QCD and Collider Physics III: W/Z transverse momenta, resummation, uPDFs

- transverse momentum of W/Z
  - perturbative region
  - Qt resummation
  - intrinsic kt
- connection to uPDFs
  - definition and features
  - advantages
- The end

- Literature:
  Ellis, Stirling, Webber: *QCD and Collider Physics*
  Field: *Applications of perturbative QCD*
  CTEQ summerschool 2000, 2003
  References in lecture

http://www-h1.desy.de/~jung/qcd_collider_physics_wise_2006

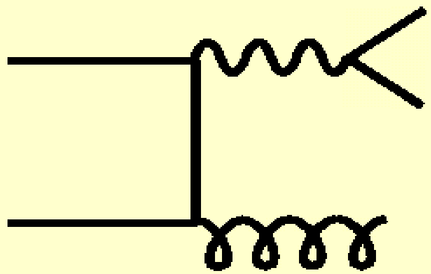
Factorization in Drell – Yan

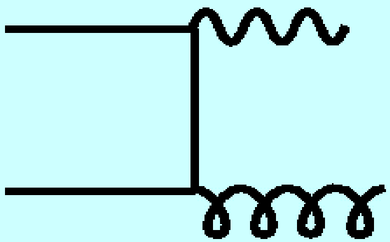
Fred Olness, CTEQ
summerschool 2003

Side Note: From $pp\to\gamma/Z/W$, we can obtain $pp\to\gamma/Z/W\to l^+ l^-$

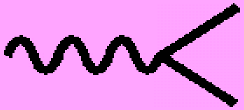
Schematically:

$$d\sigma(q\bar{q}\to l^+ l^- g) = d\sigma(q\bar{q}\to\gamma^* g) \times d\sigma(\gamma^*\to l^+ l^-)$$





$\otimes$

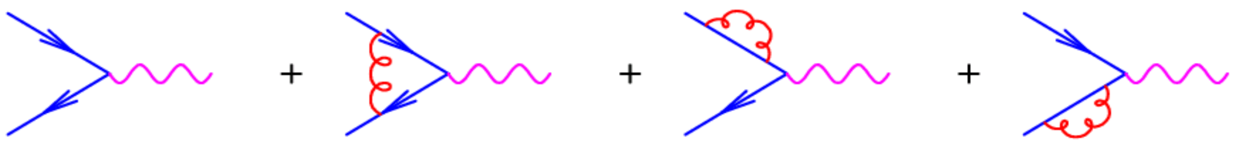


For example:

$$\frac{d\sigma}{dQ^2\, d\hat{t}}(q\bar{q}\to l^+ l^- g) = \frac{d\sigma}{d\hat{t}}(q\bar{q}\to\gamma^* g) \times \frac{\alpha}{3\pi Q^2}$$

QCD corrections for Drell – Yan I

K. Ellis, LHC lecture,
http://theory.fnal.gov/people/ellis/Talks



(a)

(b) (c)

- Calculate real correction

$$q + \bar{q} \to \gamma^* + g$$

$$|M|^2 = \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2(M^2 s)}{\hat{u}\hat{t}} \right]$$

$$= \left[\left(\frac{1+z^2}{1-z} \right) \left(\frac{-s}{t} + \frac{-s}{u} \right) - 2 \right]$$

- with $z = M^2/s, s + t + u = M^2$
- real diagrams contain collinear divergency $\hat{t} \to 0, \hat{u} \to 0$ and soft divergency $z \to 1$
- coefficient is DGLAP splitting fct:

$$P_{qq}(z) \sim \frac{1+z^2}{1-z}$$

QCD Corrections to Drell – Yan II

- Virtual emissions, integrated over z (R. Field, App. pQCD, p179ff): $q\bar{q} \to \gamma^* g$

$$\hat{\sigma}_{MG}(virtual)_{DY} = \frac{2\alpha_s}{3\pi}\hat{\sigma}_0\left[-\log^2(\beta) - 3\log(\beta) - \frac{7}{2} - \frac{2\pi^2}{3} + \pi^2\right]$$

$$(\hat{\sigma}_{MG}(real) + \hat{\sigma}_{MG}(virtual))_{DY} = \frac{2\alpha_s}{3\pi}\hat{\sigma}_0\left[\frac{4\pi^2}{3} - \frac{7}{2}\right]$$

- Define K -factor (1st order): $\hat{\sigma}^{DY}_{tot} = \hat{\sigma}_0 \times (1 + \cdots) = \hat{\sigma}_0 \times K$

$$K^{DY}(\text{1st order}) = 1 + \frac{\alpha_s}{\pi}\left[\frac{8\pi^2}{9} - \frac{7}{3}\right] = 1 + 2.05\alpha_s \sim 2$$

- compare to DIS

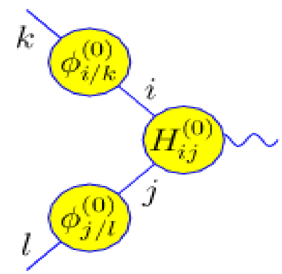
$$K^{DIS}(\text{1st order}) = 1 - \frac{\alpha_s}{\pi}$$

QCD corrections for Drell – Yan III

C.P Yuan,
CTEQ summerschool 2002

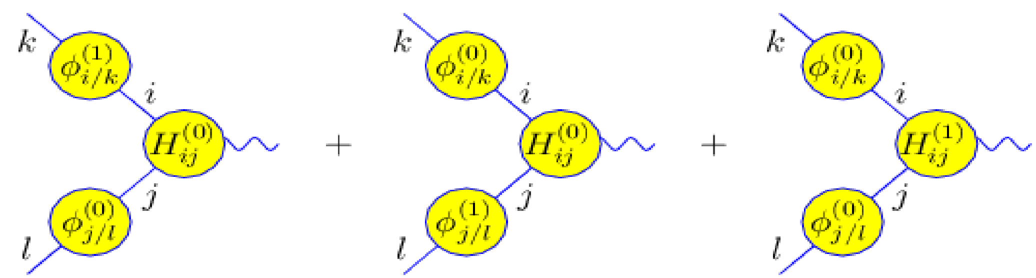
- soft divergencies cancelled by real and virtual emissions (1)
- factorise collinear divergency into renormalised parton density (2)

$\sigma^{(0)}_{kl} =$



$\Rightarrow \quad H^{(0)}_{kl} = \sigma^{(0)}_{kl}$

$\sigma^{(1)}_{kl} =$



$H^{(0)}_{ij} = \sigma^{(0)}_{ij} =$ "Born"

suppress "^" from now on

$$H^{(1)}_{ij} = \sigma^{(1)}_{ij} - \left[\sigma^{(0)}_{il}\, \phi^{(1)}_{l/j} + \phi^{(1)}_{k/i}\, \sigma^{(0)}_{kj} \right]$$

Computed from Feynman diagrams (process dependent)

Computed from the definition of perturbative parton distribution function (process independent, scheme dependent)

Factorization scheme dependent

$$\Rightarrow \quad H^{(1)}_{kl} = \sigma^{(1)}_{kl} - \left[\phi^{(1)}_{i/k} H^{(0)}_{il} + H^{(0)}_{kj} \phi^{(1)}_{j/l} \right]$$

Finite

Divergent

Measurement of W - mass

The Jacobian Peak

Fred Olness, CTEQ summerschool 2003

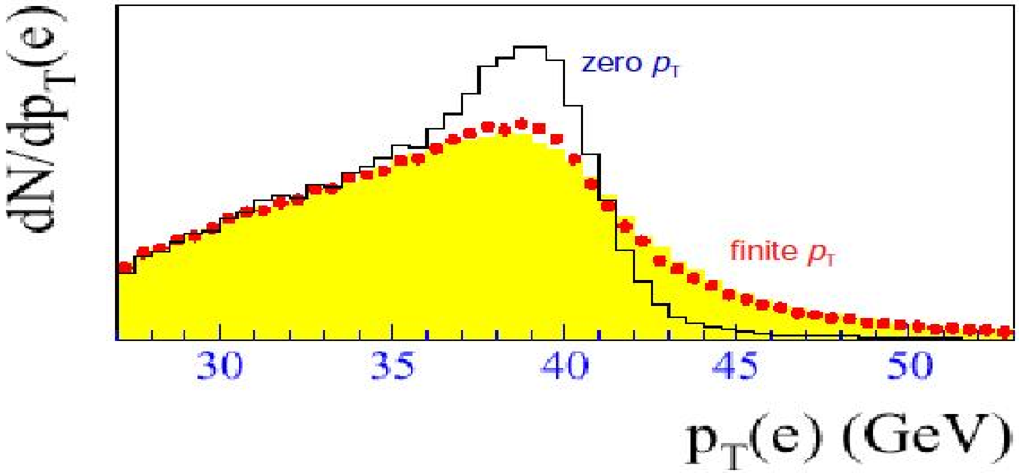
Now that we've got the picture, here's the math ... *(in the W CMS frame)*

$$p_T^2 = \frac{\hat{s}}{4} \sin^2 \theta \qquad \cos\theta = \sqrt{1 - \frac{4\, p_T^2}{\hat{s}}} \qquad \frac{d\cos\theta}{d\, p_T^2} = \frac{2}{\hat{s}} \frac{1}{\cos\theta}$$

So we discover the P_T distribution has a singularity at $\cos\theta=0$, or $\theta=\pi/2$

$$\frac{d\sigma}{d\, p_T^2} = \frac{d\sigma}{d\cos\theta} \times \frac{d\cos\theta}{d\, p_T^2} \approx \frac{d\sigma}{d\cos\theta} \times \frac{1}{\cos\theta}$$

singularity!!!



BUT !!!

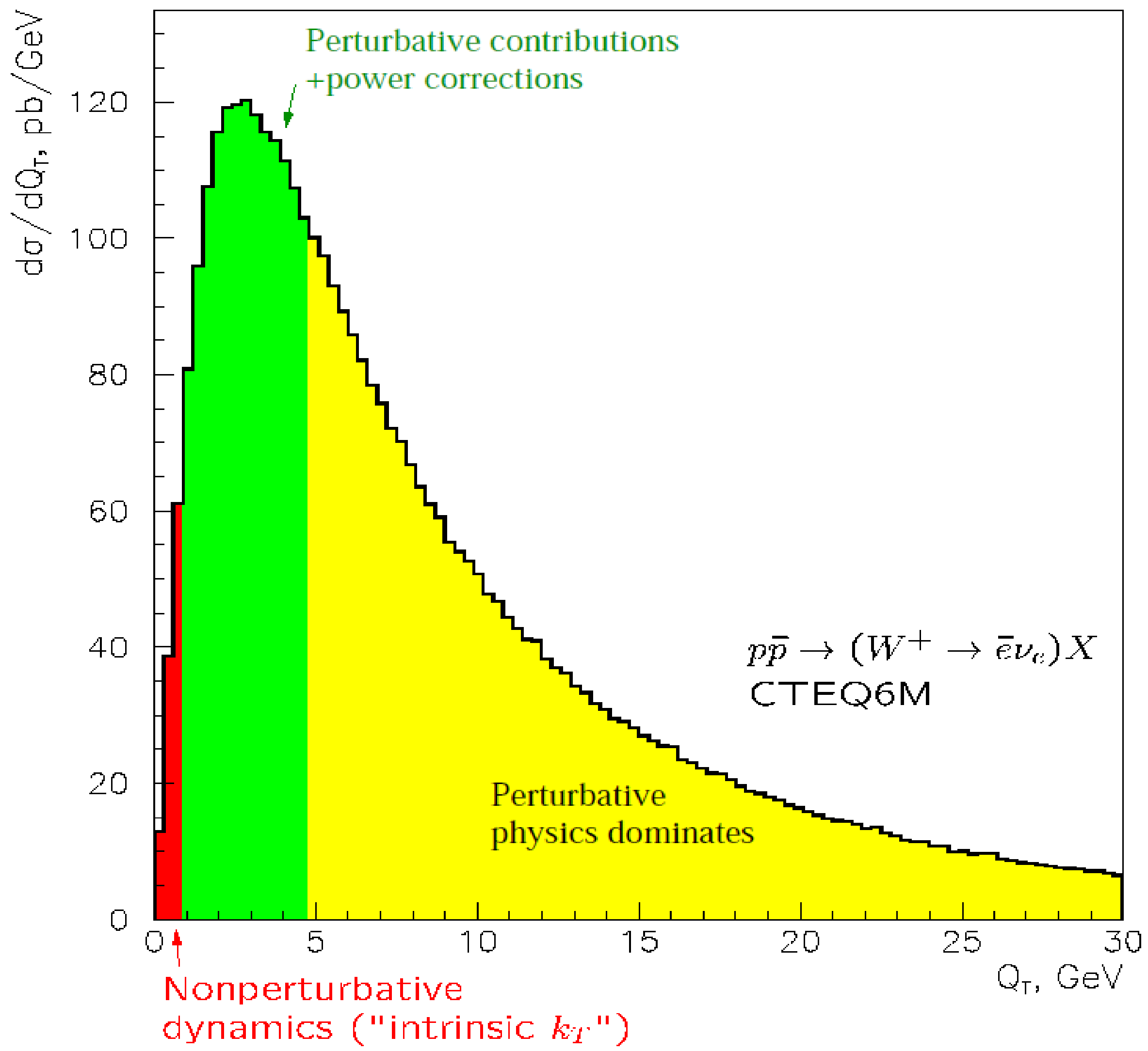
Measuring the Jacobian peak is complicated if the W boson has finite P_T.

Transverse Momentum of W/Z

The complete P_T spectrum for the W boson

Fred Olness, CTEQ summerschool 2003

The full P_T spectrum for the W-boson showing the different theoretical regions



Original References

PHYSICS REPORTS (Review Section of Physics Letters) 58, No. 5 (1980) 269–395. North-Holland Publishing Company

HARD PROCESSES IN QUANTUM CHROMODYNAMICS

Yu.L. DOKSHITZER, D.I. DYAKONOV and S.I. TROYAN

Leningrad Nuclear Physics Institute, Gatchina, Leningrad 188350, U.S.S.R.

Received 28 May 1979

Nuclear Physics B154 (1979) 427–440

SMALL TRANSVERSE MOMENTUM DISTRIBUTIONS IN HARD PROCESSES

G. PARISI

INFN, Laboratori Nazionali di Frascati, Italy

R. PETRONZIO*

CERN, Geneva, Switzerland

Received 8 February 1979

D – Y x-section at large pt

R. Field, Appl. of pQCD, p195 ff

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \frac{8}{27}\frac{\alpha^2\alpha_s}{M^2}\frac{1}{p_T^2}\int_{x_a^{min}}^1 dx_a P^{DIS}\frac{1}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2x_a x_b}\right)$$

- with

$$x_a^{min} = \frac{x_a x_2 - \tau}{x_a - x_1}$$

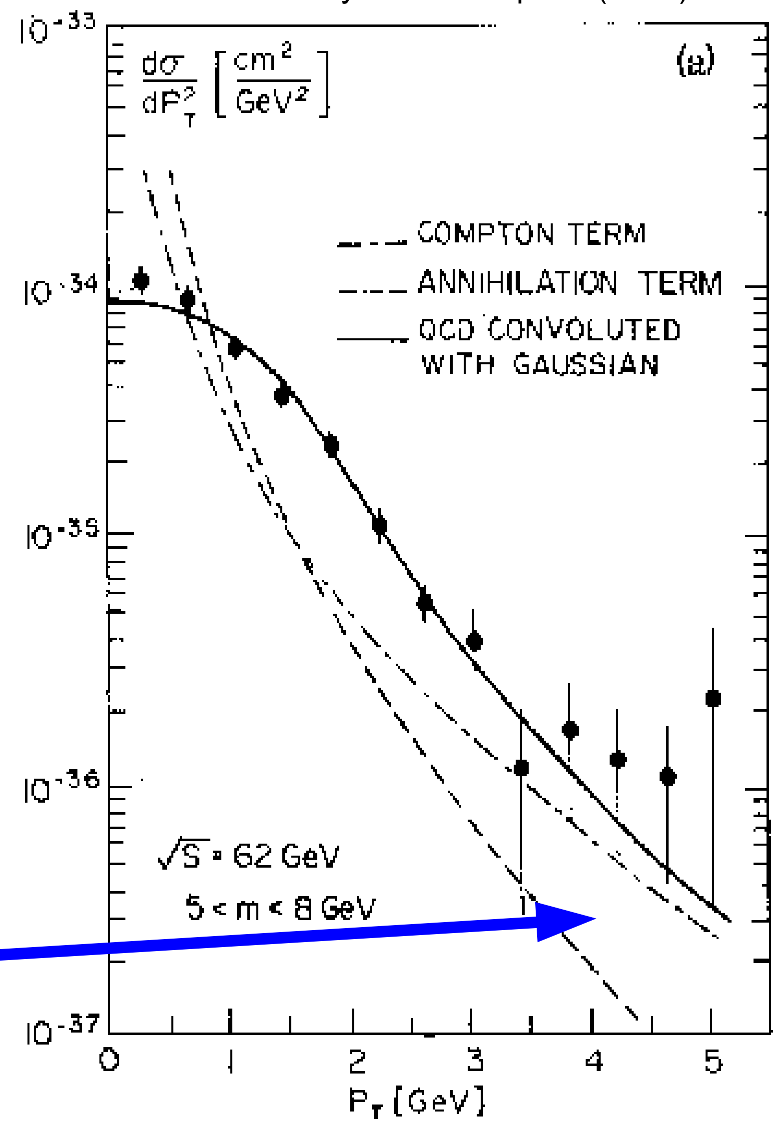
$$p_t^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

$$x_t = \frac{2p_t}{\sqrt{s}}$$

$$P^{DIS} = \sum e_q^2 \left(q_i(x_a,Q^2)\bar{q}_i(x_b,Q^2) + \bar{q}_i(x_a,Q^2)q_i(x_b,Q^2)\right)$$

- large pt

Antreasyan PRL 48 p302 (1982)



x-section at small pt

R. Field, Appl. of pQCD, p195 ff

- Evaluate integral:

$$\int_{x_a^{min}}^{1} dx_a \frac{1}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2}\right) \sim -2\log(x_t^2/4) = 2\log s/p_t^2$$

- gives then:

$$\frac{d\sigma}{d\tau dy dp_t^2} = \left(\frac{d\sigma}{d\tau dy}\right)_{born}\left(\frac{4\alpha_s}{3\pi}\frac{1}{p_T^2}\log(s/p_t^2)\right)$$

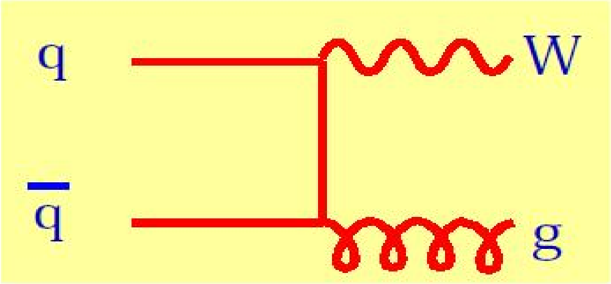
- with:

$$\left(\frac{d\sigma}{d\tau dy}\right)_{born} = \frac{4\pi\alpha^2}{9M^2}P^{DIS}$$

Pt distribution in Drell Yan

Fred Olness, CTEQ summerschool 2003

NLO P_T distribution for the W boson



In the limit $P_T \to 0$

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2}$$

singular

$$\int_0^s \frac{d\sigma}{d\tau\, dy\, dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} + O(\alpha_s)$$

finite

$$\int_0^{p_T^2} \frac{d\sigma}{d\tau\, dy\, dp_T^2}\, dp_T^2 = \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{1 - \int_{p_T^2}^{s} \frac{4\alpha_s}{3\pi} \frac{\ln s/p_T^2}{p_T^2}\, dp_T^2\right\}$$

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \left\{1 - \frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2}\right\}$$

effect of gluon emission

$$= \left(\frac{d\sigma}{d\tau\, dy}\right)_{Born} \times \exp\left\{\frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_T^2}\right\}$$

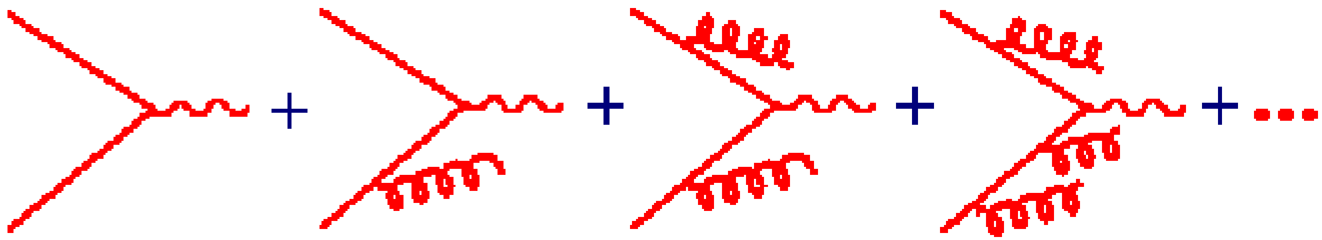
assume this exponentiates

Parisi & Petronzio, NP B154, 427 (1979)
Dokshitzer, D'yakanov, Troyan, Phy. Rep. 58, 271 (1980)

Resummation

C-P Yuan, CTEQ summerschool 2002

Diagramatically, Resummation is doing



→ Resum large $\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)$ terms

$$\left.\frac{d\sigma}{dq_T^2 dy}\right|_{q_T \to 0} \sim \frac{1}{q_T^2}\sum_{n=1}^{\infty}\sum_{m=0}^{2n-1}\alpha_S^n \ln^m\left(\frac{Q^2}{q_T^2}\right)\cdot C_m^n$$

Monte-Carlo programs ISAJET, PYTHIA, HERWIG contain these physics.

Monte Carlo approach

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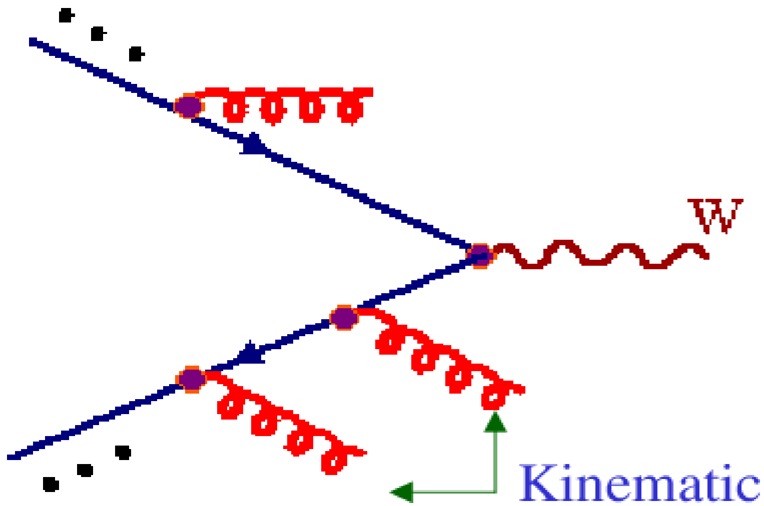

Backward Radiation
(Initial State Rad.)

Kinematics of the radiated gluon, controlled by Sudakov form factor with some arbitrary cut-off. (In contrast to perform integration in impact parameter space, i.e., b space.)

→ The shape of q_T (w) is generated. But, the integrated rate remains the same as at Born level (finite virtual correction is not included).

* Recently, there are efforts to include part of higher order effect in the event generator.

Kinematic constraints

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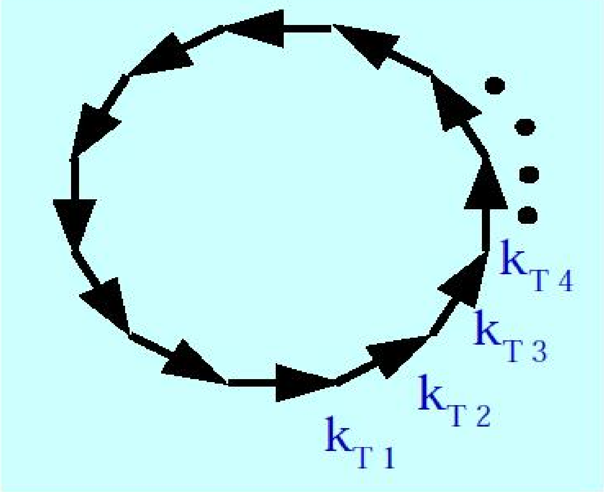
3) We assumed gluon emission was uncorrelated

$$\frac{d\sigma}{d\tau\, dy\, dp_T^2} \approx \frac{\ln s/p_T^2}{p_T^2} \times \exp\left\{-\frac{2\alpha_s}{3\pi}\ln^2\frac{s}{p_T^2}\right\}$$



This leads to too strong a suppression at P_T=0.
Need to impose momentum conservation for P_T.

A particle can receive finite k_T kicks,
yet still have P_T=0

A convenient way to impose transverse momentum conservation is in impact parameter space (b-space) via the following relation:

$$\delta^{(2)}\left(\sum \vec{k}_{ti} - \vec{p}_t\right) = \frac{1}{(2\pi)^2}\int d^2b\, \exp(-i\vec{b}\vec{p}_t)\prod \exp(i\vec{b}\vec{k}_{ti})$$

Resummation

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A Brief *(but incomplete)* History of Non-Perturbative Corrections

Original CSS: $S_{NP}^{CSS}(b) = h_1(b,\xi_a) + h_2(b,\xi_b) + h_3(b)\ln Q^2$

J. Collins and D. Soper, *Nucl.Phys.* **B193** 381 (1981);
erratum: **B213** 545 (1983); J. Collins, D. Soper, and G. Sterman, *Nucl. Phys.* **B250** 199 (1985).

Davies, Webber, and Stirling (DWS): $S_{NP}^{DWS}(b) = b^2\left[g_1 + g_2 \ln(b_{max} Q^2)\right]$

C. Davies and W.J. Stirling, Nucl. Phys. B244 337 (1984);
C. Davies, B. Webber, and W.J. Stirling, Nucl. Phys. B256 413 (1985).

Ladinsky and Yuan (LY): $S_{NP}^{LY}(b) = g_1 b\left[b + g_3 \ln(100\xi_a \xi_b)\right] + g_2 b^2 \ln(b_{max} Q)$

G.A. Ladinsky and C.P. Yuan, Phys. Rev. D50 4239 (1994);
F. Landry, R. Brock, G.A. Ladinsky, and C.P.Yuan, Phys. Rev. D63 013004 (2001).

"BLNY": $S_{NP}^{BLNY}(b) = b^2\left[g_1 + g_1 g_3 \ln(100\xi_a \xi_b) + g_2 \ln(b_{max} Q)\right]$

F. Landry, "Inclusion of Tevatron Z Data into Global Non-Perturbative QCD Fitting", Ph.D. Thesis, Michigan State University, 2001.
F. Landry, R. Brock, P. Nadolsky, and C.P.Yuan, PRD67, 073016 (2003)

"q_T resummation": $\mathcal{F}^{NP}(q_T) = 1 - e^{-\bar{a} q_T^2}$ *(not in b-space)*

R.K. Ellis, Sinisa Veseli, Nucl.Phys. B511 (1998) 649-669
R.K. Ellis, D.A. Ross, S. Veseli, Nucl.Phys. B503 (1997) 309-338

Functional Extrapolation:

J. Qui, X. Zhang, PRD63, 114011 (2001); E. Berger, J. Qiu, PRD67, 034023 (2003)

Analytical Continuation:

A. Kulesza, G. Sterman, W. vogelsang, PRD66, 014011 (2002)

Intrinsic kt

J.F. Owens, CTEQ summerschool 2000

- using Fourier transform of Delta function gives with

$$\delta^{(2)}(\sum \vec{k}_{ti} - \vec{p}_t) = \frac{1}{(2\pi)^2} \int d^2 b \exp(-i\vec{b}\vec{p}_t) \prod \exp(i\vec{b}\vec{k}_{ti})$$

- and

$$\frac{1}{(2\pi)^2} \int d^2 b \exp(-i\vec{b}\vec{p}_t) = \frac{1}{2\pi} \int b db J_0(p_t b)$$

- gives:

$$\frac{1}{\sigma_{born}} \frac{d\sigma}{dp_t} \sim \int_0^\infty b db J_0(p_t b) \exp(-S(b, M)) P^{DIS}$$

- with

$$S_{intr} = b^2 \alpha$$

- gives

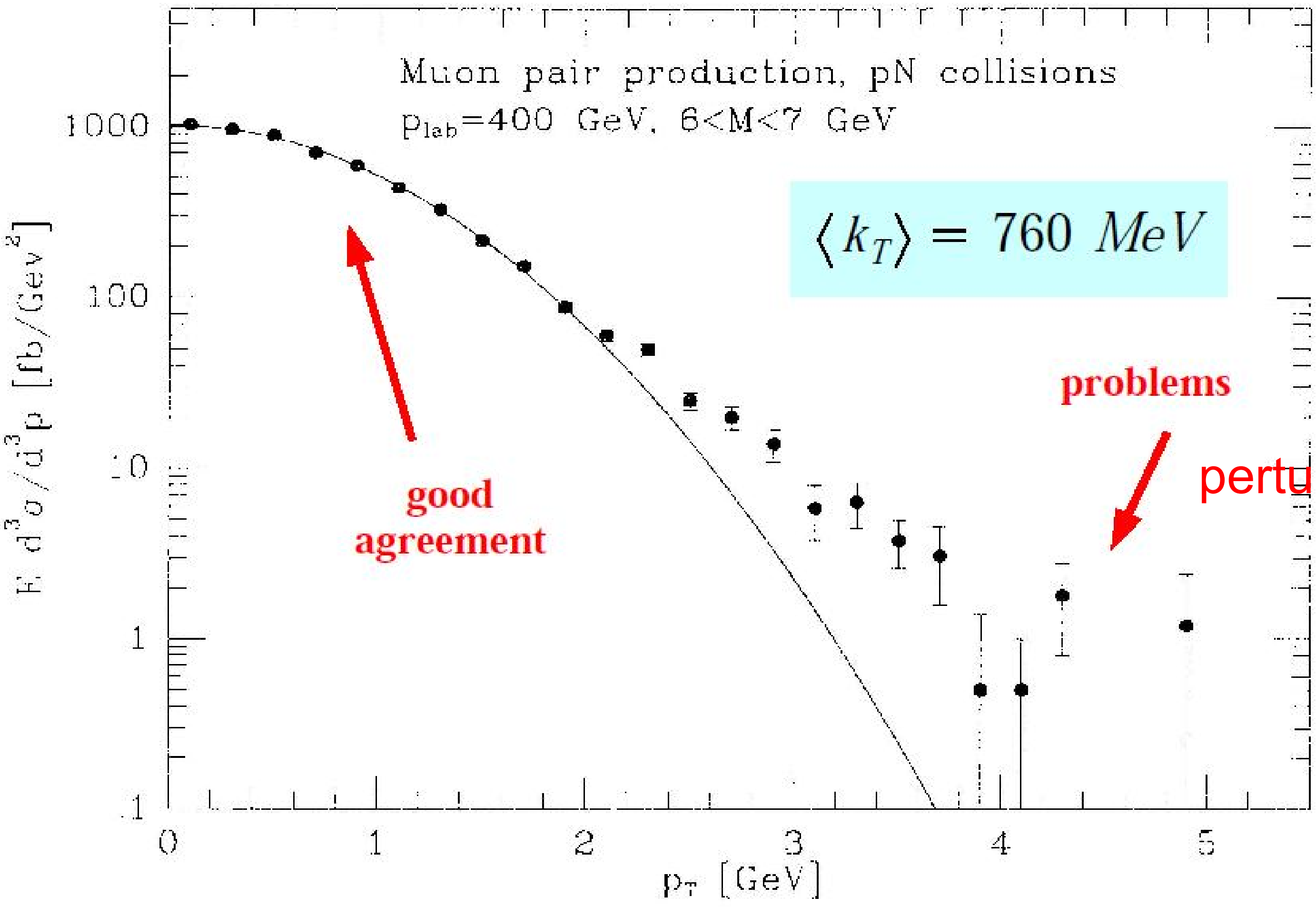
$$\frac{d\sigma}{dp_t} \sim \exp\left(\frac{-p_t^2}{4\alpha}\right) P^{DIS}$$

Intrinsic kt II

Fred Olness, CTEQ summerschool 2003

Assume a Gaussian form:

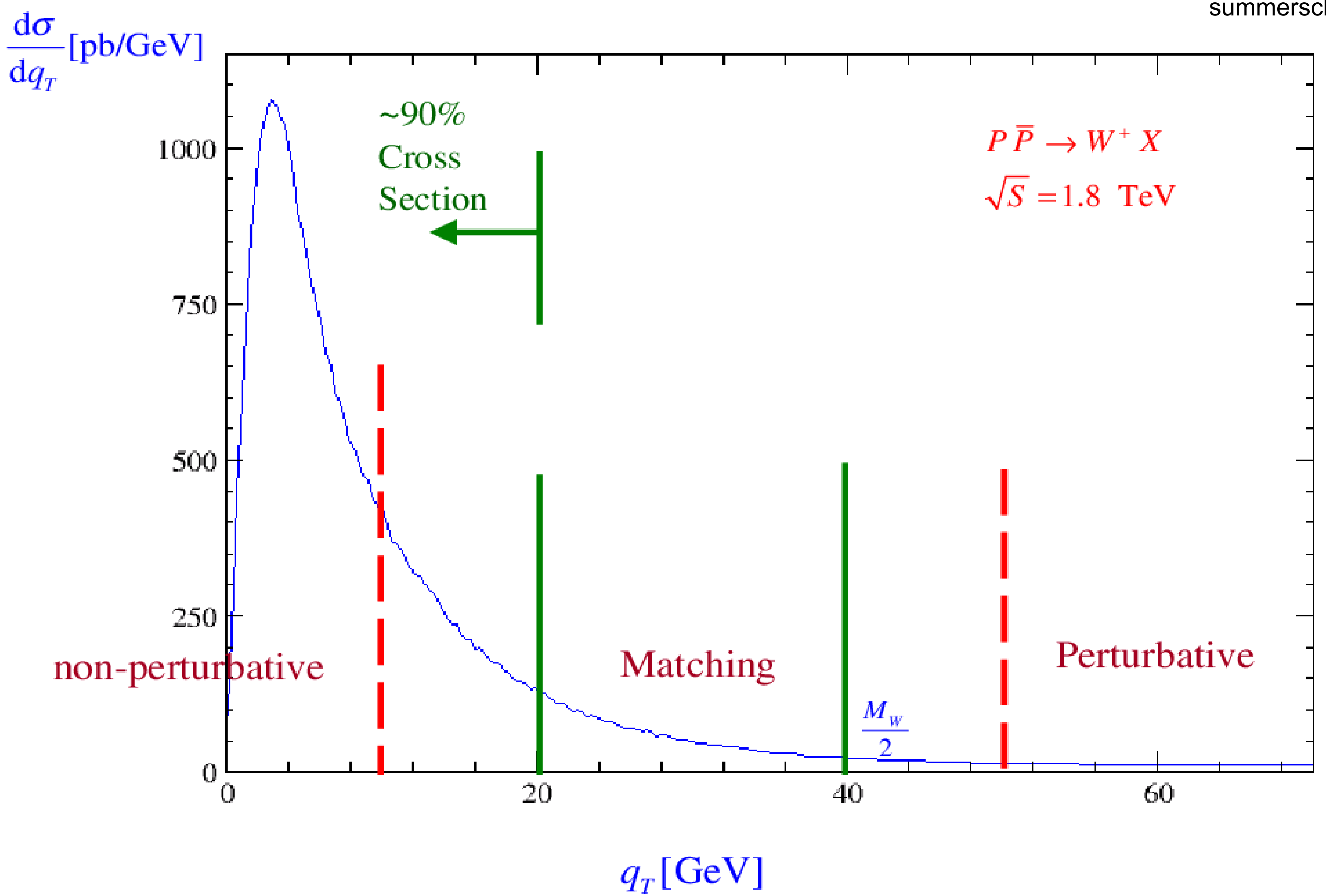
$$\frac{d^2\sigma}{d^2 p_T} \approx \sigma_0\, e^{-p_T^2}$$



perturbative $1/pt^2$

Transverse Momentum of W/Z

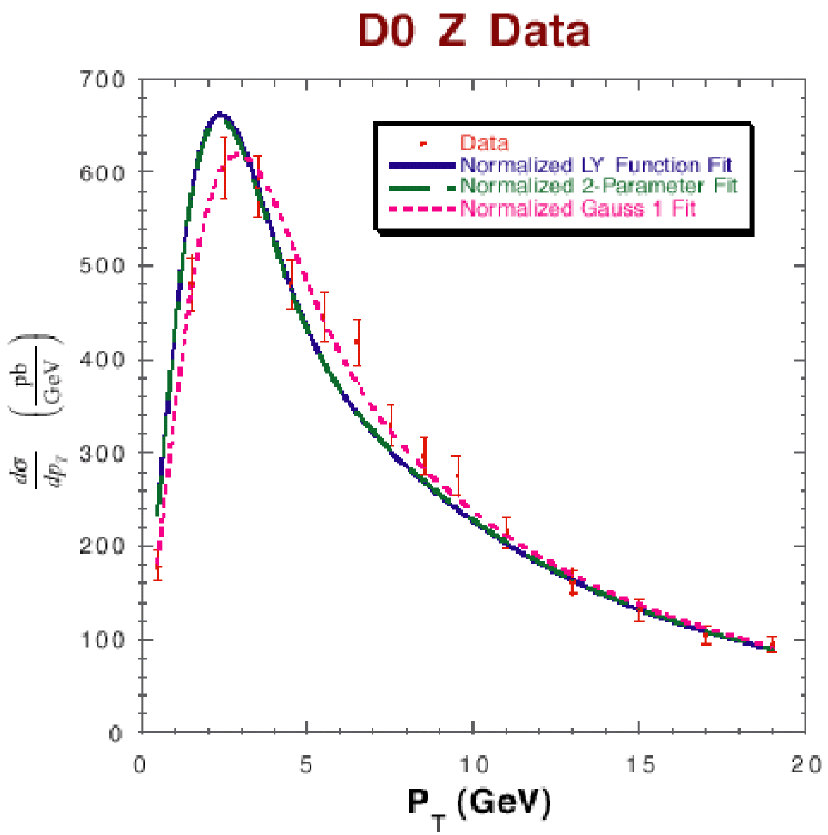
C-P Yuan, CTEQ summerschool 2002

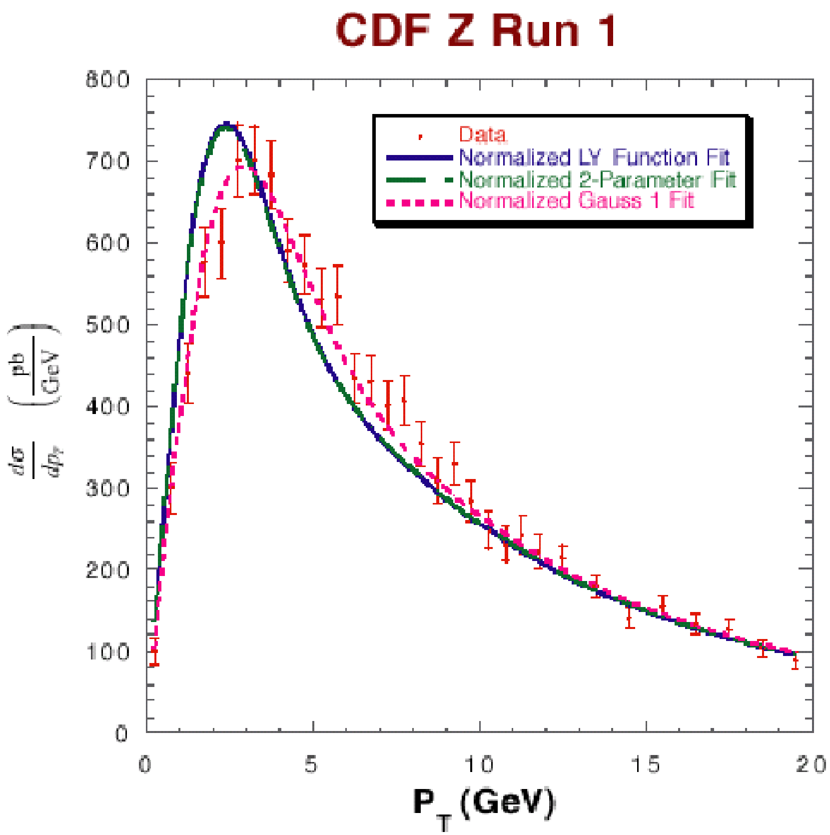


Transverse Momentum of W/Z

Fred Olness, CTEQ
summerschool 2003

We'll look at Z data where we can measure both leptons for $Z \to e^+e^-$





different $S_{NP}(b,Q)$ functions yield difference at small q_T.

Q_t - Resummation

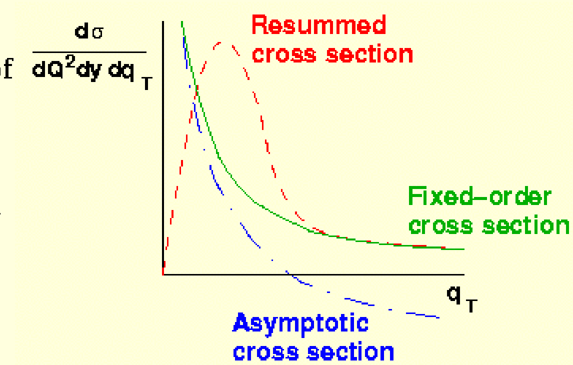
RESUMMED TRANSVERSE MOMENTUM DISTRIBUTIONS

go directly to the plotter

On this website, you can plot transverse momentum distributions for cross sections of several particle reactions. Currently, the following processes are implemented (p corresponds both to protons and antiprotons):

- Massive vector boson production: $pp \to W^{\pm}X$, $pp \to Z^0 X$
- Photon pair production: $pp \to \gamma\gamma X$
- Z-boson pair production: $pp \to Z^0 Z^0 X$
- SM Higgs boson production $pp \to H^0 X$

The output figure shows distributions $d\sigma/dQ^2 dy dq_T$ for the production of *on-shell* particles (or pairs of *on-shell* particles in the case of the $\gamma\gamma$ and ZZ production) with specified invariant mass Q , rapidity y and transverse momentum q_T in the lab frame (the center-of-mass frame of the hadron beams). You can plot resummed, fixed-order and asymptotic cross sections. For a short explanation of these quantities, visit **this page** (for a detailed explanation see, for instance, a paper by J.C. Collins, D.E. Soper and G. Sterman in Nucl. *Phys. B250, 199 (1985)).*

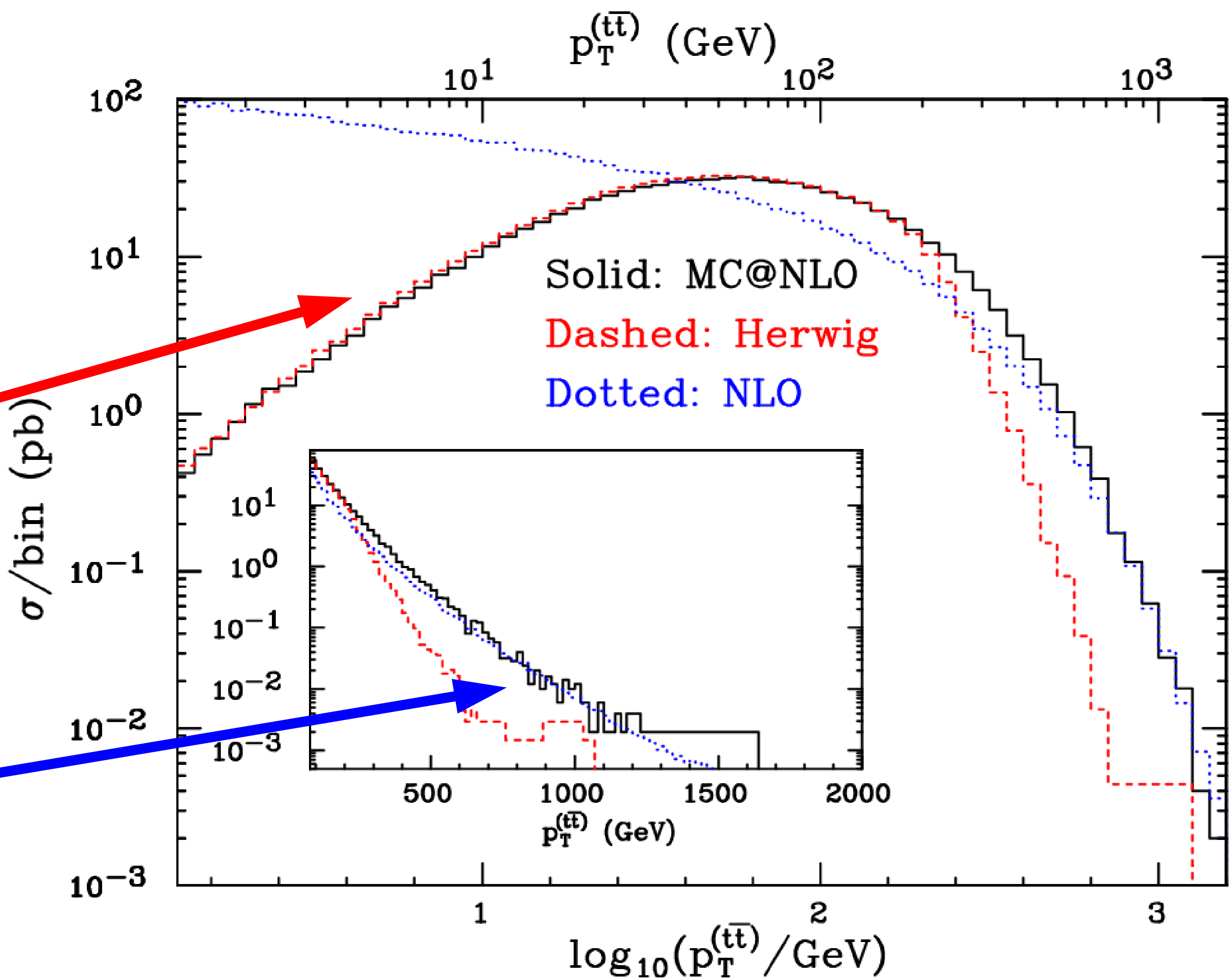


http://hep.pa.msu.edu/wwwlegacy/

Resummation in heavy quark prod.

- Compare fixed NLO calculation of top production with resummed calculation from Monte Carlo
- Similar effects at small pt are observed: Supression of xsection at small pt
- At large pt, resummation is too small, NLO is better

Frixione et al, hep-ph/035252



Defining uPDFs

- start from integral equation:

$$f(x,q) = f(x,Q_0)\Delta_s(q) + \int \frac{dz}{z} \int \frac{d^2q'}{\pi q'^2} \cdot \frac{\Delta_s(q)}{\Delta_s(q')} \tilde{P}(z) f\left(\frac{x}{z}, q'\right)$$

- use unintegrated pdfs: $\mathcal{A}(x,k,q)$

$$x\mathcal{A}(x,k_t,q) = x\mathcal{A}_0(x,k_t)\Delta_s(q) + \int dz \int \frac{d^2q'}{\pi q'^2} \cdot \Delta_s(q, f(q'))\tilde{P}(z,q',k_t)\Theta(\mathcal{O})\frac{x}{z}\mathcal{A}\left(\frac{x}{z}, k_t', q'\right)$$

because of phi integration:

$$\frac{dt}{t} \to \frac{dq^2}{q^2} \to \frac{d^2q}{\pi q^2}$$

define updf:

$$xg(x,Q) = \int \frac{d^2k_t}{\pi} x\mathcal{A}(x,k_t,Q)\Theta(Q-k_t)$$

- same as before.... but included explicitly dependence on transverse momentum k_t in addition to evolution scale q
- what are the ordering constraints $f(q')$ and $\Theta(\mathcal{O})$?
- what is the splitting function?

k_t-factorization

- need to couple gluons to photon
- use high energy (kt -) factorization:
  (Catani,Ciafaloni, Hautmann NPB 366 (1991) 135,
  Gribov, Levin, Ryskin, Phys. Rep.100 ,(1983),1,
  Collins, Ellis, NPB 360 ,(1991) ,3)

$$\sigma(\mathrm{ep} \to \mathrm{e}'\mathrm{q}\bar{\mathrm{q}}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t, \bar{q}) x_g \mathcal{A}(x_g, k_t, \bar{q})$$

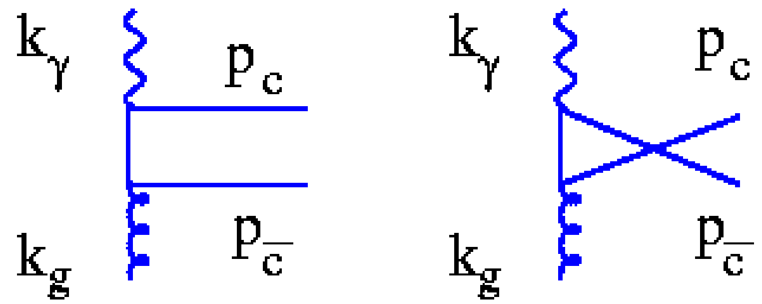
- with

$$\int^{Q^2} d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$$

- *t*-channel gluon with virtuality $k^2 = -k_t^2$ dominates the process in the high energy limit $s \gg \hat{s}$
- collinear limit obtained by:

$$\hat{\sigma}(\hat{s}, 0, Q) \cdot \Theta(Q - k_\perp)$$

“off-shell” matrix elements



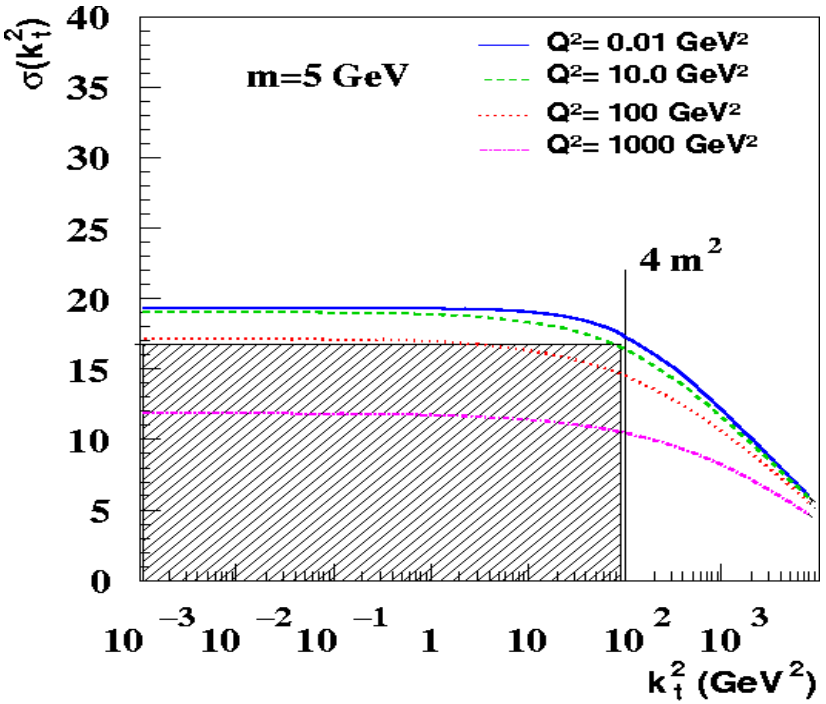
- calculation using standard Feynman rules

$$\mathcal{M}(\gamma g \to c\bar{c}) = \bar{u}(p_c) \left(\frac{\not\epsilon_\gamma \,(\not p_c - \not k_\gamma + m_c)\, \not\epsilon_g}{k_\gamma^2 - 2k_\gamma p_c} + \frac{\not\epsilon_g \,(\not p_c - \not k_g + m_c)\, \not\epsilon_\gamma}{k_g^2 - 2k_g p_c} \right) u(p_{\bar{c}})$$

- use high-energy polarization projection:

$$G^{\mu\nu} = \overline{\epsilon_g^{\mu} \epsilon_g^{*\nu}} = \frac{k_{t\,g}^{\mu} k_{t\,g}^{\nu}}{|k_{t\,g}|^2}$$

- ME is finite for $k_t \to 0$
- ME has tail to large k_t

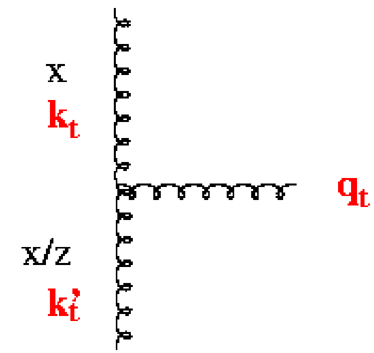


BFKL equation

J. Kwiecinski, A. Martin, P. Sutton PRD 52 (1995) 1445
Forshaw,Ross, QCD and the pomeron, p165

- Non-Sudakov form factor screens 1/z singularity,
  ..... as the Sudakov does for 1/(1-z)

$$\begin{aligned}\Delta_{ns} &= \exp\left(-\bar{\alpha}_s \int \frac{dq^2}{q^2}\int_z^1 \frac{dz'}{z'}\Theta(k_t^2-q^2)\Theta(q^2-\mu_0^2)\right)\\ &= \exp\left(\bar{\alpha}_s \log z \log\frac{k_t^2}{\mu_0^2}\right)\\ &= z^\omega \text{ with } \omega = \bar{\alpha}_s \log\frac{k_t^2}{\mu_0^2}\end{aligned}$$



$$x\mathcal{A}(x,k_t,q) = x\mathcal{A}_0(x,k_t) + \int \bar{\alpha}_s \frac{dz}{z}\Delta_{ns}\int\frac{d^2q'}{\pi q'^2}\frac{x}{z}\mathcal{A}\left(\frac{x}{z},k'_t,q'\right)$$

- here use:

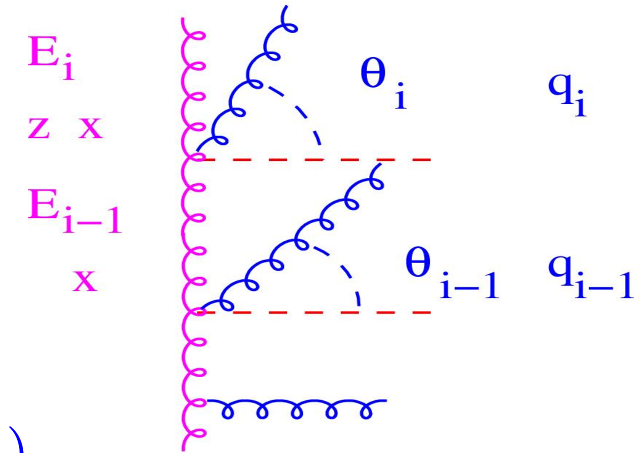
$$\vec{k}'_t = \vec{k}_t + \vec{q}$$

- recursive equation for BFKL, solve it numerically with iteration...

CataniCiafaloniFioraniMarchesini evolution

- Apply color coherence in form of angular ordering

  $\bar{q} > z_n q_n, q_n > z_{n-1} q_{n_1}, ..., q_1 > Q_0$



- with:

$$\tilde{P}(z, q, k_t) = \frac{\bar{\alpha}_s(q(1-z))}{1-z} + \frac{\bar{\alpha}_s(k_t)}{z} \Delta_{ns}(z, q, k_t)$$

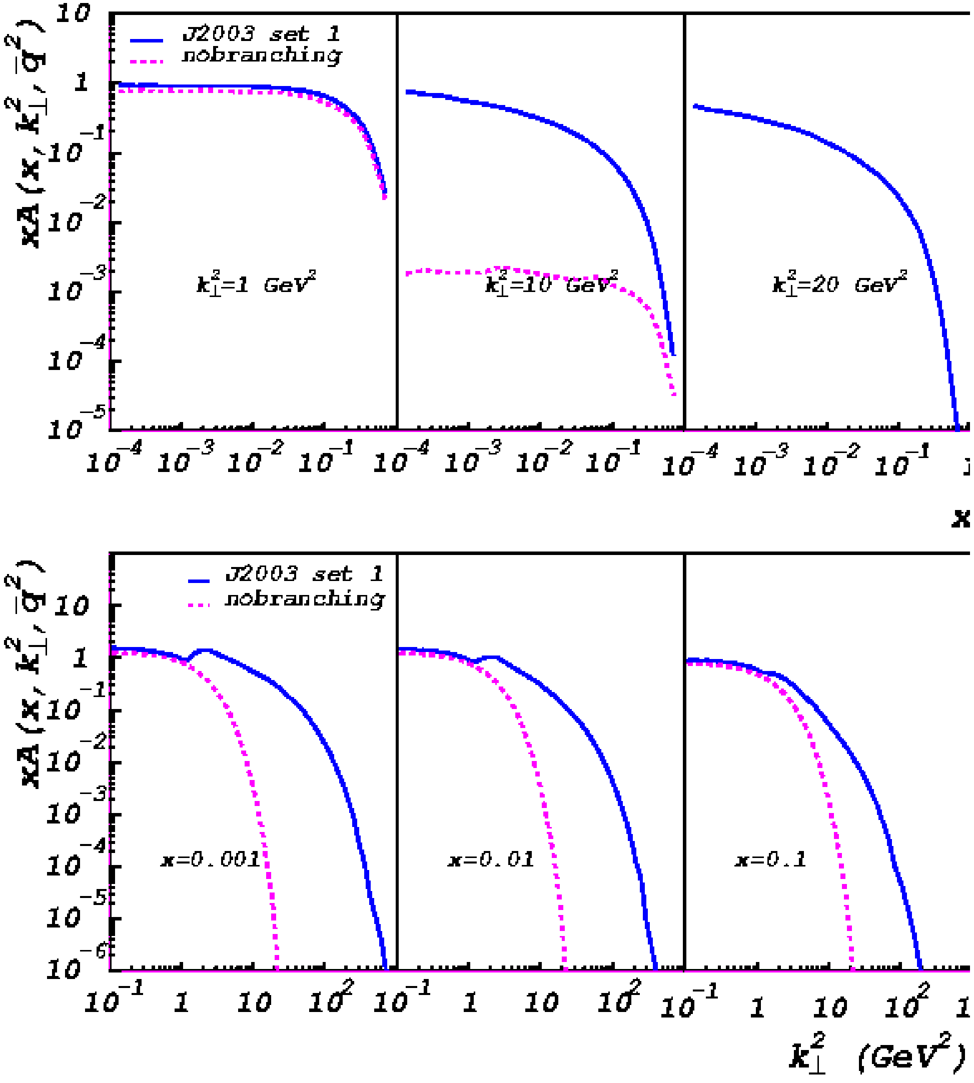
- gives:

$$x\mathcal{A}(x, k_t, q) = x\mathcal{A}_0(x, k_t)\Delta_s(q) + \int dz \int \frac{d^2q'}{\pi q'^2} \Theta(\bar{q} - zq) \cdot \Delta_s(q, zq') \tilde{P}(z, q', k_t) \frac{x}{z} \mathcal{A}\left(\frac{x}{z}, k_t', q'\right)$$

- integration much more complicated due to angular constraints

Advantage of uPDFs

$$\mathcal{A}(x,k_t,\bar{q}) = \mathcal{A}_0(x,k_t)\Delta_s(\bar{q},Q_0) + \int \frac{dz}{z}\frac{d^2q}{q^2}\Delta_s(\bar{q},zq)\cdot\tilde{P}(z,\ldots)\mathcal{A}\left(\frac{x}{z},k'_t,q\right)$$

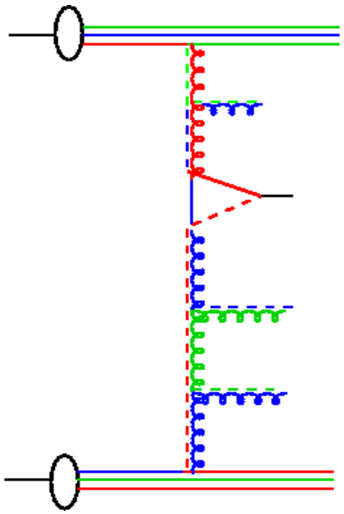


Advantage of uPDF:

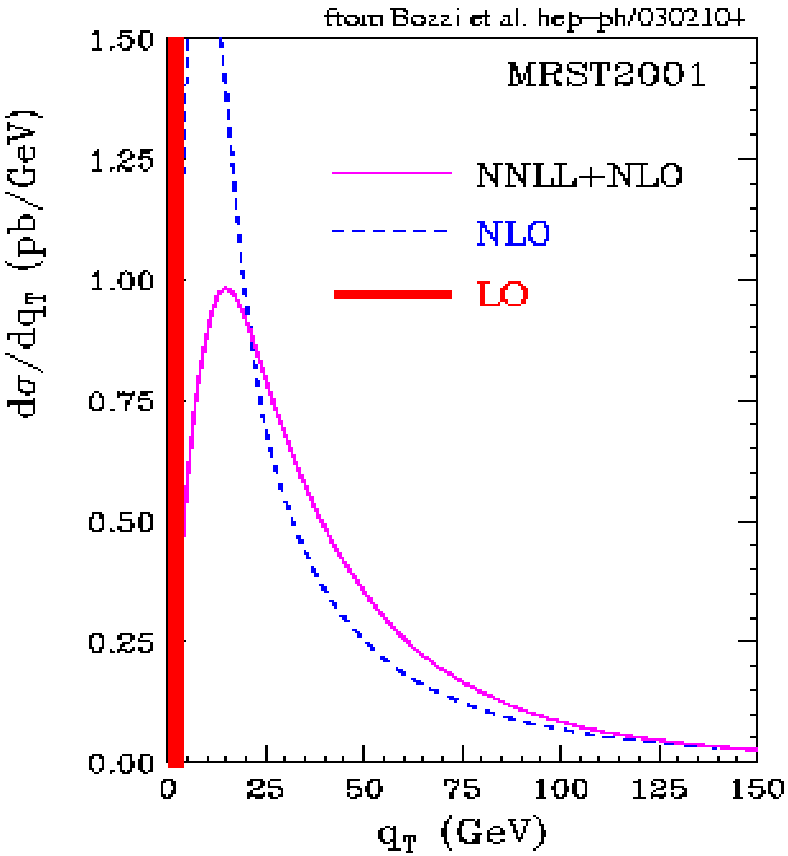
- initial condition clearly seen in small k_t region
- even at large scales *q*

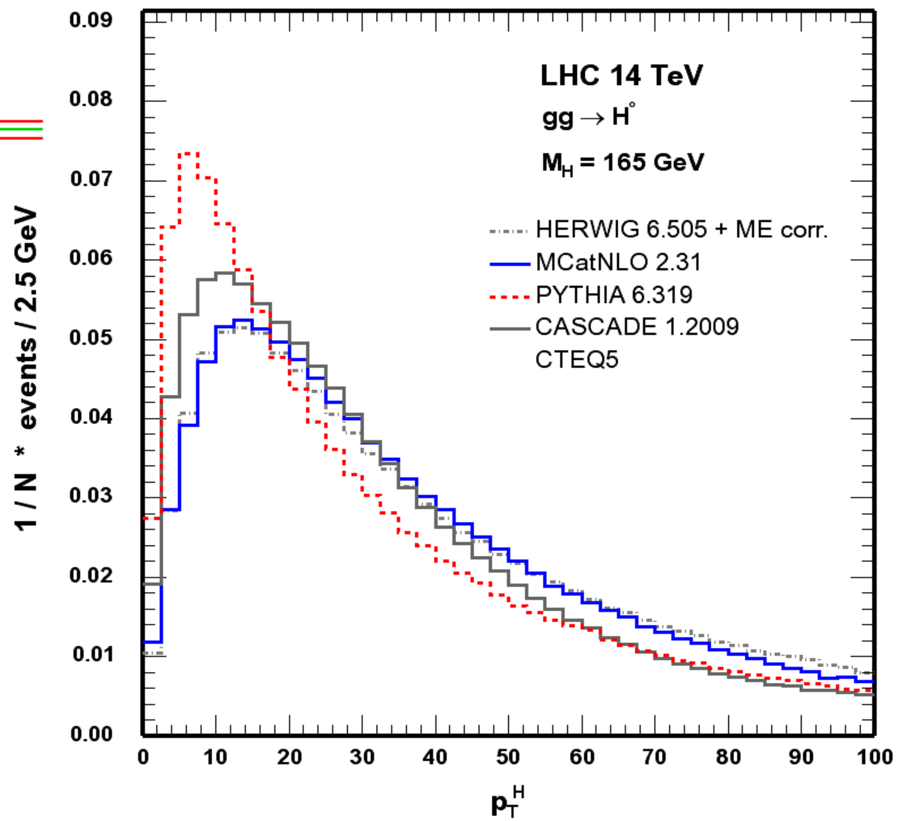
k_t effects at LHC: Higgs production

from G. Davatz, HERA – LHC workshop
hep-ph/0601012, hep-ph/0601013



$$gg \to \text{Higgs} \to W^+W^- \to l^+\bar{\nu}l^-\nu$$





The end of the winter term !

- questions ?
- please give us feedback: critics are very welcome
- lectures continue in summer term:
  - Mondays 14:00-16:00
  - start after Easter

HERA – LHC workshop

- You are welcome to participate
- some topics:
  - W/Z production for PDFs
    Drell-Yan for small x
  - Resummation effects
    pt spectrum of W/Z and Higgs
  - multiparton interactions
    discussion forum