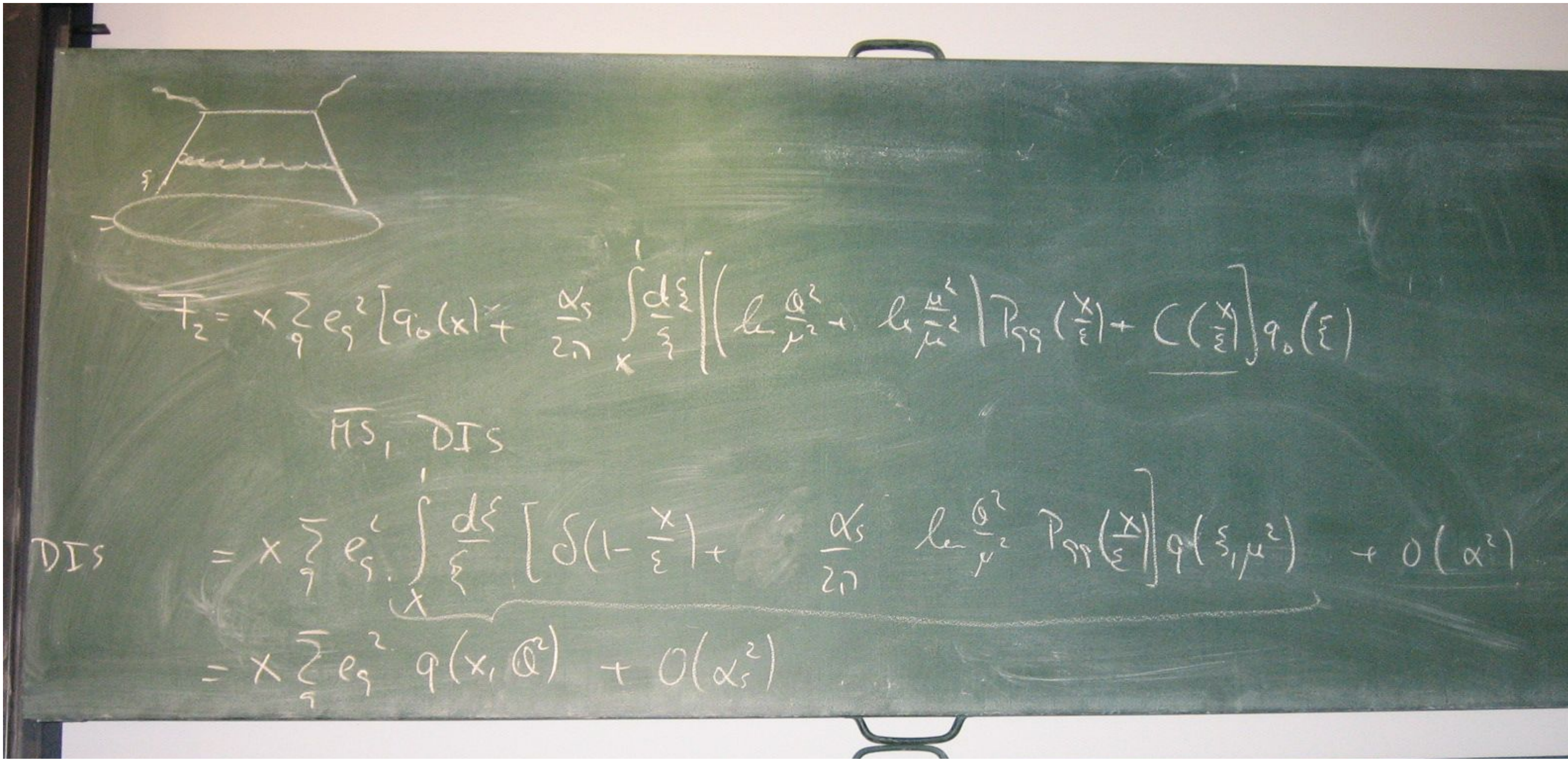
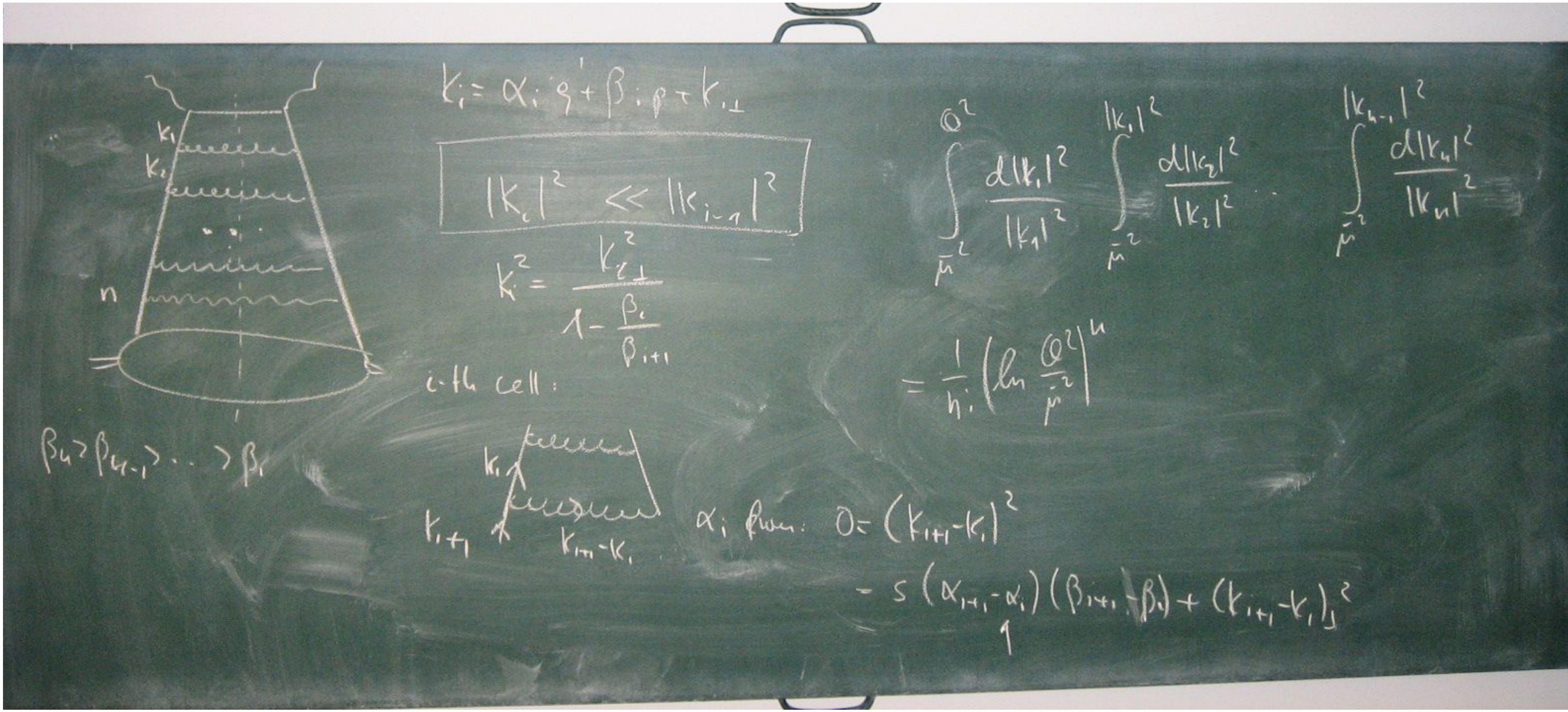

ξ
$F_2 = x \sum_q e_q^2 \left[q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left(\left(\ln\frac{Q^2}{\mu^2} + \ln\frac{\mu^2}{\hat{\mu}^2} \right) P_{qq}\left(\frac{x}{\xi}\right) + C\left(\frac{x}{\xi}\right) \right] q_0(\xi) \right)$
$\overline{MS}$, DIS
DIS $= x \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} \ln\frac{Q^2}{\mu^2} P_{qq}\left(\frac{x}{\xi}\right) \right] q(\xi, \mu^2) + O(\alpha^2)$
$= x \sum_q e_q^2 \, q(x, Q^2) + O(\alpha_s^2)$


k_i = α_i q' + β_i p + k_i⊥
|k_i|² << |k_{i-1}|²
k_i² = k²_{i⊥} / (1 - β_i/β_{i+1})
i-th cell:
β_n > β_{n-1} > ... > β_1
α_i from: 0 = (k_{i+1} - k_i)²
= s (α_{i+1} - α_i)(β_{i+1} - β_i) + (k_{i+1} - k_i)_⊥²
∫_{μ²}^{Q²} d|k_1|²/|k_1|² ∫_{μ²}^{|k_1|²} d|k_2|²/|k_2|² ... ∫_{μ²}^{|k_{n-1}|²} d|k_n|²/|k_n|²
= 1/n! (ln Q²/μ²)^n

Small x: $\beta_{i+1} \gg \beta_i$

$$|k_i|^2 = k_{i\perp}^2$$

Fixed α_s:

$$q(x,Q^2) = q(x,\mu^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\beta_1}{\beta_1}\ln\frac{Q^2}{\mu^2}\, P_{qq}\left(\frac{x}{\beta_1}\right) q(\beta_1,\mu^2)$$

$$+ \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{2!}\left(\ln\frac{Q^2}{\mu^2}\right)^2 \int_x^1 \frac{d\beta_1}{\beta_1} P_{qq}\left(\frac{x}{\beta_1}\right)\int_{\beta_1}^1 P_{qq}\left(\frac{\beta_1}{\beta_2}\right) q(\beta_2,\mu^2)\frac{d\beta_2}{\beta_2}$$

$$Q^2\frac{\partial}{\partial Q^2} q(x,Q^2)$$

$$= \frac{\partial}{\partial \ln Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\beta_1}{\beta_1} P_{qq}\left(\frac{x}{\beta}\right) q(\beta_1,\mu^2) + \left(\frac{\alpha_s}{2\pi}\right)^2 \ln\frac{Q^2}{\mu^2}\int_x^1 \frac{d\beta}{\beta} P_{qq}\left(\frac{x}{\beta_1}\right)\int_{\beta_1}^1 \frac{d\beta_2}{\beta_2} P_{qq}\left(\frac{\beta_1}{\beta_2}\right) q(\beta_2,\mu^2) + \ldots$$

$$Q^2\frac{\partial}{\partial Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\beta_1}{\beta_1} P_{qq}\left(\frac{x}{\beta_1}\right) q(\beta_1,Q^2)$$

running α_s: $\alpha_s \to \alpha_s(Q^2)$

$C_A = N$

$C_F = \frac{N^2-1}{2N}$

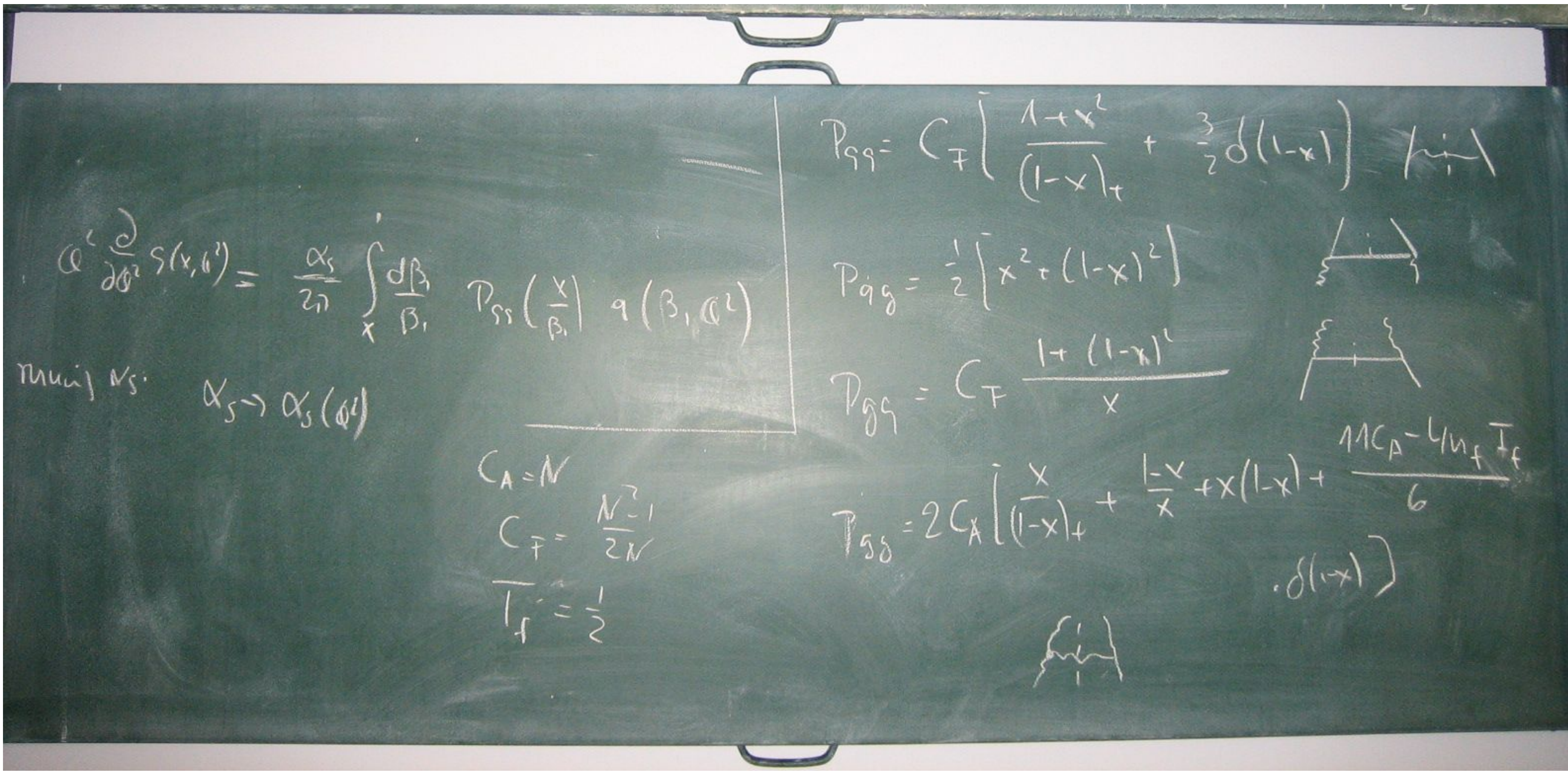
$T_F = \ldots$

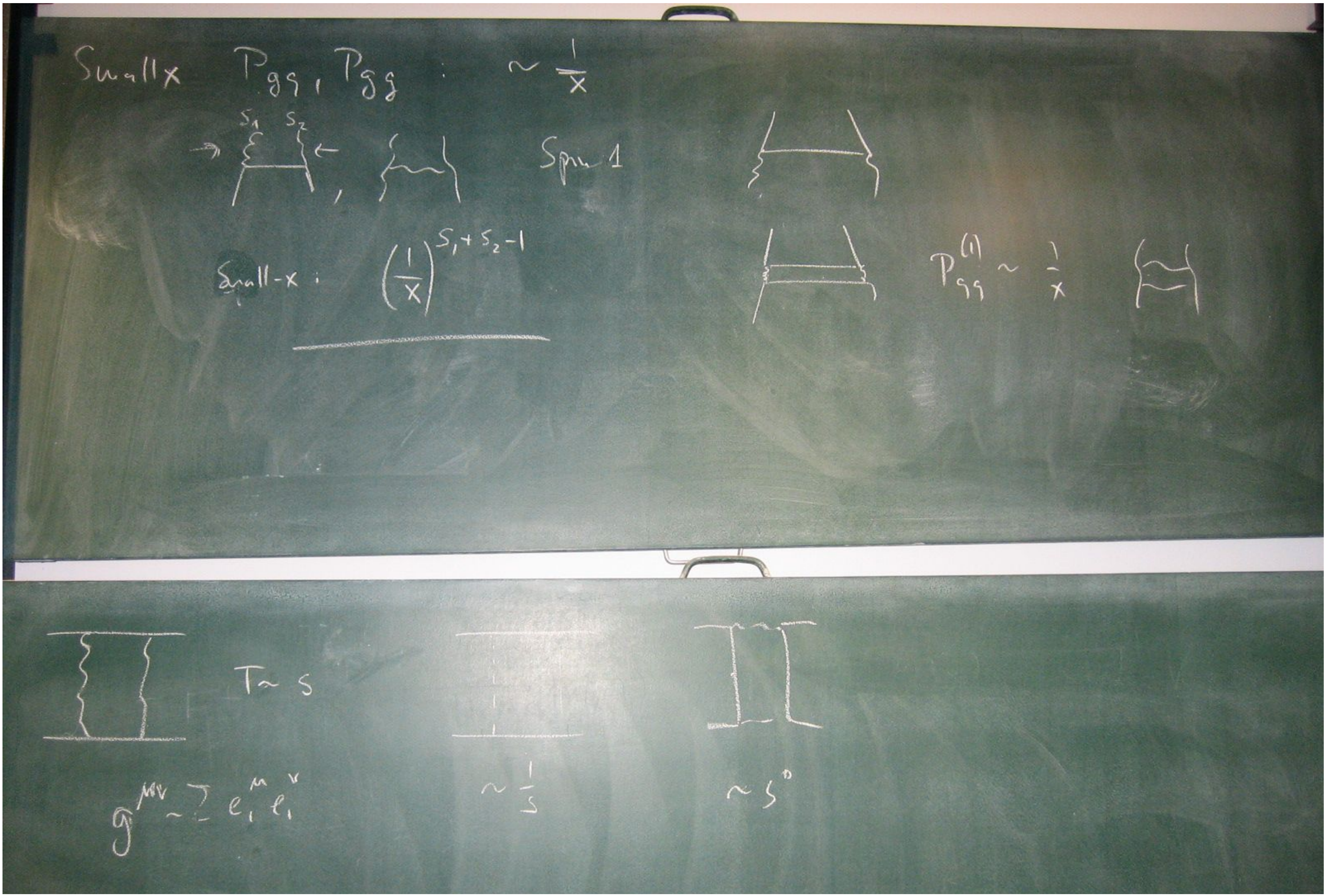
$$P_{qq} = C_F\left[\frac{\ldots}{(1-x)_+} + \frac{\ldots}{2}\delta(1-x)\right]$$

$$P_{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq} = C_F\,\frac{1+(1-x)^2}{x}$$

$$P_{gg} = 2C_A\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \frac{11C_A - 4n_f T_F}{6}\,\delta(1-x)$$

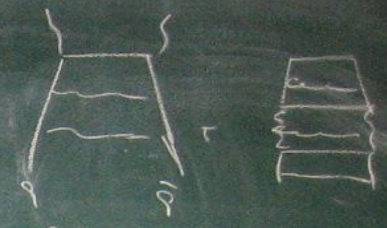

$Q^2 \frac{\partial}{\partial Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\beta_1}{\beta_1} P_{qq}\left(\frac{x}{\beta_1}\right) q(\beta_1, Q^2)$
running α_s: $\alpha_s \to \alpha_s(Q^2)$
$C_A = N$
$C_F = \frac{N^2-1}{2N}$
$T_f = \frac{1}{2}$
$P_{qq} = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right]$
$P_{qg} = \frac{1}{2}\left[x^2 + (1-x)^2 \right]$
$P_{gq} = C_F \frac{1+(1-x)^2}{x}$
$P_{gg} = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \frac{11C_A - 4n_f T_f}{6} \cdot \delta(1-x) \right]$


Smallx Pgq, Pgg : ~ 1/x
s1 s2
Spin 1
Small-x : (1/x)^(s1+s2-1)
P(1)gg ~ 1/x
T ~ s
g^μν ~ Σ e1^μ e1^ν
~ 1/s
~ s^0

$\ln Q^2 \to t$

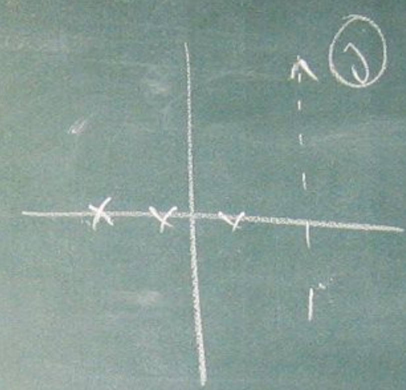
Alternative language:

$$t \frac{\partial}{\partial t} q_{NS}(x,t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{d\xi}{\xi} P_{qq}\left(\frac{x}{\xi}, \alpha_s(t)\right) q_{NS}(\xi, t)$$



$$\tilde{q}_{NS}(j,t) = \int_0^1 dx\, x^{j-1} q_{NS}(x,t)$$

$$q_{NS}(x,t) = \int_{c-i\infty}^{c+i\infty} \frac{dj}{2\pi i} x^{-j} \tilde{q}_{NS}(j,t)$$

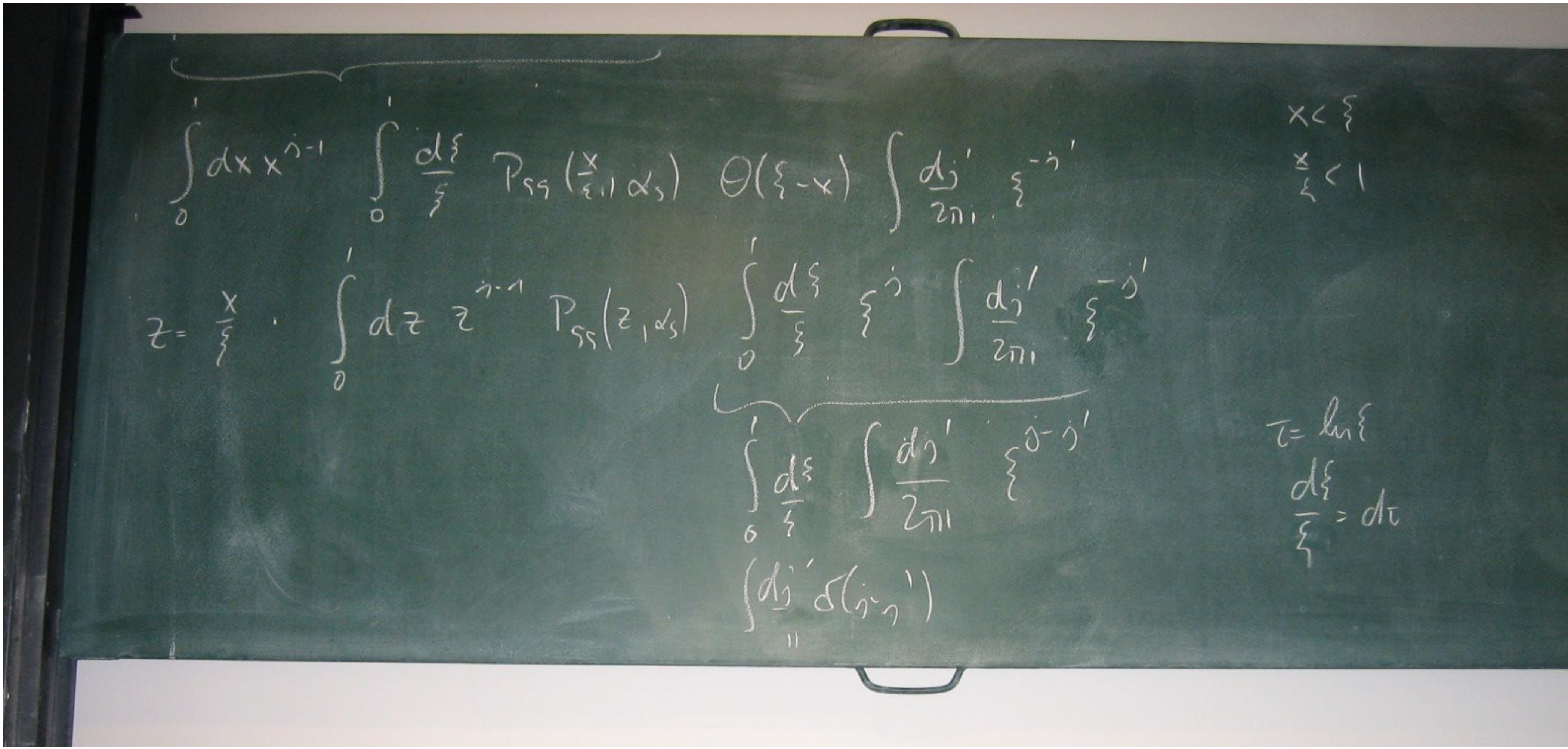


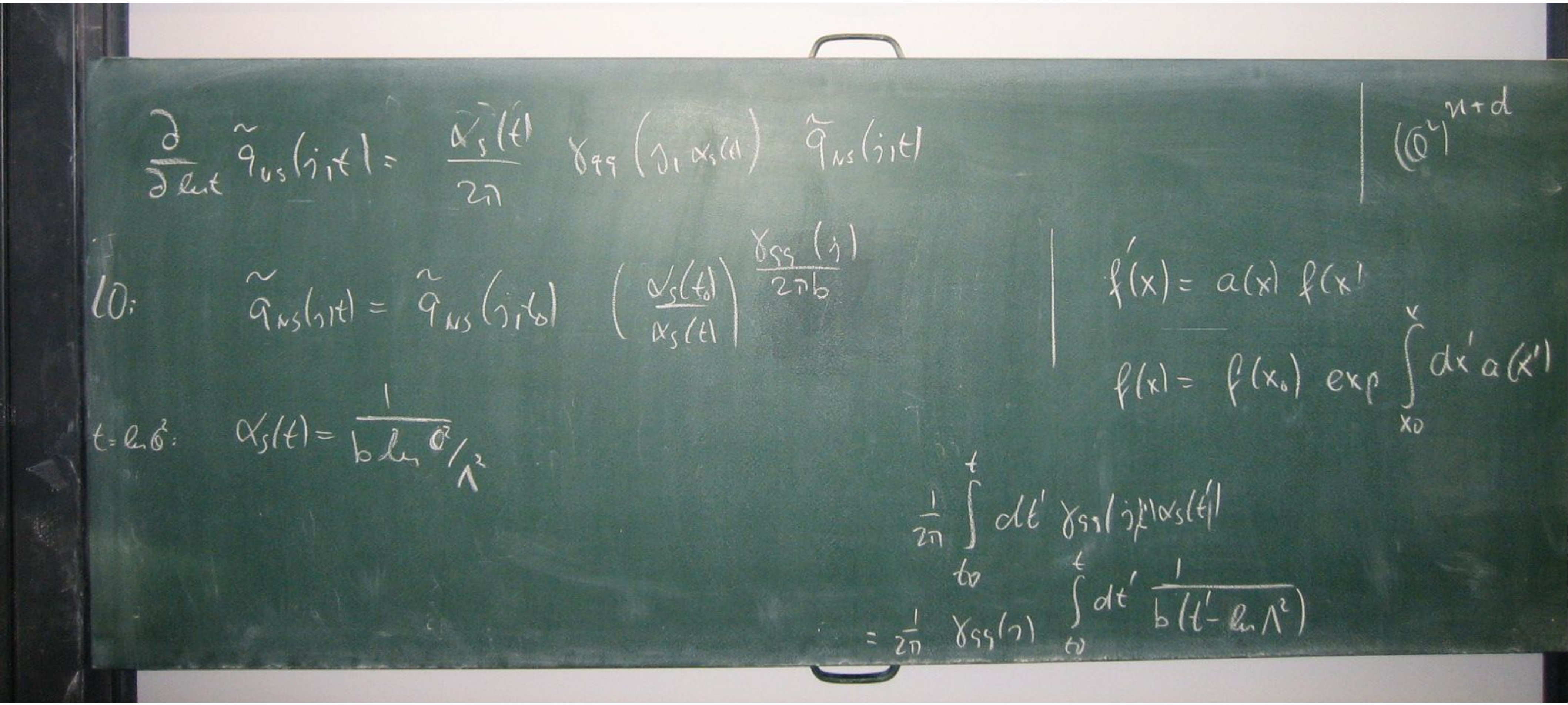
$$\int_0^1 dx\, x^{j-1}$$

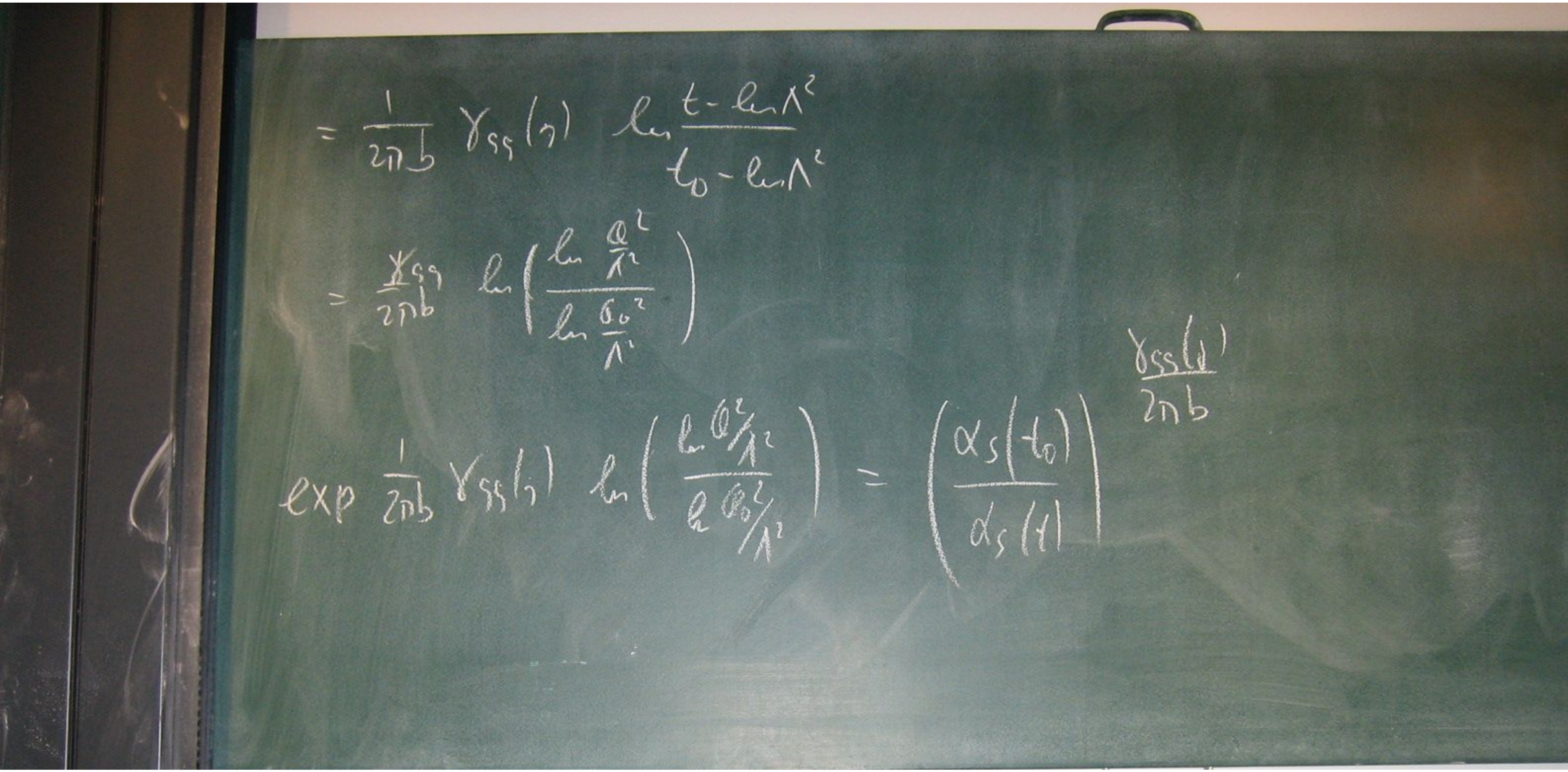
$$t \frac{\partial}{\partial t} \tilde{q}_{NS}(j,t) = \frac{\alpha_s(t)}{2\pi} \underbrace{\int_0^1 dx\, x^{j-1} \int_x^1 \frac{d\xi}{\xi} P_{qq}\left(\frac{x}{\xi}, \alpha_s\right) \int_{-i\infty}^{\infty} \frac{dj'}{2\pi i} \xi^{-j'} \tilde{q}_{NS}(j',t)}$$

$$\gamma_{qq}(j,t)\tilde{q}_{NS}(j,t) = \int_0^1 dz\, z^{j-1} P_{qq}(z,\alpha_s)\, \tilde{q}_{NS}(j,t)$$

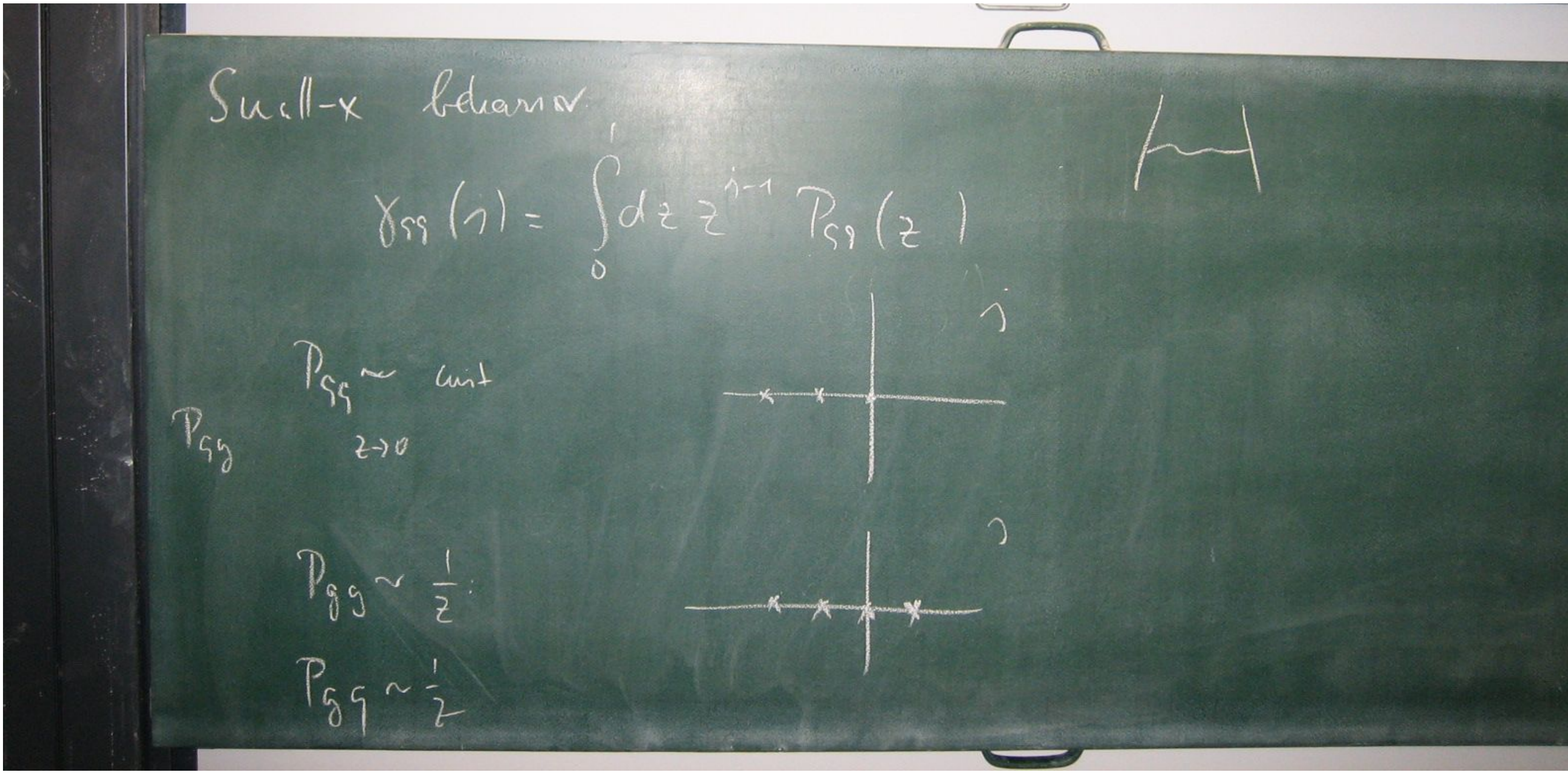
$$t \frac{\partial}{\partial t} \tilde{q}_{NS}(j,t) = \frac{\alpha_s(t)}{2\pi} \gamma_{qq}(j,t) \cdot \tilde{q}_{NS}(j,t)$$


$\int_0^1 dx\, x^{\nu-1} \int_0^1 \frac{d\xi}{\xi} P_{gg}(\frac{x}{\xi}, \alpha_s)\, \Theta(\xi - x) \int \frac{d\nu'}{2\pi i} \xi^{-\nu'}$
$x < \xi$
$\frac{x}{\xi} < 1$
$z = \frac{x}{\xi}$
$\int_0^1 dz\, z^{\nu-1} P_{gg}(z, \alpha_s) \int_0^1 \frac{d\xi}{\xi} \xi^{\nu} \int \frac{d\nu'}{2\pi i} \xi^{-\nu'}$
$\int_0^1 \frac{d\xi}{\xi} \int \frac{d\nu'}{2\pi i} \xi^{\nu-\nu'}$
$\int d\nu' \delta(\nu - \nu')$
$\tau = \ln \xi$
$\frac{d\xi}{\xi} = d\tau$


$\frac{\partial}{\partial \ln t} \tilde{q}_{NS}(j,t) = \frac{\alpha_s(t)}{2\pi} \gamma_{qq}(j, \alpha_s(t)) \, \tilde{q}_{NS}(j,t)$
$(Q^2)^{n+d}$
LO: $\tilde{q}_{NS}(j,t) = \tilde{q}_{NS}(j,t_0) \left(\frac{\alpha_s(t_0)}{\alpha_s(t)} \right)^{\frac{\gamma_{qq}(j)}{2\pi b}}$
$f'(x) = a(x) f(x)$
$f(x) = f(x_0) \exp \int_{x_0}^{x} dx' a(x')$
$t = \ln Q^2$: $\alpha_s(t) = \frac{1}{b \ln Q^2/\Lambda^2}$
$\frac{1}{2\pi} \int_{t_0}^{t} dt' \gamma_{qq}(j, \alpha_s(t'))$
$= \frac{1}{2\pi} \gamma_{qq}(j) \int_{t_0}^{t} dt' \frac{1}{b(t' - \ln \Lambda^2)}$



$= \frac{1}{2\pi b}\,\gamma_{gg}(\gamma)\ \ln\frac{t-\ln\Lambda^2}{t_0-\ln\Lambda^2}$
$= \frac{\gamma_{gg}}{2\pi b}\ \ln\left(\frac{\ln\frac{Q^2}{\Lambda^2}}{\ln\frac{Q_0^2}{\Lambda^2}}\right)$
$\exp\frac{1}{2\pi b}\,\gamma_{gg}(\gamma)\ \ln\left(\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}\right) = \left(\frac{\alpha_s(t_0)}{\alpha_s(t)}\right)^{\frac{\gamma_{gg}(\gamma)}{2\pi b}}$


Small-x behavior
$\gamma_{gg}(n) = \int_0^1 dz\, z^{n-1} P_{gg}(z)$
$P_{gg} \sim$ const
$z \to 0$
P_{gg}
n
$P_{gg} \sim \frac{1}{z}$
n
$P_{gq} \sim \frac{1}{z}$