

$$\begin{aligned}
 V_{\mu\nu}^{(1)} &= \frac{1}{4\pi} e_g^2 \frac{1}{2} g^2 C_F \int \frac{d^4k}{(2\pi)^4} \frac{2nd(q+k) \cdot 2nd(p-k)}{k^2 k'^2} \frac{1}{s} \frac{1}{(s-k^2)} \frac{1}{(s-k'^2)} \\
 &= \frac{1}{4\pi} e_g^2 g^2 \frac{1}{2} C_F \int \frac{d^4p d^4k_1}{(2\pi)^4} \frac{2nd(x - \frac{k_1^2}{s(1-\beta)})}{s(1-\beta)} \frac{2nd(s-\beta-x)}{k'^2} \frac{1}{(s-k_1^2)} \frac{1}{(s-k_2^2)} \\
 &\int \frac{dk_1}{1-\beta} = \pi \int \frac{dk_1^2}{1-\beta} = \pi \int dk_1^2 \\
 &\rightarrow \ln \left[ \gamma^{\nu} (\not{s} + \not{q}) \gamma^{\mu} \not{p} \right]
 \end{aligned}$$

$$\int \frac{dk}{s-k^2} \sim \frac{1}{s} \int \frac{dx}{x}$$

$$\int \frac{dx}{k^2} \rightarrow \int \frac{dx}{\mu^2 k^2} = \ln \frac{\mu^2}{k^2} + C(x)$$

$$V_{\mu\nu}^{(1)} = \frac{1}{4\pi} \frac{1}{2} e_g^2 g^2 C_F + \frac{1}{8\pi^2} \int d\beta d\beta' \frac{1-\beta^2}{1-\beta} \left[ \ln \frac{\mu^2}{k^2} + C(x) \right] \delta(\beta-x) \ln \left[ \gamma^{\nu} (\not{s} + \not{q}) \gamma^{\mu} \not{p} \right]$$

$$\begin{aligned}
 T_2 &= e_g^2 \left[ x \delta(1-x) + \frac{N_c}{2\pi} \ln \frac{\mu^2}{k^2} \int d\beta d\beta' \frac{1-\beta^2}{1-\beta} \delta(\beta-x) + C(x) \right] \\
 &= e_g^2 x \left[ \delta(1-x) + \frac{N_c}{2\pi} \ln \frac{\mu^2}{k^2} P_{qg}^{(1)}(x) + C(x) \right]
 \end{aligned}$$

$$\int_0^1 \frac{dx}{5x+4} \sim \frac{1}{5} \int_0^1 \frac{dx}{x}$$

$$\int_0^1 \frac{dx}{|k^2|} \rightarrow \int_0^1 \frac{dx}{\mu^2 |k^2|} = \ln \frac{\mu^2}{\mu^2} + C(x, \mu)$$

$$W_{\mu\nu}^{(1)} = \frac{1}{4\pi} \frac{1}{2} e_s^2 g^2 C_F \frac{1}{8\pi^2} \int_0^1 d\beta \beta \frac{1+\beta^2}{1-\beta} \left[ \ln \frac{\mu^2}{\mu^2} + C(x, \mu) \right] \delta(\beta-x)$$

$$F_2 = e_s^2 \left[ x \delta(1-x) + \frac{N_s C_F}{2\pi} \ln \frac{\mu^2}{\mu^2} \int_0^1 d\beta \beta \frac{1+\beta^2}{1-\beta} \delta(\beta-x) + C(x) \right]$$

$$= e_s^2 x \left[ \delta(1-x) + \frac{N_s}{2\pi} \ln \frac{\mu^2}{\mu^2} P_{qq}''(x) + C(x) \right]$$

$\text{tr}[\gamma^\nu (\not{s} \not{t}) \gamma^\mu \not{p}]$

$$0 = \int_0^1 d\beta \mathcal{P}_{gg}(\beta) = \int_0^1 d\beta [\mathcal{P}_{gg}''(\beta) + \lambda \delta(1-\beta)]$$

Conservation of probability

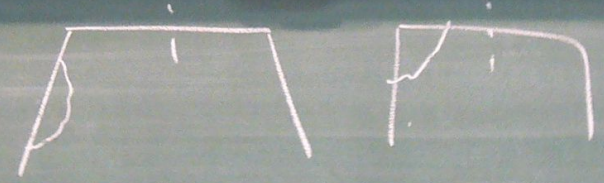
$$\begin{aligned} \mathcal{P}_{gg}' &= \frac{1+\beta^2}{1-\beta} + \lambda \delta(1-\beta) \\ &= \frac{1+\beta^2}{(1-\beta)_+} + \lambda \delta(1-\beta) \\ &\quad \lambda = 3/2 \end{aligned}$$

$$\begin{aligned} \int_0^1 dx \frac{f(x)}{(1-x)_+} &\stackrel{\text{def}}{=} \int_0^1 dx \frac{f(x) - f(1)}{1-x} \\ &= \int_0^1 dx \frac{f(x)}{1-x} - \int_0^1 dx \frac{\delta(1-x) f(x)}{1-x} \end{aligned}$$

Comments: DGLAP

(a)  $\beta \rightarrow 1$

(b)  $\ln \alpha_s \rightarrow \alpha_s$  ?



Answer: (b):  $\alpha_s \rightarrow \alpha_s(Q^2) = \frac{1}{b \ln(Q^2/\Lambda^2)}$       $b = (33 - 2n_f)/12\pi$

(a):  $\beta \rightarrow 1$  . other diagrams  $\sim \delta(1-x)$

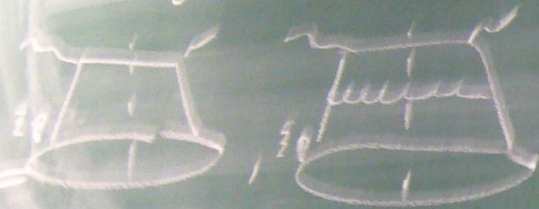
$$\mathcal{P}_{gg}''(x) \rightarrow \mathcal{P}_{gg}'(x) + \lambda \delta(1-x)$$

Final splitters, Pocher.

$$P_{q\bar{q}}^{(1)} = C_F \left[ \frac{1+\beta^2}{(1-\beta)^2} + \frac{3}{2} d(1-\beta) \right]$$

Coupling to the proton:

$$T_2^q = e_q^2 x \left[ d(1-x) + \frac{\alpha_s}{2\pi} \left( \ln \frac{Q^2}{\mu^2} P_{q\bar{q}}(x) + C(x) \right) \right]$$



$$\rightarrow T_2^p = \sum_q e_q^2 x \left[ q_0(x) + \frac{\alpha_s}{2\pi} \int_0^1 \frac{d\xi}{\xi} \left( P_{q\bar{q}}\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{\mu^2} + C\left(\frac{x}{\xi}\right) \right) q_0(\xi) \right]$$

$$q_0(\xi), \int d\xi$$

Coupling to the proton

$$\overline{F}_2^q = e_q^2 x \left[ d(1-x) + \frac{\alpha_s}{2\pi} \left( \ln \frac{Q^2}{\mu^2} P_{qq}(x) + C(x) \right) \right]$$



$$q_0(\xi), \int d\xi$$

$$\rightarrow \overline{F}_2^p = \sum_q e_q^2 x \left[ q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left( P_{qq}\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{\mu^2} + C\left(\frac{x}{\xi}\right) \right) q_0(\xi) \right]$$

$\ln \frac{Q^2}{\mu^2} + \ln \frac{Q^2}{\mu^2}$

$$\bar{\mu}^2 < |k|^2 < \frac{Q^2}{x}$$

$$\ln \frac{Q^2}{\mu^2} = \ln \frac{\mu^2}{\mu^2} + \ln \frac{Q^2}{\mu^2}$$

$$d^4k \rightarrow d^3k \rightarrow d^2k$$

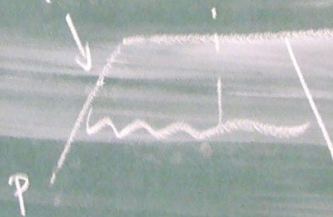
$$\bar{\mu}^2 < \mu^2 < \frac{Q^2}{x}$$

factorization scale

$$\bar{\mu}^2 \int \frac{d^4k}{|k|^2}$$

$|k|^2$  small

$$k^2 \rightarrow \frac{k_0^2}{1-\beta}$$



Dimensional.  $4 \rightarrow d = 4 - 2\epsilon$

$$\int \frac{d^4k}{|k|^2} \rightarrow \frac{d^4k}{|k|^{2+2\epsilon}}$$

$$\rightarrow \frac{1}{\epsilon}$$

$$q_0(x, \mu^2) = q_0(x) + \frac{\alpha_s}{2n} \int_x^1 \frac{d\xi}{\xi} \left( \ln \frac{\mu^2}{\xi^2} P_{qq}(\frac{x}{\xi}) + C(\frac{x}{\xi}) \right) q_0(\xi)$$

$$F_2 = x \sum_q e_q^2 \left[ q_0(x, \mu^2) + \frac{\alpha_s}{2n} \int_x^1 \frac{d\xi}{\xi} \ln \frac{\mu^2}{\xi^2} P_{qq}(\frac{x}{\xi}) q_0(\xi, \mu^2) + O(\alpha_s^2) \right]$$

$$TS: \quad \frac{1}{\xi} + \ln 4n - \gamma_E - C = \ln 4n - \gamma_E + \tilde{C}$$

$$q_0(x, \mu^2) = q_0(x) + \frac{\alpha_s}{2n} \int_x^1 \frac{d\xi}{\xi} \left( \ln \frac{\mu^2}{\xi^2} P_{qq}(\frac{x}{\xi}) + (\ln 4n - \gamma_E) \right) q_0(\xi)$$

$$F_2 = x \sum_q e_q^2 \left[ q_0(x, \mu^2) + \frac{\alpha_s}{2n} \int_x^1 \frac{d\xi}{\xi} \left( \ln \frac{\mu^2}{\xi^2} P_{qq}(\frac{x}{\xi}) + \tilde{C}(\frac{x}{\xi}) \right) q_0(\xi, \mu^2) + O(\alpha_s^2) \right]$$

||  
Coefficient function