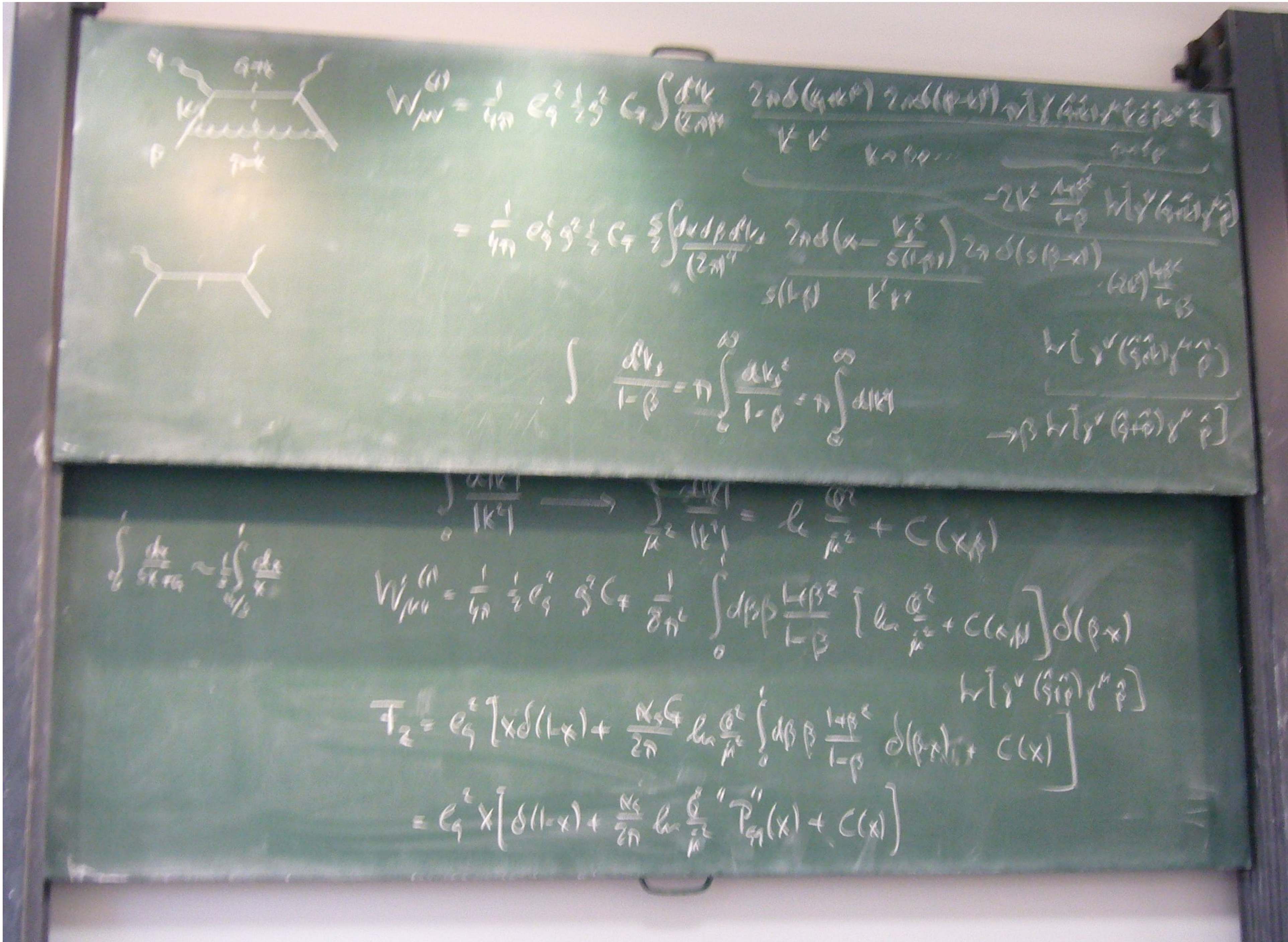
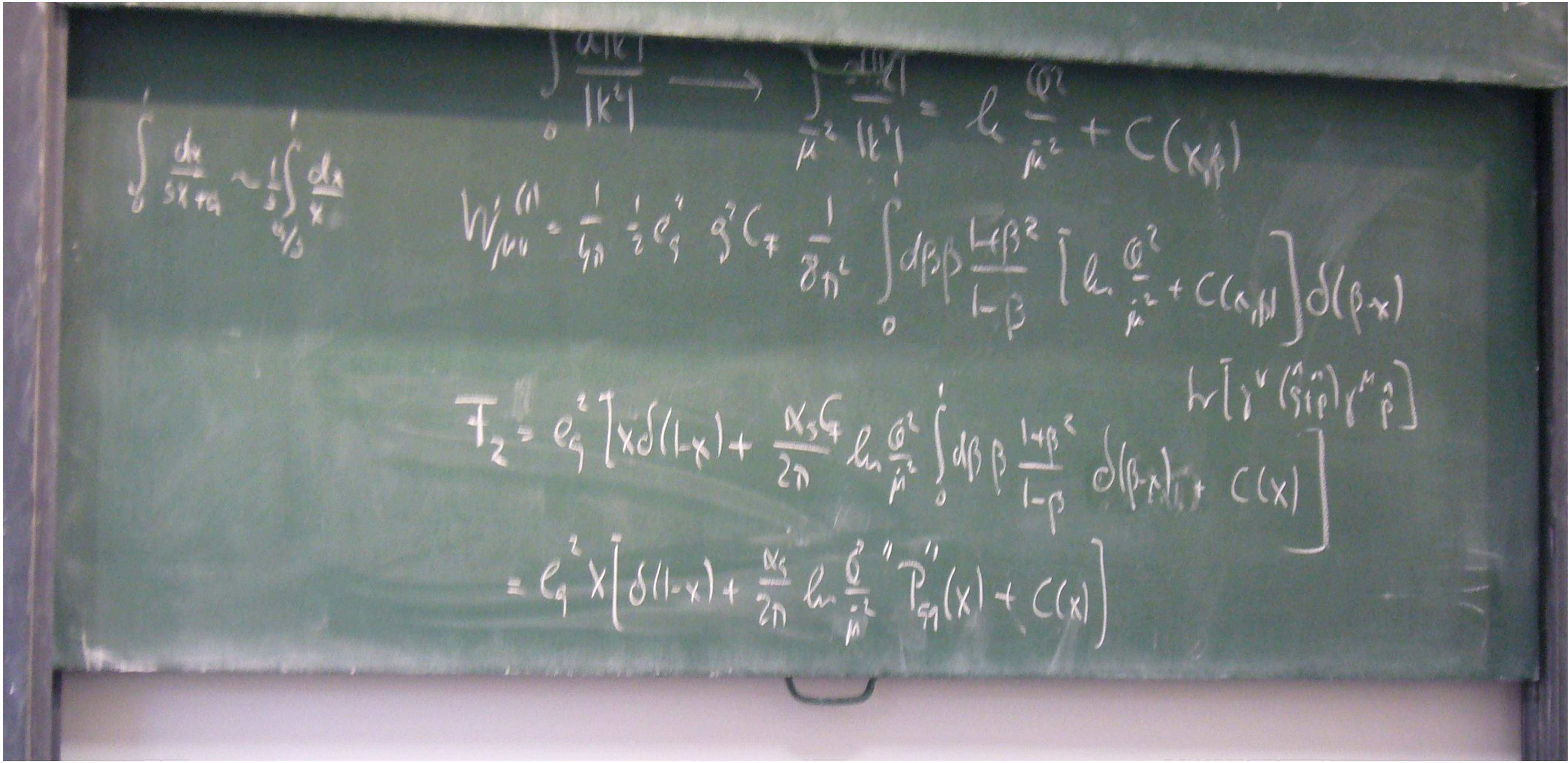

= e_q^2 x [δ(1-x) + α_s/2π ln Q²/μ² "P_qq"(x) + C(x)]



$\int_0^1 \frac{dx}{sx+a} \sim \frac{1}{s}\int_{a/s}^1 \frac{dx}{x}$
$\int_0 \frac{d|k|}{|k^2|} \longrightarrow \int_{\mu^2} \frac{d|k|}{|k^2|} = \ln\frac{Q^2}{\mu^2} + C(x,\beta)$
$W_{\mu\nu}^{(1)} = \frac{1}{4\pi}\frac{1}{2}e_q^2\, g^2 C_F \frac{1}{8\pi^2}\int_0^1 d\beta\,\beta\,\frac{1+\beta^2}{1-\beta}\left[\ln\frac{Q^2}{\mu^2}+C(x,\beta)\right]\delta(\beta-x)$
$\mathrm{tr}\left[\gamma^\nu(\hat{q}+\hat{p})\gamma^\mu\hat{p}\right]$
$F_2 = e_q^2\left[x\delta(1-x)+\frac{\alpha_s C_F}{2\pi}\ln\frac{Q^2}{\mu^2}\int_0^1 d\beta\,\beta\,\frac{1+\beta^2}{1-\beta}\,\delta(\beta-x)_+ + C(x)\right]$
$= e_q^2\, x\left[\delta(1-x)+\frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\,``P_{qq}"(x)+C(x)\right]$

$$0 = \int_0^1 d\beta \; P_{qq}(\beta) = \int_0^1 d\beta \left[{}^{"}P_{qq}{}^{"}(\beta) + \bar{\lambda}\delta(1-\beta) \right]$$

Conservation of probability

$${}^{'}P_{qq}{}^{'} = \frac{1+\beta^2}{1-\beta} + \bar{\lambda}\delta(1-\beta)$$

$$= \frac{1+\beta^2}{(1-\beta)_+} + \lambda\delta(1-\beta)$$

$$\lambda = 3/2$$

$$\int_0^1 dx \frac{f(x)}{(1-x)_+} \overset{def}{=} \int_0^1 dx \frac{f(x)-f(1)}{1-x}$$

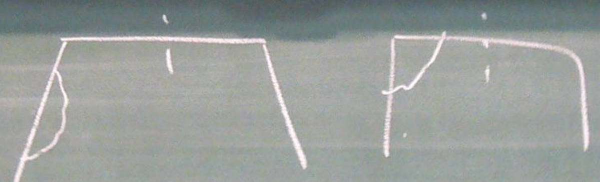
$$= \int_0^1 dx \frac{f(x)}{1-x} - \int_0^1 dx\,\delta(1-x) \frac{f(x)}{1-x}$$

DGLAP

Comments

(a) $\beta = 1$

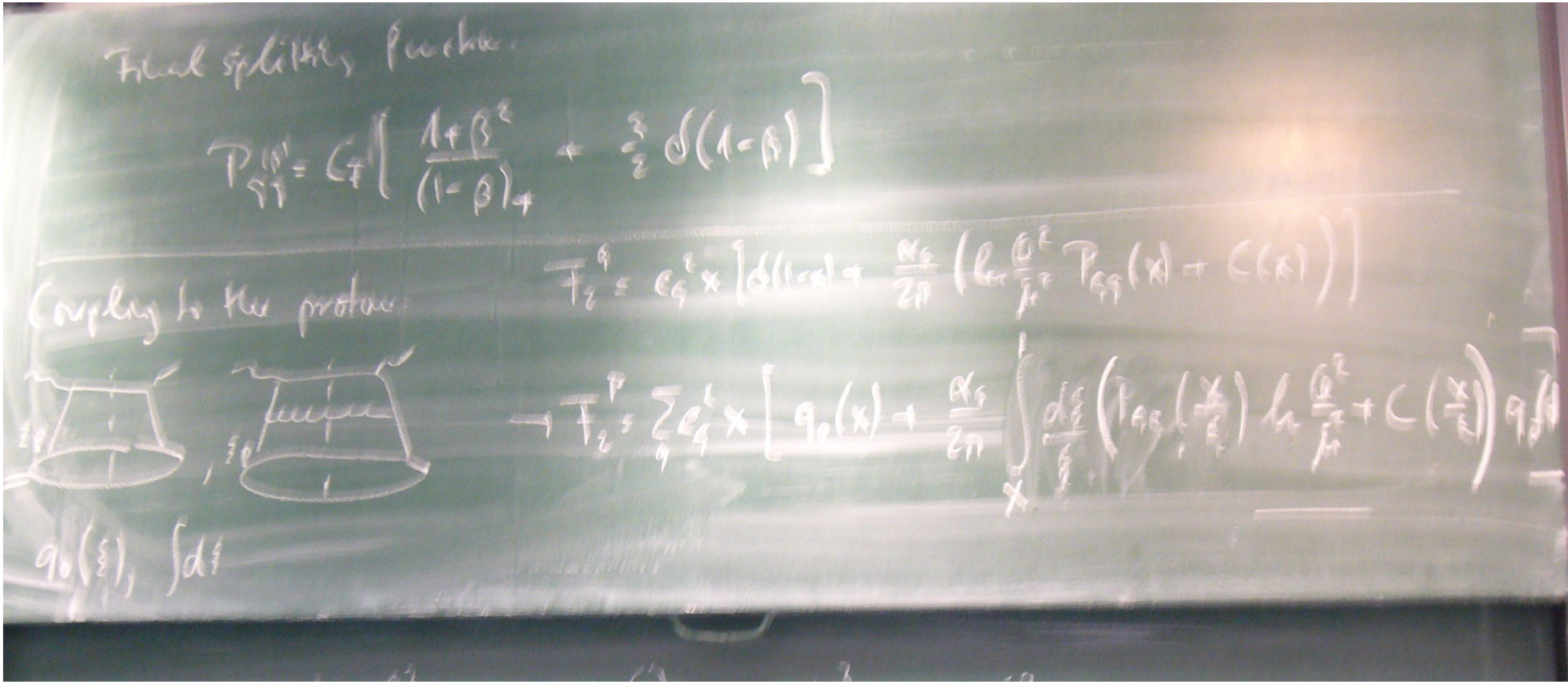
(b) running of α_s ?

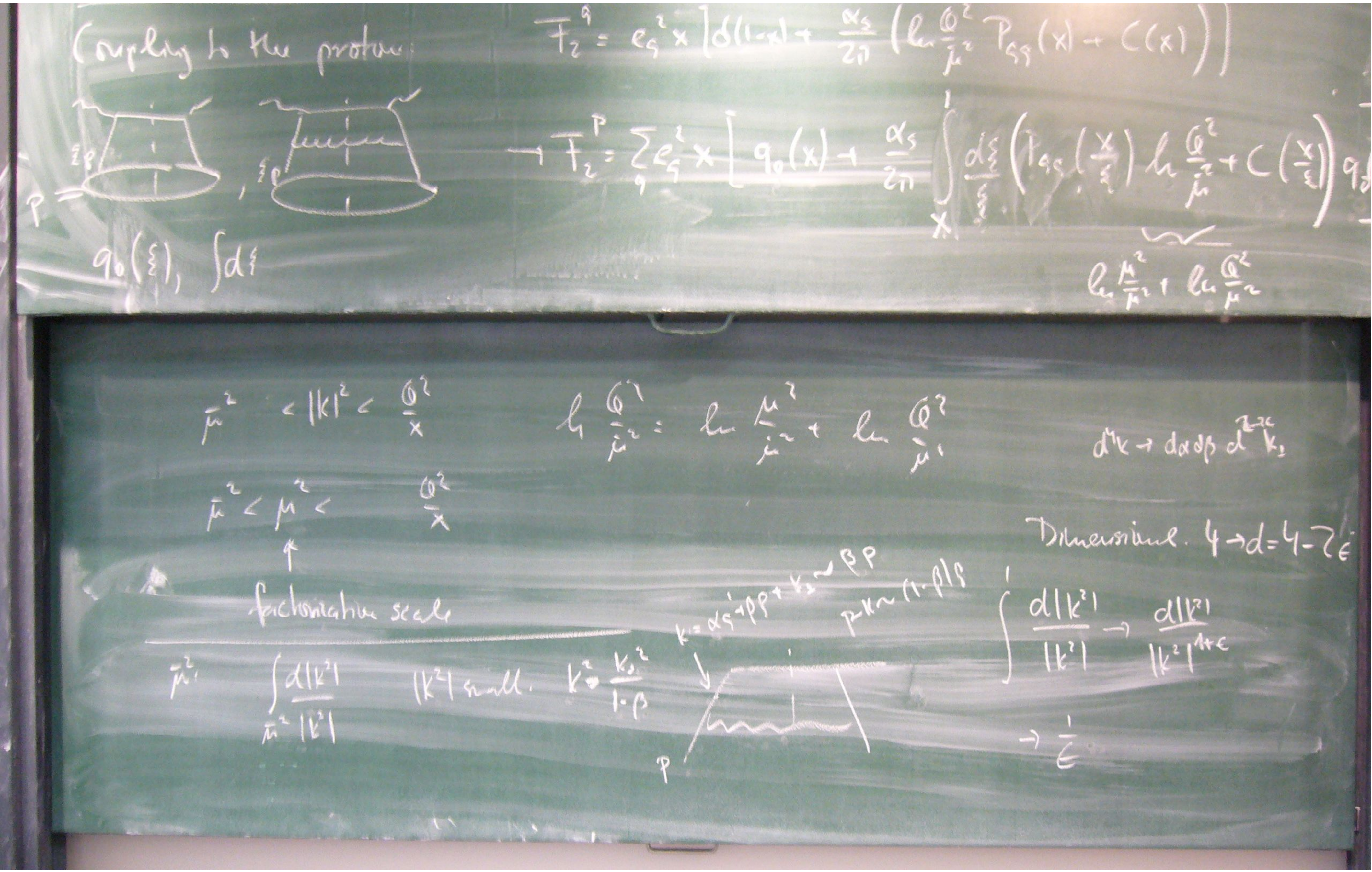


Answer: (b): $\alpha_s \to \alpha_s(Q^2) = \frac{1}{b \ln Q^2/\Lambda^2}$ $\quad b = (33-2n_f)/12\pi$

(a). $\beta \to 1$. other diagrams $\sim \delta(1-x)$

$${}^{"}P_{qq}{}^{"}(x) \to {}^{'}P_{qq}{}^{'}(x) + \lambda\delta(1-x)$$


Final splitting function:
$P_{qq}^{(0)} = C_F \left[\frac{1+\beta^2}{(1-\beta)_+} + \frac{3}{2}\delta(1-\beta) \right]$
Coupling to the proton:
$F_2^q = e_q^2 x \left[\delta(1-x) + \frac{\alpha_s}{2\pi}\left(\ln\frac{Q^2}{\mu^2} P_{qq}(x) + C(x) \right) \right]$
$\to F_2^p = \sum_q e_q^2 x \left[q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left(P_{qq}\left(\frac{x}{\xi}\right) \ln\frac{Q^2}{\mu^2} + C\left(\frac{x}{\xi}\right) \right) q_0(\xi) \right]$
$q_0(\xi)$, $\int d\xi$


Coupling to the proton
factorization scale
Dimensional. 4 → d = 4 - 2ε

$$q_0(x,\mu^2) = q_0(x) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left(\ln\frac{\mu^2}{\mu^2}P_{qq}\left(\frac{x}{\xi}\right) + C\left(\frac{x}{\xi}\right)\right) q_0(\xi)$$

$$F_2 = x\sum_q e_q^2\left[q_0(x,\mu^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\ln\frac{Q^2}{\mu^2}P_{qq}\left(\frac{x}{\xi}\right) q(\xi,\mu^2) + O(\alpha_s^2)\right]$$

$\overline{MS}$: "$\frac{1}{\epsilon} + \ln 4\pi - \gamma_E$" $\quad C = \ln 4\pi - \gamma_E + \tilde{C}$

$$q_0(x,\mu^2) = q_0(x) + \frac{\alpha_s}{2\pi}\int \frac{d\xi}{\xi}\left(\ln\frac{\mu^2}{\mu^2}P_{qq}\left(\frac{x}{\xi}\right) + (\ln 4\pi - \gamma_E)\right) q_0(\xi)$$

$$F_2 = x\sum_q e_q^2\left[q_0(x,\mu^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\underbrace{\left(\ln\frac{Q^2}{\mu^2}P_{qq}\left(\frac{x}{\xi}\right) + \tilde{C}\left(\frac{x}{\xi}\right)\right)}_{=\ \text{Coefficient function}} q(\xi,\mu^2) + O(\alpha_s^2)\right]$$