QCD and Collider Physics III:
Parton Branching and Parton Showers

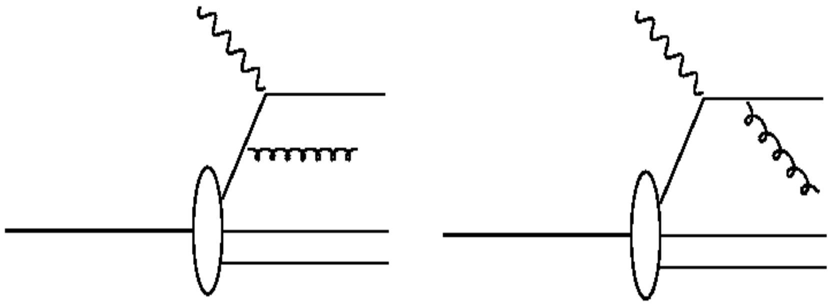
- collinear factorization: resumee
- from inclusive processes to final states
  - Kinematics of parton branching
  - collinear factorisation and DGLAP for final states
    - Sudakov form factors
    - solving DGLAP – MC example
  - some other issues of parton branching:
    - soft gluon radiation and angular ordering

- Literature:
  Ellis, Stirling, Webber: *QCD and Collider Physics*
  Dissertori, Knowles, Schmelling: *QCD - High Energy Exp and Theory*
  Dokshitzer, Khoze, Mueller, Troyan: *Basics of perturbative QCD*

http://www-h1.desy.de/~jung/qcd_collider_physics_wise_2006

Collinear Factorisation: P_{qq}

$$
\begin{aligned}
|ME|^2 &= 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{4}{3}\left[\frac{-\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} + \frac{2\hat{u}Q^2}{\hat{s}\hat{t}}\right] \\
&= 32\pi^2 \left(e_q^2 \alpha \alpha_s\right) \frac{4}{3}\frac{-1}{t}\left[\frac{Q^2(1+z^2)}{z(1-z)} + \cdots\right]
\end{aligned}
$$



QCDC OCDC

$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi}\frac{1}{k_\perp^2}\left[P_{qq}(z) + \cdots\right]$$

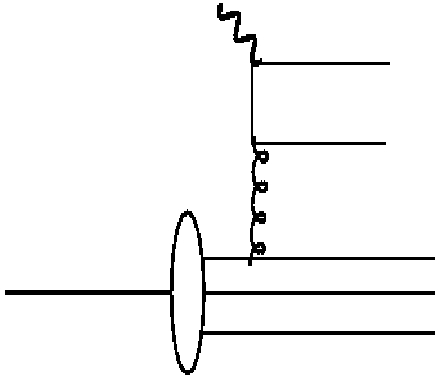
$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad \sigma_0 = \frac{4\pi^2\alpha}{\hat{s}}$$

- integrate over *kt* generates *log,* BUT what is the lower limit

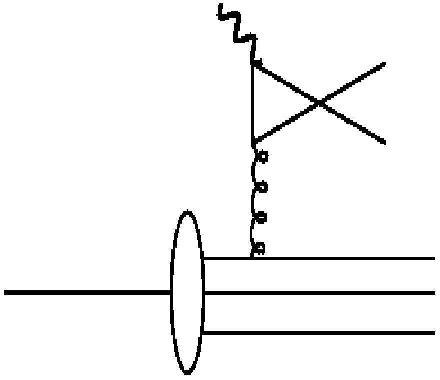
$$\sigma^{QCDC} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi}\left[P_{qq}(z)\log\left(\frac{Q^2(1-z)}{\chi^2 z}\right) + \cdots\right]$$

Collinear Factorisation: P_{qg}

$$|ME|^2 = 32\pi^2 \left(e_q^2 \alpha\alpha_s\right) \frac{1}{2} \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}}\right]$$



BGF



BGF

$$\frac{d\sigma}{dk_\perp^2} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \frac{1}{k_\perp^2} \left[P_{qg}(z) + \cdots\right]$$

$$P_{qg}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

- integrate over k_t generates *log,* BUT what is the lower limit

$$\sigma^{BGF} = \sigma_0 e_q^2 \frac{\alpha_s}{2\pi} \left[P_{qg}(z) \log\left(\frac{Q^2(1-z)}{\chi^2 z}\right) + \cdots\right]$$

Collinear factorization: DGLAP

- introduce new scale $\mu^2 \gg \chi^2$ and include soft, non-perturbative physics into renormalised parton density:

$$q_i(x,\mu^2) = q_i^0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i^0(\xi)P_{qq}\left(\frac{x}{\xi}\right) + g^0(\xi)P_{qg}\left(\frac{x}{\xi}\right)\right]\log\left(\frac{\mu^2}{\chi^2}\right)$$

- **D**okshitzer **G**ribov **L**ipatov **A**ltarelli **P**arisi equation (take derivative of the above eq):

  V.V. Gribov and L.N. Lipatov Sov. J. Nucl. Phys. 438 and 675 (1972) 15, L.N. Lipatov Sov. J. Nucl. Phys 94 (1975) 20, G. Altarelli and G. Parisi Nucl.Phys.B 298 (1977) 126, Y.L. Dokshitser Sov. Phys. JETP 641 (1977) 46

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

- BUT there are also gluons....

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- DGLAP is the analgue to the beta function for running of the coupling

Collinear factorization (part 2)

see handbook of pQCD, chapter IV, B

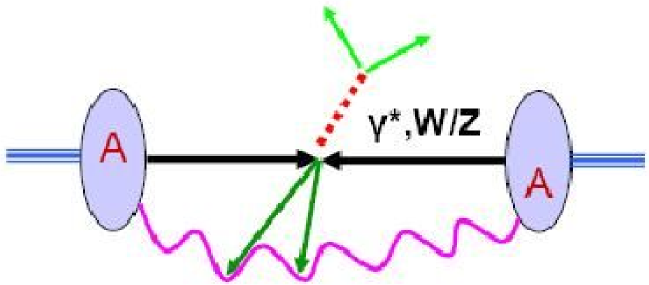
$$F_2^{(Vh)}(x,Q^2) = \sum_{i=f,\bar{f},G} \int_0^1 d\xi C_2^{(Vi)}\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \frac{\mu_f^2}{\mu^2}, \alpha_s(\mu^2)\right) \otimes f_{i/h}(\xi, \mu_f^2, \mu^2)$$

- Factorisation Theorem in DIS (Collins, Soper, Sterman, (1989) in Pert. QCD, ed. A.H. Mueller, Wold Scientific, Singapore, p1.)
  - ➔ generalisation of the parton model result
  - hard-scattering function $C_2^{(Vi)}$ is infrared finite and calculable in pQCD, depending only on vector boson *V*, parton *i*, and renormalization and factorization scales. It is independent of the identity of hadron *h*.
  - pdf $f_{i/h}(\xi, \mu_f^2, \mu^2)$ contains all the infrared sensitivity of cross section, and is specific to hadron h, and depends on factorization scale. It is universal and independent of hard scattering process.
- Generalisation: applies to any DIS cross section defined by a sum over hadronic final states .... but be careful what it really means....
- explicit factorisation theorems exist for:
  - diffractive DIS (... see above....)
  - Drell Yan (in hadron hadron collisions)
  - single particle inclusive cross sections (fragmentation functions)

Factorization is an approximation !!!

Factorization is an approximation

- ❑ Drell-Yan cross section is **NOT** completely factorized!

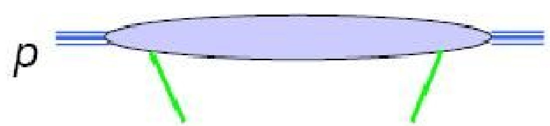


- ❖ There is **always** soft gluon interaction between two hadrons!
- ❖ Gluon field strength is **one power** more Lorentz contracted than ruler

$$\frac{d\sigma}{dQ^2} = f^{(2)} \otimes f^{(2)} \otimes \frac{d\hat{\sigma}^{(2)}}{dQ^2} + \frac{1}{Q^2} f^{(2)} \otimes f^{(4)} \otimes \frac{d\hat{\sigma}^{(4)}}{dQ^2} + \frac{1}{Q^4} F\left(\frac{Q^2}{S}\right) + \ldots$$

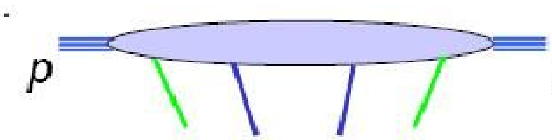
Not factorized!

$$f^{(2)} \propto \langle p|\bar{\psi}(0)\gamma^+\psi(y^-)|p\rangle, \quad \langle p|F^{+\alpha}(0)F_\alpha^{\ +}(y^-)|p\rangle$$



$$f^{(4)} \propto \langle p|\bar{\psi}(0)\gamma^+F^{+\alpha}(y_1^-)F_\alpha^{\ +}(y_2^-)\psi(y^-)|p\rangle$$

...



September 28, 2005 4 Jianwei Qiu, ISU

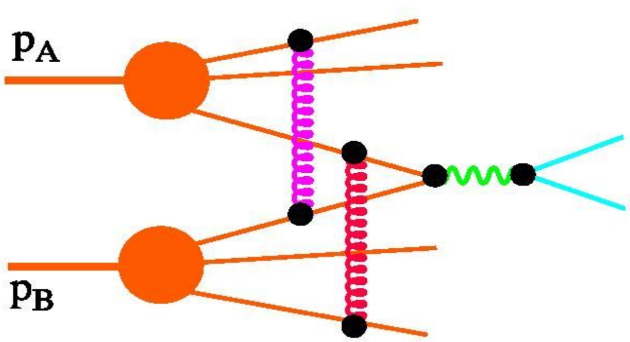
Factorization proofs and all that ...

- About factorization proofs (Wu-Ki Tung, pQCD and the parton structure of the nucleon, 2001, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 2* 887-971)

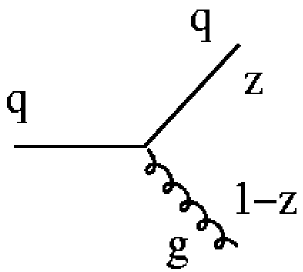
tions $F_a^{\cdot}(x, \frac{\cdot}{m}, \alpha_s(\mu))$ (a = all parton flavors). Although the underlying physical ideas are relatively simple, as emphasized in the last two sections, the mathematical proofs are technically very demanding.[7,15,19] For this reason, actual proofs of factorization only exist for a few hard processes; and certain proofs (e.g. that for the Drell-Yan process) stayed controversial for some time before a consensus were reached.[15] Because of the general character of the physical ideas and the mathematical methods involved, however, it is generally *assumed* that the attractive *quark-parton model does apply to all high energy interactions* with at least one large energy scale.

$$\frac{d\sigma}{dy} = \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B f_A^a(\xi_A,\mu) f_B^b(\xi_B,\mu) \frac{d\hat{\sigma}_{ab}(\mu)}{dy} + \mathcal{O}\left(\left(\frac{m}{P}\right)^p\right)$$

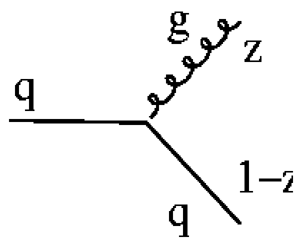
- The problem with Drell-Yan: initial state interactions...
- factorization here does not hold graph-by-graph but only for all ....



Splitting functions in lowest order

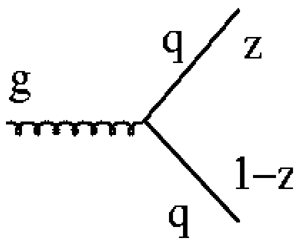


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

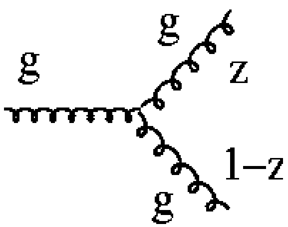


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to photon radiation from electron



$$P_{qg} = \frac{1}{2}\left(z^2+(1+z)^2\right)$$



$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

Splitting functions at higher orders

- Splitting functions have perturbative expansion in the running coupling:

$$P(z, \alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

The calculation (in a nut shell)

S. Moch, HERA-LHC workshop, June 2004

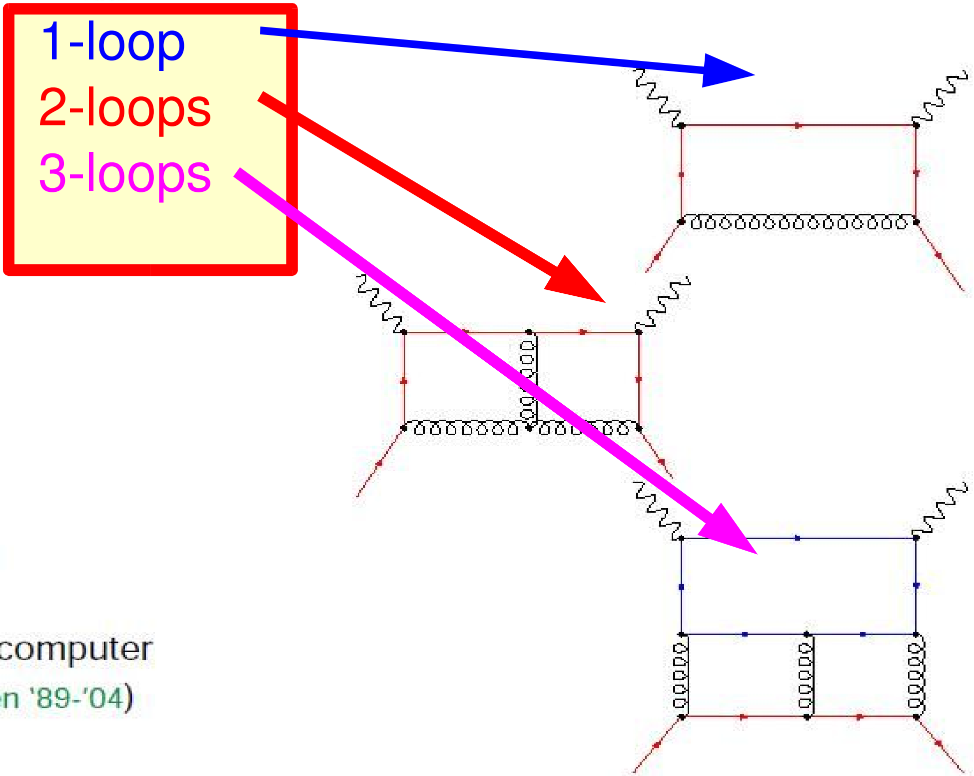
– Calculate anomalous dimensions (Mellin moments of splitting functions)
⟶ divergence of Feynman diagrams in dimensional regularization $D = 4 - 2\varepsilon$

$$\gamma_{ij}^{(n)}(N) = -\int_0^1 dx\, x^{N-1} P_{ij}^{(n)}(x)$$

– **One-loop** Feynman diagrams
⟶ in total 18 for $\gamma_{ij}^{(0)}$ / $P_{ij}^{(0)}$
(pencil + paper)

– **Two-loop** Feynman diagrams
⟶ in total 350 for $\gamma_{ij}^{(1)}$ / $P_{ij}^{(1)}$
(simple computer algebra)

– **Three-loop** Feynman diagrams
⟶ in total 9607 for $\gamma_{ij}^{(2)}$ / $P_{ij}^{(2)}$
(cutting edge technology ⟶ computer algebra system FORM Vermaseren '89-'04)



Splitting functions (cont'd)

- Splitting functions have perturbative expansion in the running coupling:

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

LO and NLO singlet splitting functions

S. Moch, HERA-LHC workshop, June 2004

$$\begin{aligned}
P^{(0)}_{\rm ps}(x) &= 0 \\
P^{(0)}_{\rm qg}(x) &= 2 n_f\, p_{\rm qg}(x) \\
P^{(0)}_{\rm gq}(x) &= 2 C_F\, p_{\rm gq}(x) \\
P^{(0)}_{\rm gg}(x) &= C_A \Big(4 p_{\rm gg}(x) + \frac{11}{3}\delta(1-x)\Big) - \frac{2}{3} n_f \delta(1-x)
\end{aligned}$$

$$P^{(1)}_{\rm ps}(x) = 4 C_F n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2\Big[\frac{8}{3}{\rm H}_0 - \frac{56}{9}\Big] + (1+x)\Big[5{\rm H}_0 - 2{\rm H}_{0,0}\Big]\Big)$$

$$\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4 C_A n_f \Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2 p_{\rm qg}(-x){\rm H}_{-1,0} - 2 p_{\rm qg}(x){\rm H}_{1,1} + x^2\Big[\frac{44}{3}{\rm H}_0 - \frac{218}{9}\Big] \\
&+4(1-x)\Big[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1\Big] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0\Big) + 4 C_F n_f \Big(2 p_{\rm qg}(x)\Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 \\
&-\zeta_2\Big] + 4x^2\Big[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2}\Big] + 2(1-x)\Big[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4}\Big] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0\Big)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4 C_A C_F \Big(\frac{1}{x} + 2 p_{\rm gq}(x)\Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1\Big] - x^2\Big[\frac{8}{3}{\rm H}_0 - \frac{44}{9}\Big] + 4\zeta_2 - 2 \\
&-7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\Big[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9}\Big] - 2 p_{\rm gq}(-x){\rm H}_{-1,0}\Big) - 4 C_F n_f \Big(\frac{2}{3}x \\
&- p_{\rm gq}(x)\Big[\frac{2}{3}{\rm H}_1 - \frac{10}{9}\Big]\Big) + 4 C_F^{\,2}\Big(p_{\rm gq}(x)\Big[3{\rm H}_1 - 2{\rm H}_{1,1}\Big] + (1+x)\Big[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0\Big] - 3{\rm H}_{0,0} \\
&+1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x\Big)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4 C_A n_f \Big(1 - x - \frac{10}{9} p_{\rm gg}(x) - \frac{13}{9}\Big(\frac{1}{x} - x^2\Big) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x)\Big) + 4 C_A^{\,2}\Big(27 \\
&+(1+x)\Big[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2}\Big] + 2 p_{\rm gg}(-x)\Big[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2\Big] - \frac{67}{9}\Big(\frac{1}{x} - x^2\Big) - 12{\rm H}_0 \\
&-\frac{44}{3}x^2{\rm H}_0 + 2 p_{\rm gg}(x)\Big[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2\Big] + \delta(1-x)\Big[\frac{8}{3} + 3\zeta_3\Big]\Big) + 4 C_F n_f \Big(2{\rm H}_0 \\
&+\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\Big[4 - 5{\rm H}_0 - 2{\rm H}_{0,0}\Big] - \frac{1}{2}\delta(1-x)\Big)\,.
\end{aligned}$$

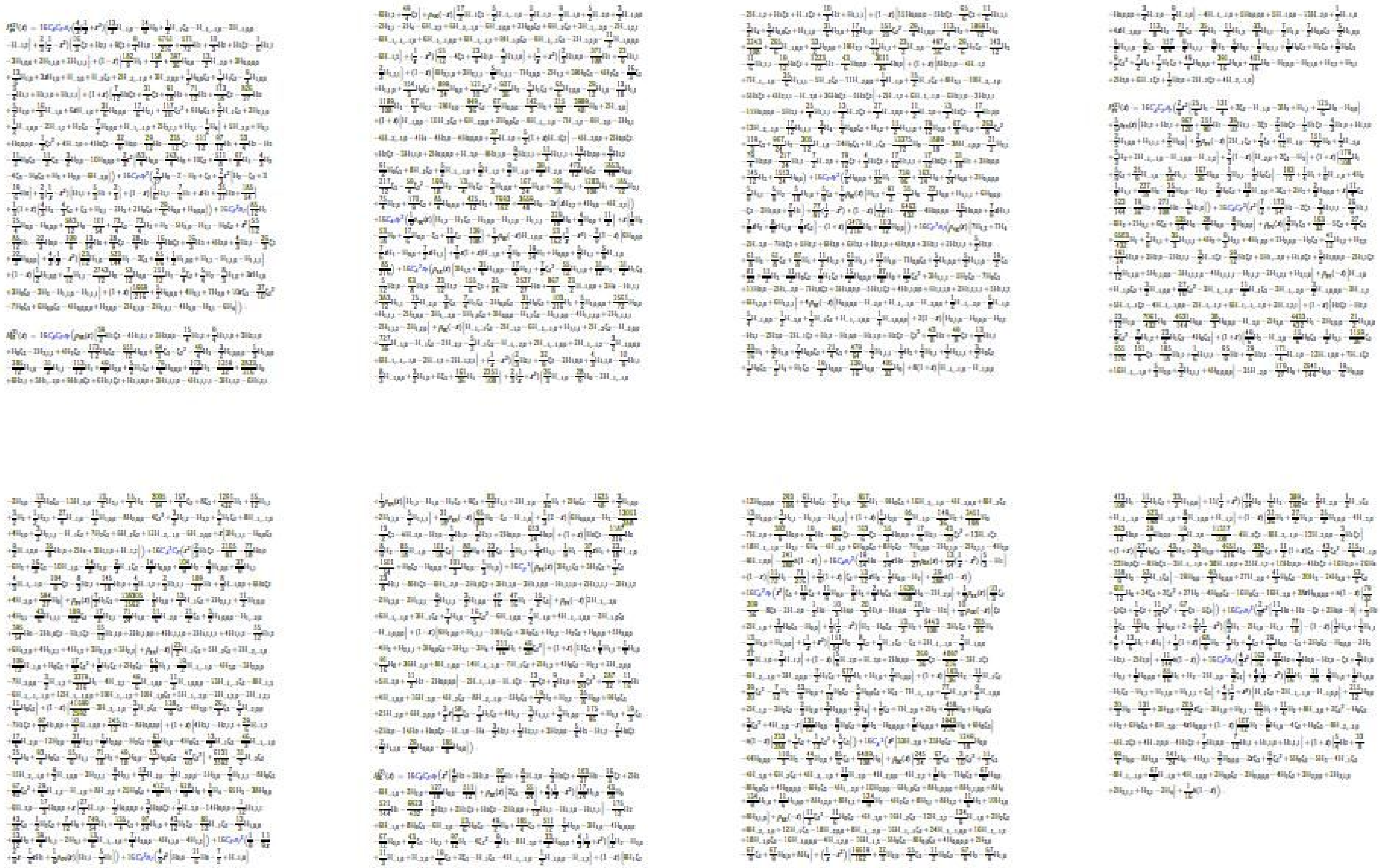
Splitting functions (cont'd)

- Splitting functions have perturbative expansion in the running coupling:

NNLO singlet splitting functions

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \ldots$$

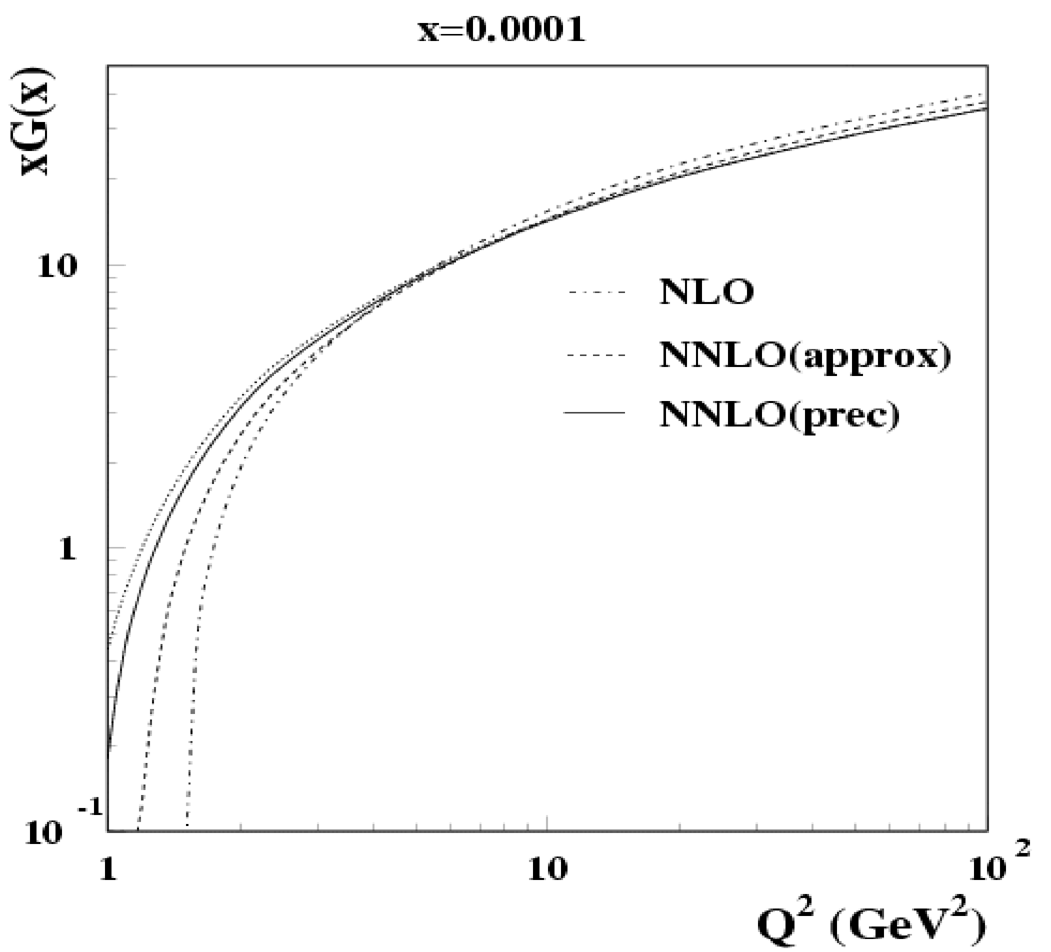
S. Moch, HERA-LHC workshop, June 2004



Gluon distribution at higher orders

- using different approximations to splitting functions results in different behavior of parton distributions
- observe negative gluon distribution at small x
- higher order corrections are important
- behavior at small/medium Q2 changes significantly when using higher order corrections ....

from HERA-LHC workshop proceedings: S.I. Alekhin



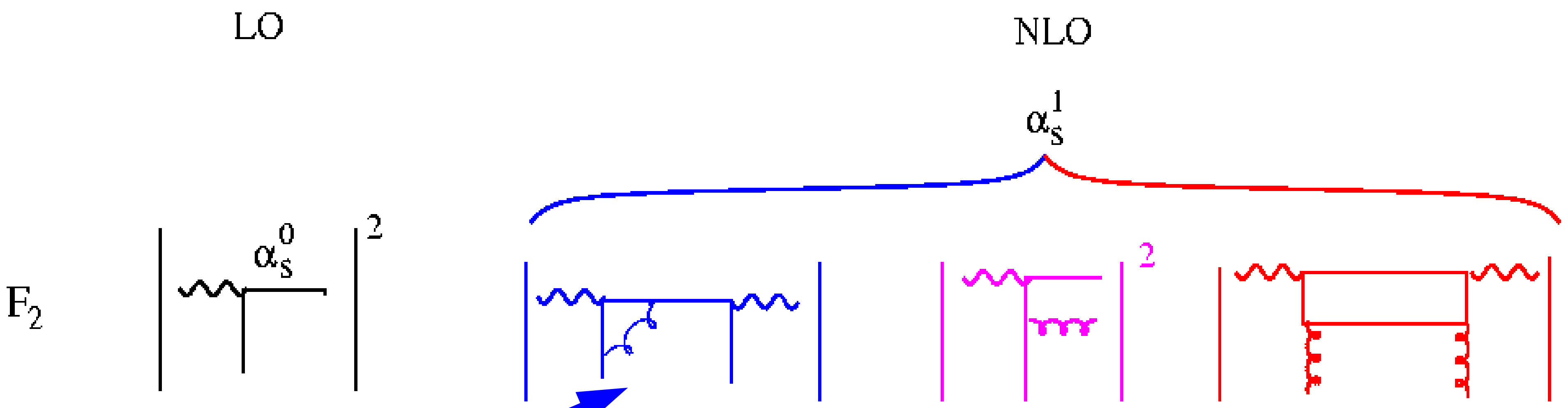
Evolution kernels – splitting functions

- some of the splitting functions are also divergent... $\frac{1}{1-z}$
- use *plus-distribution* to avoid dangerous region:

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z}$$

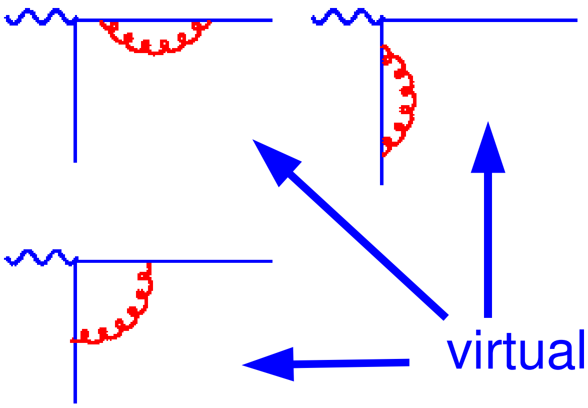
- divergence canceled by virtual corrections ...
- use splitting functions with *plus-distribution*

NLO contributions to $F_2(x,Q^2)$



virtual corrections cancel
soft singularity

$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$



virtual corrections to QPM

$$P_{qg} = \frac{1}{2}\left(z^2 + (1+z)^2\right)$$

no soft singularity

Solving DGLAP equations ...

- Different methods to solve integro-differential equations
  - brute-force (BF) method (M. Miyama, S. Kumano CPC 94 (1996) 185)

$$\frac{df(x)}{dx} = \frac{f(x)_{m+1} - f(x)_m}{\Delta x_m} \qquad \int f(x)dx = \sum f(x)_m \Delta x_m$$

  - Laguerre method (S. Kumano J.T. Londergan CPC 69 (1992) 373, and L. Schoeffel Nucl.Instrum.Meth.A423:439-445,1999)
  - Mellin transforms (M. Glueck, E. Reya, A. Vogt Z. Phys. C48 (1990) 471)
  - QCDNUM: calculation in a grid in x,Q2 space (M. Botje Eur.Phys.J. C14 (2000) 285-297)
  - CTEQ evolution program in x,Q2 space: http://www.phys.psu.edu/~cteq/
  - QCDFIT program in x,Q2 space (C. Pascaud, F. Zomer, LAL preprint LAL/94-02, H1-09/94-404,H1-09/94-376)
  - MC method using Markov chains (S. Jadach, M. Skrzypek hep-ph/0504205)
  - Monte Carlo method from iterative procedure
- brute-force method and MC method are best suited for detailed studies of branching processes !!!

Divergencies again...

- collinear divergencies factored into renormalized parton distributions
- what about soft divergencies ? $z \to 1$

treated with "plus" prescription

$$\frac{1}{1-z} \to \frac{1}{1-z}_+ \quad \text{with} \quad \int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}$$

- soft divergency treated with Sudakov form factor:

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int^{z_{max}} dz \frac{\alpha_s}{2\pi} \tilde{P}(z)\right]$$

resulting in

$$t \frac{\partial}{\partial t}\left(\frac{f}{\Delta}\right) = \frac{1}{\Delta} \int^{z_{max}} \frac{dz}{z} \frac{\alpha_s}{2\pi} \tilde{P}(z) f(x/z, t)$$

and

$$f(x,t) = \Delta(t) f(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int^{z_{max}} \frac{dz}{z} \frac{\alpha_s}{2\pi} \tilde{P}(z) f(x/z, t')$$

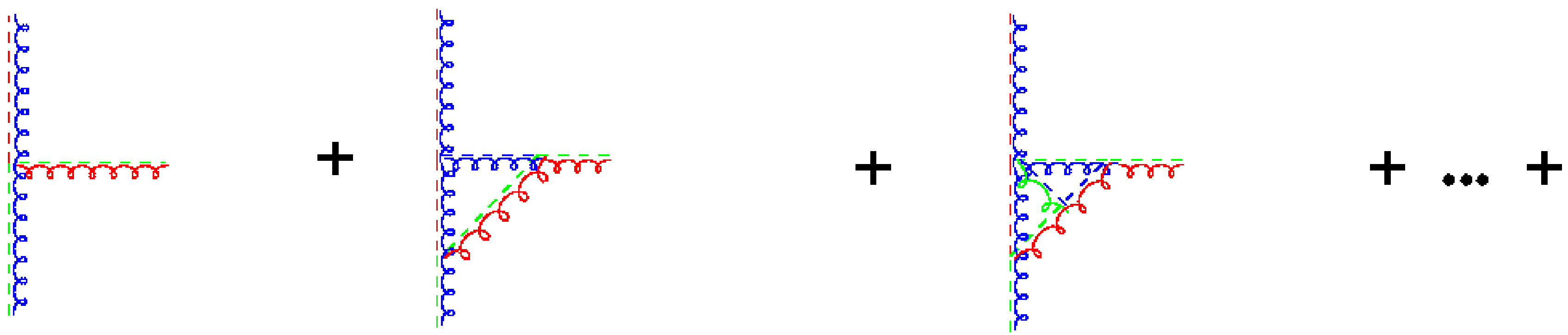
Sudakov form factor: all loop resum...

$g \to gg$ Splitting Fct $\tilde{P}(z) = \frac{\bar{\alpha}_s}{1-z} + \frac{\bar{\alpha}_s}{z} + ...$

- Sudakov form factor .... all loop resummation

$$\Delta_S = \exp\left(- \int dz \int \frac{dq'}{q'} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)$$

$$\Delta_S = 1 + \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^1 + \frac{1}{2!} \left(- \int dz \int \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 ...$$



$$\tilde{P}(z) \left[1 - \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) + \frac{1}{2!} \left(- \int \int dz \frac{dq}{q} \frac{\alpha_s}{2\pi} \tilde{P}(z) \right)^2 - ... - \right]$$

DGLAP evolution again....

- differential form: $$t\frac{\partial}{\partial t} f(x,t) = \int \frac{dz}{z}\frac{\alpha_s}{2\pi} P_+(z)\, f\left(\frac{x}{z}, t\right)$$

- differential form using f/Δ_s with

$$\Delta_s(t) = \exp\left(-\int_x^{z_{max}} dz \int_{t_0}^{t} \frac{\alpha_s}{2\pi}\frac{dt'}{t'}\tilde{P}(z)\right)$$

$$t\frac{\partial}{\partial t}\frac{f(x,t)}{\Delta_s(t)} = \int \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{\tilde{P}(z)}{\Delta_s(t)} f\left(\frac{x}{z}, t\right)$$

- integral form

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z}\int \frac{dt'}{t'}\cdot\frac{\Delta_s(t)}{\Delta_s(t')}\tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

no – branching probability from t_0 to t

Solving integral equations

- Integral equation of *Fredholm type*:
- solve it by iteration (Neumann series):

$$\phi(x) = f(x) + \lambda \int_a^b K(x,y)\phi(y)dy$$

$$\begin{aligned}
\phi_0(x) &= f(x) \\
\phi_1(x) &= f(x) + \lambda \int_a^b K(x,y)f(y)dy \\
\phi_2(x) &= f(x) + \lambda \int_a^b K(x,y_1)f(y_1)dy_1 + \lambda^2 \int_a^b \int_a^b K(x,y_1)K(y_1,y_2)f(y_2)dy_2dy_1 \\
\phi_n(x) &= \sum_{i=0}^{n} \lambda^i u_i(x) \\
u_0(x) &= f(x) \\
u_1(x) &= \int_a^b K(x,y)f(y)dy \\
u_n(x) &= \int_a^b \int_a^b K(x,y_1)K(y_1,y_2)\cdots K(y_{n-1},y_n)f(y_n)dy_2\cdots dy_n
\end{aligned}$$

with the solution: $$\phi(x) = \lim_{n\to\infty} q_n(x) = \lim_{n\to\infty} \sum_{i=0}^{n} \lambda^i u_i(x)$$

DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

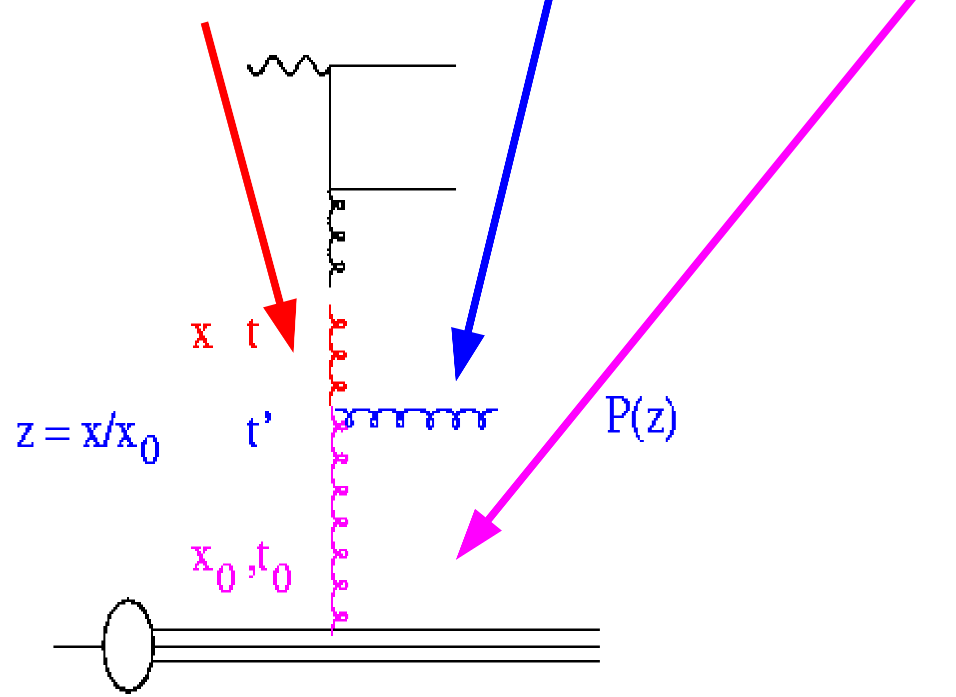
$$f_0(x,t) = f(x,t_0)\Delta(t)$$

$$f_1(x,t) = f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t')$$

from t' to t w/o branching

branching at t'

from t_0 to t' w/o branching



DGLAP re-sums leading logs...

$$f(x,t) = f(x,t_0)\Delta_s(t) + \int \frac{dz}{z} \int \frac{dt'}{t'} \cdot \frac{\Delta_s(t)}{\Delta_s(t')} \tilde{P}(z) f\left(\frac{x}{z}, t'\right)$$

- solve integral equation via iteration:

from t' to t w/o branching

branching at t'

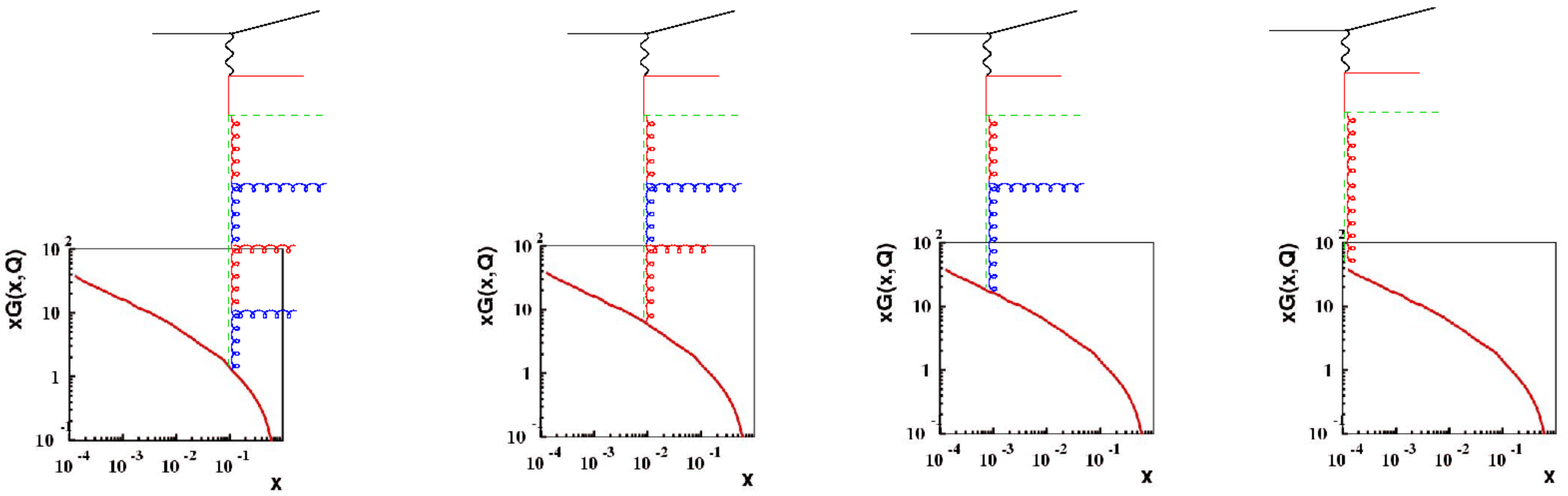
from t_0 to t' w/o branching

$$\begin{aligned}
f_0(x,t) &= f(x,t_0)\Delta(t) \\
f_1(x,t) &= f(x,t_0)\Delta(t) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \tilde{P}(z) f(x/z, t_0)\Delta(t') \\
&= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) \\
f_2(x,t) &= f(x,t_0)\Delta(t) + \log\frac{t}{t_0} A \otimes \Delta(t) f(x/z,t_0) + \\
& \frac{1}{2}\log^2\frac{t}{t_0} A \otimes A \otimes \Delta(t) f(x/z,t_0) \\
f(x,t) &= \lim_{n\to\infty} f_n(x,t) = \lim_{n\to\infty} \sum_n \frac{1}{n!} \log^n\left(\frac{t}{t_0}\right) A^n \otimes \Delta(t) f(x/z,t_0)
\end{aligned}$$

DGLAP re-sums $\log t$ **to all orders !!!!!!!!!!!!!!!!**

DGLAP evolution equation... again...

- for fixed x and Q^2 chains with different branchings contribute
- iterative procedure, spacelike parton showering

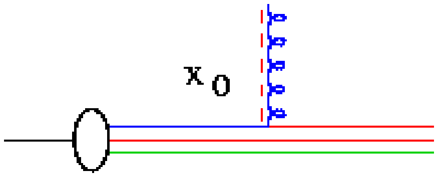


- $$f(x,t) = f_0(x,t_0)\Delta_s(t) + \sum_{k=1}^{\infty} f_k(x_k,t_k)$$

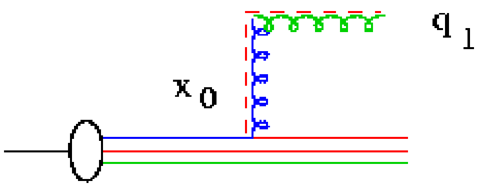
Parton showers for the initial state

spacelike (Q<0) parton shower evolution

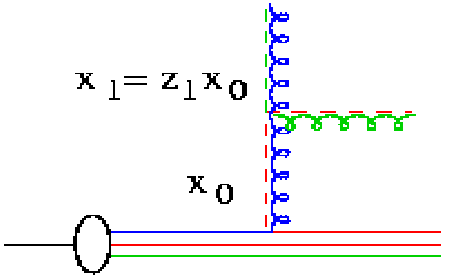
- starting from hadron (fwd evolution)
  or from hard scattering (bwd evolution)



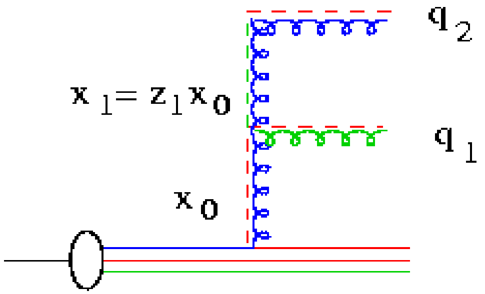
- select q_1 from Sudakov form factor



- select z_1 from splitting function



- select q_2 from Sudakov form factor

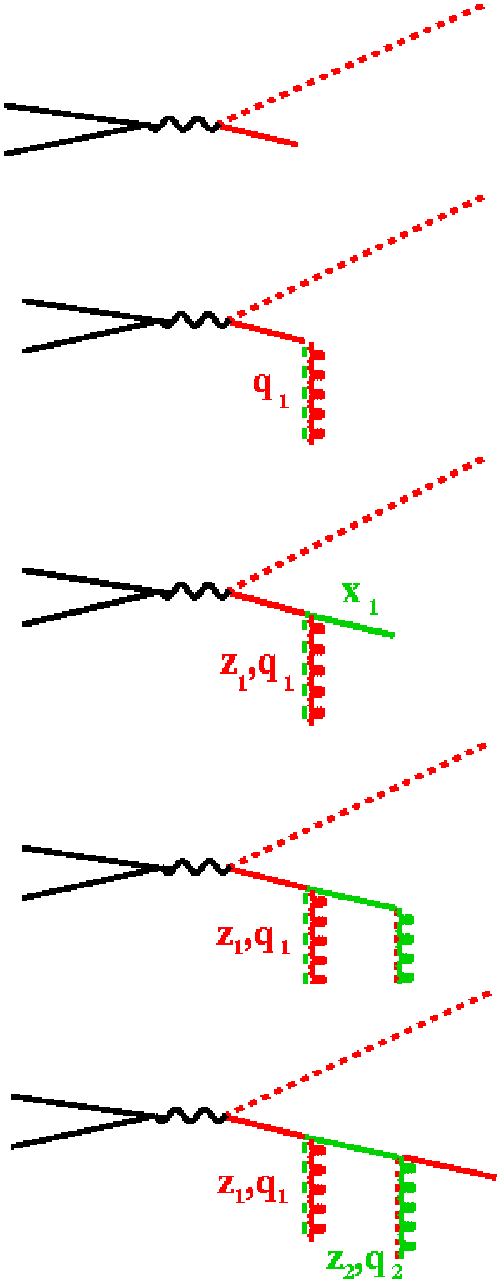


- select z_2 from splitting function
- stop evolution if $q_2 > Q_{hard}$

Parton Showers for the final state

timelike parton shower evolution

- starting with hard scattering
- select q_1 from Sudakov form factor
- select z_1 from splitting function
- select q_2 from Sudakov form factor
- select z_2 from splitting function
- stop evolution if $q_2 < q_0$

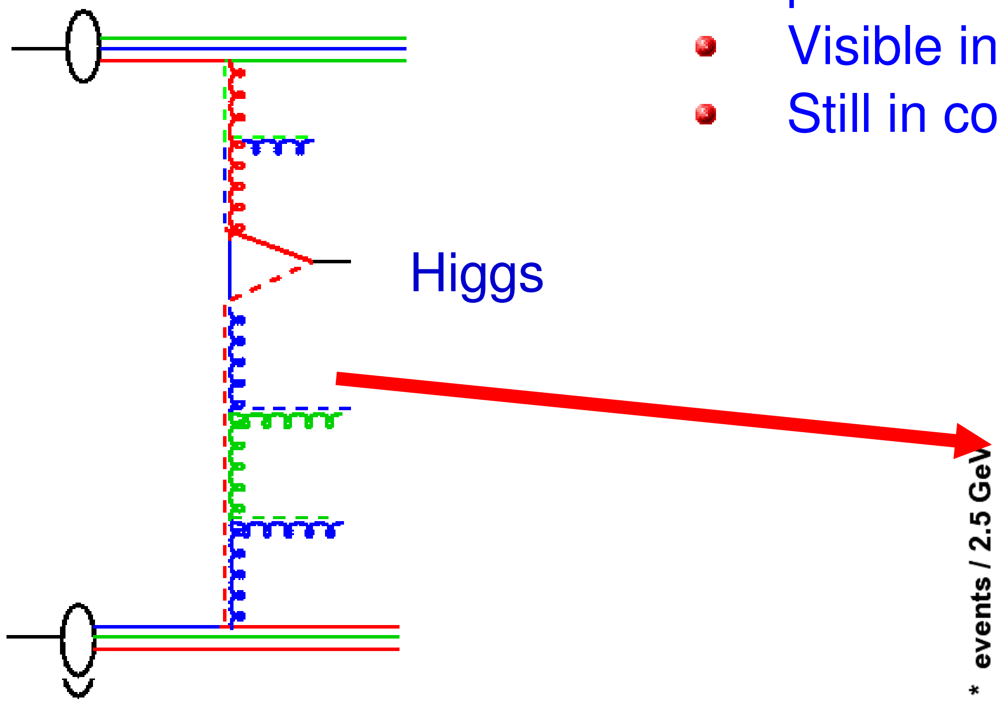


Parton Shower

- Evolution equation with Sudakov form factor recovers exactly evolution equation (with $_+$ prescription)
- Sudakov form factor particularly suited form Monte Carlo approach
- Sudakov form factor
  - ➔ gives probability for no-branching between q_0 and q
  - ➔ sums virtual contributions to all orders (via unitarity)
    - ➔ virtual (parton loop) and
    - ➔ real (non-resolvable) parton emissions
- need to specify scale of hard process (matrix element) $Q \sim p_t$
- need to specify cutoff scale $Q_0 \sim 1\ GeV$

Initial state parton shower: Higgs p_t

- Initial state parton showers generate pt of incoming partons
- Visible in pt of Higgs
- Still in collinear factorisation



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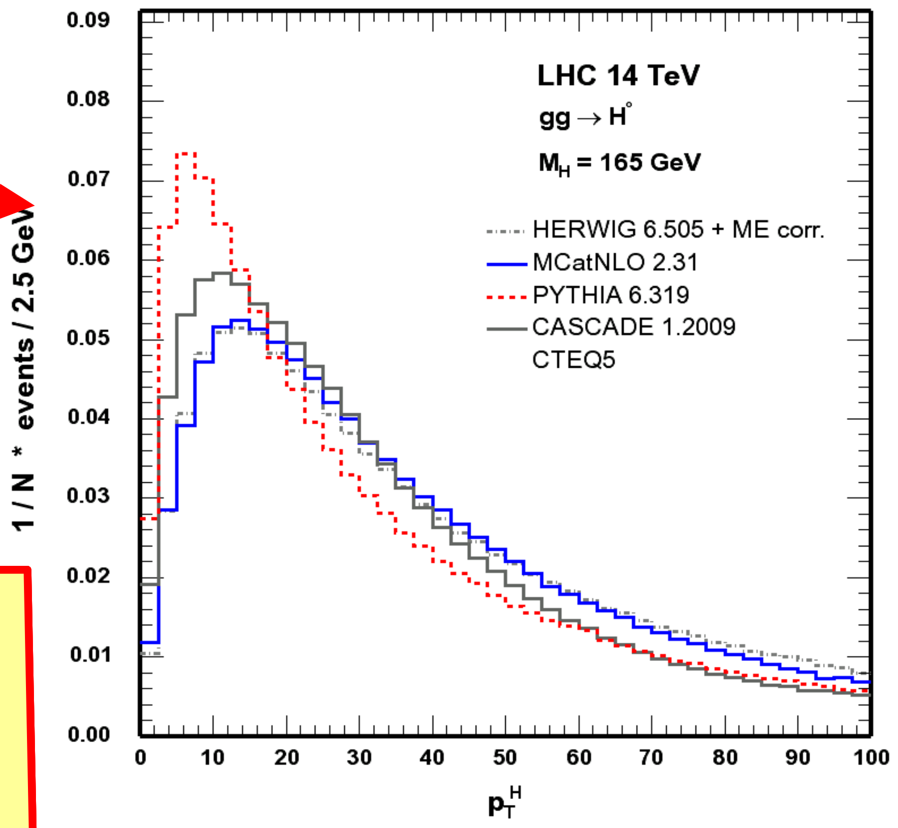

p_t distribution depends on details of initial state parton shower
➔ in producing reasonable results

Summary & Conclusion

- Collinear factorization is powerful tool to describe soft and collinear enhanced regions of phase space
- Beware .... factorization is only an approximation ...
- DGLAP formulation with Sudakov formfactor is exactly equivalent with Plus-prescription
- Sudakov form factor description has intuitive physical meaning
  - Can be used to generate parton cascade in a probabilistic approach
- Many details of parton branching can be studied explicitly:
  - choice of scale in alphas
  - evolution variable
  - effects of soft gluon enhancement ... angular ordering

- Parton branching and parton showering is important for proper description of final state observables