$2 P_1 P_2 = E^2 = s$
$\sigma_{tot} = \frac{4\pi\alpha^2}{q^2} \, 3 \sum_i e_i^2$
Lowest order
zP_1
$0 < z < 1$
q
k
$k - zP_1$
$q - k$
P_{gq}

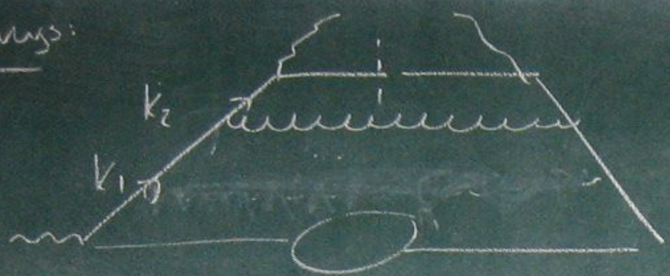

$k = \alpha p_1 + \beta p_2 + k_\perp$
$0 = (q-k)^2 = (p_1+p_2-k)^2$
$= s(1-\alpha)(1-\beta) + k_\perp^2$
$1-\beta = \frac{-k_\perp^2}{s(1-\alpha)} \quad (\beta \sim 1)$
$0 = (k - z p_1)^2 = s(\alpha - z)\beta + k_\perp^2$
$(\sim s(\alpha - z))$
$k^2 = s\alpha\beta + k_\perp^2$
$= s\left[z - \frac{k_\perp^2}{s\beta}\right]\left[1 + \frac{k_\perp^2}{s(1-\alpha)}\right] + k_\perp^2$
$\simeq sz - \cancel{k_\perp^2} + z\frac{k_\perp^2}{1-\alpha} + \cancel{k_\perp^2}$
$= sz + \frac{z}{1-\alpha}k_\perp^2$
$dk^2 = z\frac{dk_\perp^2}{1-\alpha}$
> 0 "timelike"

$$\frac{d\sigma(e^+e^- \to \gamma + X)}{dz} = \frac{4\pi\alpha^2}{q^2}\, 3 \sum_q e_q^2\, D_q^\gamma(z, \mu^2, q^2)$$

$$D_q^\gamma = \frac{\alpha}{4\pi} \int_{\mu^2}^{q^2} \frac{dk^2}{(k^2)} \int d\alpha\, \delta(\alpha - z)\; 2\,\frac{1+(1-z)^2}{z}$$

$$= \frac{\alpha}{4\pi} \ln\frac{q^2}{\mu^2} \int_z^1 \frac{dx}{x}\, \delta\!\left(1-\frac{x}{z}\right)\; 2\,\frac{1+(1-z)^2}{z}$$

More rungs:

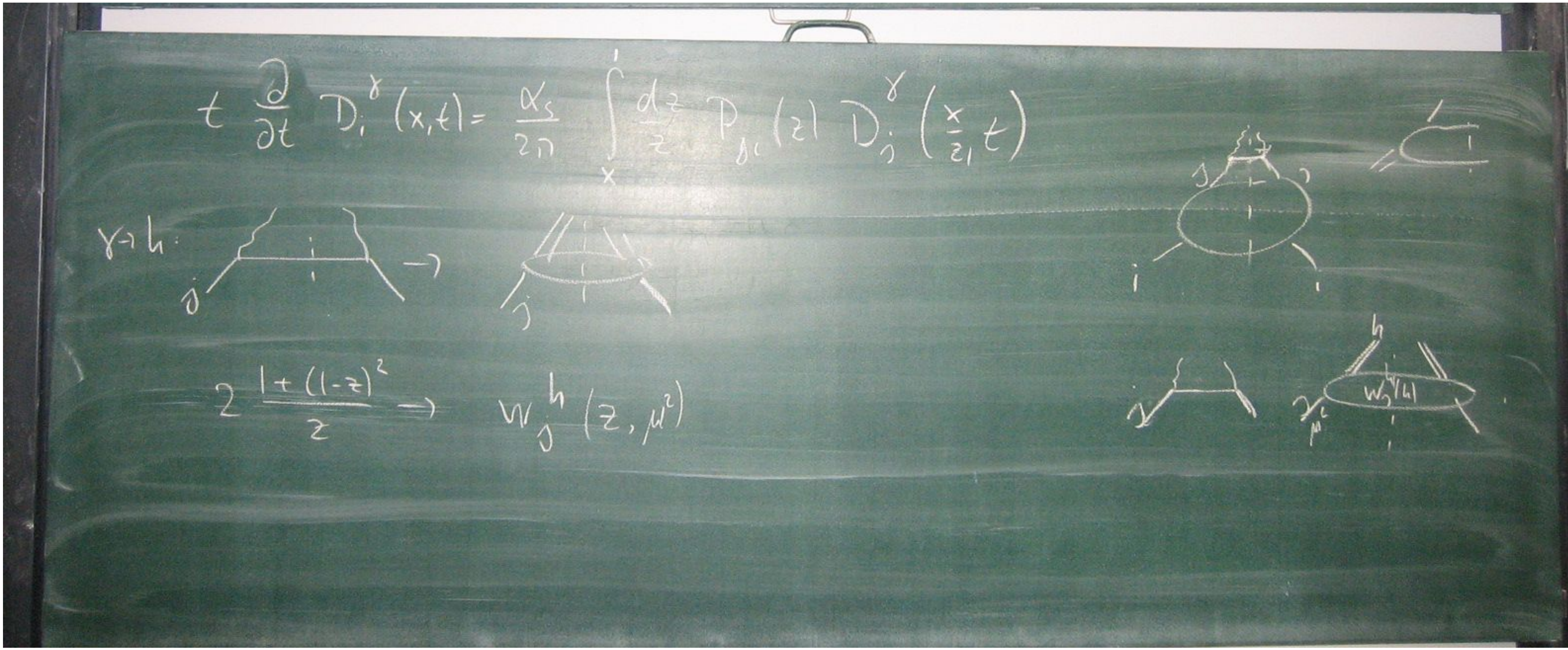


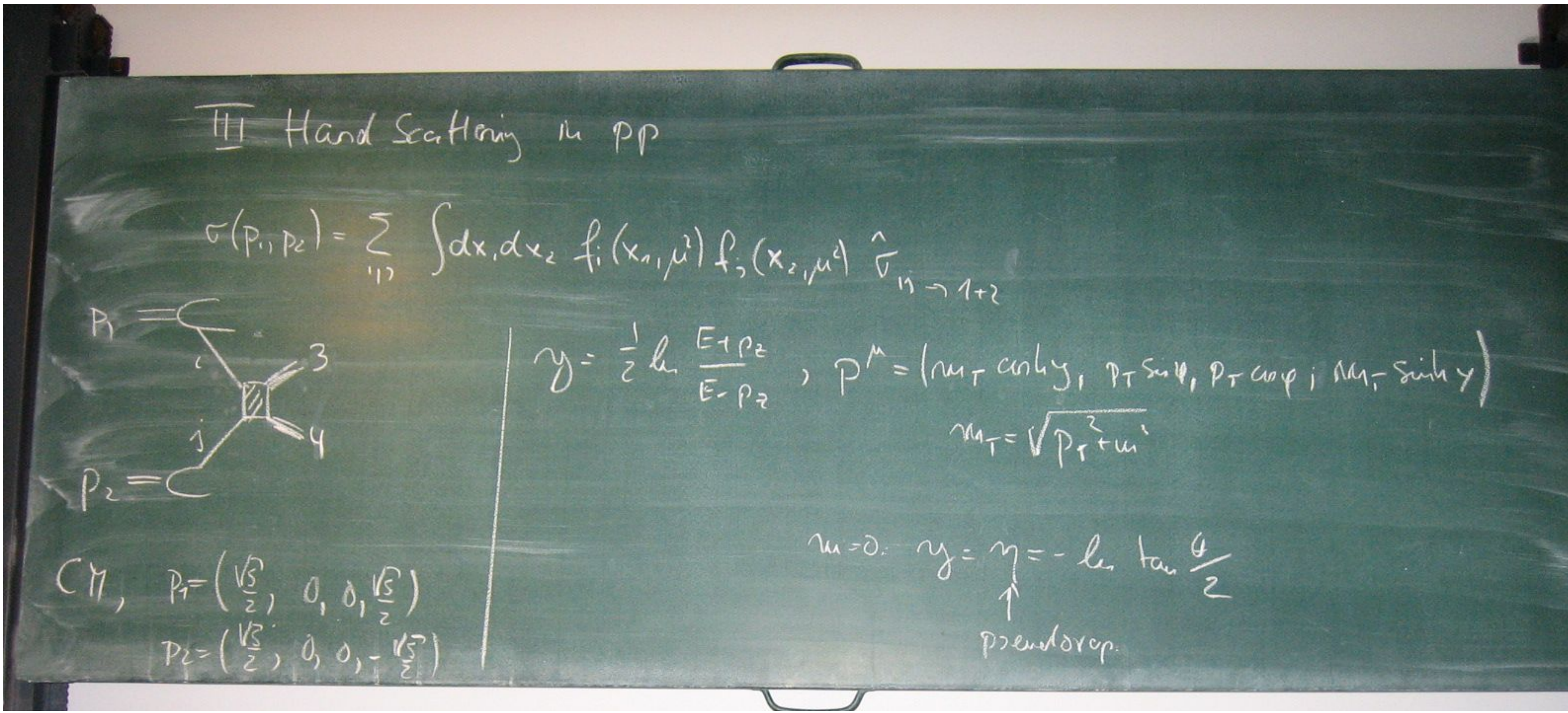
$$\int_{\mu^2}^{q^2} \frac{dk_1^2}{k_1^2} \int_{\mu^2}^{k_1^2} \frac{dk_2^2}{k_2^2} = \frac{1}{2!}\left(\ln \frac{q^2}{\mu^2}\right)^2 \qquad k_1^2, k_2^2 > \mu^2$$

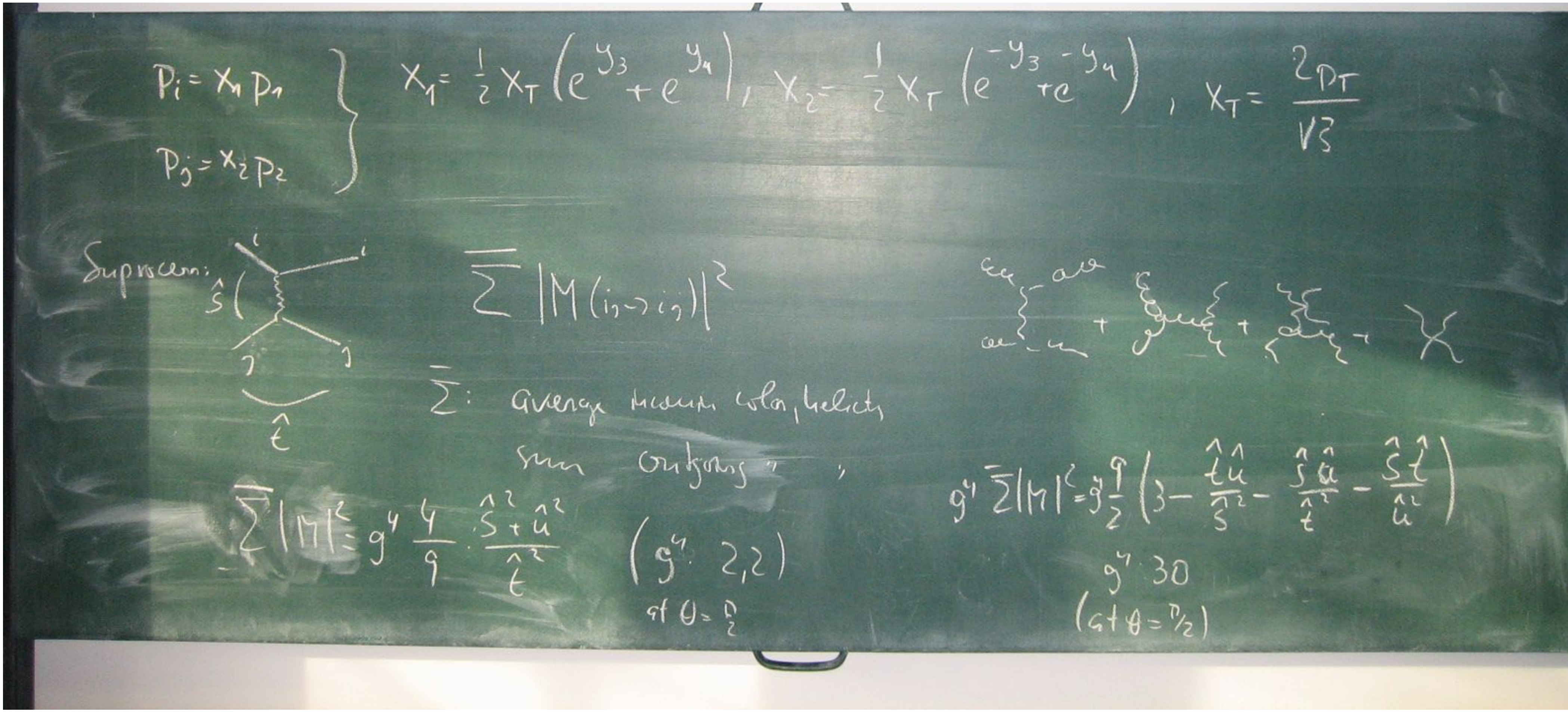
$\alpha_1 > \alpha_2$

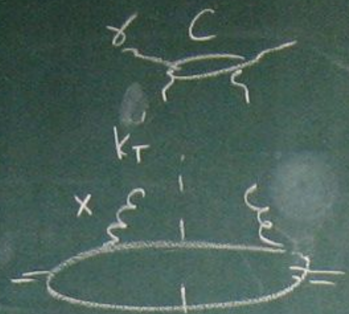
$P^{(0)}_{qq}\left(\frac{\alpha_2}{\alpha_1}\right)$ same as in DIS

$$P_{qq} = P^{(0)}_{qq} + \underbrace{\frac{\alpha_s}{2\pi} P^{(1)}_{qq}}_{\text{different from DIS}}$$


$t \frac{\partial}{\partial t} D_i^\gamma(x,t) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_{ji}(z) D_j^\gamma\left(\frac{x}{z}, t\right)$
$\gamma \to h$:
$2 \frac{1+(1-z)^2}{z} \to w_j^h(z, \mu^2)$
$w_j^{\gamma/h}$

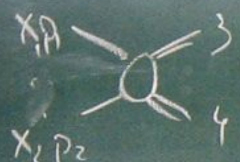

III Hard Scattering in pp
$\sigma(p_1, p_2) = \sum_{ij} \int dx_1 dx_2\, f_i(x_1, \mu^2)\, f_j(x_2, \mu^2)\, \hat{\sigma}_{ij \to 1+2}$
p_1
p_2
i
j
3
4
$y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$, $p^\mu = (m_T \cosh y, p_T \sin\varphi, p_T \cos\varphi; m_T \sinh y)$
$m_T = \sqrt{p_T^2 + m^2}$
$m=0$: $y = \eta = -\ln \tan \frac{\theta}{2}$
pseudorap.
CM, $p_1 = \left(\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\right)$
$p_2 = \left(\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\right)$


$p_i = x_1 p_1$
$p_j = x_2 p_2$
$x_1 = \frac{1}{2} x_T (e^{y_3} + e^{y_4})$, $x_2 = \frac{1}{2} x_T (e^{-y_3} + e^{-y_4})$, $x_T = \frac{2 p_T}{\sqrt{s}}$
Subprocess:
$\hat{s}$
$\hat{t}$
$\overline{\sum} |M(ij \to ij)|^2$
$\overline{\sum}$: average incoming color, helicity
sum outgoing " ;
$\overline{\sum} |M|^2 = g^4 \frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$
$(g^4 \cdot 2{,}2)$
at $\theta = \frac{\pi}{2}$
$g^4 \overline{\sum} |M|^2 = g^4 \frac{9}{2} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$
$g^4 \cdot 30$
(at $\theta = \pi/2$)



$$\int d^2k_T \; C(Q^2,k_T^2)\,\frac{1}{k_T^2}\;\mathcal{F}(x,k_T^2)$$

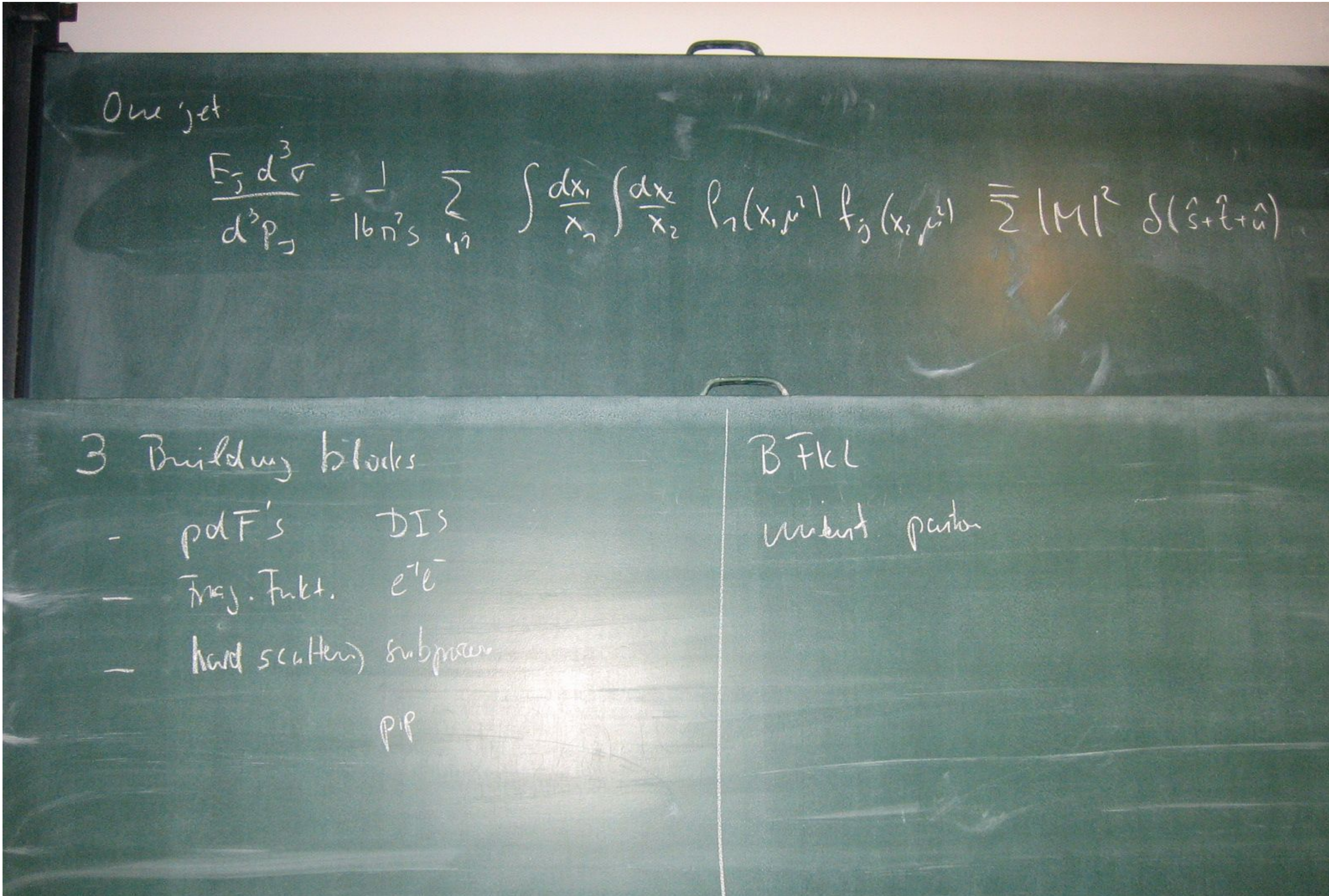
$x\to 0$ $\quad \tilde{\mathcal{F}}(x,k_T) \qquad xg(x,k^2)=\int^{k^2}\frac{dk_T^2}{k_T^2}\,\tilde{\mathcal{F}}(x,k^2)$



partonic $\quad \dfrac{E_3E_4\,d^6\hat\sigma}{d^3p_3\,d^3p_4}=\dfrac{1}{2\hat s}\,\dfrac{1}{16\pi^2}\,\overline{\sum}|M^2|\,\delta^4(p_3+p_4-x_1p_1-x_2p_2)$

$$\frac{d^3\hat\sigma}{dy_3\,dy_4\,dp_T^2}\qquad \frac{1}{16\pi\hat s^2}\,\overline{\sum}|M^2|$$

hadronic: $\quad \dfrac{d^3\sigma}{dy_3\,dy_4\,dp_T^2}\qquad \dfrac{1}{16\pi s^2}\qquad \dfrac{f_i(x_1,\mu^2)}{x_1}\qquad \dfrac{f_j(x_2,\mu^2)}{x_2}\,\overline{\sum}|M|^2$


One jet
$\frac{E_J\, d^3\sigma}{d^3p_J} = \frac{1}{16\pi^2 s}\sum_{i,j}\int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2}\, f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\,\overline{\sum}|M|^2\,\delta(\hat{s}+\hat{t}+\hat{u})$
3 Building blocks
- pdF's DIS
- Frag. Fukt. e^+e^-
- hard scattering subprocess
pp
BFKL